Invited Review Paper

From model-driven to data-driven: A review of hysteresis modeling in structural and mechanical systems

Tianyu Wang\textsuperscript{a}, Mohammad Noori\textsuperscript{b,c,*}, Wael A. Altabey\textsuperscript{d,e,*}, Zhishen Wu\textsuperscript{d,*}, Ramin Ghiasi\textsuperscript{d,f}, Sin-Chi Kuok\textsuperscript{g}, Ahmed Silik\textsuperscript{d,h}, Nabeel S.D. Farhan\textsuperscript{d}, Vasilis Sarhosis\textsuperscript{c}, Ehsan Noroozinejad Farsangi\textsuperscript{i}

\textsuperscript{a} School of Urban Construction and Safety Engineering, Shanghai Institute of Technology, Shanghai 201418, China
\textsuperscript{b} Department of Mechanical Engineering, California Polytechnic State University, San Luis Obispo, CA 93407, USA
\textsuperscript{c} School of Civil Engineering, University of Leeds, Leeds LS2 9JT, UK
\textsuperscript{d} International Institute of Urban System Engineering (IIUSE), Southeast University, Nanjing 211189, China
\textsuperscript{e} Department of Mechanical Engineering, Faculty of Engineering, Alexandria University, Alexandria 21544, Egypt
\textsuperscript{f} Structural Dynamics and Assessment Laboratory, School of Civil Engineering, University College Dublin, Dublin D04VI18, Ireland
\textsuperscript{g} State Key Laboratory of Internet of Things for Smart City, Guangdong-Hong Kong-Macau Joint Laboratory for Smart Cities, Department of Civil and Environmental Engineering, University of Macau, Macao
\textsuperscript{h} Department of Civil Engineering, Nyala University, Nyala, Sudan
\textsuperscript{i} Urban Transformations Research Centre (UTRC), Western Sydney University, NSW, Australia

A R T I C L E   I N F O

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A B S T R A C T

Hysteresis is a natural phenomenon that widely exists in structural and mechanical systems. The characteristics of structural hysteretic behaviors are complicated. Therefore, numerous methods have been developed to describe hysteresis. In this paper, a review of the available hysteretic modeling methods is carried out. Such methods are divided into: a) model-driven and b) data-driven methods. The model-driven method uses parameter identification to determine parameters. Three types of parametric models are introduced including polynomial models, differential based models, and operator based models. Four algorithms as least mean square error algorithm, Kalman filter algorithm, metaheuristic algorithms, and Bayesian estimation are presented to realize parameter identification. The data-driven method utilizes universal mathematical models to describe hysteretic behavior. Regression model, artificial neural network, least square support vector machine, and deep learning are introduced in turn as the classical data-driven methods. Model-data driven hybrid methods are also discussed to make up for the shortcomings of the two methods. Based on a multi-dimensional evaluation, the existing problems and open challenges of different hysteresis modeling methods are discussed. Some possible research directions about hysteresis description are given in the final section.

1. Introduction

Hysteresis refers to the dependence of a system on its past history. In a hysteretic system, the input–output relationship exhibits a multi-branch non-linearity, with transitions occurring between branches after input extrema \cite{1}. Hysteresis is a natural phenomenon

\* Corresponding authors.

E-mail addresses: mnoori@calpoly.edu (M. Noori), wael.altabey@gmail.com (W.A. Altabey), zsw@seu.edu.cn (Z. Wu).

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### Symbols and notation list

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<thead>
<tr>
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that is widely observed in various systems, and it has attracted the attention of researchers from different fields. In the Web of Science (WOS) core database \cite{2} (including SCIE, SSCI, A&HCI, and ESCI), a total of 120,044 papers related to “hysteresis” or “hysteretic” can be found from 1900 to 2022. Fig. 1 (a) illustrates that these papers span several disciplines, such as physics, chemistry, materials science, electrical and electronic engineering, mechanics, civil engineering, and more. Fig. 1 (b) shows the annual publication trend, indicating a recent explosive growth of interest in the field. This paper focuses on hysteresis in civil structural systems, which is an important area of hysteresis research, and specifically addresses various methods of hysteresis modeling.

A bibliometric analysis was conducted using CiteSpace 5.7R5 \cite{3} to identify the key problems in hysteresis research within civil structural systems. The analysis focused on 2,833 source documents published between 2018 and 2022 that were related to hysteresis and civil engineering. Table 1 presents the top five most frequently used keywords for each year. Upon examining the table, it becomes evident that the keywords used in papers published in different years are highly similar, primarily revolving around the study of seismic performance. This consistent use of keywords suggests a sustained emphasis on seismic performance in the hysteresis research within civil structural systems.

Fig. 2 provides the analysis results of keyword clustering from 2018 to 2022, which sheds light on the core problems addressed in hysteresis research within civil engineering. These clusters address the following key aspects:

**Research Object:** The focus of the research is on the seismic behavior, hysteretic behavior, and seismic performance (including stiffness, ductility, and energy dissipation) of various structures (such as steel structures, concrete structures) or structural components (such as columns, shear walls, beam-column joints, braces, dampers, and seismic isolation).

**Purpose of Research:** The purpose of studying hysteresis in civil engineering is to enhance nonlinear structural analysis (including seismic analysis, finite element analysis, numerical analysis) and propose methods for structural design. The aim is to improve the understanding and prediction of structural response under seismic loading.

**Research Approach:** The research is carried out through data collection from experiments (such as cyclic loading experiments), development of hysteretic models (such as the Bouc-Wen model), and conducting parametric analyses on these models. The proposed models can then be utilized for structural analysis to simulate and predict the behavior of hysteretic stress in civil engineering structures.

Hysteresis models or modeling methods serve as a crucial link between hysteretic behavior and structural nonlinear analysis. The study of hysteresis models can be traced back to the 1960s. Veletsos et al. \cite{4} proposed an elasto-plastic model to describe the hysteretic behavior of an ideal plastic system. Clough et al. \cite{5} made a significant contribution by proposing a bilinear model considering stiffness degradation in structures. Subsequently, numerous researchers have proposed different hysteretic models \cite{6-11} to describe the behavior of various structural systems or components. These models are often referred to as polynomial models since the hysteretic loops can be expressed using polynomials or piecewise polynomials. One notable characteristic of these models is that their parameters typically have well-defined physical meanings, such as the relationships to stiffness, strength, or ductility of the structure. As a result, polynomial models remain popular in the field of structural analysis, both in research and practice.

Another type of model widely applied in structural nonlinear analysis is the Bouc-Wen model and its modifications. The Bouc-Wen model was initially proposed by Bouc \cite{12,13} and later generalized by Wen \cite{14}. Subsequent work by Baber and Wen \cite{15}, Baber and Noori \cite{16} introduced stiffness degradation and pinching into the original Bouc-Wen model. Song et al. \cite{17} and Dobson, Noori, et al. \cite{24} attempted to present a more generalized Bouc-Wen model to describe asymmetrical hysteretic behavior. Aloisio et al. \cite{18} also extended the generalized Bouc-Wen model to incorporate the pinching phenomenon, however, their model lacks the mathematical tractability of Bouc-Wen-Baber-Noori model, particularly for random vibration application. Other hysteretic models, including the Duhem model \cite{19}, Preisach model \cite{20}, Prandtl-Ilsinskii model \cite{21}, Kranosel’ski-Pokrovskii model \cite{22}, and Maxwell-slip model \cite{23}, have also been applied in modeling the hysteretic behavior of structural systems in recent years. However, the utilization of these recently introduced models is relatively limited as they were not specifically designed for structural nonlinear analysis. As shown in Fig. 3, the number of papers utilizing these recent hysteretic models accounts for less than 1% of all published papers. Nevertheless, the study of these models has contributed to describing complex hysteretic behavior and improving the efficiency of structural analysis.

Hysteresis modeling is not limited to the use of parametric hysteretic models. Universal mathematical models, such as regression models \cite{24} or Boolean Algebra \cite{61}, are also employed for hysteresis modeling. These methods are referred to as data-driven methods. In contrast, the methods that utilize parametric hysteretic models are known as model-driven methods. Fig. 4 illustrates the distinctions between data-driven and model-driven approaches. In model-driven methods, parametric models are initially selected, and then hysteresis data are utilized for parameter identification. The identified models can accurately describe the hysteretic behavior...
of structural systems. Therefore, in model-driven methods, the selection of appropriate models is crucial. Hysteretic models need to incorporate the key characteristics of hysteretic behavior. Data-driven methods rely on the training process rather than model selection and parameter identification, offering more flexibility in describing hysteresis. In recent years, the rapid advancement of machine learning techniques [25] has greatly enhanced the capability of data-driven methods in capturing complex hysteresis behavior. A significant amount of research has emerged, utilizing machine learning models such as artificial neural networks [26], and support vector machines [27]. These machine learning-based approaches have shown promise in accurately characterizing hysteresis and have the potential to provide valuable insights into structural analysis and design.

Due to the complexity of hysteretic behaviors and the numerous hysterisis modeling methods available, it is essential to provide a comprehensive summary and review of these methods, particularly in the field of civil and mechanical engineering. However, the previous review papers are not comprehensive since they usually concentrate only on one or several classes of hysteretic models. For example, in Ref. [28], the authors review the Bouc-Wen model including the basic theory, the modifications of Bouc-Wen model, and its application in different areas. Duhem Model is reviewed in Ref. [19]. Sengupta and Li [29] review the hysteretic models of reinforced concrete structure including several polynomial models and Bouc-Wen class models. Hassani et al. [30] reviewed the operator based models and differential based models as well as the system identification methods. In addition, data-driven hysteresis modeling approaches are rarely covered. From the perspective of structural dynamic analysis, this paper will evaluate and compare the feasibilities of different hysteresis modeling methods in order to help in the selection of modeling method in engineering practices. The rest of the paper will be organized as follows: Section 2 will introduce the complex hysteretic characteristics exhibited by structural systems, including rate-independence, stiffness degradation, strength degradation, pinching, and asymmetry. Section 3 and Section 4

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**Table 1**
The top five most frequent keywords in published papers (2018–2022).

<table>
<thead>
<tr>
<th>Year</th>
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<th>2019</th>
<th>2020</th>
<th>2021</th>
<th>2022</th>
</tr>
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<td>hysteresis (21)</td>
<td>seismic performance (17)</td>
<td>cycling loading (14)</td>
<td>ductility (10)</td>
</tr>
<tr>
<td>Top 2 (counts)</td>
<td>seismic performance (23)</td>
<td>seismic behavior (22)</td>
<td>cycling loading (18)</td>
<td>energy dissipation (18)</td>
<td>ductility (15)</td>
</tr>
<tr>
<td>Top 3 (counts)</td>
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<td>hysteretic behavior (21)</td>
<td>seismic behavior (20)</td>
<td>ductility (14)</td>
<td>hysteresis (13)</td>
</tr>
<tr>
<td>Top 4 (counts)</td>
<td>seismic performance (63)</td>
<td>cycling loading (31)</td>
<td>seismic behavior (28)</td>
<td>energy dissipation (23)</td>
<td>hysteretic behavior (21)</td>
</tr>
<tr>
<td>Top 5 (counts)</td>
<td>seismic performance (61)</td>
<td>hysteretic behavior (21)</td>
<td>seismic behavior (28)</td>
<td>energy dissipation (22)</td>
<td>finite element analysis (22)</td>
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</table>
will discuss the hysteretic models including polynomial models, differential based models, and operator based models as well as the methods for model parameter identification. A comprehensive review of data-driven methods, including the regression model method, artificial neural networks, support vector machines, and deep learning will be provided in Section 5. These methods leverage available data to capture and describe hysteresis behavior in structural systems.

Section 6 will introduce model-data driven hybrid methods that combine the strengths of both model-driven and data-driven approaches. These hybrid methods aim to enhance the accuracy and effectiveness of hysteresis modeling. A comparison and evaluation of all hysteresis modeling methods will be presented in Section 7 with a discussion of existing problems and open challenges. Section 8 will list the main conclusions of this paper and outlines the possible future research directions in the field of hysteresis modeling in civil structural system.

2. The characteristics of structural hysteresis

In structural systems, the typical hysteretic phenomenon occurs in the relationship between the structural response and the restoring force under complex dynamic or cyclic loading conditions. This can be described using the equation of motion (EOM) for a single degree of freedom (SDOF) system:

\[ m\ddot{x}(t) + c\dot{x}(t) + f_s = f(t) \]  

(1)
where \( m \) and \( c \) are the generalized mass and damping of the structural system, \( f_s \) is the generalized restoring force. \( \ddot{x}(t), \dot{x}(t), x(t) \) are generalized acceleration, velocity, and displacement of structure, and \( f(t) \) is the external excitation.

In the elastic stage of deformation, the restoring force can be expressed as a linear function of displacement: \( f_s = k_0 x \). \( k_0 \) is the linear stiffness or the initial stiffness. However, in the plastic stage, the restoring force is not solely determined by the present displacement \( x \), but also depends on the history of displacement. This relationship between displacement and restoring force exhibits

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**Fig. 3.** The proportion of different hysteresis modeling methods.

**Fig. 4.** Difference between model-driven and data-driven methods.
typical hysteresis behavior. Hysteretic behaviors in structural systems follow the fundamental laws of hysteresis and are influenced by factors such as asymmetry and damage. The main characteristics of structural hysteresis include rate independence, asymmetry, stiffness degradation, and pinching.

2.1. Rate-independence

The rate-independence assumption in hysteresis modeling suggests that the branches of hysteretic nonlinearities are solely determined by the past extreme values of the input (displacement), while the derivative of the input with respect to time (velocity) does not affect the hysteretic behavior. This assumption implies that the rate at which the input changes over time does not influence the shape or characteristics of the hysteretic loop. As shown in Fig. 5, two different input signals, $x_1(t)$ and $x_2(t)$, can result in the same hysteretic loop because they have the same extreme values.

According to the rate-independence assumption, the relationship between structural response and restoring force can be expressed by Equation (2) as:

$$\frac{df_s}{dt} = g(x, f_s, \text{sign}(\dot{x}))\dot{x}$$  \hspace{1cm} (2)

where $g()$ is a nonlinear function.

The rate-independence assumption allows for a simplification in hysteresis modeling, as the focus can be placed on the extreme values of the input without explicitly considering the velocity-dependent effects. This assumption is supported by experimental evidence, enabling quasi-static tests to be conducted to identify the hysteretic relationship between displacement and restoring force. In this paper, most of the introduced hysteretic models adhere to this rate-independence assumption. However, it is worth noting that in recent years, there has been increased interest in studying and applying rate-dependent hysteresis in structural systems to accurately capture the hysteretic behavior, particularly under high-frequency dynamic loads [31,32]. By considering rate-dependent hysteresis, researchers aim to develop models and techniques that can better represent and simulate the behavior of structural systems subjected to dynamic loading conditions.

2.2. Complex hysteretic behaviors

The hysteresis behaviors observed in structural systems can become more complex than typical hysteresis due to various factors such as different materials, geometry, and boundary conditions. Fig. 6 illustrates three common types of complex hysteresis behaviors: asymmetry, stiffness degradation, and pinching.

2.2.1. Asymmetry

Asymmetry in hysteresis refers to the unequal response of a structure during loading and unloading cycles. This means that the hysteresis loops are not symmetric with respect to the displacement or force axes. Asymmetric hysteresis behavior can arise from various causes, such as material properties, geometric irregularities, or unbalanced loading conditions. Understanding and accurately modeling asymmetrical hysteresis behavior is essential for capturing the real-world response of structural systems. Asymmetry is an important characteristic observed in civil structures and building components, including electrical equipment [17], high-strength steel bars [33], and wire-cable vibration isolators [34]. While asymmetric hysteresis loops exist in almost every nonlinear system, in certain cases, to simplify modeling and computation, slight asymmetries may be ignored.

2.2.2. Stiffness degradation

Stiffness degradation refers to the decrease in loading and unloading stiffness of a structural system after undergoing cyclic loading.
It is considered an external manifestation of structural damage within the hysteretic behavior. Stiffness degradation is commonly observed in various types of structural systems, including steel [35,36], concrete [37,38], wood [39,40], and composite structures [41,42]. Modeling and incorporating stiffness degradation into hysteresis models are crucial for capturing the progressive changes in the structural response over time. The degree of stiffness degradation is believed to affect the energy dissipation and ductility of a structure. Generally, when the degree of stiffness degradation is small, the structure exhibits higher energy dissipation and ductility. This implies that a structure with minimal stiffness degradation can absorb and dissipate more energy during cyclic loading, allowing it to better withstand seismic or dynamic events.

2.2.3. Pinching

Pinching is a hysteresis behavior characterized by the distortion or non-convex shape of the hysteresis loop. This behavior typically occurs when the structure experiences non-linear effects such as local yielding, cracking, or contact discontinuities. Pinching can affect the symmetry, energy dissipation, and strength of the structure. Accurately capturing and modeling pinching behavior is essential for assessing the stability and performance of structural systems subjected to cyclic or dynamic loading. Pinching is an important phenomenon to consider in modeling methods for concrete and timber structures [29,43,44]. Neglecting pinching effects can lead to inaccuracies in predicting the behavior of these structures, especially in situations where cracking and closure of cracks play a significant role.

In addition to the hysteretic characteristics mentioned earlier, structural systems can exhibit other types of hysteresis behaviors that include S-shape hysteresis [45], flag-shape hysteresis [46], and kinematic hardening [47]. The presence of multiple hysteretic

Table 2
Summary of parametric hysteresis model.

<table>
<thead>
<tr>
<th>Polynomial model</th>
<th>Differential based model</th>
<th>Operator based model</th>
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<td>Pinching4 model</td>
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<td>Duhem model and its variants</td>
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Fig. 6. Complex hysteretic behaviors: (a) Asymmetry; (c) Stiffness degradation; (c) Pinching.
characteristics and the superposition of different hysteresis behaviors can introduce additional complexities and challenges in hysteresis simulation and description.

3. Parametric model for hysteresis description (Model-driven method)

Over the past few decades, numerous mathematical models have been developed to describe the hysteresis behavior of structural systems. In this section, we provide a brief review of representative parametric hysteretic models, categorizing them into three main classes: polynomial models, differential based models, and operator based models, as shown in Table 2.

3.1. Polynomial models

Polynomial models are effective in describing the hysteretic behavior of structural systems using polynomial or piecewise polynomial functions. These models provide closed-form expressions for hysteresis and establish the relationship between physical parameters and hysteresis loops. As a result, polynomial models are well-suited for structural dynamic analysis. Polynomial hysteresis models can be directly constructed from experimental data. As Fig. 7, a typical hysteresis model consists of the following components:

(a) **Backbone curve**: The backbone curve, also known as the envelope curve, is a fundamental part of the polynomial hysteresis model. Experimental data from test specimens subjected to different loads are plotted in a dimensionless coordinate system,
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typically displacement \((x)\) versus restoring force (load) \((f)\). The yielding displacement \((x_y)\) and yielding load \((f_y)\) are identified from the data. By fitting the data using piecewise polynomial functions, the backbone curve can be determined.

(b) **Standard hysteresis loop:** Similar to the backbone curve, the main characteristics of the hysteresis loops are examined based on the test data from pseudo-static tests. This analysis helps establish a standard hysteresis loop that represents the typical behavior observed in the test specimens.

(c) **Hysteretic rules:** The hysteresis loops vary with the applied loads at different loading stages. The variations in hysteresis behavior are influenced by the structural stiffness and strength. Hysteretic rules are employed to describe the degradation of stiffness and strength as the loading progresses. These rules capture the changes in hysteresis due to factors such as yielding, strength degradation, or stiffness degradation.

Polynomial models are the first class of models widely used in structural analysis. In the early 1960s, Veletsos et al. [4] and Caughey [48] proposed the elastoplastic hysteretic model and the bilinear hysteretic model, respectively. These models were based on the assumption of ideal elastoplastic materials and linearly intensified materials, allowing for direct parameter estimation from static tests. Following these early models, Clough [5] introduced the Clough model, which considered stiffness degradation. Subsequently, several hysteretic models were proposed by different researchers to capture different aspects of the displacement-restoring force relationship, including models with bilinear degrading stiffness. For instance, Takeda et al. [6] employed a trilinear curve to describe hysteresis and introduced a stiffness degradation rule. Saidimovahhed [7] and Otani [8] proposed different stiffness degradation rules based on their experimental findings, aiming to improve upon the Clough model. Sucuoglu and Erberik [9] defined a stiffness deteriorating rule based on energy dissipation. Dowell et al. [10] defined the hysteretic loop using four straight lines to represent different stages: elastic, strain-hardening, strength degradation, and residual strength. Ibarra et al. [11] proposed hysteretic models that incorporated both strength and stiffness deterioration. These models are capable of describing the hysteresis behavior of steel, wood, and reinforced concrete structures. Furthermore, for braces, which exhibit high asymmetry, researchers have conducted extensive studies and proposed specific models [49–51] tailored to capture their unique hysteretic behaviors.

Due to the huge amount of polynomial models, it is impossible to introduce every model in this paper. Therefore, four respective, and more commonly used models including Clough model [5], Takeda model [6], Modified Ibarra-Medina-Krawinkler model [11] and pinching4 model [52] are selected for the further discussions.

3.1.1. **Clough model**

The Clough model, also known as the bilinear degrading stiffness model, describes the hysteretic behavior of a structural system and incorporates stiffness degradation. Fig. 8(a) provides a visualization of the displacement-restoring force hysteretic curve associated with the Clough model. And the stiffness degradation of Clough model follows Equation (3):

\[
k_r = k_0 \left( \frac{x_{max}}{x_y} \right)^{r_c}
\]

where \(k_r\) is the unloading stiffness, \(x_{max}\) is the maximum displacement and \(x_y\) is the yielding displacement, \(r_c\) is the stiffness degradation coefficient of Clough model.

3.1.2. **Takeda model**

Takeda model is mainly used for describing the hysteresis behavior in reinforced concrete structures. Backbone curve of Takeda model consists of three lines which represent three stages of concrete deformation (Fig. 8(b)). In the first stage, structure is in the elastic stage. In the second and third stages, the structure is cracked and then yields. The stiffness degradation law of Takeda model can be expressed as:

\[
k_r = \left( \frac{f_{crack} + f_y}{x_{crack} + x_y} \right) \left( \frac{x_{max}}{x_y} \right)^{r_T}
\]
where $x_{\text{crack}}$ and $f_{\text{crack}}$ are the displacement and restoring force when the structure is cracked, $f_y$ is the yielding restoring force and $r_T$ is the stiffness degradation coefficient of Takeda model.

### 3.1.3. Modified Ibarra-Medina-Krawinkler (IMK) model

The modified IMK (Ibarra-Medina-Krawinkler) hysteresis model is a piecewise linear hysteresis model that effectively captures the degradation characteristics of structural components subjected to cyclic loading. The backbone of modified IMK model is shown in Fig. 9.

Fig. 9. The hysteretic curves of Modified IMK model: (a) Backbone curve; (b) bilinear model; (c) peak orient model; (d) pinching model.

where $x_{\text{crack}}$ and $f_{\text{crack}}$ are the displacement and restoring force when the structure is cracked, $f_y$ is the yielding restoring force and $r_T$ is the stiffness degradation coefficient of Takeda model.

3.1.3. Modified Ibarra-Medina-Krawinkler (IMK) model

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Fig. 9(a). Three different models are included in modified IMK model as: bilinear model, peak orient model, and pinching model. The hysteresis rules are presented in Fig. 9(b)–(d).

The degradation and pinching is controlled by model parameters. Take the pinching model as an example, there are a total of 25 parameters, 13 parameters for backbone curve control, 10 parameters for degradation control and 2 for pinching control. By defining appropriate parameters, modified IMK can accurately simulate the hysteresis curve of component tests. Therefore, modified IMK model has become a popular model in the nonlinear dynamic analysis of RC structures [53,54], however, the model is not suitable for random vibration analysis of RC structures due to lack of mathematical tractability suitable for random vibration or seismic loading applications.

3.1.4. Pinching4 model

The Pinching4 model is a built-in model within the OpenSees [55] software framework. It is commonly employed as a beam-column joint model to simulate the seismic response of structures. The model is designed to capture the pinching behavior observed in these joints, where the stiffness and strength degrade under cyclic loading conditions. The Pinching4 model utilizes piecewise linear curves to represent the pinching response. It considers both stiffness degradation and strength degradation as the cyclic loading history is imposed. The backbone curve of the Pinching4 model is controlled by 16 parameters, as illustrated in Fig. 10. These parameters determine the shape and characteristics of the backbone curve, which represents the behavior of the joint under ideal conditions.

Additionally, another set of 16 parameters is used to control the stiffness degradation and strength degradation. These parameters capture the deterioration of stiffness and strength due to the cyclic loading history. As the structure undergoes repeated loading cycles, both the stiffness and strength are assumed to degrade over time.

3.2. Differential based models

Differential based models are another class of models used to characterize the hysteresis of structural systems. These models describe hysteresis behaviors using differential equations. Two well-known examples of differential based models are the Bouc-Wen model and the Duhem model.

3.2.1. Bouc-Wen class models

(a) Bouc-Wen model

The Bouc-Wen model [13,14] is a differential based hysteresis model widely used to describe the smooth and continuously varying hysteretic behavior observed in civil and mechanical systems. This semi-physical model formulates the relationship between displacement and restoring force using a first-order nonlinear differential equation.

The EOM of Bouc-Wen hysteretic system is given as:

$$m\ddot{x}(t) + c\dot{x}(t) + \alpha k_0 x(t) + (1 - \alpha) k_0 z(t) = f(t)$$

(5a)
where \( z(t) \) and \( \dot{z}(t) \) are the hysteretic displacement and its time derivation, respectively; \( \alpha \) defines the proportion of nonlinear part in restoring force. The characteristic parameters \( A, \beta, \gamma \) and \( n \) describe the shape and smoothness of the hysteretic loops. For \( n \geq 1 \), this differential equation has a unique solution.

(b) Bouc-Wen-Baber-Noori model

Bouc-Wen model cannot consider the deterioration and pinching in structural systems. Therefore, Baber and Wen [15] extended the original Wen’s model to capture stiffness and strength degradation. Subsequently, in early 1980s Baber and Noori [16] improved the original Bouc-Wen model and first introduced a series model that was a combination of earlier Bouc-Wen and a slip-lock element, and later Noori and Baber introduced a single element model referred as Bouc-Wen-Baber-Noori (BWBN) model. In BWBN model, Equation (5b) is replaced by Eq. (6) as follows:

\[
\dot{z}(t) = \frac{h(z(t))}{\eta(e(t))} \left\{ A(e(t)) \dot{x}(t) - \eta'(e(t)) \left[ \beta |\dot{z}(t)| z(t) |^{n-1} - \gamma \dot{z}(t) z(t) \right] \right\}
\]

where the additional parameters \( \eta(e) \) and \( \eta'(e) \) are degradation functions that can simulate the pinching behavior as a function of total hysteretic energy and preserve strength and stiffness degradation effects. The expressions of \( A(e(t)), \eta(e(t)) \), and \( \eta'(e(t)) \) can be expressed by linear increasing functions of the absorbed hysteretic energy \( e(t) \) as:

\[
\begin{align*}
A(e(t)) &= A_0 - \delta_A e(t) \\
\eta(e(t)) &= n_0 - \delta_n e(t) \\
\eta'(e(t)) &= \eta_0 - \delta_{\eta} e(t)
\end{align*}
\]

where the constant parameters \( \delta_A, \delta_n \) and \( \delta_{\eta} \) determine the rate of stiffness and strength degradation. \( h(z(t)) \) is the “Slip-lock” element which can be expressed as:

\[
h(z(t)) = 1 - \zeta_1(e(t)) \exp \left[ \frac{[-z(t) \text{sign}(\dot{x}(t)) - q_B z_n]^2}{(\zeta_2(e(t)))^2} \right]
\]

where \( \text{sign}(\dot{x}(t)) \) is the signum function of \( \dot{x}(t) \), \( q_B \) is a constant and \( z_n \) is the ultimate value of \( z(t) \). \( \zeta_1(e(t)) \) and \( \zeta_2(e(t)) \) can be defined as:

\[
\begin{align*}
\zeta_1(e(t)) &= \zeta_0 (1 - \exp(p_n e(t))) \\
\zeta_2(e(t)) &= (\psi + \delta_{\psi} e(t))(\lambda + \zeta_1(e(t)))
\end{align*}
\]

where \( p_n, \zeta_0, \psi, \delta_{\psi} \) and \( \lambda \) are the parameters control the pinching phenomenon.

BWBN model is the first smooth hysteretic model that can accurately predict not only stiffness and strength degradation but also the pinching behavior observed in many mechanical and structural systems. The model has been widely applied in the analysis of various systems with complex hysteretic behavior [56–58]. Foliente [43] for example, used the BWBN model to describe the behavior of joints in wood structural systems. The model has also been incorporated into the OpenSees software as a material model for seismic analysis of hysteretic systems. In addition, BWBN model has been utilized in the ABAQUAS finite element software to analyze nailed joints in wood structures [59]. Furthermore, it has been employed to model the hysteretic phenomenon in magnetorheological fluid dampers [60], among other applications. Some subsequent modifications were introduced in BWBN model. Dobson and Noori [61], for the first time, introduced a Boolean Algebra based approach to model general hysteresis including pinching. They also extended the BWBN model to predict asymmetric hysteretic behavior [62] and demonstrated the application of a direct linearization scheme for random vibration of general hysteresis with pinching and asymmetry [63].

(c) Generalized Bouc-Wen (GBW) model

Song and Der Kiureghian [17] introduced a shape-control mechanism into Bouc-Wen class model and proposed the generalized Bouc-Wen (GBW) model. In GBW model, \( \dot{z}(t) \) is expressed as:

\[
\dot{z}(t) = A\dot{x}(t) - |z(t)|^{\gamma} \Psi(\dot{x}(t), x(t), z(t))
\]

where \( \Psi(\dot{x}(t), x(t), z(t)) = \beta_1 \text{sign}(\dot{x}z) + \beta_2 \text{sign}(\dot{x}x) + \beta_3 \text{sign}(xz) + \beta_4 \text{sign}(\dot{x}) + \beta_5 \text{sign}(z) + \beta_6 \text{sign}(x) \)
where $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5,$ and $\beta_6$ are the shape control parameters.

(d) Extended Generalized Bouc-Wen (EGBW) model

Aloisio et al. [18] extended the generalized Bouc-Wen model by considering asymmetry, strength degradation, stiffness degradation and pinching in a single model. In their model, Eq. (10b) is converted to Equation (11) as:

$$
\Psi_b(x(t), x(t), \dot{x}(t)) = \theta \beta_1(\epsilon) \text{sign}(\dot{x}) + \theta \beta_2(\epsilon) \text{sign}(\dot{x})
$$

$$
+ \theta \beta_3(\epsilon) \text{sign}(x) + \theta \beta_4(\epsilon) \text{sign}(x)
$$

$$
+ \theta \beta_5(\epsilon) \text{sign}(\dot{x}) + \theta \beta_6(\epsilon) \text{sign}(x)
$$

$$
\theta = (1 - \theta^+)(1 - \theta^-)
$$

$$
\theta^+ = \theta_1 \theta_2 / 2
$$

$$
\theta^- = \theta_1 \theta_2 / 2
$$

$$
\theta_1 = 0.25 \text{sign}(\dot{x}) + 0.25 \text{sign}(x) + 0.25 \text{sign}(x) + 0.25 \text{sign}(x)
$$

$$
\theta_2 = 0.25 \text{sign}(\dot{x}) + 0.25 \text{sign}(x) - 0.25 \text{sign}(x) - 0.25 \text{sign}(x)
$$

$$
\theta_3 = 1 - \text{sign}(\theta)|x_{\text{max}}| - x
$$

Aloisio et al. successfully utilize the EGBW model to simulate the asymmetric hysteretic behavior of Cross-Laminated-Timber (CLT) wall systems [64,65] and plywood-coupled Laminated veneer lumber (LVL) wall panels [65].

3.2.2. Duhem model and its variants

(a) Duhem model

Duhem model is a rate-independent differential based hysteresis model to simulate an active hysteretic behavior. The governing equation of the Duhem model is expressed as:

$$
\dot{\xi}(t) = a_0 \delta(t) \left[ |g_1(x(t)) - f_1(t)| + \dot{x}g_2(x(t)) \right]
$$

where $a_0$ is a positive constant; $f_1(t)$ and $\dot{\xi}(t)$ are the output state of the hysteretic system and its time derivation, respectively; $x(t)$ and $\dot{x}(t)$ are the input of system and its time derivation, (in structural systems they are the displacement and velocity of system) respectively; $g_1(\dot{x}(t))$ and $g_2(x(t))$ are functions of the input voltage influencing the model performance and hysteresis loop. If (i) $g_1(\bullet)$ is piecewise smooth, monotonically increasing, odd, with $\lim_{t \to \infty} g_1(\dot{x}(t))$ finite; (ii) $g_2(\bullet)$ is piecewise continuous, even, with $\lim_{t \to \infty} g_2(x(t)) = \lim_{t \to \infty} g_1(\dot{x}(t))$; and (iii) $g_2(x(t)) |g_2(\dot{x})|a_0 \exp(\alpha x(t)) \int_{-\infty}^{\infty} f(\zeta) - g_2(\zeta) |\exp(-\alpha \zeta) d\zeta$ for $\forall x > 0$ are satisfied, the differential equation can be solved. The solution is given by:

$$
\dot{\xi}(t) = g_1(x(t)) + (f_{\infty} - g_1(x_0)) \exp[-a_0(x(t) - x_0)\text{sign}(\dot{x}(t))] + \exp[-a_0 x(t)\text{sign}(\dot{x}(t))] \int_{x_0}^{x(t)} [g_2(\zeta) - g_1(\zeta)] |\exp[a_0 \zeta \text{sign}(\dot{x}(t))] | d\zeta
$$

where $\text{sign}(\bullet)$ denotes the sign function. To obtain an analytical solution, the functions $g_1(x(t))$ and $g_2(x(t))$ are assigned with polynomials based on Weierstrass theorem [66]. Mayorgoyz [67] discussed the limitation of the classical Duhem model on continuity extension. Modified version of Duhem models are introduced to develop generalized models and overcome the constraints under various applications.

The applications of Duhem model in structural dynamic analysis are very limited. Ni et al. [68] studied the random response of Duhem hysteretic systems under non-white excitation while later Jin et al. [69] studied the responses of Duhem hysteretic systems under random seismic excitation.

(b) Generalized Duhem model

The generalized Duhem model with input $x$, state $s$ and output $f_s$ consists of the following differential equations as:

$$
\dot{s}(t) = g_1(x(t), s(t)) g_2(\dot{s}(t))
$$
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\[ f_i(t) = g_i(x(t), s(t)) \]  

(14b)

where \( g_i \), \( \sigma_i \), \( \rho_i \) are nonlinear functions. There are two special forms of generalized Duhem model known as Dahl model [70] and LuGre model [71].

(c) Dahl model

The Dahl model is the nonlinear friction model which can be expressed as Eq. (15):

\[
\dot{f}_i(t) = k_0 \left| \frac{f_i(t)}{f_{co}} \right| \sin(\dot{x}(t)) \left\{ \begin{array}{ll}
1 & \text{sign} \left[ \frac{f_i(t)}{f_{co}} \right] \sin(\dot{x}(t)) \\
0 & \text{otherwise}
\end{array} \right. 
\]

(15)

where \( f_i(t) \) is the output, \( x \) is the input displacement between the two surfaces in contact, \( f_{co} > 0 \) is the Coulomb friction force, \( r_0 \) is a parameter that determines the force deflection curve, and \( k_0 \) is the initial stiffness, that is, the slope of the force–deflection curve when \( f_i(t) = 0 \). In structural and civil engineering, Dahl model is used to simulate the hysteric behavior of magnetorheological (MR) dampers [72].

(d) LuGre model

The LuGre model is given by Equation (16) as:

\[
\dot{s}(t) = x(t) - \frac{1}{\eta_L(\dot{s})(t)} \dot{\omega}(t) \\
\]

(16a)

\[
f_i(t) = a_0 s(t) + a_1 \dot{s}(t) + a_2 \ddot{s}(t) \\
\]

(16b)

where \( a_0, a_1, \) and \( a_2 \) are positive constants. There are several different expression of \( \eta_L(\dot{s}(t)) \), one of them is:

\[
\eta_L(\dot{s}(t)) = \frac{f_{co}}{a_0} + \frac{f_s - f_{co}}{a_0} \exp \left( -\frac{(\dot{s}(t))^2}{\alpha_s} \right) \\
\]

(17)

where \( f_{co} \) is the Coulomb friction force, \( f_s \) is the stiction (sticking friction) force, and \( \alpha_s \) is the Stribeck velocity. Similar to Dahl model, LuGre model is mainly used to simulate MR dampers [73,74] due to its physical prototype.

3.2.3. Vaiana–Rosati model

Vaiana–Rosati (VR) model is developed by Vaiana and Rosati [75,76] recently for the description of complex hysteretic behavior. There are two equivalent forms of VR model as differential form and analytical form. The differential form can be written as:

\[
\dot{f}_i^v(x) = \left[ k^v_{2i}(x) + \alpha^- \left( f^v_{2i}(x) + k^v_{0i} x + f^v_{0i} - f^v_{2i}(x) \right) \right] \dot{x} \\
\]

(18a)

\[
\dot{f}_i^s(x) = \left[ k^s_{2i}(x) + \alpha^+ \left( f^s_{2i}(x) + k^s_{0i} x - f^s_{0i} - f^s_{2i}(x) \right) \right] \dot{x} \\
\]

(18b)

\[
f^l_{s2i}(x) = \beta^l_1 \exp(\beta^l_2 x) - \beta^l_3 + \frac{4\gamma^l_1}{1 + \exp(-\gamma^l_2 (x - \gamma^l_3))} - 2\gamma^l_1 \\
\]

(18c)

\[
f^l_{s1i}(x) = \beta^l_1 \exp(\beta^l_2 x) - \beta^l_3 + \frac{4\gamma^l_1}{1 + \exp(-\gamma^l_2 (x - \gamma^l_3))} - 2\gamma^l_1 \\
\]

(18d)

\[
k^l_{s2i}(x) = \frac{\partial f^l_{s2i}}{\partial x} = \beta^l_1 \beta^l_2 \exp(\beta^l_2 x) + \frac{4\gamma^l_1 \gamma^l_2 \exp(-\gamma^l_2 (x - \gamma^l_3))}{1 + \exp(-\gamma^l_2 (x - \gamma^l_3))} \\
\]

(18e)

\[
k^l_{s1i}(x) = \frac{\partial f^l_{s1i}}{\partial x} = \beta^l_1 \beta^l_2 \exp(\beta^l_2 x) + \frac{4\gamma^l_1 \gamma^l_2 \exp(-\gamma^l_2 (x - \gamma^l_3))}{1 + \exp(-\gamma^l_2 (x - \gamma^l_3))} \\
\]

(18f)

where \( k^v_{0i}, f^v_{0i}, f^v_{2i}, \alpha^-, \beta^l_1, \beta^l_2, \gamma^l_1, \gamma^l_3, k^l_{0i}, \alpha^+, \beta^l_1, \beta^l_2, \gamma^l_1, \gamma^l_3 \) are the parameters which control generic loading (unloading) phase. When the hysteretic loop is symmetric, we have \( k^v_{0i} = k^l_{0i}, f^v_{0i} = f^l_{0i}, \alpha^+ = \alpha^- = \beta^l_1, \beta^l_2, \gamma^l_1 = \gamma^l_3 = \gamma^l_2 = \gamma^l_3 = \gamma^l_2 \).

VR model is utilized to describe complex hysteretic behaviors including asymmetry, pinching, S-shape, and Flag-shape by Vaiana and Rosati. This model has good potential. However, some basic problems such as parametric analysis still require further research.
3.3. Operator based models

Operator based models utilize operators to characterize hysteretic behavior in nonlinear structural systems. The hysteretic operators are superimposed with weights to describe the complex relation between displacement and restoring force. Several operator hysteretic models have been developed during the past few decades such as Preisach model [77], Prandtl-Islinskii model [21], Maxwell-slip model [78] and Kranosel’skii-Pokrovskii model [79].

3.3.1. Preisach model

Preisach model is an operator based model that employs a weighted superposition of nonlinear operators to capture the hysteretic behaviors in nonlinear systems. It provides a generalized expression to describe the relationship between the magnetic field and magnetization of a magnetic material as the parallel connection of independent relay hysterons. The relay hysteron is defined as an operator that acts on the field and produces rectangular loops. Hysteresis loop is described by the superposition of the hysterons. The model is formulated with a double integrator:

\[
f_s(t) = \int_{\alpha_p \geq \beta_p} \mu(\alpha_p, \beta_p) \gamma_{\alpha_p \beta_p}[x(t)] d\alpha_p d\beta_p
\]

where \(x(t)\) is the input; \(\alpha_p\) and \(\beta_p\) are the upper and lower thresholds of the Preisach operator, respectively; \(\gamma_{\alpha_p \beta_p}[\bullet]\) is the delayed relay operator; \(\mu(\alpha, \beta)\) is the density function; and \(f_s(t)\) is the output.

The Preisach operator is defined by the delayed relay operator given by:

\[
\gamma_{\alpha_p \beta_p}[x(t)] = \begin{cases} 
1, & x(t) \geq \alpha_p \\
-1, & x(t) < \beta_p \\
\gamma_{\alpha_p \beta_p}[x(t-\xi)], & \beta_p < x(t) < \alpha_p 
\end{cases}
\]

The previous output \(\gamma_{\alpha_p \beta_p}[x(t-\xi)]\) is \(\zeta\), where \(\zeta \in \{-1, 1\}\) is the state of the relay. The density function \(\mu(\alpha_p, \beta_p)\) is a non-negative weight function to represent the weight of each distinct hysteron. The total displacement is obtained by the superposition of the sequence of hysterons in the Preisach plane \(P = \{(\alpha_p, \beta_p) : \alpha_p \geq \beta_p, \alpha_p \leq \bar{\alpha}_p, \beta_p \geq \bar{\beta}_p\}\), where \(\bar{\alpha}_p\) and \(\bar{\beta}_p\) are the highest and lowest values of \(\alpha_p\) and \(\beta_p\). The general form of the numerical solution can be expressed as:

\[
f_s(t) = 2[F(\alpha_{p,1}, \beta_{p,1}) - F(\alpha_{p,1}, \beta_{p,1})] 
- F(\alpha_{p,1}, \beta_{p,1}) + F(\alpha_{p,1}, \beta_{p,n}) 
+ 2 \sum_{i=1}^{n-1} [F(\alpha_{p,i+1}, \beta_{p,i}) - F(\alpha_{p,i+1}, \beta_{p,i+1})] 
\]

\[
F(\alpha_{p,i}, \beta_{p,i}) = \int_{\alpha_p \geq \beta_p} \mu(\alpha_p, \beta_p) d\alpha_p d\beta_p
\]

Spanos et al. [80] considered the weight function as Equation (13) to describe the hysteretic behavior:

\[
\mu(\alpha_p, \beta_p) = \frac{k_0}{2} \delta(\alpha_p - \beta_p) - \frac{k_0}{2(f_{\max} - f_{\min})} [H(\alpha_p - \beta_p) 
- \frac{2f_{\max}}{k_0} - H(\alpha_p - \beta_p - \frac{2f_{\max}}{k_0})]
\]

where \(\delta()\) is the Dirac delta function and \(H()\) is Heaviside function.

The Preisach model offers a powerful means to assimilate arbitrary hysteresis with rate-independent behavior. Recently, Cacciola et al. [81,82] used Preisach model to realize the steady state and seismic response analysis of soil-structure interaction systems. However, there are several drawbacks to implement the model. In particular, the accuracy of the output state depends on the amount of captured data and the inverse problem of the model is computationally complicated.

3.3.2. Prandtl-Islinskii (PI) model

Prandtl-Islinskii model is considered to be a subset of the Preisach model which is applicable for hysteresis modeling in materials. This model includes two basic operators as play operator and stop operator. Play operator can be described by Equation (19):

\[
v(0) = P_s[x](0) = f_s(x(0), 0)
\]

\[
v(t) = P_s[x](t) = f_s(x(t), P_s[x](t))
\]
where \( t_i < t < t_{i+1} \), \( x(t) \) is the input of play operator which is consider to be a monotonic function in time interval \([t_i, t_{i+1}]\), and \( v(t) \) is the system output. \( f_r(x, v) \) can be expressed as:

\[
f_r(x, v) = \max(x - r_p, \min(x + r_p, v))
\]  

where \( r_p \) is the parameter of the operator, the relation between input and output of play operator can be plotted as in Fig. 11.

Stop operator can be described by Eq. (21):

\[
v(0) = S_r[x](0) = e_r(x(0)) \tag{25a}
\]

\[
v(t) = S_r[x](t) = e_r(x(t) - x(t_i) + v(t_i)) \tag{25b}
\]

where \( t_i < t < t_{i+1} \), \( e_r() \) can be expressed as:

\[
e_r(x) = \min(r_p, \max(-r_p, x)) \tag{25c}
\]

Prandtl-Ishlinskii model can be defined by play and stop operator in a continuous form as:

\[
v(t) = \int_0^R w(r_p) P_r[x](t) dr_p \tag{26a}
\]

\[
v(t) = \int_0^R w(r_p) S_r[x](t) dr_p \tag{26b}
\]

where \( w(r_p) \) is the weight function, \( R \) is the upper bound of the integral. Due to the difficulty in integral computation, discrete form is preferable in engineering practice as:

\[
v(t) = \sum_{j=1}^M w_j P_j[x](t) \tag{27a}
\]

\[
v(t) = \sum_{j=1}^M w_j S_j[x](t) \tag{27b}
\]

Classical Prandtl-Ishlinskii model is based on the constitutive relation of ideal plastic materials. Therefore, it cannot describe the complicit hysteretic loop. Several modifications are made to improve the ability of PI model in asymmetry [83,84] and rate-dependent [85] hysteresis modeling.

PI model has been applied to describe the hysteresis of structural systems and used for nonlinear structural dynamic analysis. Krejčí and Sprekels [86] used the PI model to simulate the plasticity of a beam structure. Joghataie and Farrokh [87] predicted the nonlinear response of shear structures using artificial neural network and PI model. Farrokh et al. [88] proposed a generalized PI model to describe the deterioration of structures. Wang et al. [89] use a modified PI model to simulate the asymmetry hysteric behavior. Mokhtari and Imanpour [90] utilize PI model in the hysteresis simulation of steel structure and built the digital-twin model.

3.3.3. Kranosel’skii-Pokrovskii model

Kranosel’skii-Pokrovskii (KP) model improved the Preisach model by representing the hysteretic behavior with a linear
combination of hysteresis operators. In contrast to the discontinued Preisach operator, Kranos’kii-Pokrovskii model introduced a continuous operator to describe hysteresis loops. The elementary operator of the model is a special instance of generalized play operator on the Preisach plane, which encompasses minor loops within its major loop. Herein, the Preisach plane is defined as:

\[
P = \{ p(p_1, p_2) : V^- \leq p_1 \leq p_2 \leq V^+ - a \}
\]

where \( a \) is a positive constant; \( V^+ \) and \( V^- \) are the positive and negative saturation values of input voltage \( v(t) \in C[0, T] \), respectively; and \( C[0, T] \) denotes the space of continuous piecewise monotone functions on the interval \([0, T]\). The elementary operator \( \kappa_p(v(t), \xi_p) \) is a mapping from \( C[0, T] \) to the function space of hysteresis output. Given an input voltage \( v(t) \), the operator \( \kappa_p(v(t), \xi_p) \) maps points from \( p(p_1, p_2) \in P \) to the interval \([1, 1]\):

\[
\kappa_p(v(t), \xi_p) = \begin{cases} 
\max_{p \in P} \{ \xi_p, r(v(t) - p_1) \}, & \text{if } v(t) \text{ is nondecreasing} \\
\min_{p \in P} \{ \xi_p, r(v(t) - p_1) \}, & \text{if } v(t) \text{ is nonincreasing}
\end{cases}
\]

where \( r(\bullet) \) is the Lipschitz continuous ridge function. Consequently, the hysteresis output can be obtained using the following double integral:

\[
f_s(t) = \iint_p \kappa_p(v(t), \xi_p) \mu(p_1, p_2) dp_1 dp_2
\]

where \( \mu(p_1, p_2) \geq 0 \) is the density function to weight the outputs of \( \kappa_p(v(t), \xi_p) \).

Due to the memory effect to capture all previous extremes of the hysteresis input–output behavior, the continuous operator provides more information of the nonlinearity than the Preisach operator. It turns out that the Kranos’kii-Pokrovskii model is a more
3.3.4. Maxwell-slip model

The Maxwell-slip model is a hysteresis model that was initially to describe the behavior of friction in mechanical systems. It can be seen as a generalized form of the Duhem model in an operator based formulation. This model incorporates elements similar to the stop operator in the Prandtl-Ishlinskii (PI) model. The Maxwell-slip model consists of massless linear springs and massless blocks that are subject to Coulomb friction, as depicted in Fig. 12. The model represents the hysteretic behavior observed in systems with friction, where the response depends on the relative motion between contacting surfaces. Therefore, the Maxwell-slip model can be expressed as shown in Equation (31):

\[
f_s = \sum_{i=1}^{m} f_i \tag{31a}
\]

\[
f_i = \begin{cases} 
    k_i(x - x_{b_i}) & |k_i(x - x_{b_i})|/f_i \\
    \mu N_i \text{sign}(\dot{x}) & x_{b_i} = x - \frac{\mu N_i}{k_i} \text{sign}(\dot{x})
\end{cases} \tag{31b}
\]

where \(x_{b_i}\) is the position of the \(i^{th}\) block, \(k_i\) is the stiffness of \(i^{th}\) spring, \(x\) is the displacement, \(f_i\) is the spring force, and \(N_i\) is the pressure on the \(i^{th}\) block and \(\mu\) is the friction coefficient.

4. Identification of hysteresis based on a parametric model

The process of model-based hysteresis identification can be depicted in Fig. 13. The process comprises of two main steps: the selection of an appropriate hysteresis model and the identification of model parameters based on experimental data.

The selection of a hysteresis model holds significant importance. Simple polynomial models, such as bilinear hysteresis models, can be directly from experimental data. Take the bilinear hysteresis model as an example, wherein two parameters are employed to control the model. One is the elastic stiffness \(k_0\) while the other is the post-yielding stiffness \(k_y\). Both of these two parameters can be easily obtained through either static or dynamic test.

For more complex models, such as differential based models and operator based models, parameter estimation algorithms becomes imperative. This is primarily due to the absence of explicit physical interpretations for all model parameters. The process of estimating hysteretic parameters can be formulated as an optimization problem as:

\[
\begin{array}{c}
\text{Data} \\
\text{Structural response and restoring force} \\
\text{Parameter estimation} \\
\text{Determine the parameter of hysteresis model} \\
\text{Identification result} \\
\text{Hysteresis model and model parameters} \\
\text{Characterization} \\
\text{Which hysteresis model should we use?} \\
\text{Hysteresis model} \\
\text{Such as bilinear model, Bouc-Wen model etc.}
\end{array}
\]

Fig. 13. Flow of hysteresis identification based on a parametric model.
\[
\begin{align*}
\min_{\theta} & \left( f_s(x, \theta), \tilde{f_s} \right) \\
\theta & \in \Theta
\end{align*}
\]  

(32)

where \( f_s(x, \theta) \) is the output of hysteretic model with \( x \) as input and \( \theta \) as model parameters, \( \tilde{f_s} \) is the actual restoring force (obtained by experiment or structural monitoring), and \( J(f_s(x, \theta), \tilde{f_s}) \) is the cost function which is utilized to assess the difference between model output and the actual restoring force. \( \Theta \) is domain of definition of \( \theta \). The optimization problem can be solved by different algorithms including least mean square algorithm, extended Kalman filter algorithm, and metaheuristic algorithms. Recently, there has been a development in probabilistic approaches to account for the uncertainty in structural systems.

4.1. Least mean square (LMS) algorithm

To employ the LMS algorithm for the identification of hysteretic parameters, the hysteretic model is initially represented in a regression form as Eq. (33):

\[
f_s(t) = \phi^T \theta + e(t)
\]

(33)

where \( \phi = [\phi_1(t), \phi_2(t), \ldots, \phi_n(t)] \) is the vector of regression variables, \( \theta \) is the parameter vector, \( e(t) \) is the error.

The cost function can be given as:

\[
J(\theta) = \sum_{i=1}^{N} \left( \phi^T(t_i) \theta - \tilde{f_s}(t_i) \right)^2
\]

(34)

The optimal value \( \theta^* \) can be obtained by letting the derivative of cost function equal to zero. Therefore, \( \theta^* \) can be expressed as:

\[
\theta^* = \sum_{i=1}^{N} \phi(t_i) \tilde{f_s}(t_i) \left( \sum_{i=1}^{N} \phi^T(t_i) \phi(t_i) \right)^{-1} \]

(35)

The LMS algorithm has been effectively employed in numerous studies for the purpose of hysteresis identification. Toussi and Yao [91] employed the LMS algorithm to identify the hysteresis of existing structures in the early 1980s. They utilized a polynomial hysteretic model and implemented the LMS algorithm to estimate the multinomial coefficients. Sues et al. [92] utilized LMS algorithm to identify the parameters of a degrading Bouc-Wen system. Building upon their work, Loh and Chung [93] combined the LMS with the Newton-Gauss approach to achieve parameter identification of degrading and pinched hysteretic structural systems. Kunnath et al. [94] combine least-square algorithm with Newton-Gauss approach and realize the parameter identification of degrading and pinching hysteretic structural system. Chassiako et al. [95] and Smyth et al. [96] conducted the online hysteresis identification of Bouc-Wen class model. A few years later, Lin et al. [97] developed a variable-trace-based parametric identification algorithm for hysteresis identification based on LMS algorithm. Yang and Lin [98] proposed a LMS algorithm with adaptive tracking capacity.

4.2. Kalman filter algorithm

Kalman filter algorithm can be utilized for the identification of dynamic hysteretic systems under stochastic disturbances.

![Fig. 14. Process of extend Kalman filter.](image)
Compared to the LMS algorithm, the Kalman filter algorithm is regarded as having a higher estimation accuracy due to its utilization of a two-step parameter update process involving prediction and measurement. This algorithm considers the parameters of hysteresis model as state variables. The system equation and observation equation in the extended Kalman filter algorithm are given as [99]:

**System equation:**

\[
X_{k+1} = X_k + \int_{t_k}^{t_{k+1}} g_1(X, F(t)) dt + w_k
\]

**Observation equation:**

\[
Z = g_2(X) + v
\]

where \(X = [x^T \dot{x}^T \theta_1 \theta_2 \ldots \theta_m]^T\) is the state vector which include the displacement and velocity response of structural system \(x\) and \(\dot{x}\) as well as the model parameters \(\theta_1, \theta_2, \ldots, \theta_m\), \(g_1()\) is a nonlinear function, \(F(t)\) is the matrix of external excitation varying with time, \(Z\) is the observation vector and \(g_2()\) is the observation function, \(w_k\) and \(v\) are the system and measurement noise.

The process of extend Kalman filter can be described in Fig. 14, where \(W_k\) and \(V_k\) are the covariance matrix of system and measurement noise, \(\Phi_k\) and \(H_{k+1}\) can be expressed as follows:

\[
\Phi_k = \exp\left(\Delta t \times \frac{\partial f}{\partial X} |_{X=X_k}\right)
\]

\[
H_{k+1} = \frac{\partial h}{\partial X} |_{X=X_k+1}
\]

Hoshiya and Saito [100] first applied the extended Kalman filter in structural system identification. Their research mainly focused on the linear system and nonlinear systems with a bilinear hysteresis. Yang and Ma [101] employed the constrained Kalman filter to identify the structural system with hysteresis. Yang et al. [98] proposed an adaptive Kalman filter in system identification of hysteresis systems. Wu and Symth [102,103] used unscented Kalman filter for hysteresis identification of nonlinear structural systems and conducted parameter identification of pinching hysteresis systems. Ghosh et al. [104] proposed a new variant of EKF via transversal linearization and applied it in the system identification. Yang et al. [105] applied the square root center difference Kalman filter in the system identification of RC frame structures with Bouc-Wen hysteresis. Li and Wang [106] introduced the utilization of the constrained unscented Kalman filter (CUKF) in the hysteresis identification of structural systems.

### 4.3. Metaheuristic algorithms

Metaheuristic algorithms represent a category of optimization algorithms that are employed to tackle intricate and challenging problems spanning diverse domains. They aim to identify optimal solutions by conducting searches within the solution space, employing specific searching strategies. For complex hysteretic models, LMS algorithm or Kalman filter algorithm may fall into local optimum. Metaheuristic algorithms can realize a better parameter identification of hysteretic system. Common examples of metaheuristic algorithms include: Genetic algorithms (GA), Particle swarm optimization (PSO), Simulated annealing (SA), Tabu search (TS), differential evolution (DE) algorithm, and swarm intelligence algorithms. In this review, GA and PSO will be emphatically introduced.

**Fig. 15.** Flow chart of GA for hysteretic parameter identification.
4.3.1. Genetic algorithm (GA)

GA is an intelligent optimization algorithm inspired by biological evolution. It mimics the process of natural selection, where individuals with better fitness are selected for reproduction to produce offspring for the next generation. The flowchart of GA based parameter identification is shown in Fig. 15, consisting of six phases:

**Encoding:** The model parameters vector, denoted as: \( \theta = [\theta_1, \theta_2, \ldots, \theta_n]^T \), where \( \theta \) is encoded as chromosomes or individuals. The component of \( \theta \) such as \( \theta_i \) is considered as gene and encoding by binary coding or real number coding.

**Initialization:** A population of \( N \) individuals is randomly generated as the initial population for the GA. Each individual represent a candidate solution of optimal hysteretic parameters.

**Fitness calculation:** The fitness of each individual in the population is calculated. In hysteresis identification problems, the fitness function is defined as Equation (33):

\[
J(\theta) = \sum_{i=1}^{N} (f_i(t_i, \theta) - \hat{f}_i(t_i))^2
\]

where \( f_i(t_i, \theta) \) is the restoring force outputted by hysteresis model, \( \hat{f}_i(t_i) \) is the real restoring force.

**Selection:** Several individuals with superior fitness are selected for reproduction. The selection process favors individuals with lower fitness values, increasing their chances of being chosen as parents for the next generation.

**Crossover:** The selected individuals are paired as parents. For each pair of parents to be mated, a crossover point is chosen at random from within the genes.

**Mutation:** With a low random probability, some genes of the offspring may undergo mutation, introducing small random changes to their values.

After several iterations, when the stop condition is met (usually when the fitness of best performing individual is lower than a specified value or the maximum number of iterations is reached), the best performing individual is selected as the optimal solution and the model parameters are output.

Zheng et al. [107] were the first to apply GA in the parameter identification of Bouc-Wen hysteresis system. After that a series of research is carried out for parameter identification of Bouc-Wen and Bouc-Wen-Baber-Noori class models using different modified GA methods [108–113]. Herli et al. [114] use GA for the parameters identification of Preisach model. Shakiba et al. [115] study the parameter identification of a rate-dependent PI model. Mustafa et al. [116] applied GA in the identification of Maxwell-slip hysteresis system.

4.3.2. Particle swarm optimization (PSO)

PSO proposed by Kennedy, Eberhart [117] simulates the movement of organisms in a bird flock or fish school and can be utilized for parameter identification of hysteresis models. The standard PSO in parameter identification is presented as follows:

Algorithm 1. The standard PSO algorithm.

1. **Initialization:**
   - For each particle \( i = 1 \) to \( N \) do:
     - Initialize the particle’s position with a uniformly distributed \( \theta_i \sim U(b_l,b_u) \)
     - The position of each particle is a candidate solution vector for hysteretic parameters
     - \( b_l \) and \( b_u \) are the low and upper boundary of hysteretic parameter vector \( \theta \)
     - Initialize local best known position of each particle as \( p_i = 0 \)
     - if \( J(p) < J(g) \) then update the global best known position \( g = p_i \)
     - \( J(\cdot) \) is the fitness function as Equation (33)
   - Initialize the particle’s velocity: \( v_i \sim U([b_u-b_l], [b_u-b_l]) \)

2. **While** the stop condition is not satisfied **do**:
   - for each particle \( i = 1 \) to \( N \) do:
     - for each dimension \( j = 1 \) to \( n \) do:
       - Generate: \( r_p, r_g \sim U(0,1) \)
       - update velocity as: \( v_{ij} = wv_{ij} + c_1r_p(p_{ij} - x_{ij}) + c_2r_g(g_{ij} - x_{ij}) \)
       - \( w, c_1, c_2 \) are parameters of PSO
       - update position as: \( x_{ij} = x_{ij} + v_{ij} \)
     - if \( f(p_i) < f(g) \) then update the global best known position \( g = p_i \)
   - Output optimal solution \( g \) as the identified result of model parameters

Similar to GA, PSO is mainly applied in the parameter identification of differential based and operator based hysteresis model such as Bouc-Wen class model [118–121], Duhem model [122], Preisach Model [123], Prandtl–Ishlinskii model [124], and Krasnoselskii-Pokrovskii model [125].

4.3.3. Other metaheuristic algorithms

Apart from the Genetic Algorithm (GA) and Particle Swarm Optimization (PSO), various metaheuristic algorithms have also been utilized for parameter identification of hysteresis models.

(a) Simulated annealing (SA) algorithm
Simulated Annealing (SA) is a stochastic optimization algorithm inspired by the annealing process in metallurgy, where materials are heated and then slowly cooled to remove defects and achieve a more ordered crystalline structure.

Similar as GA and PSO, in parameter identification of hysteretic model, SA is used to find an approximate solution of optimization problem as described in Eq. (28). In SA algorithm, the initial solution of hysteretic parameter identification are first generated. Then, in each step of the SA algorithm, the solution is replaced by a random “nearby” solution until the stopping criterion for iteration is met. The “nearby” solution is chosen with a probability that depends both on the difference between cost functions $\Delta J = J_i(\theta) - J_{i-1}(\theta)$ and parameter $T$ (temperature).

SA algorithm is applied in the parameters identification of different hysteretic models. At first, it is used in the parameters identification of magnetic hysteresis models [126-128]. Tian et al. [129] apply SA in the identification of Bouc-Wen model. Ling et al. [130] realize the application of SA in the parameter identification of pivot model in a reinforced concrete structure.

(b) Swarm Intelligence algorithms

Swarm intelligence algorithms are a class of metaheuristic algorithms inspired by the collective behavior of social organisms, such as ants, bees, birds, and fish. These algorithms are designed to model the decentralized, self-organized, and cooperative behaviors exhibited by these natural systems to find solutions to complex problems. In hysteresis identification problem, beside PSO, there are lots of swarm intelligence algorithms which can be utilized. These include the artificial Bee Colony algorithm (ABC) [131], bat algorithm (BA) [132], differential Evolution (DE) [133,134], fish swarm algorithm [135], grey wolf (GW) algorithm [136], and cuckoo search (CS) algorithm [137].

Expect the algorithms mentioned above, there are still numerous metaheuristic algorithms such as gravitational search algorithm [138], and charged system search [139,140] can be used for the parameter identification of parametric hysteresis model. There parameter identification is an open problem. Better optimal solutions can be obtained with the development of optimization algorithms.

4.4. Bayesian estimation

Bayesian methods have been developed for parameter estimation and uncertainty quantification in hysteretic structural systems. The basic concept of Bayesian estimation involves updating the prior distribution of parameters based on the observed measurements. Bayes’ theorem is given by Eq. (39):

$$p(\theta|\psi) = \frac{p(\psi|\theta)p(\theta)}{p(\psi)}$$  \hspace{1cm} (39)

where $\theta$ is the parameter vector of hysteretic system, $\psi$ is the measurements of hysteretic structural system, $p(\theta)$ is the prior distribution of $\theta$, $p(\theta|\psi)$ is the posteriori distribution and $p(\psi|\theta)$ is the likelihood function. The denominator is a normalizing constant such that integrating the right hand side over the parameter space yields unity.

Based on the Bayes’ theorem formula, several methods have been developed for structural system identification including Bayesian model updating [141], Bayesian fast Fourier transform [142], and Bayesian filter [143]. Li et al. [144,145] utilized Bayesian bootstrap filter in hysteretic structural identification. Muto and Beck [146] applied Bayesian updating method with model class selection technology in system hysteresis. Worden and Hensman [147] conducted parameter estimation of hysteretic system based on Bayesian inference and model selection. Liu and Au [148] model hysteretic behavior of composite walls based on Bayesian method. Green and Worden [149] used Bayesian estimation and Markov chain Monte Carlo methods to quantify the uncertainty of nonlinear systems. Erazo and Nagarajah [150] combined unscented Kalman filtering and Bayesian estimation in hysteretic identification. Yuen and Kuok [151] developed a Bayesian calibration method and successfully implemented real-time hysteretic system identification.

5. Data-driven method for hysteresis description

Data-driven methods for modeling hysteretic behaviors in structural systems rely on universal mathematical models that do not incorporate the specific characteristics of hysteresis. These methods are considered to be more flexible compared to model-driven approaches, but they require a larger amount of hysteresis data for accurate modeling. In this section, we will introduce several data-driven methods, ranging from classical regression models to emerging deep learning methods.

5.1. Regression model

The output of a hysteresis system at the current instant, denoted as $f_i(t_i)$, is related to the state at previous time instants $f_{i-1}(t_{i-1}), ..., f_{i-j}(t_{i-j})$ and the current input $x(t_{i-j})$. This relationship can be described by regression model. Commonly-used regression models include: autoregressive with exogenous inputs (ARX) model , autoregressive moving average model (ARMA) model [152,153], autoregressive moving average with exogenous inputs (ARMAX) model [154], nonlinear autoregressive moving average model with exogenous inputs (NARMAX) model [24,155,156]. They can be expressed as Equation (40) to (43) as follows:

ARX model:
\[ f_i(t_i) = \sum_{j=1}^{i} a_{ij}(t_{ij}) + \sum_{j=1}^{m} b_{ij}(t_{ij}) + \epsilon_i \] (40)

where \( a_i \) and \( b_i \) are coefficient, \( \epsilon_i \) \( \text{WN}(0, \sigma^2) \) is the Gaussian white noise with variance \( \sigma^2 \).

ARMA model:

\[ f_i(t_i) = \sum_{j=1}^{i} a_{ij}(t_{ij}) + \sum_{j=1}^{p} c_{ij}(t_{ij}) \] (41)

where \( c_j \) is the coefficient and \( \epsilon_i \) \( \text{WN}(0, \sigma_i^2) \).

ARMAX model:

\[ f_i(t_i) = \sum_{j=1}^{i} a_{ij}(t_{ij}) + \sum_{j=1}^{m} b_{ij}(t_{ij}) + \sum_{j=1}^{p} c_{ij}(t_{ij}) \] (42)

NARMAX model:

\[ f_i(t_i) = \mathcal{F}(f_i(t_{i-1}), \ldots, f_i(t_{i-p}), f_{i-1}(t_{i-1}), \ldots, f_{i-p}(t_{i-p}), \epsilon_i, \ldots, \epsilon_{i-p}) \] (43)

where \( \mathcal{F}() \) is a nonlinear function.

It should be noted that since ARX, ARMA and ARMAX are linear models, the inputs are usually processed by nonlinear functions.

5.2. Artificial neural network

The artificial neural network (ANN), also known as a multi-layer perceptron, is a popular machine learning model for the hysteresis modeling. The ANN model can be represented by Equation (44):

\[ h^1 = \phi^1(W^1 x + b^1) \] (44a)
\[ h^{i+1} = \phi^{i+1}(W^{i+1} h^i + b^{i+1}) i = 1, 2, \ldots, m - 1 \] (44b)
\[ y = \phi^{m+1}(W^m h^m + b^{m+1}) \] (44c)

where, \( x \) represents the input, \( y \) represents the output, \( h^i \) denotes the output of the \( i \)th hidden layer, \( W^i \) is the weight matrix of the \( i \)th layer, and \( b^i \) is the bias term. The activation function \( \phi \) determines the nonlinearity of each node.

As a universal approximator, ANN can be employed to approximate the relationship between the structural response and the restoring force. For instance, Ghaboussi et al. [157] utilized ANN to model the strain-stress hysteretic loops of concrete. In their ANN model, the inputs were stress, strains, and stress increments, while the output was the corresponding strain increment.

Building upon Ghaboussi et al.’s pioneering work, several studies have explored the application of ANN in hysteresis modeling. Basheer [158] employed ANN to model the hysteretic behavior of soil, while Gawin et al. [26] investigated the sorption hysteresis of concrete structures using ANN. Yun et al. [159] proposed a novel method for hysteretic modeling based on ANN. Kim et al. [160,161] utilized an ANN-based approach to model steel beam-to-column connections and bolted steel frame connections. Brewicke et al. [162] developed an ANN model to capture the hysteretic behavior of NiTiNOL and steel strands. Yan et al. [163] focused on the hysteretic performance of composite reinforced concrete beams using ANN. Some researchers [164–167] have utilized radial basis function (RBF) neural networks for hysteresis modeling.

It is worth noting that these ANN-based methods share a common objective of approximating a nonlinear function. The restoring force \( f_i(t_i) \) is expressed as a function of displacement and the hysteretic histories in Equation (45):

\[ f_i(t_i) = AN(f_i(t_{i-1}), \ldots, f_i(t_{i-p}), x(t_{i-1}), \ldots, x(t_{i-p}), \epsilon_i, \ldots, \epsilon_{i-p}) \] (45)

From the perspective of nonlinear system identification, ANN can be considered as a black box model [168] for hysteretic system. A well-trained ANN model can directly output structural system response when input external excitations. Therefore, the restoring force can be easily obtained using the Equation (46):

\[ f_i(t_i) = f_i(t_i) - M\dot{x}(t_i) - Cx(t_i) \] (46)

Early examples of ANN based system identification began in the 1990s. Masri et al. [169] and Chassiakos & Masri [170] conducted studies on the identification of structural systems with nonlinear restoring forces. However, these structural systems did not exhibit classical hysteretic behavior. Bani-Hani and Ghaboussi [171] developed an ANN model for system identification and prediction of hysteretic responses in the field of active control. Kosmatopoulos et al. [170] and Pei et al. [172] explored the use of Volterra/Wiener neural networks for identifying hysteretic structural systems. Volterra/Wiener neural networks consist of a linear filter connected in series with a neural network with linear-in-the-weights. Liang et al. [173] utilized fuzzy neural networks for system identification and hysteresis restoring forces modeling. Wu and Kareem [174] applied ANN to describe the hysteretic behavior of bridges. Lagaros and...
Papadrakakis [175] investigated the system identification of 3-D buildings under earthquake excitations and plotted hysteresis loops of displacement and restoring force. Son et al. [176] studied the system identification of nonlinear hysteretic systems with uncertainty using a Jaya-based adaptive neural network.

5.3. Least square support vector machine

Least square support vector machine (LSSVM) is a classical variant of support vector machine (SVM). Compared to SVM, LSSVM uses equality constraints instead of inequality constraints, as in ordinary SVM. Similar as ANN, LSSVM modeling the hysteretic behavior by approximating the autoregressive function in Eq. (47). The prediction function of LSSVM can be written as:

\[ f(x) = W\phi(x) + b \] (47)

where \( W \) is the weight matrix and \( b \) is the bias vector, \( \phi() \) is an nonlinear function employed to map the input into a high dimensional space.

Ji et al. [177] introduced a SVM-based feedforward controller combined with a self-tuning PID controller. Wang and Me [27] proposed two SVM-based hysteresis modeling methods. One method utilizes the hysteresis curve direction as one of the input variables, while the other method employs an improved version based on the autoregressive algorithm. Yang et al. [178] presented an LSSVM model for hysteresis modeling by incorporating an input sequences matrix to transform the multivalued mapping into a one-to-one mapping. It is worth noting that the LSSVM and SVM methods have not been widely used in structural hysteretic behavior modeling, possibly due to the popularity of other algorithms such as ANN. In terms of fitting ability, there is no significant difference between LSSVN and ANN.

5.4. Deep learning

Compared to classical ANN or LSSVM, deep learning allows for the creation of computational models that consist of multiple layers of processing. These models are capable of learning representations of data with multiple levels of abstraction [179]. This introduces several advantages in hysteresis modeling:

1. Depth of networks: Deep learning models can handle complex hysteretic behaviors by capturing hierarchical representations of data. The multiple layers allow for the extraction of more intricate features, leading to improved modeling capabilities and generalization performance.
2. Sequence-to-sequence prediction: Deep learning models can perform sequence-to-sequence predictions, which is particularly beneficial for hysteresis modeling. Unlike traditional ANN or SVM, where predictions are made step by step, deep learning models can handle the prediction of entire sequences (as Fig. 16). This helps reduce prediction errors caused by error accumulation and enhances the overall usability of the model.

5.4.1. Deep feedforward neural network

By incorporating multiple hidden layers, DFNN can learn and capture more complex patterns and features, enabling it to effectively model intricate hysteretic phenomena. The deep learning architecture enables the automatic extraction of hierarchical representations of the data, thereby enhancing the model’s capacity to generalize and make accurate predictions. Additionally, the use of specialized layers, such as normalization or dropout layers [180,181] further enhances the performance and robustness of the DFNN model.

Several research [182–185] have been carried out using DFNN for recognition of nonlinear hysteretic behavior. From those studies, it is shown that the introduction of deep learning greatly improves the ability of neural network. For instance, Amodeo et al. [185] utilized the DFNN model to accurately describe the hysteretic behavior of iron-dominated magnets.

5.4.2. Deep convolutional neural network

Deep Convolutional Neural Networks (CNNs) are renowned architectures in the field of deep learning and are widely used for processing gridded data, such as time series and images. CNNs leverage local connections and weight sharing to efficiently extract
features from the input data. Fig. 17 presents a typical architecture of CNN for hysteresis modeling. Where the input are displacement, external excitation and other related structural load or response, and the output is the restoring force.

Convolution operations are the core of CNNs. These operations are employed in parallel to extract a diverse set of hysteretic features from the input data. Additionally, to reduce computational complexity, parts of the convolution kernel can be skipped, leading to more efficient operations. The convolution operation can be mathematically represented as follows:

$$O_{i,j,k} = (X*V)_{i,j,k} = \sum_{l,m,n} X_{i-l+m,(j-1)s+m,n} V_{l,m,n,k}$$

where $O$, $X$, and $V$ are the output, input and convolution kernel, respectively.

A series of research on hysteresis based on CNN model has been conducted. Rodríguez-Torres et al. [185] applied a complex value convolutional neural network [186] to model hysteretic behavior. Additionally, CNN-based nonlinear system identification for hysteretic systems has been studied [187,188]. In this approach, the restoring force is indirectly simulated by performing the system identification of nonlinear hysteretic systems under seismic actions. Furthermore, Zhang et al. [189] proposed a physical-informed CNN model for nonlinear system identification and response prediction. This approach harnesses the capabilities of CNNs to extract features from data, while integrating the system’s physical knowledge to enhance prediction accuracy. By using CNNs in hysteresis modeling, researchers have successfully captured intricate patterns and relationships within the data. This has resulted in enhanced modeling accuracy and the ability to generalize effectively.

5.4.3. Deep recurrent neural network

Deep recurrent neural networks (RNNs) are well-suited for time series processing because they can effectively capture temporal dependencies and patterns in sequential data. RNNs have feedback connections that enable the persistence of information over time, making them highly effective for processing time-varying data, such as hysteretic behavior. A RNN model for hysteresis simulation is depicted in Fig. 18. In this model, each RNN cell accepts the input $x_i$ and the state $s_i$ and outputs the restoring force $f_i$. It indicates that the restoring force relies not only on the current input but also on the system’s history which is consistent with the definition of hysteresis.

There are several types of recurrent neural network models including Elman neural network (ENN) [190], long and short term memory (LSTM) neural network [191] and gate recurrent unit (GRU) neural network [192].

Early research on hysteresis modeling using recurrent neural networks (RNNs) often relied on the Elman Neural Network (ENN) due to its historical significance and the limited computing resources at that time. ENN has been successfully applied to model both rate-independent [193–196] and rate-dependent hysteretic behaviors [197] in various systems. Hysteretic Recurrent Neural Networks (HRNN), proposed by Veeramani et al. [194], are a modification of ENN that specifically targets hysteresis modeling. HRNN has been utilized to model the hysteretic behavior of various systems and has shown promising results [198,199]. Recent research has also applied LSTM [200–203] and GRU models [204] in hysteresis modeling. Ni et al [205] proposed a deep learning model named HysteresisNet, based on the bidirectional LSTM and GRU neural networks to predict the hysteretic behavior of RC columns under cyclic loading. Gu et al. [206] proposed a LSTM based deep learning model for modeling structural hysteresis. Zhang et al. [207] applied LSTM for system identification and improved the model’s performance by incorporating physics-based information. This approach has been successfully applied to different hysteretic structures in various studies [208–211].

5.4.4. Sequence to sequence model

The Sequence to Sequence (S2S) model, introduced by Sutskever et al. [212], is a powerful deep learning model for sequence prediction tasks. The architecture of the Seq2Seq model consists of two main components: the encoder and the decoder as Fig. 19.

Encoder: The encoder is responsible for processing the input sequence and generating a meaningful representation of it. The encoder typically uses recurrent neural networks (RNNs) such as LSTM or GRU to capture the sequential information in the input data.
Decoder: The decoder takes the representation or hidden states generated by the encoder as input and generates the output sequence. It uses another RNN, similar to the one used in the encoder, to generate the output sequence step by step.

Transformer model, proposed by Vaswani et al. [213], is a significant advancement in the field of sequence-to-sequence (Seq2Seq) models. Transformer model employs the attention mechanism to capture dependencies between elements in a sequence more efficiently. Readers can refer to Ref. [197] for more details.

Seq2Seq models have been employed to model the nonlinear behavior of materials or structures. Wang et al. [214] proposed a Seq2Seq model based on the GRU neural network and applied this model to predict responses in nonlinear hysteretic structures. The hysteretic loops can be obtained through the proposed model. Li et al. [215] introduced the attention mechanism into the Seq2Seq model and proposed an attention-based model for response prediction. Wang et al. [216] proposed a CNN-based Seq2Seq model to describe the hysteretic behavior of nonlinear materials. Wang et al. [217] introduced the multi-head attention mechanism into the Seq2Seq model, inspired by the Transformer model. They proposed an End-to-End deep learning framework for structural analysis to understand the behavior of hysteretic structures and perform structural dynamic analysis. Meng and Pei [218] applied Seq2Seq model in the hysteretic behavior modeling of saturated sand.

5.5. Other data-driven methods

In addition to the four data-driven methods discussed in Sections 5.1 to 5.4, there are several other methods in the literature and engineering practice for hysteresis modeling. In this section, a brief introduction to some of these methods will be presented, although the technical details will not be discussed.

Gaussian Processes (GP) are a probabilistic-based supervised learning method that can be used for solving regression problems. As an alternative to conventional regression methods, GP has been utilized for modeling hysteretic behaviors. It has been applied in modeling the hysteresis behaviors of actuators [219–221]. Additionally, de Oliveira Teloli et al. [222] used GP combined with the Nonlinear Auto-Regressive with eXogenous inputs (NARX) model and applied it in the hysteresis modeling of bolted structures.

Extreme Learning Machine (ELM) [223] is an emerging machine learning method that aims to improve the training efficiency of ANN models. In ELM, the model is trained by solving a linear function instead of using backpropagation and gradient descent. Therefore, ELM can be employed in online learning and fast modeling of hysteretic behavior [224–226].

Reinforcement Learning (RL) is a machine learning method concerned with how intelligent agents should take actions in an environment to maximize cumulative rewards. Kirkpatrick and Valasek [227,228] used RL to implement hysteresis modeling of shape memory alloys. Zhou et al. [229] applied RL in a fuzzy neural network and used it for hysteresis modeling.

In Section 5.1-5.4, four data-driven methods for hysteresis modeling have been introduced. However, there are still many other methods in the literatures or engineering practice. Therefore, in this section, a brief introduction of other methods will be presented. The technical details of the method will not be discussed.
6. Hybrid methods for hysteresis description

Since model-driven and data-driven methods each have their own advantages in terms of usability or flexibility, a novel strategy has been introduced to explore whether these two classes of methods can be used together to improve performance in hysteresis simulation. Scholars developed a series of novel approaches along this line. In this paper, hybrid methods are categorized. According to the different parametric models utilized in the method, they can be divided into three categories: polynomial model-data driven methods, differential based model-data driven methods, and operator based model-data driven methods.

6.1. Polynomial model-data driven method

In polynomial models, the hysteretic behavior is controlled by a parameter that has physical significance. As shown in Fig. 20, this method utilizes a data-driven algorithm to establish a relationship between data features and hysteretic parameters.

Saadat, Noori et al. [230] proposed an intelligent parameter varying approach and successfully achieved parameter estimation of a bilinear hysteretic model using an RBF neural network. Yekta et al. [231] applied artificial neural networks to model the hysteresis loop of Fe-48Ni alloys based on an ideal plastic hysteresis. Horton et al. [232] utilized deep learning methods and the bilinear modified IMK model to predict the hysteretic behavior of reduced beam section connections. Huang et al. [233] employed ANN and SVM to estimate the parameters of the pinching4 material model and a hysteretic material model.

While these studies have utilized various data-driven methods, they share a common framework. One of the main limitations of this framework is its requirement for a large amount of data. For example, in Ref. [233], the researchers had to collect 486 samples from the existing literature to build their model. In many engineering practices, the cost of a large dataset is unacceptable.

Aloisio et al. [234] proposed another method to combine polynomial models and data-driven methods. As shown in Fig. 21, the hysteretic curves is divided into six phases including the positive and backbone curves, two reloading curves and two unloading curves. For every phases, the restoring force data can be directly obtained from experimental data or by solving an optimization problem.

6.2. Differential based model-data driven methods

There are mainly two types of differential based model-data driven methods. The first one is similar to polynomial model-data driven method. It establishes the mapping relation between hysteresis data or hysteretic features and hysteretic parameter. This type of method has been used to determine the parameters of BW model [235], BWBN model [236,237], and Duhem hysteresis model [238].

The other approach is a deep fusion of the model-driven method and data-driven method. The topological structures of machine learning model (such as ANN or DL) are specially designed according to the hysteretic model. The system identification is finished through model training. This type of method significantly reduces the amount of training samples and enhances the interpretability of the proposed approach. Xie et al. [239] designed a Bouc–Wen model-based neural network (as shown in Fig. 22) and used it for hysteretic system identification. Wang and Chen [240] designed a model based on Duhem model (Fig. 23).

6.3. Operator based model-data driven method

As it is shown in Fig. 24, operator based models utilize a weighted combination of hysteretic operators to simulate the nonlinear behavior of structural systems. This form of expression is similar to some machine learning models such as ANN, ELM and, SVM. Therefore, hysteretic operators are inserted into different machine learning models as a pre-processing component and provide the basic characteristics of hysteretic system.

One example of an operator based model is the Preisach model combined with machine learning-based methods. Adly and Abd-El-Hafiz [241] used the Preisach operator as a pre-processing step and employed ANN for system identification. Similar research was conducted by Zakerzadeh et al. [242] and Srivastava et al. [243]. Ma et al. [244] proposed a Preisach model based SVM for hysteresis
Fig. 21. Six phase of hysteretic curve.

Fig. 22. Bouc–Wen model-based neural network [239].

Fig. 23. Duhem model-based neural network [240].
modeling. Farrokh [245] applied LSSVM with the Preisach model to simulate hysteretic behavior and used the LSSVM-based model in structural dynamic analysis. Ma et al. [246] developed a hybrid intelligent hysteresis model based on deep brief network, deep neural network, and the Preisach operator. Farrokh et al. [247] combined the Preisach operator with ELM to simulate different hysteretic behaviors.

Another widely used operator based model is the Prandtl-Islinskii (PI) model, which is appreciated for its physical significance as an

![Comparison of operator based hysteresis model and neural node.](image)

Table 3
Comparison of different hysteresis modeling methods.

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<th>Modeling method</th>
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ideal plastic hysteresis model. Joghataie and Farrokh [87] proposed a Prandtl-Islinskii based neural network model for hysteresis modeling and structural dynamic analysis. Farrokh et al. [88] introduced structural degradation into the Prandtl-Islinskii model and applied it in the generalized Prandtl-Islinskii based neural network model. Wang et al. [248] proposed a Prandtl-Islinskii based RNN model for hysteresis modeling. Wang et al. [89] also proposed a modified Prandtl–Ishlinskii model and fused it with ANN for the modeling of asymmetric hysteretic behavior. Hu et al. [249] integrated the PI operator into a CNN model for modeling hysteretic behavior.

7. Discussion

7.1. Evaluation of different hysteresis modeling methods

In this section, a comparison of different model-driven and data-driven hysteretic modeling methods will be presented. All of the methods mentioned before will be evaluated based on five categories and criteria, including:

1. Modeling Mechanism: How does the method model the hysteretic behavior? For example, some hysteretic models are based on the study of plastic deformation of materials, while others are based on the mechanisms of magnetization. The mechanism of the modeling method is related to whether the model parameters have certain physical meanings.
2. Capacity for Describing Complex Hysteretic Behaviors: Can the method be used to model complex hysteretic behaviors, including stiffness degradation, asymmetry, pinching, and so on?
3. Hysteresis Modeling Accuracy: What is the difference between the hysteretic behavior described by the method and that observed in experiments or monitoring?
4. Difficulty in Parameter Identification or Model Training: Is it easy to identify the model parameters (for model-driven methods) or to train the model (for data-driven methods)?
5. Feasibility in Structural Analysis: Can the proposed method be easily applied in structural analysis, especially in structural nonlinear analysis?

A comparison is presented in Table 3 based on these five evaluation criteria. In the authors’ opinion, a perfect hysteresis modeling method should be able to accurately describe various complex hysteretic behaviors and have a clear modeling mechanism that can be easily applied in structural analysis. According to this standard, there is currently no perfect method that can solve all challenges in hysteresis modeling.

7.2. Open challenges and problems of existing approaches

Based on the evaluation results, several existing problems and open challenges of different hysteresis modeling methods can be identified:

7.2.1. Model driven methods

(a) Polynomial based models

Polynomial models are based on plastic deformation and can describe complex hysteretic behaviors, such as the pinching4 model. Polynomial models are based on plastic deformation and can describe complex hysteretic behaviors, such as the pinching4 model. However, these models often have lower accuracy in modeling due to their non-smooth nature. Additionally, the rapid increase in model parameters, as seen in the pinching modified IMK model with 25 parameters or the pinching4 model with 28 parameters, poses a significant challenge in parameter identification.

(b) Differential based models

Differential based models in civil structural systems mainly include Bouc-Wen class models, including the most comprehensive type, BWBN, and Duham model and its variants. For the Bouc-Wen class model, through the continued modification and improvement to original form of BW model, some complex hysteretic behaviors can be described. However, there are still some hysteretic behaviors that are not included such as S-shape, and flag-shape. Similar to the polynomial based models, Bouc-Wen class models face the challenge of parameters with increasing numbers. The application of Duham model is quite limited especially in structural dynamic analysis. It is because that the present form of Duham model and its main variants (Dahl and LuGre model) cannot describe complex hysteretic behaviors. However, the generalized Duham model provide a universal form for hysteresis description. Further research in generalized Duham model for civil engineering applications is needed. The development of other types of differential models such as VR model provide a new way for hysteresis description. But some basic problems such as parameter analysis need to be solved.

(c) Operator based models

Operator based models describe hysteresis through the superposition or integration of hysteresis operators. The application range
of these models depends on the mechanism of the hysteresis operator. Preisach and KP operators are developed based on the magnetization mechanism, making the original models not suitable for structural analysis. By selecting appropriate weight functions, they can be adapted for structural analysis, but research in this area is limited. The PI and Maxwell-slip operators have hysteretic curves similar to the elasto-plastic model, lacking the capacity to describe hysteresis degradation or other complex behaviors. Some modifications have been proposed, introducing degradation parameters in the stop operator of the PI model, but this raises new challenges in parameter identification.

7.2.2. Data-driven methods

Data-driven methods rely on universal mathematical models to approximate hysteretic behaviors. However, their effectiveness is limited by the number of available training samples. When the training data is infinite, most data-driven methods can approximate any hysteretic behavior. Therefore, the main challenge for data-driven methods is the scarcity of training samples. Following is a partial list of possible solutions to address this challenge:

1. Optimizing the Architecture of Data-Driven Models: Researchers can explore and design more efficient and effective architectures for data-driven models to improve their modeling capabilities with limited training data.
2. Improving the Optimization Algorithm: Enhancing the optimization algorithms used in data-driven methods can lead to better convergence and improved performance, even with limited data.
3. Considering Different Data-Driven Methods: Exploring the use of different data-driven methods, such as Gaussian Processes or Reinforcement Learning, may offer advantages in dealing with limited training samples.

7.2.3. Model-data driven methods

Most of present model-data driven methods are actually using the data-driven method to realize the parameter identification. Therefore, as it can be found from Table 3, the modeling capacity, modeling accuracy and modeling mechanism of this method depends on the hysteretic model utilized. Conventional hysteretic models are not designed specifically for data-driven methods. Some novel methods based on simple physics rules might be more suitable for model-data driven method.

Model-data driven methods typically use data-driven techniques for parameter identification in existing hysteretic models. Therefore, their modeling capacity, accuracy, and mechanism are heavily influenced by the hysteretic model used. Conventional hysteretic models are not specifically designed for data-driven methods, which may limit the effectiveness of model-data driven approaches.

One possible direction to address this limitation is to develop novel methods based on simple physical rules that are more suitable for model-data driven approaches.

8. Conclusion and future research directions

8.1. Conclusion

In this study, we conducted a comprehensive review of hysteresis description and modeling methods in civil structural systems. The paper focused on two main classes of methods: model-driven and data-driven approaches, with an emphasis on their applications in structural systems. The review yields the following conclusions:

1. Model-driven methods utilize parametric hysteretic models to describe the hysteretic behavior of structures. There are three types of parametric models that were introduced including polynomial, differential based and operator based models. In these three types of models, polynomial models and part of differential models such as Bouc-Wen and BWBN class model are suitable for structural analysis due to their clear physics-based mechanisms and capacities in describing complex hysteresis.
2. To include more complex forms of hysteretic behavior in a single model, the number of parameters in a model explosively increases. Numerous parameter identification methods are introduced including LMS, Kalman filter, artificial intelligent optimization methods and Bayesian estimation.
3. Data-driven methods utilize universal mathematical models instead of specially designed hysteretic models for modeling hysteretic behavior. Regression model is an early form of data-driven model. The machine learning models including ANN, LSSVM and deep learning are utilized in hysteretic behavior modeling. The development of deep learning schemes has greatly increased the capacity of hysteresis modeling in data-driven methods and has enabled the realization of sequence-to-sequence after modeling.
4. In model-data driven methods, the prior characteristics are described by hysteresis model while the data-driven method is designed to describe the hysteretic features of sample data in model-data driven methods. Therefore, it is considered to be a potentially powerful approach in hysteresis description.

8.2. Future research directions

The hysteresis description techniques have been developed over the past half century. However, in structural and mechanical systems, there are still many problems associated with hysteresis modeling that need to be resolved. The following areas are suggested as the directions for future research based on the authors’ opinion:
1. **New hysteresis models**: There is a need for greater focus on addressing uncommon and distinctive hysteretic behaviors in structural systems, including hysteresis that is dependent on the rate of loading. New hysteretic models can be developed either by modifying classical models like the Bouc-Wen and BWBN models or by proposing entirely new mechanisms. Additionally, the parameter analysis and identification for these new models should be thoroughly investigated.

2. **New data-driven methods**: To improve the accuracy and training efficiency of data-driven methods, new AI-based approaches, including high-performance deep learning models, reinforcement learning, Graph neural network can be utilized for hysteresis modeling. Furthermore, proper optimization work is needed to determine the suitable architecture of models like Convolutional Neural Networks (CNN) for hysteresis modeling.

3. **Interpretability of data-driven methods**: The lack of interpretability in machine learning and AI-based methods is a significant concern. Developing interpretable data-driven methods can enhance the reliability of hysteresis modeling and reduce the number of samples required for modeling.

4. **Uncertainty estimation of hysteresis based on data-driven methods**: Accounting for uncertainty in hysteretic systems is crucial for reliability analysis and safety evaluations. Current data-driven methods mostly result in deterministic models. Future efforts could explore uncertainty estimation techniques, such as Bayesian estimation, to create probabilistic models for hysteretic behavior.

5. **New hybrid modeling methodologies**: The current hybrid methods for hysteretic behavior modeling are typically simple combinations of parametric models and data-driven methods. A deeper exploration of hybrid methods is necessary, and novel approaches like physics-informed theory can be applied to improve hysteresis modeling.

**Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**Data availability**

No data was used for the research described in the article.

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