CHAPTER ONE

The Problem

1.1 Introduction
Mathematics as a subject of study provides a challenge for teachers in that it constitutes a body of information consisting of abstract ideas, symbols, patterns and relationships, often divorced from everyday considerations and beyond the world of the learner.

Research in the field of mathematics education is continuing to guide teaching practice. Over the past two decades a far more complete picture of students' acquisition of skills, concepts and understanding of mathematical relationships has emerged. Insights gained by educators into learning are promoting shifts towards teaching methods which acknowledge and cater for the ways in which students learn. In adapting teaching programs to suit the student, as distinct from pressuring the student to fit the program, attention is currently focussing on individual as opposed to group outcomes (Cockcroft, 1982; Grouws, 1992).

1.2 Theories of learning in mathematics education
Fundamental to many contemporary curriculum documents, such as the NSW K-6 Mathematics Syllabus (1989), is an acceptance of a theory of teaching and learning in which students are first introduced to mathematical concepts through concrete (or manipulative) materials. This approach is hardly novel. Since the nineteenth century, scholars such as Froebel, Montessori and Pestalozzi have advocated the use of concrete materials in teaching mathematics, accepting that the mathematical structures and relationships embodied by such materials are readily apparent, and that students learn best through physical and mental activity related to their experience.
The work of major cognitive theorists of the twentieth century also supports the ideas embodied in the NSW K-6 Mathematics syllabus. Cognitive theory presumes that learning or knowledge acquisition involves individual items of information which are unified by relationships. Baroody (1987, 1989) summarises the main aspects of this theory. Understanding develops as individuals actively construct knowledge by relating new and meaningful ideas to those already in existence, or by refining relationships between ideas which were previously known but had not been integrated, thus transforming their thinking patterns.

A number of key theorists have exerted profound influences in mathematics education since the 1930s. During that decade the importance of the understanding of concepts in mathematics was first demonstrated by William Brownell. Later, the developmental theories of Jean Piaget (1948), and Jerome Bruner (1966) were formulated, involving the notion of students being motivated to act on their environment and to construct their own meaning from it. Significant efforts by Richard Skemp (1971, 1976) led to a much clearer view of the notion of understanding. Skemp defined the term ‘instrumental understanding’ to describe the understanding of learners who could successfully apply a principle (or algorithm), without knowing why the principle works. By contrast, the term ‘relational understanding’ described the understanding of learners who could both apply a principle and know why it works. Teaching for instrumental understanding often gives satisfactory short term results, but lacks intrinsic motivation for the learner and places a load on memory, so that its impact is likely to be forgotten. Teaching for relational understanding is more challenging for teachers, but provides depth and
richness in understanding of relationships, which are more likely to be retained in the learner’s memory (Skemp, 1976).

Despite differing interpretations of learning by the above theorists, commonality exists in that each theory views the experience of concrete materials and the staged development of mental ideas and abstract images from those materials as essential for students, both for the understanding of mathematics and its application to the real world.

In keeping with the work of the cognitive theorists, many teachers traditionally regard their task in mathematics as the bridging of the gap between concrete and abstract thinking. In order to facilitate learning, teachers typically provide external representations of mathematical ideas, such as concrete models, illustrations, drawings, demonstrations or computer simulations, so that mathematical understandings may be internalised. A significant and relatively recent development is a new generation of computer software, enabling the linking of physical, pictorial and symbolic representations. Simulation packages, such as *Blocks Microworld*, are now available so that students can perform on-screen manipulations on images of Dienes' Multibase Arithmetic Blocks. As the mouse is dragged and clicked by students, they can observe the dynamics of their actions affecting the composition or decomposition of the mathematics materials, and the attendant changes of mathematical symbols representing the numbers on which they are working.

Increasing research in education over the past two decades is resulting in further development or refinement of theories explaining how students learn mathematics. A relatively recent phenomenon is the rapid development of the so-called constructivist movement. Though having
its roots in Piagetian thinking, a spectrum of constructivist views is currently evolving, further explaining the nature of learning and knowledge acquisition.

The central tenet of constructivist thinking is that learners actively construct knowledge in an idiosyncratic fashion, having personal ownership of ideas which are negotiated in a social context (Ellerton & Clements, 1992). Such thinking lessens the importance (or may even deny the existence) of a central body of mathematics content existing externally to the learner, in favour of ideas that are constructed inwardly.

Constructivist thinking is currently challenging earlier theories emphasising transmission or absorption models of learning and the cognitive theorists' notion of an ordered, linear sequence from concrete to abstract ways of representing mathematical ideas. Teaching and learning in accordance with constructivist principles does not necessarily involve a transition from concrete to abstract - alternatively, it involves the presentation of as many different models or external representations of ideas as possible, so that learners may actively choose those which have relevance and meaning to them and which they are willing to adopt (Perry & Howard, 1994).

1.3 Learning styles theory
In other fields, a revival of interest in the needs of gifted and talented learners is also a feature of the NSW education system at the present period. Of interest to many educators are theories of multiple intelligences (Gardner, 1985) and of learning and teaching styles. Such theories are being increasingly well received by teachers and are likely
to be of increasing import within the domain of mathematics education in future.

The construct of learning style describes how a learner perceives, interacts with, and responds to stimuli in different learning contexts (Keefe, 1987, cited in Pithers & Mason, 1992). Learning style is useful in identifying the methods by which people prefer to receive information from their environment and undertake their learning - whether by visual, auditory or kinesthetic means. In recent times use of the term learning style has largely supplanted the earlier term of cognitive style which broadly described a similar range of behaviours.

At present an evolving pedagogy of mathematics is taking into account the styles or patterns in which individual students learn, and the teaching methodologies (or styles) which are created to match student learning styles. Proponents of learning styles theory (Keefe, 1987, 1989; Midkiff & Thomasson, 1993) accept that no single learning event or teaching style is likely to maximise learning for every student and, therefore, programs must be designed to suit particular student characteristics or aptitudes. A multi-sensory approach to teaching is advocated, involving students handling objects and equipment, and watching, listening and talking through ideas. Seeing, hearing, touching and feeling enables learning to occur, with learners demonstrating preference for receiving information by visual, auditory or kinesthetic means or a combination of these means.

Recent research into learning (or cognitive) styles and teaching methodologies which have been developed to match them has documented improvements in the areas of student achievement, ease of

The advantages of teaching according to an individual student's preferred way of receiving information is also long recognised by many teachers, who have often observed previous negative results of mismatches of teaching and learning style in their classrooms. Such mismatches include the teaching of mathematics by auditory or visual methods to learners who prefer to receive information kinesthetically, or phonetic instruction in reading to students who possess high visual responses. When a single, inappropriate teaching methodology is imposed upon the learner, the result is often the inhibition or total restriction of learning, and the growth of negative attitudes.

Perceptive classroom practitioners thus recognise that their students learn in individual and unique ways, with associated particular preferences and behaviours. These educators are flexible in constructing teaching programs according to the ways in which students learn, maximising learning outcomes by structuring activities to accommodate different learning styles, as skills, concepts and understandings are developed. The matching of a particular teaching style to a student's learning style is a recipe for success, according to these educators.

Despite the attractiveness of the notion of matching teaching and learning styles, a major obstacle currently exists in implementing programs based on this approach. It is often overlooked that the foundations of learning styles theory lie in the quick and accurate
identification of student learning styles, and that without valid identification of learning style the choice of an appropriate teaching style becomes a meaningless enterprise. The paucity of instruments available to help teachers in the identification of learning styles of younger students is currently hampering the implementation of learning styles theory, and much work remains to be done in locating and developing instruments or methodologies to quickly and accurately help teachers in this task.

1.4 Statement of the problem and methodology of the investigation

Reflection upon the preceding theories of learning mathematics and issues involving use of computers in mathematics education and the role of teaching and learning styles, leads to the conclusion that further research is warranted within these fields.

This investigation, therefore, aims to contribute new and potentially useful information to existing or emerging bodies of knowledge. It is devised to focus on different modes of representation in communicating mathematical knowledge, to investigate the use of computer software in linking physical, pictorial and symbolic representations, and to further identify the role of learning styles and appropriate teaching styles in mathematics.

In detail, this investigation seeks to answer the questions of:

• whether computer software can act as a transition device in the integration of concrete, pictorial and symbolic representations of numeration concepts; and
whether teaching programs which are constructed in accordance with the preferred learning style of students lead to increased achievement by students.

It is presumed that, if the computer is truly a transition device linking concrete and symbolic representations of ideas, students using computer simulations will exhibit superior performance in mathematics when compared to those receiving alternative teaching treatments. Similarly, if the exact matching of learning style and teaching style is possible, the greatest gains in achievement will occur when students undertake a program of learning which is carefully matched to the particular style in which they prefer to receive information and undertake their learning. Specifically, students exhibiting strong visual preferences will benefit most by working with images provided by the computer; students with strong auditory preferences will benefit most by expository teaching, answering questions and listening to teachers' verbal explanations; and students with strong kinesthetic preferences will benefit most from "hands-on" teaching styles, in which they manipulate materials.

In establishing the procedures and treatment for this mainly quantitative investigation, students with preferences for a single learning style were first identified and then allocated to one of three teaching treatments: either, a group in which instruction took place predominantly by means of a computer using a simulation software package; a group in which instruction was given predominantly by teacher exposition and questioning; or a group in which instruction took place predominantly as students manipulated concrete materials. Pretesting and posttesting of students took place, so that the achievement outcomes of different student groups could be measured and compared, according to their
method of instruction, learning style preferences, gender and grade levels.

By attempting to control for the variable of learning style in this study it is anticipated that many of the difficulties associated with previous studies involving the use of computers and concrete materials can be overcome.

1.5 The context of the study
The pilot sample for this investigation was a group of Year 6 students at a state primary school situated in Greater Western Sydney. The primary functions of the pilot study were to trial a suitable instrument to assess students' preferred learning styles; to develop materials to measure students' achievements; to establish different teaching treatments; to identify appropriate computer software which would aid the transition from concrete to symbolic modes of representation of mathematical ideas; and to develop the methodology used in the analysis of results.

As an outcome of the pilot study a number of refinements in sampling and methodology were implemented, before the major investigation was carried out. These refinements included: the screening of a much larger sample of students in order to determine preferences for a single learning style; the use of a standardised instrument in pre and posttesting; and, the assignment of the researcher as the teacher of each of the groups involved, in order to improve consistency of treatment and enhance the validity and reliability of results.
The selected site for the final investigation was a second state primary school in western Sydney, approximately 20 kilometres to the east of the school used in the pilot study. The school is situated in a relatively disadvantaged socio-economic area with a large multicultural population.

Permission to conduct the investigation was gained from the Principal and the Director of Teaching and Learning of the NSW Department of School Education. Parents gave written consent to their children participating in research activities.

1.6 Assumptions involved in the investigation

A number of major assumptions are involved in this investigation. The notion of a continuum or ramp from concrete to abstract modes of representation of mathematical ideas is fundamental to the use of computer software as a linking device between the modes. If it can be proven that such a continuum does not exist, then computer simulations of mathematical ideas become at best a series of alternate representations, which are non-linear and have no recognisable stages.

A second set of major assumptions centres around the issue of learning style. It is necessary that the instrument used in the investigation to identify individual learning style preferences is valid and reliable and that the individual's preference for receiving information can be identified solely by the administration of a simple quiz. The identification of individual style preferences also requires a high degree of self awareness and reflectivity on behalf of the individual, which may not always be present in young students. In addition, it is assumed that the preference for a particular learning style must also be consistent
throughout different learning contexts and not alter over a reasonable period of time, including the time available between the pretest and posttest.

Finally, this investigation is also founded on a belief, held by many teachers and researchers, that learning style and teaching style can be accurately matched. That is, it is valid and feasible to devise a specific type of teaching program that caters accurately and exclusively for a single learning style. The manipulation of concrete materials in a program devised for strong kinesthetic learners, or a reading scheme employing a whole word approach for strong visual learners, are examples of such approaches already employed commonly by teachers, according to the assumption that exact correspondence of teaching and learning style exists.

The following chapters of this work contain a detailed examination of the research literature relating to the problem being investigated, information relating to the design of the investigation, an analysis of the data collected and a final interpretation of results.

Chapter Two  A Review of Related Literature
Chapter Three  The Conduct of the Investigation
Chapter Four  Results and Discussion
Chapter Five  Conclusions of the Investigation

A list of the references cited in the investigation and an Appendix are also included.
CHAPTER TWO
A Review of Related Literature

2.1 Introduction
A number of key issues in mathematics education have emerged in recent years and are presently continuing to evolve. Concerns such as the way in which mathematical ideas may be meaningfully represented and understood, the relevance for teachers of so-called constructivist theories of learning, the role of computers and the importance of learning styles theory in mathematics education are currently the subjects of much critical research.

This chapter contains an examination of the above fields of research literature, in order to clarify the present situation and detail the foundations of the problem investigated by this researcher. The review is presented in the following sections:

2.2 The representation of mathematical ideas
2.3 Constructivism and the instructional representational approach
2.4 The role of concrete materials in mathematics education
2.5 Technology and mathematics education
   2.5.1 Technology and curriculum
   2.5.2 Technology and concrete materials
2.6 Learning styles
   2.6.1 Learning styles and mathematics education
   2.6.2 The definition of learning style
   2.6.3 Categories of learning style
   2.6.4 Learning style diagnosis
   2.6.5 Teaching programs and student learning style
2.7 Summary
2.2 The representation of mathematical ideas

And the vision that was planted in my brain still remains ...

(Simon, 1968, [song])

Human beings demonstrate the ability to utilise complex sets of skills and to manipulate powerful ideas, either concretely, by the manipulation of objects in their environment, or abstractly, when those objects are not present. A characteristic of human experience is also a quest to understand the world in which we live, by employing such tools as reflective thinking.

Over many years cognitive learning theorists, such as Piaget (1948) and Bruner (1966), have sought to understand how people make sense of their experience and to explain how knowledge is structured by the brain. A distinction was made between mental operations, which are intangible, and physical operations which can be observed. The notion of a continuum between concrete and abstract functioning served to link the reality of everyday activity and activity in the mind.

According to the notion of the existence of a continuum between the concrete and the abstract, cognitive theorists believe that learners extend their understanding of concepts by transforming ideas gained through interactions with concrete materials into semi-concrete representations, such as pictures, images or diagrams, finally internalising the concepts into abstract thought by means of symbols. In describing ideas in the process of construction, the term "modes of representation" was first used by Bruner (1966) to signify various stages of the continuum - the enactive mode (involving manipulations of objects), the iconic mode (involving visual imagery) and the symbolic mode (using words and symbols).
More recent research has refined the continuum of conceptual learning in the representation of mathematical ideas (Heddens, 1986; Underhill, 1977). Four stages along the continuum are identified - the concrete level, the semi-concrete level, the semi-abstract level and the abstract level. In the semi-concrete level pictures of real objects are used rather than the objects themselves, whereas the semi-abstract level involves symbols or pictures which do not resemble the objects that they represent.

Traditionally, teaching mathematics has involved the communication of knowledge by the bridging of the gap between the physical and abstract worlds of the learner.

Activities involving pictures of objects, textbook illustrations, models and the overhead projector, and drawings, as well as demonstrations by teachers and peers, can smooth the transition between concrete and abstract functioning. (Heddens, 1986, p.15)

Such activities are teacher-centred and generally do not acknowledge student participation in the manner and style of learning.

The effect of the previous learning theories on teachers and curriculum developers has led to the widespread use of concrete materials over the past three decades, particularly from preschool to the Year 3 level, and the acceptance of the belief that students learn better when concrete materials are used (Hart, 1994). Manipulation of concrete materials is believed to be a crucial first step along a path leading towards the relational understanding of mathematical ideas (Skemp, 1971, 1976), based on a critical assumption that those ideas are primarily embodied in
the manipulatives and have to be later internalised by the learner. As Perry and Howard (1994) state:

The use of manipulatives also relies on a theoretical belief that the mathematics to be learned is somehow captured in the manipulatives and what the learner has to do is 'discover' this mathematics and transfer its material representation into a conceptual and symbolic representation. (p.489)

The focus of much current interest and debate in education is the assumption that concrete materials embody mathematical concepts which are waiting to be acted upon and absorbed by the learner.

2.3 Constructivism and the instructional representational approach

A contemporary and contrasting theory of learning, which accepts that mathematics knowledge is constructed by individual learners, as opposed to being directly imparted, has been termed the constructivist perspective (Ellerton & Clements, 1992; Wright, 1992). This term is generally not helpful, due to its multiplicity of meanings or differing interpretations by researchers, such as "radical" or "social" constructivism (Ellerton & Clements, 1992).

Nevertheless, the central tenets of constructivism can be defined. As a general theory, constructivism acknowledges that people personally negotiate meaning by creating different mental representations of knowledge. Active cognitive processing ensures that the learner's personal mathematical knowledge is continually being modified. Understanding of mathematical ideas in terms of relationships occurs by a process of reflection, as connections between different ideas are made.
Reference to a traditional method of teaching called "the instructional representational approach" (Cobb, Yackel & Wood, 1992), helps to clarify the constructivist interpretation of the learning of mathematical concepts. Within such an approach the term representation refers to both internal representations in the mind of the learner and external representations present in the environment. Three major features of the instructional representational approach are identified by the authors:

1. The overall goal of instruction is to help students construct mental representations that correctly or accurately mirror mathematical relationships located outside the mind in instructional representations.
2. The method for achieving this instructional goal is to develop transparent instructional representations that make it possible for students to construct correct internal representations.
3. External instructional materials presented to students are the primary basis from which they build their mathematical knowledge. (p.4)

Cobb, Yackel and Wood cogently define the limitations of the instructional representational approach. Among these is "the learning paradox" described by Bereiter (1985) and recognised earlier by Holt (1982). The learner's transition from the manipulation of concrete materials to pictorial or mental images of concrete materials, and finally to using symbols, involves an assumption that mathematics is a property of the learning materials, and that students will actively construct the correct internal representation from the materials that they are manipulating. But this assumption implies that students' learning will be triggered by the mathematical relationships they must construct before they have constructed them. Teachers who understand the relationships involved in Dienes' Base 10 Arithmetic Blocks, for example, are working from a position of knowledge of how numbers work, yet the relationships involved are hardly apparent to students who are at a more
fundamental stage of learning number concepts. As Cobb, Yackel and Wood (1992) pertinently ask: "How then ... is it possible for them to recognise mathematical relationships that are developmentally more advanced than their current internal representations?" (p.5).

It is by encouraging students to make the links between the different types of representation, as they generalize the corresponding aspects of each system, that this question is answered. A process of negotiation of differing interpretations of individuals and the teacher must occur, applying individual and collective activity to the representational view of mind. Such a position is often described as social constructivism:

...teachers might in practice challenge the assumptions that give rise to the learning paradox by attempting to see beyond their expert interpretations of instructional materials. They then might consider the various ways that students actively interpret the materials as they engage in genuine mathematical communication in the social context of the classroom. The materials would then no longer be used as a means of presenting readily apprehensible mathematical relationships but would instead be aspects of a setting in which the teacher and the students explicitly negotiate their differing interpretations as they engage in mathematical activity. (Cobb, Yackel & Wood, 1992, p.5)

2.4 The role of concrete materials in mathematics education
Concrete materials (also termed manipulative materials or manipulatives) are variously defined as "models that incorporate mathematical concepts, appeal to several senses and can be touched and moved around by students" (Hynes, 1986, p.11), or "objects which represent mathematical ideas that can be abstracted through physical involvement with the objects" (Young, 1983, cited in Kennedy, 1986, p.6).
The value of concrete materials in making learning meaningful to students is long recognised, from the work of 19th and 20th century scholars such as Froebel, Pestalozzi and Montessori, to the learning theories of Piaget (1948) and Bruner (1966). The success of approaches involving manipulatives is indicated in reviews of research (Sowell, 1989; Suydam, 1984), which suggest that mathematics achievement is enhanced by the use of concrete materials across a variety of topics, and at every grade, achievement and ability level.

The current NSW K-6 Mathematics Syllabus (NSW Department of Education, 1989) recognises the success of approaches using concrete materials used by many teachers and indicated by overseas research findings. A major investigation occurred in the early 1980s in the United Kingdom, where the Committee of Inquiry into the Teaching of Mathematics in Schools (Cockcroft, 1982) recommended that activity-based instruction and student-interaction should be considered as alternatives to traditional instruction, with its heavy emphasis on 'teacher talk'. The findings of the Cockcroft committee provided further justification for the approach adopted by the NSW Mathematics Syllabus. Concrete materials were a necessary basis in the construction of mathematical understandings by learners of all ages because of:

- the learner's need to work from the concrete to the abstract with most new mathematical concepts;
- the need to provide a variety of experiences in a series of alternative settings;
- the confidence and enjoyment shown by students when given opportunities to use materials;
- the need to set the learning experiences in contexts that are relevant to the student's world;
- the need to cater for different learning styles. (NSW Department of Education, 1989, p.17)
Other significant beliefs of educators regarding the use of concrete materials are outlined by Hart (1994). The "accepted wisdom" is that:

Children find it easier to learn formal mathematics if the introduction is through concrete materials.
If children can solve a problem using materials and then move on to a formalisation, they can easily return to the use of the materials.
If a child is having trouble solving a problem formally then one should suggest that he takes blocks/materials to help.
The lead up to the formalisation through a wealth of practical experiences is appreciated by the child and forms the basis from which understanding comes. (p.4)

Consideration of these beliefs and a reading of the background material and teaching/learning units of the K-6 Mathematics Syllabus document indicates that the process of teaching which is prescribed is firmly rooted amongst the ideas of the instructional representational approach.

Although concrete approaches to teaching mathematics are widely advocated and are also supported by successful classroom practice, research regarding the efficacy of concrete materials is, however, not entirely unequivocal (Kaput & Pattison-Gordon, 1987; Labinowicz, 1985; Lesh, Post & Behr, 1987). Anxieties of teachers due to perceived control and management problems, demonstrations in which most students act as passive observers, difficulties in representing large numbers and lack of variety in representations are all factors reported as limiting the success of concrete approaches.

One critical problem recognised is in relating the use of Base 10 Arithmetic Blocks to the written algorithm. For some students the connections between the actions on the material and the actions of representation by formal mathematical notation are often masked,
because the cognitive load imposed during activity with the material is overwhelming (Kaput, 1989). In particular, the manipulation of Base 10 material to perform the sequence of steps involved in processing an algorithm, and simultaneously recording progress in symbols on paper is too demanding a task for many students. Links between the different types of representation of mathematical ideas are consequently at risk.

Hart (1994) reports further evidence of a missing link existing between concrete and symbolic modes. In one research project, after a three month period students did not remember the practical experiences leading up to the formalisation of a mathematical idea. The overall effect of learning was not uniform for all students. Critically, the concrete materials given to the students often did not facilitate the standard form of the subtraction algorithm that teachers were attempting to develop.

Perry and Howard (1994) outline an opposite problem in that standard approaches to teaching number algorithms can actually constrain students' individual approaches, as the links between materials, language and symbols are made. It is recognised that the approach taken to teaching number with Base 10 material, advocated by many curriculum documents such as the NSW K-6 Mathematics Syllabus, is just as didactic as the logicist tradition of teaching mathematics by manipulating symbols.

It is clear from findings such as the former that the transition from concrete to symbolic modes in mathematics education is problematic. Teachers need to be aware of barriers which may occur and need to develop new ways to help learners make the links between the
manipulation of concrete material and the writing of symbols to represent their manipulations. As Hart (1994) counsels:

Before we recommend to teachers that they use manipulatives we should advise them to view the appropriateness and limitations of the materials for the purpose of leading to and authenticating a part of formal mathematics. (p.6)

Social constructivist approaches involving the negotiation of meaning between teachers and students are one way that the difficulties reported by Hart and Perry and Howard may be overcome.

2.5 Technology and mathematics education

2.5.1 Technology and curriculum

Anyone who presumes to describe the role of technology in mathematics education faces challenges akin to describing a newly active volcano - the mathematical mountain is changing before our eyes, with myriad forces operating on it and within it simultaneously. (Kaput, 1992, p.515)

The rapid transformation of modern society due to the influence of information technology has not been completely mirrored within mathematics curriculum. Despite widespread use of computers as word processing machines and information retrieval devices in some Key Learning Areas, schools have been generally slow to react to the opportunities offered by the availability of computers for teaching mathematics.

Many reasons can be advanced for the slow rate of progress in utilising computers to teach mathematics, including: the limited availability of machines due to cost factors; a shortage of suitable software; a lack of skilled staff willing to train and enthuse their peers; the paucity of inservice programs; an over-emphasis on programming; the inordinate
amount of time which has to be spent by classroom practitioners in designing lessons, debugging programs and mastering hardware systems; a bewildering array of models and technologies of ever-increasing power; and the "traditional inertia" of the mathematics curriculum (Balla & Gow, 1987; Czernezkyj, 1990; Kaput, 1992).

The role of the computer in mathematics education is also hampered because the body of thinking which defines how the computer can best help to achieve the objectives of the curriculum is also confused and unclear (Penter, 1986). There is a time lag evident in the implementation of powerful machines in schools and the gaining of an understanding by teachers and researchers of how these tools may best be used.

In a climate of uncertainty, fundamental educational issues are being re-evaluated. Under consideration are matters such as how computers are altering the way educators think about learners, learning and subject matter; the effects of computers on the social and organisational structure of schools and classrooms; and various pedagogical strategies (Kaput, 1992).

Additionally, despite commonly held views about computers and technology being mechanistic and neutral, the learning situation which utilises them is not free of values considerations, as educators employ various software according to their beliefs and biases. Sensitive practitioners are also aware of cultural or religious differences which may be relevant to the use of computers in schools, and of the need to evaluate the effectiveness of computers in particular social settings.
In an era of accelerating technological change, relentless multinational marketing, and rapidly obsolescent hardware and software, a myriad of questions regarding computers thus exist. At this point in time there is a distinct danger that questions about computers may never be satisfactorily answered, even though their use is a continuing feature in education.

In spite of such pessimism an attempt must nevertheless be made to clarify the objectives and purposes which can be served by computers. A number of useful paradigms which help educators understand various computer applications in mathematics education are defined by MacDonald, Atkin, Jenkins and Kemmis (1977). The four paradigms are:

1. The instructional paradigm
2. The revelatory paradigm
3. The conjectural paradigm
4. The emancipatory paradigm

Mastery of content by the learner is the key concept of the instructional paradigm. The role of the computer is in presentation of relatively simple content and task prescription. Student motivation is provided by fast feedback. The underlying assumption of the instructional paradigm is of a conventional body of subject matter with an hierarchical structure. The use of games and drill and practice packages to reinforce skills and concepts already acquired is associated with this paradigm.

Advocates of software conforming to the instructional paradigm point to the fact that drill using a computer is more effective than by more conventional means and that instructional packages encourage
confidence in teachers who are first coming to terms with the technology (Messing, 1986). The popularity of such programs with students is widely evident.

Critics of the instructional approach argue that drill and practice is useless unless there is a concrete basis of understanding on which memorisation can be built, and that over-emphasis on drill and practice programs may distort learning, or interrupt the sequence of learning from concrete to abstract stages (NSW Department of Education, 1989).

The notion of discovery, intuition and exploration of ideas underlies the revelatory paradigm. Within a child-centred approach, opportunities are provided for learning by doing. Computer software associated with this paradigm consists of simulations allowing reference to topics which normally would be beyond the reach of students, due to expense, danger or amount of time taken. An approximation of the real world is provided by the program, allowing the user to input variables on which the program operates to provide a possible outcome. The computer is therefore acting as mediator between the student and a hidden model of some real-life situation. As students interact with the model hidden within the computer, they develop a feeling for the behaviour of the model under various circumstances and are led to discover certain rules which govern it. For example, users of a software package entitled The Factory (Sunburst) are challenged to create a product by designing rotations, punches and stripes to duplicate a given object, mimicking the stages of a manufacturing process. The sequence in which the operations are performed is crucial and can be difficult to visualise without off-computer help. Paper squares help students to model the steps involved, which are then re-created on the computer screen.
Although software which embodies the relevatory program does concentrate on students and their relation with the subject as portrayed by the computer, a number of disadvantages are evident. The student is not totally in control as the situation is software-driven, or the simulation itself may not be perfect. As Rushby (1979) states: "care must be taken to ensure that the orderly model presented in the simulation does not so oversimplify the reality it sets out to explicate that it defeats its own object" (p.9).

The third and possibly most exciting paradigm identified by MacDonald et al. is the conjectural paradigm. The key concept is the manipulation of ideas and hypothesis testing as the computer functions as a tool for models, programs and plans, according to a problem-solving oriented theory of learning. Students are totally in control of the situation and can construct understanding for themselves.

Open-ended software conforming to the conjectural paradigm has been developed to teach concepts of geometry and shape to students, by allowing them to build and investigate their own models (Rushby, 1979). A square can be represented using the Logo programming language, developed by Logo Computer Systems Inc. (Davidson, 1982), by drawing a line of one unit length and turning it four times to the right, through an angle of ninety degrees. Through Logo investigations, a wide variety of patterns can be created and the students can construct their own knowledge of shapes. The conjectural paradigm is generally in accordance with constructivist ideas of teaching and learning and is at the core of many current primary mathematics curricula which emphasise problem solving (Australian Education Council, 1991;

The final paradigm is emancipatory in nature in that its key concept is the use of the computer as a means of reducing the workload of the student and supporting the teacher in the management of learning. It is claimed that the capacity of the computer to perform rapid, accurate calculation and information handling reduces the non-essential work needed to attain learning objectives. The use of databases, spreadsheets, statistical software in data analysis, applications programs incorporating word processing, CD-ROM programs and problem-generating software are all associated with the emancipatory paradigm at work. Though it is claimed that the use of such packages liberates students in making learning more relevant, efficient and stimulating, there is also the certainty of an initial increase in teacher workload in mastering the technology and applying it successfully.

The preceding educational paradigms are useful in that they provide educators with a conceptual framework with which to view the use of computers in mathematics education, especially in the light of developing evidence that the "traditional inertia" and other factors which have acted to hamper the use of computers are presently being overcome (Kaput, 1992). A new field of research is emerging which is investigating the integration of computer software with physical objects, facilitating the transition from concrete to abstract thinking by overcoming many of the obstacles associated with the instructional representational approach. Many examples of quality educational software conforming largely to the revelatory and conjectural paradigms are available and are discussed in the following section.

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2.5.2 Technology and concrete materials

Computers can also provide a link between concrete manipulation and abstract processing. (NSW Department of Education, 1989, p.37)

The use of the computer as a communication tool which enables the linking of physical, pictorial or symbolic representations is recognised by the NSW K-6 Mathematics Syllabus (1989). On-screen simulations involving the use of "cybernetic manipulatives" (Kaput & Pattison-Gordon, 1987) provide a unique and dynamic link between the concrete and the symbolic, and are familiar to and popular with learners who have grown up in an audio-visual age.

Despite recognising that the computer allows students to construct knowledge by generalising the corresponding aspects of different systems in a social context, the K-6 Syllabus does not directly detail how links between concrete and abstract modes of representation may be achieved. A brief mention of the possibilities of using computers is provided within the support statements of the syllabus, but the issue of how the curriculum may be influenced (or ultimately transformed) by computers and constructivist ideas is not addressed.

To enable a more complete understanding of how the link between the concrete and abstract may be achieved, it is necessary to point to a steadily increasing body of research which has evolved since the mid 1980s and which indicates how many of the past difficulties associated with the use of concrete materials (referred to in Section 2.4) may be overcome (Berlin & White, 1986; Brown, 1990; Kaput, 1992; Lesh, Post & Behr, 1987; Newton, 1988; Perl, 1990; Thompson, 1992). Studies by these researchers demonstrate how mathematical knowledge
may be constructed by using concrete materials in conjunction with on-screen activities, or by manipulating representations cybernetically.

In a study conducted by Perl (1990) students who worked with concrete materials and computer software demonstrated much greater sophistication in classification and logical thinking than did a control group working only with concrete materials. An earlier study by Bieck and Wilson (1988, cited in Perl, 1990) concluded that software and concrete materials used together can be more effective than either used alone. Year 3 and 4 students were divided into 2 groups, one group using software and the other using water and containers to solve the same problems. Students pouring real water gained a better understanding of the step-by-step processes involved through enforced record keeping, while students using the software concentrated on finding ways to solve the problems and solved them faster.

A useful categorisation of software enabling the synthesis of concrete materials and the computer is developed by Perl (1990). Four categories are recognised including modelling, mirroring, manipulating and managing types of software.

When using modelling software, the software plays the central role and the concrete material serves as an aid to visualising the concepts developed at the computer. Brown (1990) advocates the importance of teachers constructing problem-solving activities so that exploratory work with concrete materials and specific computer software can be integrated to produce more effective learning, in contrast to the typical classroom scenario where investigations using concrete materials and computers occur as disjoint activities.
For example, work with rotating, punching and decorating cardboard shapes is undertaken as students simulate the concrete processes carried out on components on a factory assembly line. The students then move to investigate the program called *The Factory*, involving a semi-abstract or symbolic representation of concrete items, in which particular components have to be produced on a simulated assembly line according to certain specifications. Skills of visual reasoning and the understanding of the mathematical concept of rotation are developed. The problem-solving strategies of working backwards, identifying subgoals, guessing, checking and refining, in particular, are emphasised, together with the freedom to choose different methods of solution.

An approach in which modelling software aids the transition from Pattern block play to Logo programming is described by Newton (1988):

> Pattern blocks are a natural medium for stimulating spatial and geometric thinking. The blocks seem to resonate with children's ideas. With Logo those same ideas can appear in bright colours on a computer screen or printout. The abstract, formal notions of mathematics and programming grow quite naturally when rooted in the fertile ground of manipulatives. Piaget spent years trying to teach us this. (p.6)

Newton describes activities in which students create pattern block designs and then reproduce them on the screen using Logo. This develops mathematical thinking through the use of programming skills employing goals, subgoals and sub-subgoals. Students can construct a program for a specific design, organise it as a procedure and then link it into more complex procedures towards the goal of the completed design. The creation of geometric designs with pattern blocks and the writing of programs to simulate the designs on the computer screen also
lead to students developing new styles of thinking. As their ideas become more tangible, students are better able to think about those ideas. As they reflect on their ideas they are led towards more abstract modes, and move towards the Piagetian level of formal operations.

Mirroring software involves the display of objects that look like corresponding manipulatives and are used in exactly the same way. Various programs such as *Gertrude's Secrets* (Sunburst) and *Hands-on Math* (Ventura) mirror a range of equipment such as attribute blocks, Base 10 material and geoboards.

Mirroring programs have several advantages in overcoming many of the difficulties associated with the use of concrete materials. They can help bridge levels of abstraction between concrete and symbolic constructs, and they can automatically monitor the learning that is occurring. The software also provides an endless and flexible supply of materials, often supporting actions and manipulations not possible with real world materials. Packages which mirror Base 10 Arithmetic Blocks, for example, have appeal to teachers in that they are obviously easier to manage and clear away after use than their concrete analogues.

A recent powerful mirroring program has been developed by Thompson (1991). *Blocks Microworld* is a screen simulation of Base 10 Arithmetic Blocks, in which mathematical ideas and operations can be simultaneously represented by diagrams of the material and the attendant mathematical symbols for the structures. As students transform the screen images by mouse-driven manipulations, there are changes in the word structures and mathematical notation provided by the computer, corresponding to the new structure which has been
created. Students are provided with the choice of working with place value terms (hundreds, tens, ones), "wood" terms (flats, longs and shorts) or mathematical symbols as they build structures representing numbers on the computer screen.

In utilising Blocks Microworld "the student is exposed to working with words and symbols while thinking in images, to make the important connection between concrete representations of numerical value and traditional symbolic arithmetic" (Thompson, 1991, p.37). In this way difficulties associated with the learning paradox (cited in Section 2.3) may be overcome as students actively construct their own knowledge.

Figure 2.1 shows the pattern in which Base 10 material may be represented on the screen when using Blocks Microworld. The attendant representation is described either numerically by words or symbols or by using the "wood" terms of blocks, flats, longs and shorts. In this instance the number 432 has been represented.

![Fig.2.1 - On-screen representation of the number 432 using Blocks Microworld](image)

If, for example, Blocks Microworld is used to subtract 135 from 432 using the decomposition method, a student performs a series of
transformations by clicking various images on the screen, which are then "unglued" by the program, representing the notion of trading. As the decomposition proceeds, the string of numbers above the images of the material is accordingly transformed. First one of the longs is decomposed into ten shorts (Fig.2.2).

![Fig.2.2 - Decomposition of one ten into ten ones.](image)

This is followed by the removal of five shorts from the subsequent group of 12 shorts (Fig.2.3).

![Fig.2.3 - After removal of five ones.](image)

Now one of the flats is unglued so that enough longs are available to remove three longs (Fig.2.4 and Fig.2.5).

![Fig.2.4 - Decomposition of one hundred into ten tens.](image)

![Fig.2.5 - After removal of three tens.](image)
Finally, one flat is removed giving the answer of 297 (Fig. 2.6).

![Diagram](image_url)

Fig.2.6 - After removal of one hundred.

Although the subtraction algorithm used in the previous example is standard to many curricula, including that of NSW, non-standard approaches can be encouraged, contingent with students constructing their own mathematical knowledge and negotiating meaning in a social context.

A study performed by Thompson (1992) involved two groups of learners, one group using the Base 10 material and the other using on-screen representations of them. The computer users demonstrated superior performance, measured by written tests and structured interviews. Thompson contended that the superior performance was largely a result of the lessening of the cognitive load involved, by use of a system which enables students to make the immediate link between concrete representations and numeric representations, as they perform parallel sequences of actions simultaneously on the two systems.

The disadvantages of mirroring programs such as *Blocks Microworld* are: they provide a powerful disincentive to hands-on experience (NSW Department of Education, 1989); that time must be invested in learning the keyboarding skills to manipulate the images; and that the speed at which such programs run is often frustratingly slow. It may also be more difficult for the teacher to monitor the progress of student
learning using machines than by the usual hands-on methods (Perl, 1990). Whether such disadvantages are outweighed by the ability of Blocks Microworld to effect the transition from the concrete to the abstract is worthy of further investigation.

The third category of software defined by Perl is manipulating software, in which the software itself serves as a manipulative. Advanced graphical interfaces allow users to touch, stretch and move what is on the screen, rather than devices which are off it. Manipulating software is of particular advantage for higher level topics in secondary mathematics. A program called Calculus (Broderbund) allows students to drag a mouse along a curve and watch the slope-line appear at each point the mouse touches. Simultaneously, an algebraic expression in a window on the screen changes to reflect the new curve.

Other recent software products allow the development of spatial visualisation skills by offering experiences with rotational geometry. Future development of manipulating software for primary students will offer more tactile approaches in the linking of concrete and symbolic mathematical representations.

Software which itself drives the manipulative materials involved, working together as a single product, is called managing software and constitutes the final category outlined by Perl. Lego TC Logo exemplifies how a real world object in the form of a robot-like device can be computer controlled. With Lego TC Logo users build machines with Lego Technics and connect them via an interface box and wiring to a computer that speaks a dialect of LogoWriter. They then enter simple commands to turn motors on and off, reverse their direction, or
respond to feedback from specially mounted touch and light sensors. There is a wealth of mathematics involved in the design, construction and implementation phases.

Mirroring, modelling, manipulating and managing types of software thus have a powerful role to play in communicating meaning in mathematics education. Technology offers the possibility of assisting learners to build up the maximum number of representations, both concrete and imagistic, and to construct knowledge by making the connections between the different modes.

2.6 Learning styles

2.6.1 Learning styles and mathematics education

If a man does not keep with his companions, perhaps it is because he hears a different drummer. Let him step to the music which he hears, however measured or far away. (Thoreau, undated, cited in Butler, 1993, p.126)

Amongst the references cited in Section 2.5.2 was a study by Berlin and White (1986), who investigated the influence that concrete manipulations and computer activities had on students' transitions from concrete understandings to abstract thinking.

There were 3 different levels of the treatment condition in the Berlin and White study: activities using only concrete materials; activities using only computer simulations; and, thirdly, activities which involved a combination of concrete materials and computer simulations. Students were randomly assigned to groups by gender and grade level and investigated spatial tasks, involving the recognition and duplication of design, the recognition and extension of pattern and orientation and discrimination.
According to Berlin and White (1986), the influence that concrete manipulations and computer activities had on the students' transitions from concrete understandings to abstract thinking was unclear. The evidence suggested that not all students were influenced in the same manner. In general, the concrete and computer activities had different effects on students according to their socio-cultural background and their gender, but the subjects did not react in the same way, nor achieve equally well with the different modes of learning activities. It was apparent that some students did better with concrete objects, while others did better with activities which involved computer use.

Differences in achievement in relation to gender, as identified in the Berlin and White study, continue to be the subject of much debate in mathematics education (Leder & Forgasz, 1995). In particular, differing spatial abilities of girls and boys are noted and it is suggested that, on certain mathematical tasks, spatial skills relate more closely to performance for females rather than males, though the relationship between spatial ability and mathematics achievement is a complex one (Leder & Forgasz, 1995). Much controversy is generated by attempts of researchers to explain gender differences in spatial ability and achievement as being biologically determined (Moir & Jessel, 1991), in opposition to views which reject biological determinants in favour of a complex interaction of societal, school, cognitive and affective variables influencing performance (Leder, 1992; Leder & Forgasz, 1995).

The explanation of the achievement differences observed in the 1986 Berlin and White study, however, was not in terms of gender or spatial ability. Berlin and White postulated that the preferred learning styles of the students was an uncontrolled variable, acting to hamper their
findings in regard to the mathematics achievement of the different groups. It was concluded that links between the use of computers and concrete materials needed much further investigation. Their suggestion was that teachers should devise mathematical experiences for students that take into account their preferred learning style and then provide extension and elaboration in other learning styles. It was suggested that grouping according to preferred learning style, which would offer an alternative to traditional grouping by ability or achievement, would be advantageous in terms of increasing educational outcomes.

The work of Berlin and White and others indicate that the field of learning style is currently receiving new emphasis, though it has been the subject of research in education at various times throughout the past century (Keefe, 1987).

A number of different conceptualisations of learning style, each with its own definitions and methods of assessment, are dealt with by a large and complex body of research findings from the 1950s onwards (Gregorc, 1979; Keefe, Monk, Letteri, Languis & Dunn, 1989; Klein, 1951; Kogan, 1971, 1976; Kolb, 1976; Letteri, 1980; Witkin, 1967).

Learning style is "a generic term, an umbrella concept, and a name for recognising individual learning differences" (Butler, 1993, p.9). Research into learning style has been hampered by the fact that different researchers have focused on a number of widely different domains of learning style and different labels have been applied to describe its various aspects. Consequently, the construct of learning style requires clear definition.
2.6.2 The definition of learning style

Do you see what I hear? (Speer, 1979, p.22)

Reviews of earlier research detail many of the efforts in terms of understanding learning styles (Keefe, 1987, 1988; Saracho, 1988, 1989; Speer, 1979). Learning style is variously defined as:

how a student learns and likes to learn. (Keefe, 1988, p.2)

a relatively consistent pattern of perception, interaction with and response to stimuli in a particular learning environment. (Keefe, 1987, cited in Pithers & Mason, 1992, p.61)

characteristic cognitive, affective, and physiological behaviours that serve as relatively stable indicators of how learners, perceive, interact with, and respond to the learning environment. (National Association of Secondary School Principals, 1979, p.7)

Whilst these descriptions clarify definition of the term by referring to characteristics of the individual, further confusion results when the term "cognitive style" is associated with or used interchangeably with learning style. Cognitive style is reported as:

individual differences in modes of perceiving, remembering and thinking. (Kogan, 1971, p.246)

the information processing habits which represent a person's typical modes of perceiving, thinking, remembering and problem solving. (Messick, 1969, p.7)

the person's mode of understanding, thinking, remembering, judging and solving problems. (Saracho, 1989, p.75)

an individual's characteristic and consistent approach to organising and processing information. (Tennant, 1988, cited in Riding & Douglas, 1993, p.298)
The cognitive style of a learner is relatively in-built and is a static characteristic (Riding & Cheema, 1991; Riding & Douglas, 1993). Riding and Cheema contend that all previous descriptions of cognitive style can be broadly grouped into two principal cognitive styles - the so-called Wholist-Analytic style, referring to a learner processing information during thinking into wholes or parts, and the Verbal-Imagery style, referring to a learner representing information by verbal means or in mental images. Further, these two styles may be regarded as independent of one another in that one learner may be both an imager and a wholist, another may be an imager and an analytic, or another may be a verbaliser and a wholist, and someone else may be a verbaliser and an analytic.

Analysis of the previous definitions of learning style and cognitive style leads to clarification in the use of the two terms. Learning style is a term which is typically used in either of three ways: it may be used generically, describing a range of ideas; it may be used congruently with the term cognitive style; or it may subsume cognitive style and include other factors such as affective and physiological behaviours.

Recent interpretation of a wide range of research indicates that the term cognitive style has largely been supplanted by the term learning style. Whilst other descriptions of learning style are more current, for the purposes of this study the definition of learning style according to the National Association of Secondary School Principals (NASSP) (1979) is adopted, due to the comprehensive way it incorporates cognitive, affective and physiological factors in describing behaviours.
As a term which recognises learning differences across three domains, learning style indicates overall the manner in which information is being processed and is concerned with the manner or preference of performance rather than intellectual ability evidenced. Crucially, it is determined by the way a student tends to seek meaning, and the way in which information is personally filtered.

2.6.3 Categories of learning style

There are children playing in the street who could solve some of my top problems in physics, because they have modes of sensory perception that I lost long ago. (Oppenheimer, undated, cited in McLuhan & Fiore, 1967, p.93)

A number of major categories define different learning styles. Some students are observed to prefer to learn verbally by listening and reading, while others prefer information that is more visual, in the form of pictures, diagrams or graphs. A third major classification is that of learners who need the sensory input provided by tactile experience, as they manipulate objects or demonstrate their high need for being mobile in learning situations. These three classifications have been thus termed auditory, visual and kinesthetic learning styles.

Further characteristics of auditory, visual and kinesthetic learners are outlined by Keefe (1989) and Midkiff and Thomasson (1993) and are presented in Table 2.1.

Classification of students in terms of learning style is an appealing construct for teachers in order to explain classroom phenomena such as: students who move their lips while reading silently (likely to be auditory learners); students who are acute observers of minor changes in the classroom environment (likely to be visual learners); or students who
leave their desks often to go to the pencil sharpener (likely to be kinesthetic learners) (Midkiff & Thomasson, 1993).

<table>
<thead>
<tr>
<th>Auditory learners</th>
<th>Visual learners</th>
<th>Kinesthetic learners</th>
</tr>
</thead>
<tbody>
<tr>
<td>Need to hear information from lectures, tapes and discussions. Like to listen to others talk about ideas, concepts and objects. Learn through auditory repetition- remember what they have heard. Favour the sequential presentation of information. Often repeat to themselves what they are trying to learn.</td>
<td>Need to see information and watch presentations involving pictures, graphs, diagrams and visual media. Like to view and inspect material. Learn by forming pictures of objects or concepts- remember what they have seen. Favour wholistic presentation of information. Often close their eyes to reconstruct a picture of what they remember.</td>
<td>Need to feel and experience objects and concepts by handling, modelling or building. Like to move around while completing tasks. Learn by utilising fine muscle skills and/or a whole body response- remember what they have done. Favour hands-on approaches and direct involvement. Often exhibit a strong emotive response to what they are trying to learn.</td>
</tr>
</tbody>
</table>

In comparing the learning styles of children with adults, a major difference is apparent. In adults, the three styles usually function cooperatively, with a preference for one discernible style or a combination of styles, whilst in children, preference seems to evolve from kinesthetic in early childhood to visual and auditory modes in late primary years (Bruner, Oliver & Greenfield, 1966; Keefe, 1987; Sperry; 1973). Teaching approaches emphasising the use of concrete materials in mathematics in the early years are thus seen as not conflicting with teaching according to learning style and are also in accordance with the instructional representational approach and cognitive theories involving the transition from concrete to symbolic modes of representation.

2.6.4 Learning style diagnosis

Learning style diagnosis opens the door to personalising education on a rational basis. It gives the most powerful leverage yet available to educators to
Various instruments are available in order to assist teachers and researchers in identifying the ways students learn across a number of major domains of learning style.

In the cognitive domain of learning style, a major dimension is field independence/field dependence (FDI). This measures "a continuum of an analytic as opposed to a non-analytic way of experiencing the environment" (Keefe, 1987, p.17). The notion of the field dependent learner is of a person who views situations as a whole, as opposed to a learner who views situations as consisting of discrete parts (field independent).

FDI is widely studied and many tests are developed to determine this dimension, such as the Embedded Figures Test (Witkin, Oltman, Rasking & Karp, 1971). In this test a simple two dimensional geometric figure is shown to a learner who is then asked to identify a matching figure by tracing it within the bounds of a more complex figure. The field dependent child usually completes the task without difficulty. Field dependent students engage more often in activities with others and with concrete objects, while field independent students are more involved in restructuring their environment and are likely to learn analytically.

Other significant instruments allow the determination of aspects of learning style (or cognitive style), including: the Learning Style Inventory (LSI) designed by Dunn, Dunn and Price, (1975,1978,1979,1981,1985,1986); the Canfield Learning Styles
Inventory (Canfield, 1983); the Verbaliser-Visualiser Questionnaire (Richardson, 1977); and the NASSP Learning Style Profile (1986).

The Cognitive Styles Analysis (CSA) constructed by Riding (1991) allows the classification of learners in two domains of cognitive style. Learners are said to exhibit either a Wholist-Analytic Style or a Verbal-Imagery Style. The CSA is quick and easy to administer and is self-scoring. Riding argues that the CSA is preferable to other methods of assessing cognitive style as it assesses “the fundamental underlying mechanisms which determine behaviour” (1991, p.299), as opposed to less direct methods using questionnaires, which assess styles by examining the strategies which arise from them.

For the purposes of this investigation the NASSP Learning Style Profile (LSP) was regarded as a superior instrument, as it could be applied across three domains of learning, was the most comprehensive and was practical to administer at classroom level. The profile was developed during the mid 1980s by the NASSP and a task force of researchers in various American universities.

The LSP consists of a number of independent subscales representing cognitive skills, perceptual responses, and study or instructional preferences. The subscales and their meanings are summarised by Keefe (1989) as the following:

**Individual Learning Style Profile**

**PART A**

**Cognitive Skills - General approach to processing information**

**Analytic** - The ability to identify simple figures hidden in a complex field; to analyse information; to use the critical element in a problem in a different way.
Spatial - The ability to identify a geometric shape, remember it, and discriminate it from other patterns. Also, the ability to rotate objects in the imagination.

Discrimination - The ability to visualize the important elements of a task; to focus attention on required detail and avoid distractions; to do what is asked.

Categorisation - The ability to take some risk in classifying information, using reasonable (vs. vague) criteria to form accurate, complete and organised categories...

Sequential Processing - The ability to process information sequentially taking items one step at a time. The ability to perceive the separate elements of experience; to respond to information verbally.

Simultaneous Processing - The ability to respond to new information visually and spatially; to grasp the entire meaning of an experience at once; to perceive the separate elements of a whole and see the bigger picture when only the parts are available.

Memory - The ability to remember discrete bits of information; to detect and remember subtle changes in information.

PART B
Perceptual Responses - Initial response to verbal information
Visual - Preference for processing information initially by seeing it. Visual learners like to receive input from pictures and visual media.

Auditory - Preference for processing information initially by hearing it. Auditory learners who like to listen to others talk about experience. Auditory information is presented sequentially and relies on short-term memory.

Emotive - Preference for processing information by kinesthetic (physical) means or reacting with feeling. A bodily response may take the form of rocking, doodling, note taking or sketching.

PART C
Study and instructional preferences - Personal preferences for the motivational and environmental elements of style.
Persistence orientation - Willingness to work at a task beyond the required time, to withstand discomfort, and to face the prospect of failure.

Verbal risk - Willingness to verbalise, to state opinions even if others disagree.

Manipulative preference - Preference for learning or instruction through hands-on activities.
Time preference - Preference for studying in the early morning, late morning, afternoon or evening.

Verbal-spatial preference - Preference for learning or instruction through verbal or spatial activities.

Grouping preference - Preference for learning or instruction in whole class, small group, or with one other student.

Posture preference - Preference for formal vs. informal study arrangements.

Mobility preference - Preference for moving about and taking breaks while studying vs. staying in one place till the task is finished.

Sound preference - Preference for a quiet study area vs. one with background sound.

Lighting preference - Preference for bright vs. lower lighted study area.

Temperature preference - Preference for studying in a warm vs. cool environment.

For the purposes of this investigation, a particular aspect of the LSP was most suitable. The perceptual response subscales enabled determination of a learner's preference for the initial processing of information in three domains, by either visual, verbal or kinesthetic means. The perceptual response subscales could be easily administered and were necessary in view of the precise nature of the problem under investigation.

2.6.5 Teaching programs and student learning style

Identification of individual learning style makes possible the development of teaching programs which cater for the ways in which individual students learn. A 15-year history of research claims that student achievement, ease of learning and self-concept improves when teaching programs are developed to take into account the learning style of students (Andrews, 1990; Bauer, 1991; Butler, 1993; Hill, 1987; Kyriacou, 1994; Wheeler, 1983).
Studies by Riding (1993) and Riding and Buckle (1990) found that highly visual students, or 'Imagers' learnt best from pictorial representations, whereas those students who preferred to work in the verbal mode of representation, the 'Verbalisers', learnt best when given verbal presentations. Imagers also learnt much better and made greater use of drawings when information was presented in a text-plus-picture mode rather than a totally verbal mode.

The notion of tailoring teaching programs to suit the learning styles of individual students is also termed teaching through "modality strengths" (Barbe & Milone, 1980; Barbe, Swassing & Milone, 1988).

Modality strengths are defined as the individual's perceptual channels which are most efficient for processing information. Individuals internalise their own native language according to sensory input from their environment, by certain perceptual responses which are either visual, auditory or kinesthetic or combinations of these responses (Barbe & Milone, 1981; Barbe, Swassing & Milone, 1988; Keefe, 1989; Keefe & Monk, 1988).

Modality strengths are seen as distinct from modality preferences, or the preferred channels for the way individuals receive information. (Barbe & Milone, 1980). However, most individuals prefer to use their strongest perceptual channel and there is a strong correlation of modality strengths with modality preferences (Keefe, 1989).

In contributing to the evidence regarding the efficacy of teaching through modality strengths, Wheeler (1983) identified the perceptual preferences of learning disabled second grade students using the
Learning Style Inventory (Dunn, Dunn & Price, 1981). Three different sensory approaches (visual, auditory and kinesthetic) were used in instructing each student, with one approach being matched to each student's strongest modality and the other two mismatched. Analysis of the data revealed a significant interaction between perceptual strength and a matched instructional approach.

The ability of students with learning disabilities to reliably describe their learning preferences is detailed by Hill (1987), who also found a significant interaction between the students' modality preferences and the instructional method. Similar findings were recorded by Bauer (1991) who found that students with learning disabilities and emotional problems achieved more strongly when introduced to mathematics topics with a dual modality tactile/visual approach.

Differences in student behaviour, general improvements in attitude towards learning, the elimination of classroom discipline problems and increased achievement on standardised tests are also reported by Andrews (1990), as a result of the development of a learning styles program in a low socio-economic, underachieving North Carolina elementary school.

Increased reading and mathematics standardised achievement and attitude test scores among underachievers who were taught through their learning style strengths are also recently reported in a review of research by Kyriacou (1994). Such underachievers included African-American students, special education students and others considered generally at risk.
Despite the positive research findings regarding teaching through modality strengths (Andrews, 1990; Bauer, 1991; Butler, 1993; Hill, 1987; Kyriacou, 1994; Wheeler, 1983), the approach is nevertheless the subject of some controversy. In a review of research regarding the use of learning style instruments Bonham (1988) states, "specifically, evidence is weak that learning will increase if the teacher or learner chooses an instructional method supposed to match the learner's style" (p.12).

Butler (1993) identifies many further limitations in matching teaching style to learning style including: the narrowing of thinking by precluding learning in other styles, the discouraging of experiments with new ways of learning, the hampering of exploration of new fields of ability or interest, and the blocking of personal growth and development by confining each student to one correct approach.

Channelling of information by the teacher towards a single modality strength or preference is thus restrictive and does not recognise the fact that learning is a complex process. Teaching to a single style is contrary to eclectic approaches typically utilised by experienced teachers, and ignores the success of existing methods. By concentrating too heavily on the needs of the individual, the role of content and context in mathematics learning is also overlooked.

Kavale and Forness (1987) report negative findings in a large-scale investigation of the existence of empirical evidence to justify modality teaching. A meta-analysis of approximately 250 studies with elementary and secondary school students gives only weak support for modality
teaching in terms of increased performance of subjects. The authors conclude:

Only slightly more than half of the experimental subjects realised any gain on standardised outcome measures from instructional techniques designed to capitalise on their modality preferences, and, even when performance improved, the levels were quite modest and only slightly better than those achieved by chance... (Kavale & Forness, 1987, p. 234)

In explaining their findings Kavale and Forness question the basic premise of modality teaching. Classifications of teaching methods and instructional materials as either primarily auditory, visual or kinesthetic are not feasible, as certain modalities cannot be limited or controlled in respect to others. From the evidence:

... it appears that all modalities are strongly involved in the learning process, and academic achievement without the inclusion of all modalities is virtually impossible for a child not suffering a sensory deficit (visual or auditory impairment). (Kavale & Forness, 1987, p. 237)

The authors conclude that future research efforts "would be better directed at improving the more substantive aspects of the teaching/learning process" (Kavale & Forness, 1987, p. 238). Arguments of substance over style or style over substance indicate that further research is warranted to attempt to resolve this question.

2.7 Summary

The way learners construct mathematical ideas is at the core of much contemporary research in mathematics education (Ellerton & Clements, 1992; Wright, 1992). Methods by which students internalise mathematical knowledge and broaden their conceptions by making the links between the different ways knowledge is represented are the
subject of much investigation and debate. The constructivist perspective accepts that knowledge is not directly imparted but is constructed idiosyncratically by learners.

Much evidence exists as to the efficacy of using concrete materials in the development of understanding in mathematics (Suydam, 1984; Sowell, 1989). The use of teaching approaches involving concrete materials is widely advocated (Australian Education Council, 1991; Australian Education Council, 1994; Grouws, 1992; NSW Department of Education, 1989). At the same time, however, difficulties associated with the implementation of concrete materials within an instructional representational approach are identified (Cobb, Yackel & Wood, 1992; Kaput 1987; Labinowicz, 1985; Lesh, Post & Behr, 1987).

Computers are not yet exerting a profound influence in transforming the mathematics curriculum (Balla & Gow, 1987; Czernezkyj, 1990; Kaput, 1992). Yet it is clear that they have an important role to play. One of these roles is in helping learners link the different modes of representation of mathematical knowledge - concrete, pictorial and symbolic (NSW Department of Education, 1989). By negotiation of different mathematical ideas in a social context, difficulties associated with the instructional representational approach may be overcome.

In other fields of research attention is being given to learning styles and teaching through modality strengths, or the way individuals prefer to receive information. Advocates of such programs claim that individual learning style can be identified by the use of valid assessment instruments and that student achievement, ease of learning and self-concept improves when teaching programs are developed to take into account the learning style of students (Andrews 1990; Bauer, 1991;
Butler, 1993; Hill, 1987; Kyriacou, 1994; NASSP, 1986; Wheeler, 1983). Critics of such approaches claim that there is only weak evidence for such modality teaching and that teaching and learning should be viewed in the context of substance and not style (Kavale & Forness, 1987).

From an extended literature review of the previous fields of research it is apparent that further research is needed into some crucial issues. Accordingly, this investigation will focus on different modes of representation of mathematical knowledge in examining the use of the computer to link concrete, pictorial and symbolic representations, and will also seek to determine the viability of matching teaching style to learning style in mathematics education.
CHAPTER THREE

The Conduct of the Investigation

3.1 Introduction

The methodology of this investigation involves quantitative research into the effects of the method of instruction, learning style, gender and grade levels on the achievement of the members of the group studied.

A pilot study was undertaken in order to investigate two major questions of the defined problem. These questions were:

- whether computer software can act as a transition device in the integration of concrete, pictorial and symbolic representations of numeration concepts; and
- whether teaching programs which are constructed in accordance with the preferred learning style of students lead to increased achievement by students.

The first of the major study questions arose primarily as an attempt to replicate the findings of Thompson (1992), who observed the superior performance of students using the computer package *Blocks Microworld* in performing number operations, compared to a group of students who received traditional instruction in mathematics. Thompson contended that the higher achievement of the computer users was due to the lessening of the cognitive load on the students, by providing on-screen representations of mathematical structures with corresponding numeric representations. This allowed students to make a more immediate link between concrete and symbolic representations. Given the small sample of students in the original Thompson study and the relatively short timeframe in which investigations were conducted, it was determined by this
researcher that Thompson's findings were worthy of further investigation.

The second major study question arose largely as an attempt to resolve the controversy regarding modality teaching, or the notion of tailoring teaching programs to suit the learning styles of individual students. If the exact matching of teaching style is possible, then the greatest gains in achievement should occur when students undertake a program of learning which is carefully matched to the particular style in which they prefer to receive information and undertake their learning.

Associated with each of the major study questions of the investigation is the analysis of possible gender and age differences in achievement.

Research into the major study questions of the pilot study commenced with the development of objectives, the selection of subjects and setting, the identification of a suitable instrument to determine students' preferred learning styles, the development of instruments to measure students' achievements, the establishment of teaching treatment groups, the identification of software which would facilitate the transition from concrete to symbolic modes of representation of mathematical ideas, and the development of the methodology used in the analysis of results.

3.2 Pilot study

3.2.1 Objectives

The objectives of the pilot study were:

- to determine a suitable instrument to be used in the identification of student learning styles;
- to design and administer suitable pretests and posttests in order
to measure achievement outcomes;
• to develop appropriate instructional methodology for each of the
teaching treatment groups used;
• to identify which scales of the learning style instrument were
most closely associated with the instructional methodology to be
used;
• to evaluate suitable software which could be used to aid students
in making the transition from concrete to symbolic modes of
representation;
• to analyse the interactions between the method of instruction and
the learning style of students; and
• to recognise limitations in procedures and methodology which
could be eliminated in the major investigation to follow the pilot
study.

3.2.2 Subjects and setting
The subjects for the pilot study were 27 Year 6 students (12 boys and 15
girls) from a single class in a state primary school situated in a
relatively disadvantaged socio-economic area of Greater Western
Sydney. Achievements by the Year 6 student body on basic skills
numeracy tests conducted by the Department of School Education
indicated that it was performing at a level slightly below the mean level
of achievement for schools in NSW.

3.2.3 Instruments used in the pilot study
The conduct of the study first involved the selection and administration
of an instrument to identify the preferred learning style of individual
students. After a survey of a number of different instruments, the
NASSP Learning Style Profile (LSP) (Keefe, Monk, Letteri, Languis &
Dunn, 1989) was chosen as a most contemporary, comprehensive, relevant and practical instrument to administer at classroom level. The LSP consists of a large number of independent subscales representing cognitive skills, perceptual responses, and study or instructional preferences.

The perceptual response subscales of the LSP, developed from the Edmonds Learning Style Identification Exercise (ELSIE), were used in this study (Appendix 1.1). In field tests of 1 500 and 5 000 students, the subscales evidenced acceptable reliability (0.51 to 0.48), validity (0.64 to 0.51) and internal consistency (0.57 to 0.48) (Keefe & Monk, 1988).

Three perceptual response subscales are contained in the LSP - the visual, auditory and emotive subscales which allow the categorisation of learning preferences of students as visual, auditory or kinesthetic. Visual learners have a preference for processing information initially by seeing it, whereas auditory learners prefer to process information initially by hearing it and emotive learners prefer to process information by kinesthetic (physical) means or reacting with feeling.

Administration of the instrument involved individuals responding to a list of 20 words read to them by their teachers. These words represented certain ideas and objects in terms of generating visual, kinesthetic or auditory responses. As each word was read aloud by the teacher, students were asked whether they derived meaning by visualising an image, hearing the sound of the word alone or gaining a feeling in response to the word. The subjects chose one of the three modalities for each word and marked a score sheet with their
preferences. A higher score on each subscale indicated a greater initial preference for that response.

Student academic performance in the pilot study was monitored by pre and posttesting, by means of parallel forms of a 70 item decimal numeration test designed, administered and scored by the researcher (Appendix 1.2). Due to the large number of items in the instrument, reliability, validity, and internal consistency measures for this test were not calculated. This weakness of the pilot study instrument was later overcome by the adoption of a properly standardised instrument in the major investigation.

3.2.4 Procedures and treatment

Three different teaching treatment groups, involving different ways of representing mathematical ideas to students, were utilised in the pilot study. The first teaching treatment group, designated as the computer instruction group, received instruction which was primarily visual, using images and symbols provided by the computer simulation package Blocks Microworld, selected as being most suitable for the purposes of the study. In utilising Blocks Microworld, students were able to solve verbal and written problems by constructing on-screen representations of decimal numbers and observing the mathematical symbols corresponding to those numbers, which were provided by the program. One Apple Macintosh computer was available for each group of three students involved in the treatment.

A second teaching treatment group, referred to as the expository instruction group, received instruction which was predominantly symbolic or abstract in nature in that it utilised only spoken language
and reference to symbols by the teacher, aimed at explaining the Base 10 numeration system as effectively as possible. Instruction by such teacher-centred "chalk and talk" methods, involving exposition and questioning by the teacher and the manipulation of symbols by the teacher and students, is a feature of many traditional classrooms today, especially at the middle to upper levels in primary schools.

Instruction given to the third teaching treatment group was predominantly concrete, in that it involved students using Dienes' Base 10 Arithmetic blocks to represent and investigate decimal numbers and numeration ideas. Students worked informally in pairs or alone during lessons, solving verbal and written problems posed to them by the teacher. The term 'concrete instruction group' is used to refer to this treatment group in future.

The procedures and treatment adopted in the pilot study enabled the investigation of three core hypotheses, developed from the major study questions of the defined problem. It was predicted that:

- students undertaking a course of study in decimal numeration using the computer package Blocks Microworld would demonstrate superior performance on written tests over students undertaking the same course by means of expository teaching methods, or the use of Base 10 concrete materials;
- students receiving instruction which was matched to their preferred learning style would demonstrate superior performance on written tests compared to those students whose preferred learning style was not matched to a particular type of instruction; and
- performance was related to gender.
Specifically, in relation to the second hypothesis, it was predicted that the achievements of visual learners receiving computer instruction, auditory learners receiving expository instruction and kinesthetic learners receiving concrete instruction would be greater than those learners whose preferred learning style was not matched to any particular type of instruction.

The three different types of learning style (visual, auditory and kinesthetic) identified by the LSP were useful in that they corresponded exactly with three contrasting modes of representation of teaching treatment groups (computer instruction, expository instruction and concrete instruction). This allowed the assessment of the efficacy of matching specific types of teaching with the preferred learning style of students within a 3x3x2 design using factorial analysis of variance.

Once the learning style preferences of the 27 Year 6 students in the study were determined, each student was classified as primarily either a visual, auditory, or kinesthetic learner, according to their strongest perceptual response expressed on the LSP. Three students from each learning style category were allocated at random to each of the three teaching treatment groups to be utilised in the study, either: computer instruction, expository instruction or concrete instruction. Ultimately, nine individual subgroups containing three students each could be identified for the purposes of instruction and analysis (Table 3.1).

The subgroups were:
Subgroup one - visual learners receiving computer instruction;
Subgroup two - visual learners receiving expository instruction;
Subgroup three - visual learners receiving concrete instruction;
Subgroup four - auditory learners receiving computer instruction;  
Subgroup five - auditory learners receiving expository instruction;  
Subgroup six - auditory learners receiving concrete instruction;  
Subgroup seven - kinesthetic learners receiving computer instruction;  
Subgroup eight - kinesthetic learners receiving expository instruction;  
Subgroup nine - kinesthetic learners receiving concrete instruction.

Table 3.1 - The design used in the pilot study.

<table>
<thead>
<tr>
<th>Visual learning style</th>
<th>Computer instruction group</th>
<th>Expository instruction group</th>
<th>Concrete instruction group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subgroup one</td>
<td>Subgroup four</td>
<td>Subgroup five</td>
<td>Subgroup six</td>
</tr>
<tr>
<td>Subgroup seven</td>
<td>Subgroup eight</td>
<td>Subgroup nine</td>
<td></td>
</tr>
</tbody>
</table>

n=3 for each subgroup; total n=27; Year 6 students  
Instruction topic: decimal numeration

Each of the major instructional groups of nine students was taught by a different teacher during 10 consecutive daily sessions of 40 minutes over a period of two weeks. Total instructional time was about seven hours. Regular meetings and protocols issued to the participating teachers ensured consistency and uniformity in the content of lessons presented to the students. The lesson content was drawn from the Fractions and Decimals substrand of the K-6 Mathematics Syllabus (NSW Department of Education, 1989) (Appendix 1.3).

3.2.5 Results of the pilot study

After scoring and collation of data from the pilot study, ANOVA (factorial) analyses were carried out in order to assess the effects of the methods of instruction and learning style. The analysis also included the consideration of gender differences.
A 3x3x2 factorial analysis of variance found no statistically significant differences at the .05 probability level (pretest, posttest by gender, style and treatment). There was no difference in the performances of the three groups according to the measures administered. The findings of the earlier Thompson study (1992) were not supported by the pilot study - learners using computer simulations of Base 10 material did not necessarily outperform those using the actual blocks or those receiving expository instruction. No evidence was thus found to support the contention that the software package involved provided a necessary link between concrete and symbolic modes of representation of mathematical ideas.

In relation to the expressed learning style preferences of students, a slight treatment effect was observed with the group receiving instruction by concrete materials. Those students expressing a preference for a kinesthetic style of learning did best in the Base 10 materials group, though the differences found were not statistically significant. No significant differences in performance were identified for visual learners receiving computer instruction or auditory learners receiving expository instruction. Thus no evidence was found to support the hypothesis that learners would perform best with the teaching treatment that was in accord with their learning style (Lewis, 1994).

3.3 The major investigation
Despite the achievement of the majority of the aims and objectives of the pilot study (Section 3.2.1), the findings of the study were not conclusive. It was determined that further investigation of the effects of the method of instruction and the learning style of students was needed.
The principal focus was the re-investigation of the three former core hypotheses. Once again it was predicted that:

- students undertaking a course of study in decimal numeration using the computer package Blocks Microworld would demonstrate superior performance on written tests over students undertaking the same course by means of expository teaching methods, or the use of Base 10 concrete materials;
- students receiving instruction which was matched to their preferred learning style would demonstrate superior performance on written tests compared to those students whose preferred learning style was not matched to a particular type of instruction; and
- performance was related to gender.

The second core hypothesis of the major investigation involved identical detail to that of the pilot study: that the achievements of visual learners receiving computer instruction, auditory learners receiving expository instruction and kinesthetic learners receiving concrete instruction would be greater than those learners whose preferred learning style was not matched to any particular type of instruction.

A crucial feature of the major investigation was the many refinements which occurred as a result of the pilot study. These included:

- the initial screening of 294 students using the learning style instrument;
- a focus on the identification of students who showed an extreme preference for a single learning style;
- the selection of students who showed an extreme preference for a single learning style as the final subjects of the investigation;
• the use of a larger sample (36 students in three treatment groups) which was governed by the number of students expressing extreme preference for a single learning style;
• the use of a properly standardised instrument for pre and posttesting;
• the assignment of the same teacher (the researcher) to each of the three teaching groups in order to improve consistency in treatment.

3.3.1 Subjects and setting
The selected site for the major investigation was a second state primary school in western Sydney, approximately 20 kilometres to the east of the school used in the pilot study. The school was situated in a disadvantaged socio-economic community and had a large multicultural population. Basic skills numeracy tests administered by the Department of School Education indicated that the students involved in the investigation were performing on average at a level below their counterparts in the rest of NSW.

3.3.2 Instruments used in the major investigation
Identification of learning style preferences was again undertaken using the Perceptual Response subscales of the NASSP Learning Style Profile (LSP) (Keefe et al., 1989). This instrument was retained due to its demonstrated, reliability, validity and internal consistency (Keefe & Monk, 1988) and its simplicity and ease of administration which was experienced during the pilot study.

The LSP was administered by the classroom teachers of Year 3 to Year 6 classes because of their familiarity with their students and the large
sample size (294 students). A training session for teachers in the administration of the LSP was conducted by the researcher prior to testing. Before administering the LSP, teachers were requested to give students practice in visual, auditory and kinesthetic responses to trial items in the context of a word game.

After a number of informal games sessions, which involved clarifying and practising the type of perceptual responses involved, the instrument was administered. Students chose one of the three modalities for each word read aloud to them by the teacher and marked a score sheet with their preference. Students experiencing language or learning difficulties or who were unwilling were not required to take part in the survey.

Student academic performance in the major investigation was monitored by pretesting and posttesting by means of a modified form of the M1 Mathematics Test (ACER, 1977) (Appendix 1.4). The M1 test was chosen as it reflected the scope and sequence of objectives for the topic of Numeration in the NSW K-6 Mathematics Syllabus, used as the basis of study in the major investigation. The M1 test was also considered reliable and internally consistent according to the Kuder-Richardson formula (KR 20 of 0.88) (ACER, 1972) and was administered and scored by the researcher.

3.3.3 Procedures and treatment
A total of 294 students across the school were initially tested for their learning style preference using the LSP (Keefe & Monk, 1990). The learning styles instrument yielded raw scores of three different learning style preferences (visual, auditory and kinesthetic) which were scored and collated by a research assistant.
Raw scores for visual, auditory and emotive responses were converted to standard scores using a table of norms contained in the examiner's manual of the LSP (Keefe & Monk, 1990). This enabled the classification of particular perceptual responses as weak, average or strong.

According to the findings of the LSP, 71 of the 294 students surveyed (24%) were identified as expressing a strong preference for a single learning style, with 223 remaining students (76%) showing preference for combinations of two or all three learning styles (bimodal or multimodal preferences).

Consultation with school staff took place in order to select students who were most likely to be co-operative and committed to the final investigation. Students with perceived language difficulties or who were considered likely to provide behavioural problems were excluded at the request of the principal. In all, 36 students were finally selected from the 71 students originally identified as expressing a strong preference for a single learning style. The students ranged in age from nine to twelve years: there were 15 Year 3 students; nine Year 4 students; eight Year 5 students; and four Year 6 students.

In a similar fashion to the pilot study, after 36 students had been chosen according to their learning style preferences, three teaching treatment groups (computer instruction, expository instruction and concrete instruction) were established, each consisting of 12 students. Students were allocated at random so that each instruction group contained four students indicating preference for a particular learning style (either
visual, auditory or kinesthetic). The design of the final investigation is outlined in Table 3.2.

The subgroups were:
Subgroup one - visual learners receiving computer instruction;
Subgroup two - visual learners receiving expository instruction;
Subgroup three - visual learners receiving concrete instruction;
Subgroup four - auditory learners receiving computer instruction;
Subgroup five - auditory learners receiving expository instruction;
Subgroup six - auditory learners receiving concrete instruction;
Subgroup seven - kinesthetic learners receiving computer instruction;
Subgroup eight - kinesthetic learners receiving expository instruction;
Subgroup nine - kinesthetic learners receiving concrete instruction.

Table 3.2 - The design used in the final investigation.

<table>
<thead>
<tr>
<th>Learning Style</th>
<th>Computer Instruction Group</th>
<th>Expository Instruction Group</th>
<th>Concrete Instruction Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visual</td>
<td>Subgroup one</td>
<td>Subgroup two</td>
<td>Subgroup three</td>
</tr>
<tr>
<td>Auditory</td>
<td>Subgroup four</td>
<td>Subgroup five</td>
<td>Subgroup six</td>
</tr>
<tr>
<td>Kinesthetic</td>
<td>Subgroup seven</td>
<td>Subgroup eight</td>
<td>Subgroup nine</td>
</tr>
</tbody>
</table>

n=4 for each subgroup; total n=36; Years 3, 4, 5 and 6 students
Instruction topic: numeration

Students in the computer group worked in pairs on Apple Macintosh computers as only six machines were available. The first session for students in the computer group was allocated to free exploration of the software package, so that students could gain familiarity with its use and the researcher could evaluate students' responses and recognise any problems or limitations occurring (Perl, 1990). Student access to Blocks Microworld was not allowed outside of the lessons conducted by the researcher.
To ensure consistent treatment each group of students was taught in turn by the researcher during 10 consecutive daily sessions of approximately 40 minutes over a two week period. Total instructional time was about seven hours.

In view of the mixed ages of the subjects, the lesson content was drawn from the Numeration strand of the K-6 Mathematics Syllabus (NSW Department of Education, 1989). Table 3.4 details the scope and sequence of content and some sample exercises which were incorporated in the instructional program and the M1 assessment instrument.

The normal format of lessons was for the researcher to provide a brief review of ideas and skills previously experienced, to discuss new ideas and skills, and then provide each of the three groups of students with an identical worksheet containing suggested learning tasks. A sample worksheet used in the investigation is provided in Appendix 1.5.

Students were also encouraged to complete a workbook during the 10 lessons of the investigation. The workbook served as a record of student responses to learning tasks given on worksheets and also as a means of detailing reflections and feelings as they reacted to the lesson material.

In common with most other learning style instruments, the LSP involves a self-report inventory whose accuracy is dependent on subjects knowing themselves and being willing and able to reveal that knowledge (Bonham, 1988). It was decided to compare and contrast the responses gained by the use of the learning style instrument with students' own opinions regarding their preferred learning style.
During the introductory lessons the researcher engaged each group of students in discussion involving examples of the different ways in which

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Content</th>
<th>Computer instruction group</th>
<th>Expository instruction group</th>
<th>Concrete instruction group</th>
</tr>
</thead>
</table>
| One    | • Whole number numeration to one million  
        • Building and/or writing numbers to ten thousand | • Build the blocks on the screen and write the numerals for the number: 3 blocks, 4 flats, 3 longs and 6 shorts. Build the number on the screen and then write it in words: 1 348 | • Write the number that has 3 thousands, 4 hundreds, 3 tens and 6 ones. | • Build the number and then write it in words: 1 348 |
| Two    | • Reading and writing numerals in words  
        • Reading and writing words in numerals | | • Write the numeral in words: 1 348 | |
| Three  | • Making numbers with mixed digits | Build the largest possible number on the screen using the digits: 7, 2, 4, 5, 2, 0 | Write the largest possible number using the digits: 7, 2, 4, 5, 2, 0 | Build the largest possible number using the digits: 7, 2, 4, 5, 2, 0 |
| Four   | • Arranging numbers from smallest to largest  
        • Arranging numbers from largest to smallest | Build the numbers on the screen in order from smallest to largest: 272, 927, 792, 972 | Arrange the numbers in order from smallest to largest: 272, 927, 792, 972 | Build the numbers in order from smallest to largest: 272, 927, 792, 972 |
| Five   | • Sequences involving missing numbers | Build each number on the screen and work out the missing numbers: 876, 865, 854, —, —, 821 | Write the missing numbers: 876, 865, 854, —, —, 821 | Build each number and work out the missing numbers: 876, 865, 854, —, —, 821 |
| Six    | • Face and place value of digits in numbers | Build the number 7 457 on the screen. How many tens in 7 457? | How many tens in 7 457? | Build the number 7 457. How many tens in 7 457? |
| Seven  | • Writing numbers more or less than certain numbers | Build the numbers on the screen that are: 10 more than 4 999; 100 less than 4 027 | Which numbers are: 10 more than 4 999; 100 less than 4 027 | Build the numbers that are: 10 more than 4 999; 100 less than 4 027 |
| Eight  | • Using zero as a place holder | Build the following numbers on the screen: 104, 110, 2 017, 3 510 Which kind of block is missing? | Write how many hundreds, tens and ones in the following: 104, 110, 705, 900 | Build the following numbers: 104, 110, 2 017, 3 510 Which kind of block is missing? |
| Nine   | • Expanded form | Show the following number on the screen: 3 000 + 400 + 50 + 3 | Write the expanded form of 3 453 | Build the expanded form of 3 453 |
| Ten    | • Revision | See previous examples | See previous examples | See previous examples |
individuals were able to learn, focussing on preferences for a particular style of learning, or combination of styles.

After the discussions the students were encouraged to write their responses in their workbooks by completing the sentence stem "I like to learn by...". Student responses in the workbooks were then compared with the learning style preferences indicated by the assessment instrument.

Throughout the investigation, maximum effort was undertaken by the researcher to establish a warm and non-threatening atmosphere in each learning context.

3.3.4 Analysis of results

After scoring and collation of data from the investigation, factorial analyses of variance were carried out, allowing the simultaneous comparisons of the effects of the method of instruction and learning style on student achievement. In view of the findings of Berlin and White (1986) that concrete and computer activities had different effects on students according to their gender, a comparison of the effects of gender was included. It was also determined that the effects of grade levels on achievement would be investigated, according to the evidence that preference in learning styles in students seems to evolve from kinesthetic in early childhood to visual and auditory modes in late primary years (Bruner, Oliver & Greenfield, 1966; Keefe, 1987; Sperry; 1973).

A detailed analysis of the results and a discussion of findings of the major investigation, involving the use of software, the learning styles
instrument, the use of student workbooks and various statistical analyses follows in Chapter 4.
CHAPTER FOUR
Results and Discussion

4.1 Introduction
This chapter reports the results of a quantitative investigation of the effects of the method of instruction, learning style, gender and grade levels on the achievement of the members of the group studied. The use of the software package Blocks Microworld, the Learning Styles Profile and student workbooks are also evaluated.

Results of the major investigation and a discussion of the findings in the light of the major study questions and hypotheses are presented in this chapter in the following sections:

4.2 Use of the mirroring program Blocks Microworld
4.3 The learning styles instrument survey
4.4 Analysis of student workbooks
4.5 Statistical analyses
    4.5.1 The pretest
    4.5.2 The posttest
    4.5.3 Factorial analysis
4.6 Summary of findings
4.7 Discussion

4.2 Use of the mirroring program Blocks Microworld
The major problems or limitations often associated with the use of mirroring programs (Perl, 1990) such as lack of keyboarding skills by students, slow running of software, and difficulties in monitoring progress, were not experienced with the use of the software package Blocks Microworld. From the first lesson onwards, it was apparent that the machines used ran the software at an acceptable speed and that the
students possessed satisfactory keyboarding skills in order to manipulate the images. The monitoring of student progress was also easily achieved, due to the arrangement of the machines in two parallel rows of three machines in the laboratory.

A major advantage in using Blocks Microworld, receiving favourable comments from many students, was the ability of the package to physically compose and decompose representations of numbers on the screen, by the actions of "gluing" and "ungluing" blocks. This feature provided a dynamic quality to the operation which was much appreciated by the students and lessened the cognitive load imposed on them. Because students could physically compose and decompose on-screen images of blocks, the need to introduce additional Base 10 material for the purpose of trading was eliminated, and the confusion which often surrounds this “extra” Base 10 material was removed.

One difficulty regarding the efficacy of concrete materials which is noted in the research literature is the problem of representing large numbers with Base 10 material (Kaput & Pattison-Gordon, 1987; Labinowicz, 1985; Lesh, Post & Behr, 1987). During the investigation a similar type of minor problem was encountered involving the representation of five, six and seven digit numbers using Blocks Microworld. The relatively small screen of the Macintosh soon became overcrowded by the many blocks which, although superimposed by the program, were very difficult to distinguish. The solution to the problem of representing large numbers, however, was handled adeptly by the students, using a built-in facility of the software. Just as when using real Base 10 blocks, it was possible to alter the reference of the numeration system from where a single Base 10 cube represented the
number one, to where the same cube represented one thousand, so that the numbers became easier to structure and manipulate.

4.3 The learning styles instrument survey

The three perceptual response subscales of the LSP were used to survey 294 students in Years three, four, five and six at the school in order to determine their learning style preferences. 71 students were identified by the LSP as expressing a strong preference for a single learning style and the final sample for the investigation was drawn from this group.

Amongst the group of 71 students there were 42 boys and 29 girls. By grade there were 23 Year 3 students, 21 Year 4 students, 11 Year 5 students and 16 Year 6 students. Differences in the number of students in each Year category existed due to the sampling technique of the preferred methodology which was employed. Preferences for a single learning style were unequally distributed across the group: visual methods of receiving information were preferred by 12 students (17%), 29 students (41%) preferred auditory methods, and 30 students (42%) preferred the kinesthetic approach.

The fact that 76% of participants were identified by the LSP as expressing multimodal preferences for the manner in which they receive information indicates support for the view that, in late primary years, preference by students for a range of different learning styles evolves from an earlier dependence on a single learning style, which is usually kinesthetic (Bruner, Oliver & Greenfield, 1966; Keefe, 1979; Sperry; 1973).
Of the 24% of students who indicated a single learning style preference, a narrow majority indicated kinesthetic preferences for receiving information, in accordance with the above findings; almost as many students indicated auditory preferences; and only a few students showed preference for visual learning. In an audio-visual age, where television and video are ubiquitous, the relatively small numbers of students preferring visual stimuli and the much larger group preferring auditory information is initially surprising. However, if it is considered that the students in the survey are providing responses in their school environment as a product of their previous experiences, the results can be explained - the principal mode of classroom instruction is traditionally by auditory methods, containing a heavy emphasis on listening to the teacher, and the use of visual media is secondary. The students in the survey are thus likely to have been schooled to listen and not to see.

The results of this investigation also support the findings that students preferring a single learning style are likely to be younger (Bruner, Oliver & Greenfield, 1966; Keefe, 1979; Sperry, 1973). Among the 36 students finally selected, nearly half were Year 3 students and there were relatively few Year 6 students. The final numbers were 15 Year 3 students; nine Year 4 students; eight Year 5 students and four Year 6 students.

Table 4.1 shows the distribution of learning style preferences revealed by the perceptual response scales of the LSP across the different grade levels. From a study of table 4.1 it is clear that, as students become older, preference evolves from a single learning style to a combination of styles.
Table 4.1 - Learning style perceptual preferences and student grade levels

<table>
<thead>
<tr>
<th>Learning Style</th>
<th>Year 3 Students</th>
<th>Year 4 Students</th>
<th>Year 5 Students</th>
<th>Year 6 Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visual learning style</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Auditory learning</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Kinesthetic learning</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.1 also reveals that the perceptual preferences of the Year 3 students are almost equally distributed across the three learning styles. The number of students preferring the auditory mode (n=6) is surprising, given the previous findings that the preference in early childhood years is usually for kinesthetic learning (Bruner, Oliver & Greenfield, 1966; Keefe, 1979; Sperry, 1973). Further investigation of the role of the school environment and culture in determining learning style preferences is warranted here.

4.4 Analysis of student workbooks

At the beginning of the major investigation students indicated their learning style preference by completing the sentence stem "I like to learn by..." in their workbooks. An analysis of their written responses was undertaken in order to categorise the preferred learning styles disclosed by individual students and correlate them with the information gained by the administration of the learning style instrument.

Despite the fact that the 36 students (19 boys and 17 girls) involved in the final investigation were chosen on the basis of their preference for a single style revealed by testing, an analysis of their written responses found that the majority (56%) expressed multimodal learning style preferences. Bimodal preferences were expressed by 10 students
(28%), another 10 students (28%) expressed trimodal preferences and only 16 students (44%) expressed a preference for a single learning style.

Of the 10 students who expressed bimodal preferences in their workbooks, only five students (50%) included one of the preferences which matched their learning style identified by the LSP. Four of the five students who matched at least one learning style preference were in the auditory group.

When the learning styles of the 16 students who showed preferences for a single learning style were correlated with the preferences indicated by the LSP, only five students (31%) had learning style preferences which matched those which were identified by the instrument. Four of the five students whose styles matched were kinesthetic learners and the remaining student was in the visual preference category.

An extremely poor correlation was thus found to exist between the learning style preferences expressed in writing by the students and the preferences revealed by the use of the LSP. The implications of these findings are discussed further in Section 5.4 of this work.

4.5 Statistical analyses

4.5.1 The pretest

Statistical analysis of results commenced with an investigation of the performance of different grade groups on the pretest. Due to the age differences and the prior experiences of the students involved it was expected that performance on the pretest would increase with grade level.
Analysis of variance was used to test for significant differences between groups according to grade level on the pretest. A one-way ANOVA found a significant difference between the performance of groups according to grade level ($F_{3,35} = 7.83, p < 0.01$). A Tukey-B post-hoc test also found that the scores for Year 4, Year 5 and Year 6 students were significantly different from the scores for Year 3 students at the 0.05 level of probability.

Pretest mean scores and standard deviations for each grade group are listed in Table 4.2.

<table>
<thead>
<tr>
<th>Group</th>
<th>Count</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 3</td>
<td>15</td>
<td>19.13</td>
<td>4.94</td>
<td>1.28</td>
</tr>
<tr>
<td>Year 4</td>
<td>9</td>
<td>28.56</td>
<td>8.31</td>
<td>2.77</td>
</tr>
<tr>
<td>Year 5</td>
<td>8</td>
<td>31.50</td>
<td>7.03</td>
<td>2.48</td>
</tr>
<tr>
<td>Year 6</td>
<td>4</td>
<td>29.50</td>
<td>7.77</td>
<td>3.88</td>
</tr>
<tr>
<td>Total</td>
<td>36</td>
<td>25.39</td>
<td>8.39</td>
<td>1.40</td>
</tr>
</tbody>
</table>

*p ≤0.05

The performance of students generally increased with grade level, although the Year 6 students were outperformed by Year 5 students. It would thus appear that more Year 5 students had entered the stage of formal operations than their Year 6 counterparts, although the Year 6 sample size may be too small in this instance to generalise these results.

4.5.2 The posttest

Analysis of variance was then repeated for the posttest, in order to investigate the achievements of the different grade groups and gain a measure of the effectiveness of the teaching involved as well as the students' ability to cope with increasingly difficult content material.
A one-way ANOVA found a significant difference between the performance of groups according to grade level (F 3,35 = 7.59, p< 0.01). A Tukey-B post-hoc test also found that the scores for Year 4, Year 5 and Year 6 students were significantly different from the scores for Year 3 students at the 0.05 level of probability.

Posttest mean scores and standard deviations for each grade group are listed in Table 4.3.

<table>
<thead>
<tr>
<th>Group</th>
<th>Count</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 3</td>
<td>15</td>
<td>21.93</td>
<td>6.31</td>
<td>1.63</td>
</tr>
<tr>
<td>Year 4</td>
<td>9</td>
<td>32.56</td>
<td>7.49</td>
<td>2.50</td>
</tr>
<tr>
<td>Year 5</td>
<td>8</td>
<td>33.25</td>
<td>6.90</td>
<td>2.47</td>
</tr>
<tr>
<td>Year 6</td>
<td>4</td>
<td>32.25</td>
<td>5.80</td>
<td>2.30</td>
</tr>
<tr>
<td>Total</td>
<td>36</td>
<td>28.25</td>
<td>8.42</td>
<td>1.40</td>
</tr>
</tbody>
</table>

*p ≤ 0.05

The significant differences in the performances of Year 4, Year 5 and Year 6 students that were found as a result of the pretest and posttest indicate the likelihood of a ceiling effect operating during the investigation. The Year 3 student group achieved significantly less well than the Year 4, Year 5 and Year 6 groups, whose mean scores were clustered together, and whose students were more likely to be affected by knowledge of concepts involved or abilities developed before the investigation had begun. Once again the Year 5 students performed better than their Year 6 counterparts, indicating that achievement was not directly related to age in this instance.

4.5.3 Factorial analysis

Having investigated achievement according to grade level, the next level of analysis sought to investigate differences in performance between teaching treatment groups (computer instruction, expository instruction
and concrete instruction) and learning styles (visual, auditory and kinesthetic).

A 3x3x2 factorial design was chosen because it allowed for the simultaneous comparison of the effects of multiple independent variables and facilitated the testing of the three major hypotheses (main effects) in the investigation, that:

- students undertaking a course of study in decimal numeration using the computer package *Blocks Microworld* would demonstrate superior performance on written tests over students undertaking the same course by means of expository teaching methods, or the use of Base 10 concrete materials;
- students receiving instruction which was matched to their preferred learning style would demonstrate superior performance on written tests compared to those students whose preferred learning style was not matched to a particular type of instruction; and
- performance was related to gender.

In the 3x3x2 design there were three independent variables: learning style (three types, controlled), method of instruction (three types, manipulated) and gender (two levels). Comparisons of the effects of the multiple independent variables and a number of interactions between variables were made using analysis of variance (ANOVA). There were three main effects (treatment, learning style and gender); three first-order interactions (treatment by learning style, treatment by gender and learning style by gender); and one second-order interaction (treatment by learning style by gender).
The main effect for treatment involved comparing the three treatment means, learning style differences were analysed by comparing the learning style means and differences in gender involved comparing the means of male and female students. Testing of main effects and interactions took place by means of F ratios.

Factorial analysis of variance was carried out on the pretest and on the posttest. Table 4.4 lists the results of the factorial analysis of variance on the pretest and Table 4.5 lists the results for the posttest.

Table 4.4 - ANOVA table showing pretest by treatment, style and gender

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Sum of squares</th>
<th>df</th>
<th>Mean square</th>
<th>F</th>
<th>Sig of F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main effects</td>
<td>223.25</td>
<td>5</td>
<td>44.65</td>
<td>.59</td>
<td>.71</td>
</tr>
<tr>
<td>TREAT</td>
<td>33.40</td>
<td>2</td>
<td>16.70</td>
<td>.22</td>
<td>.81</td>
</tr>
<tr>
<td>STYLE</td>
<td>178.27</td>
<td>2</td>
<td>89.14</td>
<td>1.18</td>
<td>.33</td>
</tr>
<tr>
<td>GENDER</td>
<td>.30</td>
<td>1</td>
<td>.30</td>
<td>.01</td>
<td>.95</td>
</tr>
<tr>
<td>2-Way Interactions</td>
<td>496.65</td>
<td>8</td>
<td>62.08</td>
<td>.82</td>
<td>.60</td>
</tr>
<tr>
<td>Treat-Style</td>
<td>339.04</td>
<td>4</td>
<td>84.76</td>
<td>1.12</td>
<td>.38</td>
</tr>
<tr>
<td>Treat-Gender</td>
<td>134.97</td>
<td>2</td>
<td>67.48</td>
<td>.89</td>
<td>.43</td>
</tr>
<tr>
<td>Style-Gender</td>
<td>153.10</td>
<td>2</td>
<td>76.55</td>
<td>1.01</td>
<td>.38</td>
</tr>
<tr>
<td>3-Way Interactions</td>
<td>386.82</td>
<td>4</td>
<td>96.71</td>
<td>1.28</td>
<td>.32</td>
</tr>
<tr>
<td>Treat-Style-Gender</td>
<td>386.82</td>
<td>4</td>
<td>96.71</td>
<td>1.28</td>
<td>.32</td>
</tr>
<tr>
<td>Explained</td>
<td>1099.72</td>
<td>17</td>
<td>64.69</td>
<td>.85</td>
<td>.63</td>
</tr>
<tr>
<td>Residual</td>
<td>1364.83</td>
<td>18</td>
<td>75.82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2464.55</td>
<td>35</td>
<td>70.42</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p ≤0.05

A study of both tables shows that there are no significant differences observed in the value of the F ratios for either effects or interactions. It was found that:

- the means for the three treatment groups (computer instruction, expository instruction and concrete instruction) did not differ significantly;
• the means for the three types of learning style (visual learners, auditory learners and kinesthetic learners) did not differ significantly;
• there was no significant difference found with regard to gender;
• the interaction between treatment and learning style was not significant;
• the interaction between treatment and gender was not significant;
• the interaction between learning style and gender was not significant; and
• the interactions between treatment, learning style and gender were also not significant.

Table 4.5 - ANOVA table showing posttest by treatment, style and gender

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Sum of squares</th>
<th>df</th>
<th>Mean square</th>
<th>F</th>
<th>Sig of F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main effects</td>
<td>164.32</td>
<td>5</td>
<td>32.66</td>
<td>.38</td>
<td>.86</td>
</tr>
<tr>
<td>TREAT</td>
<td>30.46</td>
<td>2</td>
<td>15.22</td>
<td>.18</td>
<td>.84</td>
</tr>
<tr>
<td>STYLE</td>
<td>131.49</td>
<td>2</td>
<td>55.74</td>
<td>.75</td>
<td>.49</td>
</tr>
<tr>
<td>GENDER</td>
<td>27.54</td>
<td>1</td>
<td>27.54</td>
<td>.32</td>
<td>.58</td>
</tr>
<tr>
<td>2-Way Interactions</td>
<td>422.97</td>
<td>8</td>
<td>52.87</td>
<td>.61</td>
<td>.76</td>
</tr>
<tr>
<td>Treat-Style</td>
<td>297.42</td>
<td>4</td>
<td>74.36</td>
<td>.85</td>
<td>.51</td>
</tr>
<tr>
<td>Treat-Gender</td>
<td>80.86</td>
<td>2</td>
<td>40.43</td>
<td>.46</td>
<td>.64</td>
</tr>
<tr>
<td>Style-Gender</td>
<td>140.29</td>
<td>2</td>
<td>70.15</td>
<td>.81</td>
<td>.46</td>
</tr>
<tr>
<td>3-Way Interactions</td>
<td>359.40</td>
<td>4</td>
<td>89.85</td>
<td>1.03</td>
<td>.42</td>
</tr>
<tr>
<td>Treat-Style-Gender</td>
<td>359.40</td>
<td>4</td>
<td>89.85</td>
<td>1.03</td>
<td>.42</td>
</tr>
<tr>
<td>Explained</td>
<td>907.75</td>
<td>17</td>
<td>53.38</td>
<td>.61</td>
<td>.84</td>
</tr>
<tr>
<td>Residual</td>
<td>1569.00</td>
<td>18</td>
<td>87.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2476.75</td>
<td>35</td>
<td>70.76</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p ≤0.05

Due to the lack of significance between means, and the lack of effects and interactions noted it is not possible to reject the null hypotheses implicit in the investigation.
4.6 Summary of findings

The findings of this investigation may be summarised as follows:

- The disadvantages associated with the use of other mirroring programs by students were not experienced with the use of the package *Blocks Microworld*.
- The majority of the students surveyed by the LSP were found to express preferences for multimodal learning.
- Those students who expressed a preference for a single learning style tended to be the younger participants.
- An extremely poor correlation was found to exist between the learning style preferences expressed personally by the students and the perceptual preferences revealed by the learning style instrument used in the investigation.
- The performance of Year 4, Year 5 and Year 6 students was significantly better than that of Year 3 students on the pretest and posttest. These performance differences are further discussed in Section 5.2.3.
- Year 5 students outperformed their Year 6 counterparts on the pretest and posttest.
- No significant differences in the performance of instruction groups were found using a factorial analysis of variance.
- There were no significant differences in the performances of learning style groups, or according to gender.
- There were no effects or interactions identified between treatment, learning style and gender.

4.7 Discussion

The principal focus of this investigation was the examination of three core hypotheses that:
• students undertaking a course of study in decimal numeration using the computer package Blocks Microworld would demonstrate superior performance on written tests over students undertaking the same course by means of expository teaching methods, or the use of Base 10 concrete materials;
• students receiving instruction which was matched to their preferred learning style would demonstrate superior performance on written tests compared to those students whose preferred learning style was not matched to a particular type of instruction; and
• performance was related to gender.

According to the findings of this investigation, students who received instruction by means of screen images of Base 10 numeration materials did not necessarily outperform those students who received instruction using real-life Base 10 material, or those students who received instruction by expository teaching.

No evidence was found to support the matching of teaching and learning style. The achievement outcomes which were evaluated showed that students with strong preferences for kinesthetic learning did not benefit most from "hands-on" teaching; students with strong visual preferences did not benefit most by working with computer images; and that students with auditory preferences did not benefit most by simply listening to the teacher's explanations.

No evidence was found either to support the hypothesis that performance was related to gender. Girls did not necessarily outperform boys in the investigation, or vice versa.
Due to the total lack of treatment effects or interactions found within the various types of factorial analyses applied and reported in Sections 4.5.3, it thus possible to reject the core hypotheses of this investigation, though some caution is necessary due to the small sample size involved. The relatively small final sample size in this investigation is explained as the result of the reduction of original large number of subjects following testing and rigid categorisation of their abilities and individual differences.

Despite the smallness of the sample involved in the investigation, there are, nevertheless, a number of important associated findings, worthy of further discussion and investigation. These include:

• the implications of the use of computer packages such as Blocks Microworld in the representation of mathematical ideas;
• grade differences in the performance of students which were revealed in relation to different types of instruction and learning style preferences;
• the viability of matching teaching style to the learning style preferred by students; and
• the use of instruments in assessing student learning style.

Further discussion of findings and the major conclusions of the investigation are reported in Chapter 5.
CHAPTER FIVE
Conclusions of the Investigation

5.1 Introduction
This investigation was devised primarily to seek answers to the questions of:

- whether the computer can act as a transition device in the integration of concrete, pictorial and symbolic representations of mathematics ideas; and
- whether teaching programs which are constructed in accordance with the preferred learning style of students lead to increased achievement by students.

As a result of the data collected and analysed in the investigation a number of major conclusions which have important implications for future research may be drawn. These conclusions are presented in this chapter in the following sections:

5.2 The computer in mathematics education
5.2.1 The transition between concrete and symbolic representations
5.2.2 The continuum between concrete and symbolic representations
5.2.3 Grade differences in the achievement of instructional groups

5.3 The matching of teaching style and learning style

5.4 The use of instruments to assess learning style
5.4.1 Post hoc analysis

5.5 Recommendations arising from the investigation
5.2 The computer in mathematics education

From the findings of this investigation, it is clear that the software package *Blocks Microworld* (Thompson, 1991) does not suffer from many of the disadvantages evident in the use of other types of mirroring programs. The program was observed to run at an acceptable speed, the screen images could be easily manipulated by students, the need for exchanging one type of Base 10 material for another was eliminated, and progress could be readily monitored by the teacher.

A minor limitation in the use of *Blocks Microworld* does exist in regard to the representation of very large numbers, in that the images constructed are restricted by the lack of available screen space. However, such difficulties are also experienced by students using real Base 10 material when constructing three-dimensional representations of very large numbers, so that *Blocks Microworld* users are not necessarily disadvantaged by comparison. It was significant, moreover, that the students using *Blocks Microworld* evidenced the ability to confidently overcome the space limitations imposed by the program, and to thus recognise inherent advantages of the software. Accordingly, it is clear that *Blocks Microworld* does offer the prospect of useful supplementary representations of numeration and operations ideas for the primary mathematics classroom, and that some of these representations are unique to the program.

5.2.1 The transition between concrete and symbolic representations

*Blocks Microworld* is also promoted as a communication tool which helps students make connections between concrete and abstract thought, by linking physical, pictorial and symbolic representations of
mathematical ideas. In a study by Thompson (1992), students using Blocks Microworld demonstrated superior performance in mathematics over a group of students using Base 10 material, measured by written tests and structured interviews. Thompson concluded that the superior achievements of the computer users was a result of a lessening of the cognitive load involved in the bridging of concrete and symbolic representations, by means of the screen images and symbols which were simultaneously provided.

No evidence was found in this investigation, however, to support the findings of the Thompson study in relation to the use of Blocks Microworld. It was found that the achievements of all major instructional groups were equal, and that use of Blocks Microworld gave students no attendant advantages in achievement over other means of instruction employed in the investigation. As far as the use of Blocks Microworld is concerned, therefore, no support is therefore given to the assertion that "computers can provide a link between concrete manipulation and abstract processing" (NSW Department of Education, 1989, p.37).

A number of reasons may be advanced to explain the comparable learning outcomes of the major instructional groups involved in the investigation. It may well be that the ability of the teacher in providing transparent communication of the ideas involved was the predominant factor in promoting meaningful learning, rather than the use of any particular tool, or way of representing a mathematical idea. The various methods of communicating knowledge which were utilised in the investigation would then be equally valid, in that they each allowed students to connect with and build on their previous knowledge of
numeration ideas, promoting thought and reflection by explicitly linking the particular representation to existing informal understandings, in true constructivist fashion.

That these different representations of mathematical ideas may be equivalent has been tellingly recognised by Baroody (1989):

...it does not necessarily follow...that children must manipulate something concrete and reflect on physical actions to construct meaning. It does suggest that they should manipulate something familiar and reflect on these physical or mental actions. The particular medium (objects, pictures of objects, or video displays of pictured objects) may be less important than the fact that the experience is meaningful to pupils and that they are actively engaged in thinking about it. (p.5, author's emphases)

Another factor in the equal learning outcomes achieved by the major instructional groups may be the maturational levels of the students, and the prior understandings of numeration developed in their previous schooling. Given that the topic of numeration and place value constitutes the foundations of much mathematical learning, is strongly emphasised in the NSW K-6 Mathematics Syllabus, and receives the widespread attention of teachers, many of the students in the investigation may well be beyond the stage of concrete operational thought in this topic. If these students have entered the stage of formal operations (according to Piagetian theory), they are already processing numeration ideas in an abstract fashion, and instruction by visual or kinesthetic methods, without offering choice according to their learning preferences, may not have been effective, appropriate or highly regarded by them.
5.2.2 The continuum between concrete and symbolic representations

The equal learning outcomes achieved by instructional groups in the investigation may also cast some doubt on whether a continuum between concrete and abstract (or symbolic) representations of mathematical ideas actually exists (Heddens, 1986; Underhill, 1977). The findings reinforce a current view in mathematics education recognising that different representations of ideas exist, but disputing or denying the measured linear transition between concrete and symbolic modes for all learners (Hart, 1994; Perry & Howard, 1994). Rather than existing as a transition device between the concrete and the symbolic, it may be more appropriate to view the computer as a tool which can effect independent and alternative representations.

Some constructivist theorists (Cobb, Yackel & Wood, 1992) dispute the existence of a learning continuum or ramp from concrete to abstract, advocating instead the individual and idiosyncratic representation of knowledge, according to an internal reality generated by the person's own needs and dispositions. Mathematical knowledge for these learners is not acquired by sequentially manipulating concrete materials, viewing computer images, listening to the teacher, or writing symbols, based on a belief that mathematics is in some way inherent in these media. Alternatively, knowledge for these learners is constructed by means of social discourse in situations that have meaning for them, using an appropriate representation or variety of representations - the so-called social constructivism (Ellerton & Clements, 1992).

The results of this investigation largely support a social constructivist position, highlighting the limitations of the instructional representational
approach. An inflexible view of a sequence of representations from concrete to abstract and the resultant rigid structuring of teaching programs based on an acceptance of this view is not appropriate for the contemporary mathematics classroom. Rigid practices must be replaced by flexible programs in which the teacher facilitates learning by providing a menu of different representations of ideas that are real to students, and helps negotiate their different meanings by promoting social discourse.

It is not a matter of holding a child using concrete apparatus until abstraction is possible, as if this were a linear procedure, but rather of always recognising the need to slip in and out of representations in exactly the same way as particular examples are used to enhance the meaning of a generalisation. (Burton, 1990, p.341)

5.2.3 Grade differences in the achievement of instructional groups
The superior performance of Year 4, 5 and 6 students compared to Year 3 students in the investigation is recognised and may be explained by the same combination of factors discussed in Section 5.2.1 of this work. Previous experience of content material and differences in maturational and cognitive levels are likely to be operating to explain the increased achievement of older students in this investigation. Year 5 and Year 6 students, in particular, were much more likely to have been affected by the prior learning of numeration and place value concepts, regarded by teachers as the foundation on which understandings of number ideas are built, and given appropriate emphasis in teaching. Many of the older students participating are already abstract thinkers in their own right, and can be expected to perform more strongly than Year 3 students, although the slightly greater test achievements of Year
5 students, compared to their Year 6 counterparts, is contrary to expectations. This anomaly may be explained, however, by the fact that the Year 6 sample (four students) is much smaller than that of the other grades.

5.3 The matching of teaching style and learning style
The results of this investigation also do not support the matching of teaching and learning styles, or so-called modality teaching, in which particular teaching programs are developed to accurately and entirely account for particular learning styles of students. No evidence was found that students with strong preferences for kinesthetic learning benefited most from concrete instruction, that students with strong visual preferences benefited most by computer instruction, or that students with auditory preferences benefited most by expository instruction.

These findings reinforce the assertions of Kavale and Forness (1987) that the basic premise of modality teaching cannot be sustained. Classification of teaching methods and instructional materials as either primarily auditory, visual or kinesthetic is not possible, for a particular instructional method cannot directly and entirely correspond to a single learning style. It is an oversimplification of the perception and learning process to believe that certain modalities can be limited or controlled in respect to others, for, unless they are sensorily impaired, most students are bimodal or multimodal, using a combination of modalities in processing information in learning contexts, and are never entirely or solely auditory, visual or kinesthetic learners at any point in time.
Thus, within the group utilising computers in the investigation, visual images were present on the screen, but kinesthetic inputs occurred by mouse-driven manipulations, and students communicated verbally as they negotiated the meaning of the different exercises. Within the group utilising concrete materials, kinesthetic inputs were gained as students handled materials, but mental images were involved in the internalisation of understandings of the relationships between the various types of blocks, and auditory inputs occurred as students talked to one another during activities. And finally, within the group receiving expository teaching, there was auditory input in the language of instruction and discussion, but there were significant visual and kinesthetic inputs involved in the reading and writing of symbols on the chalkboard and in student workbooks. Teachers may well be able to present mathematical ideas in particular ways, but they cannot control the manner in which students will process those ideas.

Kavale and Forness (1990) provide a further fundamental argument against modality teaching: "Of primary importance is the concept of modality itself. This elusive concept is not easily delineated" (p.357). Given an acceptance of perception and learning as a complex process, the categorisation of students as individuals having learning style preferences which are innately and firmly established becomes problematic. A simpler and more viable alternative is to classify the responses of learners in a number of contexts, or learning tasks in which particular behaviours are manifested, indicating certain learning style preferences. These learning style preferences will vary according to the particular context, or learning situation. It is therefore more productive to talk of a person solving a particular problem by visual strategies, than to typecast that person as a "visual learner."
Emphasis on individuals' responses to situations, as opposed to classifying individuals themselves, is a feature of the SOLO Taxonomy (Structure of Observed Learning Outcomes) of Collis and Biggs (1991), who provide a view of learning or problem solving as being multimodal, where the freedom of learners to move between the different representations of ideas (or different learning styles) in different learning contexts provides depth and richness to their thinking.

Acceptance of a multimodal view of learning precludes modality teaching, so that the intuitive notion of many educators of matching teaching styles to individual learning styles is a false one. At the least, teachers should be cautious about regarding a single teaching style as always most effective for certain student learning styles. As many practitioners already know, eclectic or differentiated approaches to instruction incorporating different teaching styles are likely to be more widely successful than those which emphasise a particular style, at the expense of others.

Further disadvantages of modality teaching are enumerated by Butler (1993). Modality teaching is limiting and ultimately negative in its focus, given that it narrows thinking by precluding learning in other styles, discourages experiments with new ways of learning, hampers the exploration of new fields of ability or interest, and blocks personal growth and development by confining each student to one allegedly correct approach to learning.

The limitations of modality teaching were recognised in the teaching of content items during the investigation, in the particular representation of
certain mathematical ideas. For example, although the teaching of the expanded form of four digit numbers was made more transparent by the use of concrete material than by the use of symbols alone, the use of the same material to solve problems involving missing three or four digit numbers was difficult to structure meaningfully and was vague and cumbersome for students. In this instance it was probably more meaningful for students to have worked with symbols alone.

The implications for classroom instruction using learning styles thus centre around a multisensory approach to the teaching of mathematics concepts and skills. Instruction can be given initially according to a student's preferred learning style, but it is critical that the instruction should be later reinforced through other learning style modalities, allowing the possibility of more complete cognitive growth and development.

Students need choices so that they can work in their own styles, but they also need to try new approaches and entertain new ideas. Teachers who never demand that students experience different styles may never see all the style range of particular students. Students need to experience many ways to know and think about themselves. But students will not venture into uncomfortable styles if teachers do not validate their natural style first. (Butler, 1993, p.125)

One way of organising and managing mathematics learning, according to a multisensory approach, would be for the teacher to structure problems and allow children to choose the manipulatives or media which would facilitate their solution, whether by means of concrete materials, computer images or pencil and paper. Alternatively, a more structured approach would involve allocating students according to their learning style to one of three separate stations, involving the
manipulation of concrete materials, computer images or pencil and paper, with rotation of students through each station on subsequent days, so that instruction within other learning styles is facilitated.

It is not merely sufficient, however, to cater for different learning styles of students by various instructional methods. Crucially, teachers must focus on the metacognitive aspects of learning style in their praxis, as indicated in the previous quotation by Butler (1993). Students must be encouraged to actively reflect on mathematics ideas in different ways, to switch to different modes of thinking or ways of receiving information when necessary, and to constantly monitor their learning style preferences in different contexts. As detailed in Section 5.2.2, reflective ways of thinking about mathematical ideas can be fostered in a social context, by students informally and formally discussing with the teacher and each other the different ways in which they learn as they undertake mathematical investigations and problem solving.

5.4 The use of instruments to assess learning style
The NASSP Learning Style Profile (LSP) used in this investigation (Keefe et al., 1989) was chosen on the basis that it was simple to administer and was purportedly valid, reliable and internally consistent (Keefe & Monk, 1988). It would appear, however, that this instrument contains a number of recognisable weaknesses.

Due to the extremely poor correlation which was found to exist between the learning style preferences expressed by students in writing and the preferences revealed by the use of the learning style instrument, the reliability and validity of the instrument may be questionable. Bonham (1988) recommends overcoming such difficulties by disclosing the
results of testing and allowing students to change results or placements that seem correct to them, but this course of action was not a part of this investigation.

The accuracy of the instrument also depends on the subjects' understanding of what they are required to do in the test and being prepared to reveal the knowledge required. Although protocols were issued to teachers giving clear instructions to practise test procedure by games and discussion before giving the test, some students may have remained unclear about what was actually required of them, or were unwilling to participate when testing occurred.

There is some doubt, also, regarding the kinesthetic (emotive) scale of the learning style instrument. Keefe (1989) explains that emotive response to information is either kinesthetic or emotional. Kinesthetic learners require a whole body response by taking notes, drawing diagrams or doodling while information is being presented, whereas emotive learners react in terms of their feelings, usually towards the presenter and not the information provided. It is questionable whether a reaction to a test item in terms of creating a feeling about a word dictated is equivalent to a preference for learning by hands-on experience. This is another potential source of weakness for the investigation. Further investigation of the relationship between kinesthetic and emotional responses is warranted.

In all, using the benefit of hindsight, it could be argued that allocation to certain instructional groups as a result of scores attained by a one-shot administration of the test was an unsound educational procedure and that future efforts to identify learning style of students should centre around
the combined use of observational methods, interviews and testing. If testing is regarded as important in the determination of learning style, such tests should be comprehensive enough in future to include both perceptual preference scales and a means of evaluating cognitive skills in terms of learning style.

5.4.1 Post hoc analysis
In view of the apparent weaknesses associated with the use of the learning style instrument, it was determined that a post hoc analysis of the reliability of the Perceptual Response subscales of the LSP used in the investigation should be conducted. Data for 294 cases was processed, with 130 cases involving missing values being rejected. The Cronbach alpha reliability coefficient which was calculated was found to be low (alpha = .5225, standardised item alpha = .5232). These reliability coefficients confirm the existence of inconsistencies in the measurement of perceptual response in the investigation, and therefore further limit the findings in relation to learning style. It is indicated that more suitable and complete instruments to assess learning style should be utilised in future studies.

5.5 Recommendations arising from the investigation
The prime focus of this investigation was to determine whether the computer can be used as a transition device to integrate concrete, pictorial and symbolic representations of mathematics ideas, and whether teaching programs which are constructed in accordance with the preferred learning style of students lead to increased achievement by students. Though the investigation is small scale and the data limited, these hypotheses are rejected according to the results which were found.
The conclusions that have been drawn previously in accord with the findings of the investigation have important implications for classroom teaching and future research.

As far as classroom teaching is concerned it is recommended that:

- the software tool *Blocks Microworld* be used as a supplementary representation of numeration and operations ideas, rather than a necessary link between concrete and symbolic modes of representation;
- teachers do not impose the use of particular manipulatives on students or restrict their style of instruction to any one representation;
- teaching methods and instructional materials should not be classified as either primarily auditory, visual or kinesthetic in nature;
- learning style be regarded as context specific and not fixed within the individual.

If modality teaching is to be used in future, it is also recommended that:

- learning style preferences be established by a combination of interview, observation and testing;
- where practical, initial instruction be given according to the strongest learning style preference and supplementary instruction be provided according to other learning styles;
- students be allowed to change placements within learning style groupings in the classroom;
- reliability studies of learning style instruments be carried out before they are adopted, even if those instruments have been previously standardised.
In terms of future educational research, further investigation is indicated into the role of the computer, different types of instruction, and the use of learning styles in the representation of mathematical ideas by students. These efforts in research could well be directed towards:

- study of the viability of computers in allowing the representation and manipulation of mathematical ideas, in terms of determining the suitability of particular software packages and ideal combinations of on-screen and off-screen activities using manipulatives;
- the reliable and valid identification of learning style using a combination of observations, interviews and testing;
- an examination of the effects of instruction on learning style in relation to learners solving particular problems in terms of responses, rather than being categorised as certain types of individuals;
- an examination of the effects of learning style, in relation to the achievement of learners in solving particular problems in mathematics, with particular attention being given to gender and grade differences;
- a large-scale study in which learning style is identified using a combination of observations, interviews and testing; instruction is given according to the strongest learning style preference; supplementary instruction is provided according to other learning styles and the achievement outcomes are evaluated.

In conclusion, if the attempt to describe the role of technology in mathematics education is the challenge of describing the evolution of a volcano (Kaput, 1992), then perhaps the continuing history of research into the nature and role of teaching and learning styles is akin to
wandering through a minefield. Increased efforts in research are warranted as a result of the findings and recommendations of this investigation, so that the development of the volcano can be recorded and understood, and the minefield can be successfully negotiated.
REFERENCES


**Sources of cited software**

Broderbund Software
P.O. Box 12947
San Rafael, California, 94913-2947

Intellimation Library for the Macintosh
P.O. Box 1530
Santa Barbara, California, 93116-1530

LEGO Dacta
555 Taylor Road
Enfield, Connecticut, 06082

Sunburst Communications
101 Castleton Street,
Pleasantville, New York, 10570

Ventura Educational Systems
3440 Broken Hill Street,
Newbury Park, California, 91320
APPENDIX

1.1 Learning style quiz based on perceptual response subscale of the NASSP Learning Style Profile (Keefe et al., 1989)

1.2 Decimal numeration test used in pilot study

1.3 Lesson content of pilot study

1.4 Modified form of MI Mathematics test used in major investigation

1.5 Sample worksheet used in major investigation
1.1 LEARNING STYLE PREFERENCE QUIZ

INSTRUCTIONS TO TEACHERS:

Words to be read to students at 5 second intervals

Say the following to the students before reading the words:

Some words will be read to you. After you first hear the word draw a ring around the word PICTURE if you see a PICTURE, draw a ring around the word SOUND if you hear a SOUND, or draw a ring around the word FEELING if the word gives you a FEELING.

1. SUMMER
2. CHICKEN
3. LIAR
4. BEAUTIFUL
5. FIVE
6. READ
7. BABY
8. ENEMY
9. STORY
10. OCEAN
11. DOWN
12. RUNNING
13. LAW
14. FRIEND
15 SWIM
16. POOL
17. GOD
18. KILL
19. HOUSE
20. HAPPY
LEARNING STYLE PREFERENCE ANSWER SHEET

NAME: ----------------------------------
CLASS: ___
DATE OF BIRTH: ----------------------
GENDER: MALE/FEMALE

INSTRUCTIONS TO STUDENTS:

Some words will be read to you. After you first hear the word draw a ring around the word PICTURE if you see a PICTURE, draw a ring around the word SOUND if you hear a SOUND, or draw a ring around the word FEELING if the word gives you a FEELING.

<table>
<thead>
<tr>
<th></th>
<th>PICTURE</th>
<th>SOUND</th>
<th>FEELING</th>
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</tbody>
</table>
1.2  DECIMALS TEST 1

Name: ___________________________ Class: __________

Date: _________  Age: _____ years _____ months

1. What does 7.6 mean?
   Ring the best answer from the following:
   a) seventy-six
   b) seven remainder six
   c) seven hundred and six
   d) seven and one six
   e) seven and six tenths

2. This clock shows the digital time 8:59 or one minute to nine.
   a) Is this a decimal number? _______________________
   b) Explain how you can tell _______________________

3. Write in words how you would say these decimal numbers:
   a) 0.7 __________________________
   b) 0.48 __________________________

4. Write the decimals for the following numbers:
   a) seven tenths _________
   b) thirty-four hundredths _______
   c) two and sixty-seven hundredths _______

5. Ring the BIGGEST of the three numbers:
   a) 0.7 or 0.48 or 0.57
   b) How can you tell it is the biggest?
6. Read the following scales and write your answers in the boxes provided. Give all your answers as decimals.

a) [Diagram of scale with point at 2.5]

b) [Diagram of scale with point at 2.4]

c) [Diagram of scale with point at 7.9]

d) [Diagram of scale with point at 8.9]

e) [Diagram of scale with point at 2.8]

f) [Diagram of scale with point at 2.9]

g) [Diagram of scale with point at 3.5]

h) [Diagram of scale with point at 5.6]

i) [Diagram of scale with point at 8.7]
7. This square stands for one square unit.

Look at the areas shaded and write your answers as DECIMALS.

a) The area shaded is \( \boxed{\text{square units}} \) square units.

b) The area shaded is \( \boxed{\text{square units}} \) square units.

c) The area shaded is \( \boxed{\text{square units}} \) square units.

8. Shade the squares to show the following decimal numbers:
   a) seven tenths

   b) twenty-four hundredths
9. Ring the BIGGEST number in each group:
   a) 5.436 or 5.47 or 56
   b) 6.78 or 45.6 or 345
   c) 3.52 or 3.6 or 3.25
   d) 4.07 or 4.17 or 4.7
   e) 15.29 or 15.90 or 15.92

10. Ring the BIGGEST number in each pair.
    Write SAME if you think they are the same size.
    a) 76 or 760 ________
    b) 76 or 0.76 ________
    c) 0.5 or 0.50 ________
    d) 0.7 or 0.07 ________
    e) 0.65 or .65 ________

11. Ring the number nearest in size to 0.16
    0.1 0.2 15 0.21 10
    Ring the number nearest in size to 2.08
    209 2.9 2.05 2.1 20.9

12. Write down the next two numbers in the sequence as decimals.
    a) 0.2, 0.3, 0.4, ____, ____ (adding on 0.1 each time)
    b) 0.1, 0.3, 0.5, ____, ____ (adding on 0.2 each time)
    c) 0.92, 0.94, 0.96, ____, ____ (adding on 0.02 each time)
    d) 1.13, 1.12, 1.11, ____, ____ (taking away 0.01 each time)

13. Write down the numerals for the following numbers in decimal form.
    a) five and one tenth ______
    b) six and seventeen hundredths ______
    c) one hundred and thirty-five and fifty-one hundredths ______
14. Write down the following decimal numbers in words.
   a) 5.6
   b) 23.82
   c) 16.09

15. $5 + 2 = 7$
    A story which goes with this sum is:
    John had 5 records. His father gave him 2 more for his birthday. So now he has 7 records altogether.
    Write your own story to go with this sum:
    $4.6 + 5.3 = 9.9$

16. Write answers as decimals for the following.
   a) $5 + 0.2$
   b) $0.3 + 1.4$
   c) $1.13 + 0.22$
   d) $2.7 + 1.16$

17. Write answers as decimals for the following.
   a) $1 - 0.3$
   b) $1.9 - 0.4$
   c) $1.68 - 0.32$
   d) $3.9 - 1.26$
18. Write answers as decimals for the following.
   a) $6.2 \times 10$ $$
   b) 2.3 \times 100 $$
   c) $27 \div 10$ $$
   d) 34 \div 100 $$
   e) $1.1 \times 5$ $$
   f) 3 \div 6 $$
   g) $6 \div 3 $$

19. Ring the number you think is nearest in size to the correct answer. ESTIMATE your answer - DO NOT WORK IT OUT.
   a) $15.65 + 4.22$ 1.99 or 19.9 or 199 or 1999
   b) $24.75 \div 3.62$ 2.111 or 21.1 or 211 or 211.1
   c) $16.2 \div 4.1$ 0.03 or 0.3 or 3 or 30 or 300
   d) $3.9 \times 5.2$ 0.02 or 0.2 or 2 or 20 or 200

20. Add the following.
   $$0.06 + 0.03 + 0.7 + 0.72 + 3.64 +$$
   $$0.03 0.07 0.3 0.45 8.41$$

21. Subtract the following.
   $$0.08 - 0.32 - 9.4 - 6.48 - 0.7 -$$
   $$0.03 0.05 8.7 4.46 0.41$$

22. Multiply the following.
   $$0.3 \times 1.2 \times 2.6 \times 2.12 \times 2.27 \times$$
   $$2 3 3 4 2$$
1.3 Scope and Sequence for lessons on Decimals

2 lessons per week (Tuesdays and Thursdays) over a five week period
Weeks 4-9
Term 2

<table>
<thead>
<tr>
<th>DAY</th>
<th>COMPUTER GROUP - Ed</th>
<th>BLOCKS GROUP - Clem</th>
<th>TRADITIONAL GROUP - Therese</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1. Introduction to software - modelling of whole numbers</td>
<td>1. Revision of Base 10 numeration concepts - place value charts 2. Building numbers after changing the value of the &quot;one&quot;</td>
<td>1. Revision of Base 10 numeration concepts - reading and writing numbers in terms of thousands, hundreds, tens and ones</td>
</tr>
<tr>
<td>2</td>
<td>1. Using the &quot;flat&quot; as one. Decimal numeration concepts - modelling hundredths</td>
<td>1. Using the &quot;flat&quot; as one. Decimal numeration concepts - modelling hundredths</td>
<td>1. Introducing the idea of tenths in written form. The right hand side of the number as the whole numbers and the numbers to the left as parts of the number. Link with money notation.</td>
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<td>3</td>
<td>1. Decimal numeration concepts - modelling hundredths and tenths</td>
<td>1. Decimal numeration concepts - modelling hundredths and tenths</td>
<td>1. Introducing the idea of hundredths in written form.</td>
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<td>1. Decimal numeration concepts - modelling hundredths and tenths</td>
<td>1. Decimal numeration concepts - modelling hundredths and tenths</td>
<td>1. Introducing the idea of hundredths in written form.</td>
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<td>5</td>
<td>Decimal numeration concepts - comparing and ordering hundredths and tenths Distinguishing significant and non-significant zeros</td>
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E. Lewis
ACU
4.5.93
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<th>Decimal computation concepts - comparing and ordering hundredths and tenths</th>
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<td>6</td>
<td>Distinguishing significant and non-significant zeros Number patterns of decimal numbers</td>
<td>Distinguishing significant and non-significant zeros Number patterns of decimal numbers</td>
<td>Distinguishing significant and non-significant zeros Number patterns of decimal numbers</td>
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<td>Decimal computation addition of tenths and hundredths in decimal form</td>
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<td>Decimal computation multiplication of tenths and hundredths in decimal form Multiplication by 10 and 100</td>
<td>Decimal computation multiplication of tenths and hundredths in decimal form Multiplication by 10 and 100</td>
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<td>Decimal computation - mixed examples</td>
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Counting, Numeration, and Place Value

Directions
For all questions on this test several answers are given, but only one of these answers is correct. You are to choose the answer you think is correct.

Below are some questions like those in the test. Wait until you are told how to answer the questions in this test.

Make no marks in this booklet. Thank you.

Practice Examples

P1  Tom had 25 toy cars. He lost 10 cars. How many cars did he have left?
   A  10  
   B  15  
   C  25  

P2  Which shape has been shaded?
   A  △  
   B  □  
   C  ◆  
   D  ◆  

P3  △ stands for a number.
    △ + △ = 10
    △ stands for
    A  2  
    B  5  
    C  10  
    D  20  

P4  How many large squares are drawn?
    A  3  
    B  6  
    C  more than 6  

Do not turn this page until you are told to do so.
1. Which one of these pictures shows that 13 means 1 ten and 3 ones?
   A
   * * * *
   * * * * * * *
   B
   * * * * * * * * *
   * * * * *
   C
   * * * * * * * * *
   * * * * * * * * *
   D
   * * * * * * * * *
   * * * * * * * * *

2. Which of these numbers means 7 tens?
   A 7
   B 700
   C 70
   D 107

3. Another name for 28 is
   A 8 tens and 2 ones.
   B 28 tens.
   C 2 tens and 2 ones.
   D 2 tens and 8 ones.

4. Which of the following is a numeral for sixty-nine?
   A 96
   B 69
   C 609
   D 6009

5. Which of the following shows the smallest number you can make with the digits 5, 9, 6?
   A 659
   B 965
   C 596
   D 569

6. How many 0s do you need to use with 5 to write the numeral for five hundred?
   A 2
   B 1
   C 5
   D 10

7. Thirty tens is another name for
   A 30
   B 3
   C 0
   D 300

8. This peg stands for 10
   This peg stands for 1
   The pegs at the right stand for
   A 4
   B 31
   C 13
   D 40

9. 90, 190, 290, 390, ____, 590, 690
   The missing number is
   A 400
   B 490
   C 580
   D 90

10. The picture shows
    |   |   |   |   |   |   |   |   |
    |   |   |   |   |   |   |   |   |
    A 3 9
    B 3 3
    C 2 9
    D 9 3

11. In 3,834 the 8 has a value of
    A eight thousands.
    B eight hundreds.
    C eight tens.
    D eight ones.

12. The number which is one less than 1,000 is
    A 901
    B 190
    C 9,999
    D 999

13. Which picture shows 100?
    A
    B
    C
    D

3
14. How many tens are there in 180?
A 80
B 10
C 8
D 18

15. Which one of these numerals has the digit 2 in the hundreds place?
A 1,273
B 2,419
C 612
D 428

16. Which set of numerals shows counting by fives?
A 537, 539, 541, 543, 545
B 525, 528, 531, 534, 537
C 511, 515, 519, 523, 527
D 527, 532, 537, 542, 547

17. The picture shows
A 5
B 32
C 50
D 23

18. Another name for one thousand and sixty-three is
A 1,063
B 10,063
C 163
D 100,063

19. Which abacus shows 104?
A
B
C
D

20. Which digit has the greatest place value in the numeral 1,460?
A 0
B 4
C 6
D 1

21. 876, 865, 854, ____, ____, 821
The missing numbers are
A 863, 874
B 855, 856
C 843, 832
D 865, 876

22. Fifteen hundred and fifty may be written as
A 150,050
B 1,550
C 15,050
D 1,505

23. One more than 44,499 is
A 444,991
B 44,500
C 144,499
D 44,498

24. Sally is standing on the twenty-ninth step from the bottom.

Which step A, B, C, or D is the thirty-first step?

GO ON TO THE NEXT PAGE
26 The picture shows

A 631 C 529
B 345 D 136

32 When two more noughts are put after the numeral 1,000 it shows a number
A ten times as large.
B a hundred times as large.
C a thousand times as large.
D the same size.

33 How many tens are there in 2,080?
A 2070
B 280
C 208
D 28

34 Another name for 6.73 is
A six hundred and seventy-three
B six and seventy-three hundredths
C six and seventy-three tenths
D six seventy-three

35 One more than XXIX is
A XXIXI
B XXX
C IXXIX
D XXXI

36 The number shown by point X is
A 3,942
B 4,000
C 4,018
D 3,980

37 Which row shows its numbers in order of size?
A 279, 927, 792, 972
B 972, 729, 792, 279
C 792, 927, 297, 297
D 972, 792, 729, 297

38 Which sentence is true?
A 1,468,239 is larger than 1,486,932
B 1,468,239 is the same size as 1,486,932
C 1,468,239 is smaller than 1,486,932

GO ON TO THE NEXT PAGE
39 Which of the following numbers is nearest in value to 605?
A 650
B 595
C 640
D 506

40 Which of the following shows the number which is 100 times as large as 90?
A 190
B 1,900
C 9,000
D 90,000

41 If 36 is the fourth number shown on this line, the eighth number would be
A 44
B 96
C 84
D 108

42 The number which is 100,000 less than 3,612,486 is
A 3,712,486
B 2,612,486
C 3,602,486
D 3,512,486

43 The number nearest in size to 0.29 is
A 0.1
B 0.2
C 0.3
D 0.4

44 To make 60 into six hundred thousand you need to multiply it by
A 10
B 100
C 1,000
D 10,000

45 The place value of the 4 in 5.45 is
A 4 tens
B 4 hundredths
C 4 tenths
D 4 thousandths

46 How many hundreds could you make out of forty tens?
A 4
B 0
C 40
D 400

47 Which numerical expression names the largest number?
A 60000 + 100 + 6
B 8000 + 300 + 60 + 5
C 60000 + 30 + 6
D 300 + 300 + 300 + 6

48 Which statement is true?
In the number 978,765,104 there are
A 9,787 millions.
B 9,787 ten thousands.
C 9,787 hundred thousands.
D 97 millions.

49 Which place on the number line would occupy?
A P
B Q
C R
D S

50 The expanded form of 23,306 is
A 2000 + 300 + 30 + 6
B 20 000 + 3 000 + 30 + 6
C 20 000 + 3 000 + 300 + 6
D 2 300 + 3 000 + 6

LOOK BACK OVER YOUR WORK
1.5
Computer lesson 2
Worksheet 1

Build the blocks on the screen and write numerals for each number.

1. 3 blocks 4 flats 3 longs 6 minis
2. 4 blocks 7 longs 5 longs 2 minis
3. 1 flat 5 longs 1 mini
4. 2 blocks 7 flats 5 minis
5. 6 blocks 4 longs
6. 9 blocks 6 flats 2 longs 7 minis
7. 4 flats 7 longs
8. 8 blocks 8 longs 6 minis
9. 3 blocks
10. 5 blocks 2 flats 8 longs
11. 4 blocks 4 flats and 1 mini
12. 1 blocks 3 longs and 2 minis
13. 3 blocks 4 flats and 1 mini
14. 7 blocks 2 flats 2 longs
15. 5 blocks
16. 9 blocks 7 flats 8 longs and 4 minis
17. 8 blocks 0 flats 0 longs and 0 minis
18. 2 blocks 0 flats 0 longs and 8 minis
19. 6 blocks 6 flats 8 longs and 2 minis
20. 1 long and 0 minis
MODES OF REPRESENTATION OF IDEAS, COMPUTERS AND LEARNING STYLES IN K-6 MATHEMATICS

A Master of Education (Honours) Thesis presented to the Faculty of Education The University of Western Sydney Nepean

by

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PLEASE NOTE

The greatest amount of care has been taken while scanning this thesis,

and the best possible result has been obtained.
ABSTRACT

Constructivist theories of mathematics learning suggest that learners personally negotiate meaning by creating different mental representations of mathematical knowledge. They may then progress towards relational understandings of mathematical ideas by making connections between the different modes of representation. Partnerships between concrete materials and computer software offer a way of enhancing the value of both approaches and have benefits in making learning more powerful by the integration of concrete, pictorial and symbolic modes of representation.

This thesis investigates the use of the computer as a transition device in linking different modes of representation of mathematical knowledge. A particular software package was chosen and three teaching treatments were devised, corresponding to three different modes of representation of knowledge. Computers were used to provide a pictorial treatment, Dienes' Base 10 Arithmetic Blocks were used to provide a treatment which was predominantly concrete, and teacher exposition (explanation and manipulation of mathematical symbols) provided a treatment which conformed to symbolic instruction.

The investigation also focusses on the viability of constructing teaching programs which are matched to specific student learning styles, according to the notion that the greatest gains in achievement should occur when students are exposed to teaching which is carefully matched to the style in which they prefer to receive information and undertake their learning. A learning style instrument was used to identify students with strong preferences for a particular learning style - either visual,
kinesthetic or auditory. Students from each learning style category were then allocated to one of the three teaching treatments (computer instruction, concrete instruction, or expository instruction). Instruction took place over 10 consecutive daily sessions of 40 minutes duration.

Quantitative analysis of the data collected found no significant statistical differences in achievement outcomes of any treatment groups. The findings of this investigation do not support the contentions that the computer is a necessary link between concrete and symbolic modes of representation of knowledge, or that programs in which teaching style is matched to the preferred learning style of students lead to increased attainment by students.
CERTIFICATE OF ORIGINALITY

I certify that the substance of this thesis has not already been submitted for any degree and is not currently being submitted for any other degree.

I certify that any help in preparing this thesis, and all sources used, have been acknowledged in this thesis.

Signature ____________________________
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