APPLICATION OF EPS GEOFOAM IN ATTENUATING GROUND VIBRATIONS DUE TO VIBRATORY PILE DRIVING

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BSc. Eng. (Hons.)

This thesis is submitted for the degree of

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This thesis is dedicated to;

My lovely wife, Nidharshi Warusavitharana,

and

My beloved parents, Seetha Igalagama and Shantha Ekanayake.
Declaration

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Degree: Ph.D.

I certify that the work presented in this thesis is, to the best of my knowledge and belief, original, except as acknowledged in the text, and the material has not been submitted, either in full or in part, for a degree at this or any other institution.

I certify that I have complied with the rules, requirements, procedures and policy relating to my higher degree research award of the University of Western Sydney.

----------------------------------------------

Author’s Signature
Acknowledgements

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Abstract

Modern day urban construction activities are largely carried out adjacent to existing buildings due to scarcity of land for construction. In order to utilise the available land in the most efficient way, often high-rise buildings are constructed necessitating pile foundations to transfer large design loads to strong and deep soil layers below the ground surface. Although a number of methods are available to install pile foundations, in urban areas several factors need to be taken into consideration when selecting the suitable method. Due to the proximity of new and existing structures, noise disturbance and damages to existing nearby structures resulting from pile installation should be kept to a minimum. In that respect, vibratory pile driving is the most suitable pile installation method for urban construction activities. However, ground vibrations induced by vibratory pile driving may cause damages to existing structures depending on the proximity and sensitivity of the structure. Hence, it is necessary to take proper mitigation measures against vibratory pile driving induced ground vibrations. A possible remedy is to use in-filled wave barriers with concrete, bentonite, water or expanded polystyrene (EPS) geofoam, which can diminish the construction induced vibrations. EPS geofoam offers a number of advantages over other fill materials because of its light weight, cost effectiveness, energy absorbing characteristics, efficiency in terms of construction time and ease of handling. There have been many research studies carried out to investigate the mechanical behaviour of EPS geofoam. However, the full potential of EPS geofoam is yet to be realised. Therefore this thesis aims to investigate the severity of ground vibrations induced by vibratory pile driving and effectiveness of EPS geofoam wave barriers in protecting nearby structures. These investigations are carried out using both two- and three-dimensional finite element models developed based on the Arbitrary-Lagrangian-Eulerian approach. They are discretised in both space and time to capture the wave propagation within ground.
First, a finite element formulation of a constitutive model developed to simulate mechanical behaviour of EPS geofoam based on an extended version of the Drucker Prager yield criterion is presented. The formulation is implemented in ABAQUS/Explicit finite element program and verified with data available from triaxial tests conducted on two varieties of EPS geofoam manufactured in Australia and Korea. Also results from the proposed formulation is compared with four other constitutive models found in the literature to confirm the suitability of this model in simulating experimentally observed EPS geofoam behaviour.

Wave propagation due to vibratory and resonant pile driving was investigated using a two-dimensional finite element model. First, the proposed model is verified using field data found in the literature and then it is used to simulate vibratory and resonant pile driving. Finite element mesh is truncated at the boundaries of the finite domain using wave transmitting boundaries for both shear and dilation waves. A parametric study was conducted by varying the amplitude and the frequency of the driving forces pertinent for commercially available driving rigs, soil rigidity index and soil material damping. The influence zones for different structures are derived by considering the Peak Particle Velocity (PPV) limits given in a number of design standards.

In the next part of the thesis, the efficiency of water and EPS geofoam in-filled wave barriers is investigated using a three-dimensional finite element model. The finite element formulation developed at the first stage of the research is used to simulate the mechanical behaviour of EPS geofoam. The proposed model is first verified using field data available for attenuation of ground vibrations in free field and at the presence of an EPS geofoam wave barrier. Then the model was used to conduct a parametric study varying the physical properties of the wave barrier: depth, length, width and location, as well as the frequency of the vibratory source. Outcomes of this investigation proved that EPS geofoam is the most efficient fill material in attenuating ground vibrations.

The effect of EPS geofoam wave barriers in attenuating ground vibrations during vibratory pile driving is then studied using a three-dimensional dynamic finite element model. An advanced finite element model is developed to facilitate deep penetration of
the pile from the ground surface avoiding mesh distortions. An EPS geofoam wave barrier is installed between the driven pile and the nearby existing pile. The efficiency of EPS geofoam wave barrier in protecting the existing pile was investigated by observing the maximum bending moment developed in the existing pile during vibratory pile driving. A parametric study was conducted by varying the geometric parameters of the EPS geofoam wave barrier and properties of the soil medium to evaluate the significance of these parameters in attenuating ground vibrations.

Finally, the research is concluded with a summary of major conclusions, important design criteria and recommendations for future research on the use of EPS geofoam wave barriers in attenuating ground vibrations.
# Table of Contents

Declaration  
Acknowledgements  
Abstract  
Table of Contents  
List of Figures  
List of Tables  
List of Symbols

1 Introduction  
1.1 Background  
1.2 Objective  
1.3 Structure of Thesis  
1.4 Publications

2 Literature Review  
2.1 Introduction  
2.2 Ground vibration propagation  
2.3 Ground vibration propagation during pile driving  
2.4 Mitigation measures against ground vibration propagation  
2.5 Characteristics of EPS geofoam  
2.5.1 Application of EPS geofoam  
2.5.2 Constitutive models for EPS geofoam  
2.5 Gaps in our understanding

3 Wave transmitting boundaries used in numerical modelling  
3.1 Introduction  
3.2 Non-local wave transmitting boundaries
# Table of Contents

3.3 Local wave transmitting boundaries .......................................................... 50
3.4 Numerical examples .................................................................................. 53
  3.4.1 Axisymmetric wave propagation ......................................................... 54
  3.4.2 Two-dimensional plane-strain wave propagation ................................. 58
3.5 Conclusion ................................................................................................. 60

4 Numerical simulation of EPS geofoam behaviour ........................................ 61
  4.1 Introduction .............................................................................................. 61
  4.2 Constitutive models for EPS geofoam ....................................................... 62
  4.3 Finite element formulation ....................................................................... 64
  4.4 Implementation of the constitutive model in ABAQUS ............................ 66
  4.5 Extracting material properties for the constitutive model ....................... 69
  4.6 Verification of finite element formulation .............................................. 72
    4.6.1 Triaxial tests by Wong and Leo (2006) .............................................. 72
    4.6.2 Triaxial tests by Chun et al. (2004) .................................................. 76
  4.7 Deformation of the EPS geofoam sample during loading .................... 77
  4.8 Comparison of Wong and Leo (2006) with other constitutive models .... 79
    4.8.1 Findley model .................................................................................... 79
    4.8.2 LCPC model ...................................................................................... 80
    4.8.3 Bilinear model .................................................................................. 81
    4.8.4 Hyperbolic model ............................................................................ 82
  4.9 Conclusion ............................................................................................... 84

5 Free-field ground vibration propagation ..................................................... 85
  5.1 Introduction .............................................................................................. 85
  5.2 Numerical model simulating ground vibration propagation .................. 86
  5.3 Verification of numerical model ............................................................. 91
  5.4 Numerical modelling of pile driving ....................................................... 93
    5.4.1 Vibratory pile driving ....................................................................... 94
    5.4.2 Resonant pile driving ....................................................................... 94
  5.5 Effect of soil rigidity index ..................................................................... 100
  5.6 Effect of material damping of soil ......................................................... 102
Table of Contents

5.7 Conclusion.......................................................................................................................... 110

6 Attenuation of ground vibrations using in-filled wave barriers 112
  6.1 Introduction...................................................................................................................... 112
  6.2 Numerical modelling of ground vibration propagation.................................................. 113
  6.3 Verification of the numerical model.................................................................................. 115
  6.4 Material models for in-filled wave barriers................................................................. 118
    6.4.1 EPS geofoam............................................................................................................ 119
    6.4.1 Water....................................................................................................................... 119
  6.5 Comparison of free-field ground vibrations with open and in-filled wave barriers........... 120
  6.6 Influence of in-filled wave barrier size on wave attenuation........................................ 121
    6.6.1 EPS geofoam filled wave barriers............................................................................ 122
    6.6.2 Water filled wave barriers..................................................................................... 127
  6.7 Conclusion...................................................................................................................... 134

7 Application of EPS geofoam in attenuating ground vibrations during vibratory pile driving 137
  7.1 Introduction...................................................................................................................... 137
  7.2 Numerical model for vibratory pile driving................................................................... 138
  7.3 Numerical simulation of vibratory pile driving............................................................. 140
  7.4 Performance of EPS geofoam wave barriers in attenuating ground vibrations............ 143
    7.4.1 Effect of geometry of the wave barrier in different soil types......................... 143
    7.4.2 Effect of barrier location....................................................................................... 148
    7.4.3 Effect of soil density............................................................................................... 149
    7.4.4 Effect of Poisson’s ratio....................................................................................... 150
    7.4.5 Effect of rigidity index........................................................................................... 151
  7.5 Conclusion...................................................................................................................... 152

8 Summary and conclusion 155
  8.1 Summary......................................................................................................................... 155
  8.2 Conclusion....................................................................................................................... 156
  8.3 Recommendation for future research............................................................................. 160
<table>
<thead>
<tr>
<th>References</th>
<th>R-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appendix</td>
<td>A-1</td>
</tr>
</tbody>
</table>
List of Figures

2.1 Vibration attenuation for different soil types (Dym, 1976)............................................ 12
2.2 Vibration attenuation with distance in wave lengths (Gutowski and Dym, 1976)................................................................. 13
2.3 Variation of peak particle velocity with the distance from the source (New, 1990)...................................................... 14
2.4 Possible wavefronts, generated from a driven pile (Attewell and Farmer, 1973)................................................................. 17
2.5 Seismic wave propagation in pile driving (Woods, 1997)................................. 18
2.6 Site information at bridge site in Pusan, South Korea (Kim and Lee, 2000).... 19
2.7 3D transducer time records for friction pile driving induced vibrations (Kim and Lee, 2000).............................................. 19
2.8 Stress strain curve for EPS geofoam with density of 21 kg/m3 under short-term unconfined axial compression loading (Horvath, 1994).............................. 29
2.9 Schematic configuration of the centrifuge tests, Murillo et al. (2009)............. 30
2.10 Attenuation effects of wave barriers, Murillo et al. (2009).............................. 31
2.11 Comparison of compressive strengths (Hazarika, 2006).................................. 36
2.12 Comparison of different constitutive models (Hazarika, 2006)...................... 38
2.13 Deviator stress vs. Axial strain (Wong and Leo, 2006)................................. 39
2.14 Volume change vs. Axial strain (Wong and Leo, 2006)................................. 40
3.1 Axisymmetric boundary condition proposed by Deeks and Randolph (1994) for; (a) Shear Wave and (b) Dilation Wave................................. 51
3.2 Local time domain transmitting boundary condition proposed by Du and Zhao (2010) for two-dimensional elastic wave propagation.......................... 52
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3</td>
<td>Schematic diagram of the axisymmetric finite element model</td>
</tr>
<tr>
<td>3.4</td>
<td>Amplitude vs. Time history of the dynamic load</td>
</tr>
<tr>
<td>3.5</td>
<td>Displacement vs. time history for shear wave propagation</td>
</tr>
<tr>
<td>3.6</td>
<td>Displacement vs. time history for dilation wave propagation</td>
</tr>
<tr>
<td>3.7</td>
<td>Schematic diagram of the plane-strain finite element model</td>
</tr>
<tr>
<td>3.8</td>
<td>Vertical displacement vs. time history for plane-strain wave propagation</td>
</tr>
<tr>
<td>4.1</td>
<td>Axisymmetric finite element mesh used to simulate EPS geofoam block with an aspect ratio of 1:1 (triaxial tests by Wong and Leo 2006)</td>
</tr>
<tr>
<td>4.2</td>
<td>Variation of deviatoric stress with axial strain for different confining pressures</td>
</tr>
<tr>
<td>4.3</td>
<td>Variation of model parameter K with confining pressure (Wong and Leo, 2006)</td>
</tr>
<tr>
<td>4.4</td>
<td>Variation of volume change with axial strain for different confining pressures</td>
</tr>
<tr>
<td>4.5</td>
<td>Comparison of finite element results with triaxial test data (Chun et al. 2004)</td>
</tr>
<tr>
<td>4.6</td>
<td>Laboratory observed volumetric contraction of EPS geofoam at different levels of axial compression</td>
</tr>
<tr>
<td>4.7</td>
<td>Deformed shape of samples from finite element formulation at axial strain 18, 30 and 50% under confining pressures 0, 20, 40 and 60 kPa for EPS geofoam samples (Frictional contact)</td>
</tr>
<tr>
<td>4.8</td>
<td>Comparison of finite element results based on the Wong and Leo (2006) model with previous constitutive models</td>
</tr>
<tr>
<td>5.1</td>
<td>Tip of the pile embedded in soil</td>
</tr>
<tr>
<td>5.2</td>
<td>Axisymmetric model for pile installation</td>
</tr>
<tr>
<td>5.3</td>
<td>Ground settlement at 0.5 m below the surface level after penetrating 4.6 m</td>
</tr>
<tr>
<td>5.4</td>
<td>Deformed shape of the mesh after penetrating 4.6 m</td>
</tr>
<tr>
<td>5.5</td>
<td>Dimensionless amplitude curve for vibrators operating at 40 Hz</td>
</tr>
</tbody>
</table>
List of Figures

5.6 Variation of particle velocity through the medium when the pile tip is 4 m below the ground surface, for vibratory system MRZV_12S operating at 28H. 97
5.7 PPV variation with depth for Vibrator MRZV_12S operating at 28 Hz............ 98
5.8 PPV variation for a vibratory driven pile..................................................... 99
5.9 PPV variations for a resonant vibrator driven pile........................................... 101
5.10 PPV variation for vibratory driving with varying soil stiffness.................... 102
5.11 PPV variation for resonant driving with varying soil stiffness..................... 104
5.12 PPV variation with material damping for vibratory driving ($G/s_u = 285$)...... 105
5.13 PPV variation with material damping for resonant driving ($G/s_u = 285$)...... 108
5.14 Upper and lower bounds of PPV variation for vibratory pile driving............. 109
6.1 Three-dimensional finite element model simulating ground vibration Propagation.............................................................................................................................. 115
6.2 Normalised vertical peak particle velocities with vibratory source operating at (a) 40 Hz and (b) 50 Hz.............................................................. 117
6.3 Normalised vertical peak particle velocities for vibration attenuation from EPS geofoam in-filled trench................................................................. 118
6.4 Normalised peak particle velocities for vibration attenuation from open and in-filled wave barriers............................................................................. 121
6.5 Variation of normalised peak particle velocities with depth of EPS geofoam wave barrier...................................................................................................... 122
6.6 Variation of normalised peak particle velocities with length of EPS geofoam wave barrier.......................................................................................... 124
6.7 Variation of normalised peak particle velocities with width of EPS geofoam wave barrier............................................................................................ 124
6.8 Variation of normalised peak particle velocities with location of EPS geofoam wave barrier.......................................................................................... 125
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.9</td>
<td>Variation of normalised peak particle velocities with different frequencies of vibratory source</td>
<td>126</td>
</tr>
<tr>
<td>6.10</td>
<td>Variation of normalised peak particle velocities with depth of water filled wave barrier</td>
<td>127</td>
</tr>
<tr>
<td>6.11</td>
<td>Variation of normalised peak particle velocities with length of water filled wave barrier</td>
<td>128</td>
</tr>
<tr>
<td>6.12</td>
<td>Variation of normalised peak particle velocities with width of water filled wave barrier</td>
<td>129</td>
</tr>
<tr>
<td>6.13</td>
<td>Variation of normalised peak particle velocities with location of water filled wave barrier</td>
<td>130</td>
</tr>
<tr>
<td>6.14</td>
<td>Variation of normalised peak particle velocities with different frequencies of vibratory source</td>
<td>131</td>
</tr>
<tr>
<td>6.15</td>
<td>Effect of depth of the wave barrier in attenuating ground vibrations</td>
<td>132</td>
</tr>
<tr>
<td>6.16</td>
<td>Effect of length of the wave barrier in attenuating ground vibrations</td>
<td>133</td>
</tr>
<tr>
<td>6.17</td>
<td>Effect of width of the wave barrier in attenuating ground vibrations</td>
<td>133</td>
</tr>
<tr>
<td>6.18</td>
<td>Effect of frequency of the vibratory source in attenuating ground vibrations</td>
<td>134</td>
</tr>
<tr>
<td>7.1</td>
<td>Conical tip of the pile embedded in soil</td>
<td>139</td>
</tr>
<tr>
<td>7.2</td>
<td>Plane section of the 3D finite element model through the plane of the driven pile and the existing pile</td>
<td>142</td>
</tr>
<tr>
<td>7.3</td>
<td>Three-dimensional finite element model used for vibratory pile driving</td>
<td>142</td>
</tr>
<tr>
<td>7.4</td>
<td>Variation of maximum bending moment developed along the existing pile in Ariake clay for varying (i) depth (ii) length and (iii) width of wave barrier</td>
<td>144</td>
</tr>
<tr>
<td>7.5</td>
<td>Variation of maximum bending moment developed along the existing pile in Bangkok clay for varying (i) depth (ii) length and (iii) width of wave barrier</td>
<td>145</td>
</tr>
<tr>
<td>7.6</td>
<td>Variation of maximum bending moment developed along the existing pile in Singapore clay for varying (i) depth (ii) length and (iii) width of wave barrier</td>
<td>146</td>
</tr>
</tbody>
</table>
List of Figures

7.7 Variation of maximum bending moment developed along the existing pile at varying barrier locations.......................................................... 149
7.8 Variation of maximum bending moment developed along the existing pile with varying soil density......................................................... 150
7.9 Variation of maximum bending moment developed along the existing pile with varying Poisson’s ratio....................................................... 151
7.9 Variation of maximum bending moment developed along the existing pile with varying rigidity index (RI)................................................. 152
List of Tables

2.1 Absorption coefficients (Dym, 1976)........................................................................... 11
2.2 Ground vibration attenuation, D, for different soil types (Dym, 1976)............. 12
2.3 Damage risk to ordinary dwelling houses with varying ground conditions
    (Langefors and Kihlstrom, 1978).................................................................................... 15
2.4 Case histories of vibration induced settlements (Drabkin et al., 1996)............ 16
2.5 Geometric damping coefficients for various sources (Kim and Lee, 2000)...... 20
2.6 AASHTO: Maximum acceptable vibration levels to prevent structural damage
    (Jones and Stokes, 2004).............................................................................................. 25
2.7 Swiss Standards (SN640312): Maximum acceptable vibration levels to prevent
    structural damage (Jackson et. al., 2007).................................................................... 25
2.8 Eurocode 3: Maximum acceptable vibration levels to prevent structural damage
    (EN 1993)....................................................................................................................... 25
3.1 Wave transmitting boundary coefficients for Deeks and Randolph (1994)
    model............................................................................................................................... 51
3.2 Wave transmitting boundary coefficients for Du and Zhao (2010) model........ 52
3.3 The values for coefficients kl, cl and ml for shear (N = 4) and dilation (N = 3)
    wave transmitting boundaries......................................................................................... 53
4.1 EPS geofoam properties extracted for finite element modelling of triaxial tests
    by Wong and Leo (2006)............................................................................................... 72
4.2 EPS geofoam properties extracted for finite element modelling of triaxial tests
    by Chun et al. (2004)..................................................................................................... 76
7.1 Soil properties (Tanaka et al., 2001)............................................................................ 141
# List of Symbols

**Roman**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D</td>
<td>Three-dimensional</td>
</tr>
<tr>
<td>$A_0$</td>
<td>Wave amplitude at a distance $r_0$ from the source, without isolation system</td>
</tr>
<tr>
<td>$A_i$</td>
<td>Wave amplitude with isolation system</td>
</tr>
<tr>
<td>$A(r)$</td>
<td>Decayed amplitude of the wave</td>
</tr>
<tr>
<td>$A_{RR}$</td>
<td>Amplitude reduction ratio</td>
</tr>
<tr>
<td>$a$</td>
<td>Parameter depends on plastic deviatoric strain</td>
</tr>
<tr>
<td>$b_1$</td>
<td>Linear bulk viscosity damping coefficient</td>
</tr>
<tr>
<td>$b_2$</td>
<td>Quadratic bulk viscosity damping coefficient</td>
</tr>
<tr>
<td>$C_e$</td>
<td>Elastic stiffness matrix</td>
</tr>
<tr>
<td>$C_j$</td>
<td>Parameter signifying the phase speeds in a given direction</td>
</tr>
<tr>
<td>$c$</td>
<td>Velocity of the propagating wave</td>
</tr>
<tr>
<td>$c_d$</td>
<td>Dilation wave velocity</td>
</tr>
<tr>
<td>$c_l$</td>
<td>Dimensionless dashpot coefficient</td>
</tr>
<tr>
<td>$c_s$</td>
<td>Shear wave velocity</td>
</tr>
<tr>
<td>$D$</td>
<td>Dimensionless barrier depth (Chapter 2)</td>
</tr>
<tr>
<td></td>
<td>Diameter of the pile (Chapter 5)</td>
</tr>
<tr>
<td>$D'$</td>
<td>Elasticity matrix</td>
</tr>
<tr>
<td>$d$</td>
<td>density of EPS geofoam (Chapter 2)</td>
</tr>
</tbody>
</table>
List of Symbols

\( d_s \) Average thickness of the specimen

\( E \) Young’s modulus

\( E_t \) Tangential modulus

\( E_i \) Initial tangential modulus

\( e^p \) Deviatoric component of plastic strain

\( f \) Frequency

Yield function (Chapter 2)

\( f_{\text{max}} \) Maximum frequency

\( G \) Shear modulus

\( g \) Plastic potential function

\( H \) Thickness of the sand layer (Chapter 2)

Hardening parameter (Chapter 4)

\( h \) Normalised hardening modulus

\( I_t \) Internal force vector

\( J_c(t) \) Creep compliance parameter

\( K \) Hardening function of the material (Chapter 2)

Parameter defining the flow rule and dilatancy behaviour of EPS geofoam

(Chapter 4)

\( K_0 \) Bulk modulus

\( k_0 \) Dimensionless spring coefficient

\( L_c \) Characteristic length of an element

\( L_{\text{max}} \) Maximum length of an element
**List of Symbols**

- $m$ Dimensionless material parameter
- $m_{0_r}$ Dimensionless Findley material parameter
- $m_F$ Dimensionless Findley material parameter
- $m_l$ Dimensionless mass coefficient
- $N$ Order of boundary condition
- $n$ Geometric damping coefficient (Chapter 2)
- $N$ Number of boundaries (Chapter 3, Section 3.2)
- $n_F$ Dimensionless Findley material parameter
- $P_t$ Applied load vector
- $p$ Mean stress
- $p_H$ Hugoniot pressure
- $p_{bv1}$ Linear bulk viscosity pressure
- $p_{bv2}$ Quadratic bulk viscosity pressure
- $q$ Deviatoric stress
- $R$ Dimensionless source-barrier distance (Chapter 2, Section 2.5.1)
  Unconfined compressive strength (Chapter 2, Section 2.5.2)
  Hardening variable (Chapter 4, Section 4.2)
- $r$ Distance from the source
- $r_b$ Radial distance to the boundary
- $S_0$ Static stiffness
- $s$ Deviatoric stress tensor
- $s_u$ Undrained shear strength
List of Symbols

\( t \) \hspace{1cm} \text{Time}

\( t_0 \) \hspace{1cm} \text{One hour}

\( t_n \) \hspace{1cm} \( n \)th time step

\( U_p \) \hspace{1cm} \text{Particle wave velocity}

\( U_s \) \hspace{1cm} \text{Shock wave velocity}

\( \dot{u} \) \hspace{1cm} \text{Tangential velocity}

\( u^N \) \hspace{1cm} \( N \)th displacement or rotation degree of freedom

\( V \) \hspace{1cm} \text{Velocity}

\( V_D \) \hspace{1cm} \text{Dilation wave velocity}

\( V_R \) \hspace{1cm} \text{Rayleigh wave velocity}

\( V_S \) \hspace{1cm} \text{Shear wave velocity}

\( W \) \hspace{1cm} \text{Dimensionless barrier width}

\( w_n \) \hspace{1cm} \text{Vibration amplitude at a distance of } r_n \text{ from the source}

\( \dot{w} \) \hspace{1cm} \text{Normal velocity}

\( X \) \hspace{1cm} \text{Distance to the boundary in } x \text{ direction}

\( X_c(t) \) \hspace{1cm} \text{Average decrease in thickness of specimens over time } t

\( y \) \hspace{1cm} \text{Settlement of the specimen}

**Greek**

\( \alpha \) \hspace{1cm} \text{Absorption coefficient/material damping coefficient}

\( \alpha_R \) \hspace{1cm} \text{Mass proportional damping factor}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Dilatancy of the material</td>
</tr>
<tr>
<td></td>
<td>Parameter defining rate of hardening</td>
</tr>
<tr>
<td>( \beta_R )</td>
<td>Stiffness proportional damping factor</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>Settlement of the sand layer</td>
</tr>
<tr>
<td>( \Delta t )</td>
<td>Time increment</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Strain</td>
</tr>
<tr>
<td>( \varepsilon_1 )</td>
<td>Axial strain</td>
</tr>
<tr>
<td>( \varepsilon_3 )</td>
<td>Confining strain</td>
</tr>
<tr>
<td>( \varepsilon_{0i} )</td>
<td>Dimensionless Findley material parameter</td>
</tr>
<tr>
<td>( \varepsilon_c )</td>
<td>Time dependent component of strain after time ( t )</td>
</tr>
<tr>
<td>( \varepsilon_i )</td>
<td>Immediate strain</td>
</tr>
<tr>
<td>( \varepsilon_{iF} )</td>
<td>Dimensionless Findley material parameter</td>
</tr>
<tr>
<td>( \varepsilon_q )</td>
<td>Deviatoric strain</td>
</tr>
<tr>
<td>( \varepsilon_v )</td>
<td>Volumetric strain</td>
</tr>
<tr>
<td>( \varepsilon^p )</td>
<td>Plastic strain</td>
</tr>
<tr>
<td>( \varepsilon_{qp} )</td>
<td>Deviatoric plastic strain tensor</td>
</tr>
<tr>
<td>( \varepsilon_v^p )</td>
<td>Plastic volumetric strain increment</td>
</tr>
<tr>
<td>( \varepsilon_x^p )</td>
<td>Plastic normal strain in ( x ) direction</td>
</tr>
<tr>
<td>( \varepsilon_y^p )</td>
<td>Plastic normal strain in ( y ) direction</td>
</tr>
<tr>
<td>( \varepsilon_z^p )</td>
<td>Plastic normal strain in ( z ) direction</td>
</tr>
</tbody>
</table>
List of Symbols

$\varepsilon_{xy}^p$ Plastic shear strain in $x$-$y$ plane

$\varepsilon_{yz}^p$ Plastic shear strain in $y$-$z$ plane

$\varepsilon_{zx}^p$ Plastic shear strain in $z$-$x$ plane

$\bar{\varepsilon}^p$ Volumetric component of the plastic strain

$\dot{\varepsilon}_{vol}$ Volumetric strain rate

$\Gamma_0$ Grüneisen ratio

$\varphi$ Internal friction angle

$\eta$ Loss factor (Chapter 2)

Nominal volumetric compressive strain (Chapter 6)

$\lambda$ Wave length

Lamé’s first constant (Chapter 3)

Plastic scaling factor (Chapter 4)

$\lambda_{\text{min}}$ Minimum wave length

$\lambda_R$ Rayleigh wave length

$\lambda_{R_{\text{min}}}$ Minimum Rayleigh wave length

$\nu$ Poisson’s ratio

$\rho$ Density

$\rho_0$ Initial density

$\sigma$ Normal stress

$\sigma_1$ Axial stress

$\sigma_{1y}$ Value of $\sigma_1$ when the material starts to yield
### List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_3$</td>
<td>Confining stress</td>
</tr>
<tr>
<td>$\sigma_d$</td>
<td>Damping stress</td>
</tr>
<tr>
<td>$\sigma_{sy}$</td>
<td>Findley material parameter</td>
</tr>
<tr>
<td>$\sigma_{m_r}$</td>
<td>Findley material parameter</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>Stress corresponding to the onset of yielding</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>Normal stress in $x$ direction</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>Normal stress in $y$ direction</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>Normal stress in $z$ direction</td>
</tr>
<tr>
<td>$\sigma_n^{initial}$</td>
<td>Initial stress at $n^{th}$ step</td>
</tr>
<tr>
<td>$\sigma_n^{final}$</td>
<td>Final stress at $n^{th}$ step</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Shear stress</td>
</tr>
<tr>
<td>$\tau_{xy}$</td>
<td>Shear stress in $x$-$y$ plane</td>
</tr>
<tr>
<td>$\tau_{yz}$</td>
<td>Shear stress in $y$-$z$ plane</td>
</tr>
<tr>
<td>$\tau_{zx}$</td>
<td>Shear stress in $z$-$x$ plane</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>First natural mode</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Fraction of critical damping</td>
</tr>
</tbody>
</table>

### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALE</td>
<td>Arbitrary Lagrangian Eulerian</td>
</tr>
<tr>
<td>AASHTO</td>
<td>American Association of State Highways and Transportation Officials</td>
</tr>
<tr>
<td>EPS</td>
<td>Expanded Polystyrene</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>EXP</td>
<td>Experimental</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite element model</td>
</tr>
<tr>
<td>$PPV$</td>
<td>Peak Particle Velocity</td>
</tr>
<tr>
<td>$PPV^*$</td>
<td>Normalised peak particle velocity</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Background

Rapid population growth and industrial development over the last few decades has drastically increased the demand for infrastructure in urban areas, driving the modern day construction industry to utilise the land available adjacent to existing buildings. Consequently, major cities are becoming more compact with construction activities being carried out in close vicinity to existing buildings. Although it has made the utilisation of available land optimum, it has not come without a cost.

Construction induced ground vibrations from adjacent sites have become a major threat to the structural health of nearby existing substructures and subsequently buildings supported by them, as many are not planned to withstand such conditions. A six-storey office building in Shanghai was collapsed in 2003 during construction of a metro line 20 m underground. This incident occurred due to ground vibrations and deformations due to the underground metro line construction. It caused a severe damage to the community, which resulted in evacuation of people and disruption to the services in the vicinity.

Rectification of damaged foundations and buildings, and the resulting delays to the construction activities are highly expensive and extremely inconvenient. Hence introduction of proper engineering methods to eliminate, or at least to reduce, the effects
of construction induced ground vibrations, has become a priority among the geotechnical engineers dealing with urban construction. This will benefit the construction industry to move forward with minimum impact to the rest of the society. Although different construction activities generate vibrations of different nature and intensities, in this thesis attention is focused on vibratory pile driving. Vibratory pile driving is used in urban construction activities due to less noise and ground disturbances caused during installation, compared to other pile installation methods.

Protective trenches or wave barriers are often used to mitigate the construction induced stress wave propagation in ground and open protective trenches are the most effective wave isolators. However, open trenches are vulnerable to localised collapse and they can lead to numerous maintenance issues while construction continues. A possible remedy to the aforementioned construction hardship of protective trenches is to use a suitable fill material in the wave barrier, which can support the trench as well as to diminish the construction induced vibrations. EPS geofoam has a special place among traditional fill materials used for wave barriers in modern day construction industry due to its light weight, high strength to weight ratio, volume contraction under deviatoric compressive loading, strain hardening beyond yielding and ease of use. In addition, EPS geofoam has been widely used in embankment construction, slope stabilization, retaining wall structures and bridge approaches. However, the potential of EPS geofoam as an engineering material in geotechnical engineering applications has not been fully realised.

The main focus of this research is to investigate the applicability of Expanded Polystyrene (EPS) geofoam as a fill material in wave barriers with an especial emphasis on vibratory pile driving as mentioned previously. A major barrier in utilising EPS geofoam as a wave barrier is due to lack of proper understanding of the most efficient geometry of the barrier and influence zone around the vibratory source. This research will investigate those issues and provide solutions and guidelines for practicing geotechnical engineers. These outcomes are of immense value to the construction industry in terms of reducing construction delays and litigation issues arising from damages to nearby existing buildings during construction activities.
1.2 Objective

In the past few decades many research studies have been carried out to understand the mechanical behaviour of EPS geofoam and its utilisation as a wave barrier. However, there are no proper guidelines available for practising geotechnical engineers to use EPS geofoam as wave barriers. Hence, the objective of this research is to develop a set of design charts to use EPS geofoam as a wave barrier against the ground vibrations with a special emphasis on vibratory pile driving. The main objectives of this research are therefore to:

1. Conduct a comprehensive literature review on wave propagation due to vibratory pile driving, mechanical behaviour of EPS geofoam and use of in-filled wave barriers in mitigating ground vibrations in general.
2. Investigate the wave transmitting boundaries available in the literature to efficiently truncate the finite domain used for the analysis.
3. Develop a finite element formulation for a constitutive model that can predict the mechanical behaviour of EPS geofoam and verify the model against experimental data available in the literature.
4. Develop a two-dimensional axisymmetric dynamic finite element model to simulate deep penetration of a pile from the ground surface avoiding large mesh distortions inherent for deep penetration problems.
5. Investigate the free-field ground vibration propagation induced by vibratory pile driving and determine the influence zones based on vibration sensitivity of different types of structures specified in design specifications.
6. Develop a three-dimensional finite element model to compare the effectiveness of different fill materials used in wave barriers and conduct a parametric study to determine the effect of geometric properties of wave barrier in attenuating ground vibrations.
7. Develop a three-dimensional finite element model to simulate vibratory pile driving from the ground surface, avoiding large mesh distortions inherent for deep penetration problems. Using this model, investigate the applicability of EPS geofoam...
geofoam as a wave barrier to protect existing foundations from vibratory pile driving induced ground vibrations.

8. Produce a set of design charts for practicing geotechnical engineers to use EPS geofoam as a wave barrier against vibrations induced from vibratory pile driving and other vibratory sources.

1.3 Structure of the Thesis

- Chapter 2
  This chapter consists of a comprehensive review of literature available for construction induced ground vibration propagation. The review focuses mainly on vibratory pile driving induced ground vibrations. The mitigation methods available to reduce the effect of ground vibration propagation on existing structures are then discussed. An elaborate discussion about the experimental studies on EPS geofoam: full scale field experiments and laboratory experiments as well as the numerical studies carried out by various researchers to understand the mechanical behaviour of EPS geofoam are presented. Gaps in our understanding of the behaviour of EPS geofoam and necessity of this study are discussed at the end of the Chapter 2.

- Chapter 3
  A comprehensive review of wave transmitting boundary conditions found in the literature is presented in Chapter 3. Wave transmitting boundaries are used to truncate an infinite domain when analysing soil-structure interaction problems involving wave propagation using dynamic finite element analysis procedures. Numerous approaches, which have been introduced to simulate non-reflecting boundary conditions in the past few decades, are discussed in this chapter. Performance of several wave-transmitting boundaries is then investigated by implementing them in the finite element program ABAQUS/Explicit.

- Chapter 4
  Development of a finite element formulation of a constitutive model based on the hardening plasticity to predict the mechanical behaviour of EPS geofoam is
presented in Chapter 4. This constitutive model is an extension of the Drucker Prager yield criterion. The finite element formulation is based on the explicit integration scheme and is implemented in ABAQUS/Explicit. The finite element formulation is verified using triaxial test data found in the literature (Wong and Leo 2006 and Chun et al. 2004) for two varieties of EPS geofoam. Performance of the constitutive model is then compared with four other constitutive models found in the literature for EPS geofoam.

- Chapter 5
This chapter presents a numerical study for vibratory driving of closed-ended piles. Two-dimensional (2D) free field vibration propagation was simulated using a set of axisymmetric finite element models to replicate the wave propagation during vibratory pile driving and to investigate the effect of wave propagation on the surrounding ground. A parametric study was performed to determine the influence of change in pile driving force on the far field using different operating frequencies and amplitudes of the driving force, and rigidity index and material damping of the surrounding soil. The impact on the far field is discussed comparing the peak particle velocity distributions with the specifications given by the American Association of State Highways and Transportation Officials (AASHTO), Swiss Standard SN640312 and Eurocode 3. The upper and lower bounds for the peak particle velocity distributions around a vibratory driven closed-ended pile are presented to determine the influence zones for different types of nearby structures.

- Chapter 6
The effect of EPS geofoam and water filled wave barriers on ground vibration attenuation is presented in this chapter. A three-dimensional finite element model is developed to study the efficiency of different fill materials in attenuating ground vibrations in general. The model is first verified using data from full-scale field experiments, where EPS geofoam has been used as a fill material in wave barriers. Then the same model has been used to evaluate the efficiency of open
trenches, water filled wave barriers and EPS geofoam filled wave barriers on attenuation of ground vibrations. A parametric study is carried out to investigate the effect of geometric properties of the wave barriers on attenuating the ground vibration propagation. Design charts are developed to predict the degree of vibration attenuation depending on the distance from the vibration source, fill material used in the wave barrier and geometry of the wave barrier.

- **Chapter 7**
  Efficiency of EPS geofoam in-filled wave barriers on ground vibration attenuation during vibratory pile driving in different soil types is presented in this chapter. A three-dimensional finite element model developed based on the Arbitrary-Lagrangian-Eulerian approach has been adopted to simulate the deep penetration of the pile from the ground surface. The mechanical behaviour of EPS geofoam is simulated using a constitutive model developed based on the hardening plasticity, which is an extension of the Drucker Prager yield criteria described in Chapter 4. A parametric study is carried out to investigate the effect of geometric properties of the wave barrier on attenuating the ground vibration propagation, which is quantified by evaluating the reduction in bending moments generated on an existing nearby pile.

- **Chapter 8**
  This chapter presents the conclusions derived from the present work and recommendations for future studies that can be carried out to expand the utilisation of EPS geofoam as a wave barrier in attenuating vibratory pile driving induced ground vibrations.

### 1.4 Publications

The following collection of papers, which have either been published or submitted to peer-reviewed journals or conferences, describes outcomes of this thesis.
Journal Publications


Conference Publications

Australia New Zealand Conference on Geomechanics, Melbourne, Australia, pp. 1135-1140. **Finalist of the best paper competition.**


Chapter 2

Literature Review

2.1 Introduction

Ground vibrations generated from construction activities can cause severe damages to surrounding structures unless proper mitigation measures are applied. Open trenches can be considered as the most effective way of minimising the ground vibration propagation (Celebi et al., 2009). However, practical applications are limited to shallow depths due to localised collapse of unsupported trench walls while construction continues. A possible remedy to the aforementioned adversity in using open trenches is to use a suitable fill material in the trench such as concrete, bentonite or EPS geofoam. EPS geofoam offers a number of advantages over other fill materials because of its light weight, cost effectiveness, energy absorbing characteristics, efficiency in terms of construction time and ease of handling. Overall, these fill materials will assist to improve the stability of trench walls but most importantly, they should be efficient in diminishing the propagation of construction induced vibrations.

This chapter presents a comprehensive literature review for ground vibrations induced by vibratory pile driving. Since the main aim of this thesis is to investigate application of EPS geofoam as a fill material, a detailed discussion on constitutive behaviour of EPS geofoam is also included in this chapter.
2.2 Ground vibration propagation

Ground vibrations are induced by different construction activities such as ground compaction, underground drilling, surface blasting and pile driving. Pile driving has become a major source of construction induced ground vibrations, as construction of high-rise buildings requires pile foundations to transfer their design loads to ground below. Driven piles have relatively smaller diameters and are prefabricated and driven into ground using a pile driver. Three techniques are widely used to install driven piles: vibratory pile driving, impact pile driving and pile jacking. Vibratory pile driving and impact pile driving are the major contributors in pile driving induced ground vibration propagation.

Intensity, frequency and location of the vibration source can vary the severity of the damage caused by ground vibrations from minor architectural damages to severe structural damages. Vibration intensities are attenuated with increasing distance from the vibration source and consequently their effect on sensitive structures. Dym (1976) discussed about vibration attenuation considering geometric damping and material damping. Geometric damping occurs due to the expansion of the wave front with increasing distance from the wave source and material damping occurs due to the various material parameters of the soil medium (Kim and Lee, 2000).

Peak particle velocity (PPV) can be used to discuss the intensity of vibrations propagating in the ground. According to Athanasopoulos and Pelekis (2000), the strains generated by propagating ground waves are proportional to the particle velocity of the medium. They summarised different approaches applied in numerous studies to derive PPV from the particle velocity measured in vertical, horizontal and transverse directions as follows:

(i) The peak value of velocity, out of velocities measured in three mutually perpendicular directions.

(ii) The peak value of the velocity in vertical direction.
(iii) The square root of the sum of squares (SRSS) of peak values of velocity in three mutually perpendicular directions.

(iv) The peak value of true vector sum (TVS) of velocities in three mutually perpendicular directions.

The fourth approach listed above is recognised as the most appropriate approach to describe the intensity of the vibration.

Stress waves generated due to ground vibrations decay with the increasing distance from the vibration source. According to Dym (1976), the decay in amplitude can take the form of,

\[ A(r) = A_0 e^{-\alpha (r-\bar{r})} \]  \hspace{1cm} (2.1)

where \( A(r) \) is the decayed amplitude of the wave at any distance \( r \) from the wave source, \( A_0 \) is the amplitude of the wave at a distance \( r_0 \) from the source and \( \alpha \) is the absorption coefficient. Dym further mentioned that this form is in contrast with the cylindrical propagation of waves, where \( A(r) \) varies with \( 1/\sqrt{r} \). The absorption coefficients for different types of soils are given in Table 2.1.

**Table 2.1. Absorption coefficients (Dym, 1976)**

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>( \alpha ) (m(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water-Saturated Clay</td>
<td>0.04-0.12</td>
</tr>
<tr>
<td>Loess and Loessial Soil</td>
<td>0.1</td>
</tr>
<tr>
<td>Sand and Silt</td>
<td>0.04</td>
</tr>
</tbody>
</table>

The ground vibration attenuation is defined by,

\[ D = -10 \log \left( \frac{A(r)}{A(r_0)} \right)^2 \]  \hspace{1cm} (2.2)
where \( A(r_0) \) is the amplitude of ground vibration given by Equation (2.2) at the source, which is the same as \( A_0 \). Table 2.2 shows ground vibration attenuation \( D \) in different types of soils.

**Table 2.2 Ground vibration attenuation, \( D \), for different soil types (Dym, 1976)**

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>( (r - r_0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100 ft</td>
</tr>
<tr>
<td>Clay</td>
<td>10.4</td>
</tr>
<tr>
<td>Loess</td>
<td>26.0</td>
</tr>
<tr>
<td>Sand</td>
<td>10.4</td>
</tr>
</tbody>
</table>

Figure 2.1 shows the variation of attenuation, \( D \), given by Equation (2.2) or dissipation of energy with distance from the source.

![Figure 2.1. Vibration attenuation for different soil types (Dym, 1976).](image)

A review published about propagation of ground vibrations (Gutowski and Dym, 1976) mentioned that the US Bureau of Mines analysed vibration data gathered from numerous quarry sites to determine the propagation of ground vibrations generated due to blasts. Based on the results, the US Bureau of Mines had come up with a propagation law of the form given by,
where $V$ is the peak particle velocity, $x$ is the distance measured from the source, and $k$ and $\beta$ are constants determined by a standard regression analysis. The propagation laws were different in the radial, vertical and transverse directions. The constant $\beta$ in the radial, vertical and transverse directions are -1.63, -1.73 and -1.28, respectively. Referring to Attewell and Farmer (1973), Gutowski and Dym (1976) further mentioned that vibration propagation due to pile driving in a wide range of soil types has the form of Equation (2.3), with $\beta$ equal to -0.87. After studying various vibration sources, Gutowski and Dym (1976) presented Figure 2.2 showing expected attenuation vs. distance in wave lengths, for various types of soils.

![Figure 2.2. Vibration attenuation with distance in wave lengths](Gutowski and Dym, 1976).

Many researchers (New, 1990; Athanasopoulos and Pelekis, 2000; Kim and Lee, 2000; Amick and Gendreau, 2000; Hao et al., 2001a; Hao et al., 2001b; Khandelwal and Singh, 2007; Uysal et al., 2008) studied the ground vibration propagation and attenuation characteristics using the peak particle velocity.

New (1990) studied the ground vibrations caused by construction activities. He categorised construction induced vibrations into two categories:
(i) Vibrations based on the construction method and the propagation characteristics of the site and
(ii) Vibrations based on the type of the structure and the sensitivity of the local population to disturbance

In New’s study, wave velocities are summarised for different types of rocks referring to Langefors and Kihlstrom (1978). It is mentioned that irrespective of the peak particle velocity and frequency developed at the vibration source, the ground distortion occurs on a structure founded on soil is significantly higher than that on rock. He pointed out that architectural and structural damages may occur to structures founded on soil and rock from traffic induced vibrations, when peak particle velocity exceeds 5 mm/sec and 10mm/sec respectively (Wiffin and Leonard, 1971). New (1990) then summarised the dissipation attenuation data from different sources, as shown in Figure 2.3.

![Figure 2.3. Variation of peak particle velocity with the distance from the source (New, 1990).](image-url)
Damage to ordinary dwellings at different soil conditions and vibration velocities are summarised in Table 2.3. Drabkin et al. (1996) summarised case studies available in the literature for vibration induced settlements, as shown in Table 2.4. In Table 2.4, there are two methods given as one-layer method and ten-layer method, which are used to estimate the settlement of 3-40 m thick layer of sand using settlement characteristics of triaxial test specimens of 150 mm height. The one-layer method considers the zone of interest as a single layer with the settlement assessed at the middle of the layer where the ten-layer method considers ten layers of equal height and settlement is evaluated at the middle of each layer and the settlements of each layer are then summed to get the settlement of the soil deposit. Drabkin et al. (1996) assumed that the settlement, $\Delta$, of a layer with thickness, $H$, is directly proportional to the settlement, $y$, of the tested specimen as shown in Equation (2.4),

$$\Delta = \frac{y}{150} H$$  \hspace{1cm} (2.4)

where $y$ and $H$ are given in millimetres.

**Table 2.3 Damage risk to ordinary dwelling houses with varying ground conditions (Langefors and Kihlstrom, 1978).**

<table>
<thead>
<tr>
<th>Compression wave velocity (m/s)</th>
<th>Sand, shingle, clay, underground water level</th>
<th>Moralne, slate, soft limestone</th>
<th>Hard limestone, quartzy sandstone, gneiss, granite, diabase</th>
<th>Type of damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>300-1500</td>
<td>2000-3000</td>
<td>4500-6000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vibration velocity (mm/s)</td>
<td>4-18</td>
<td>35</td>
<td>70</td>
<td>No noticeable cracks</td>
</tr>
<tr>
<td></td>
<td>6-30</td>
<td>55</td>
<td>110</td>
<td>Insignificant cracking (threshold value)</td>
</tr>
<tr>
<td></td>
<td>8-40</td>
<td>80</td>
<td>160</td>
<td>Cracking</td>
</tr>
<tr>
<td></td>
<td>12-60</td>
<td>115</td>
<td>230</td>
<td>Major cracks</td>
</tr>
</tbody>
</table>
Table 2.4 Case histories of vibration induced settlements (Drabkin et al., 1996).

<table>
<thead>
<tr>
<th>Investigated Sites</th>
<th>Vibration amplitude (mm/s)</th>
<th>Settlement (mm)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Observed in-situ</td>
<td>Calculated</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>One-layer scheme</td>
<td>10-layer scheme</td>
</tr>
<tr>
<td>Back Bay (Leathers, 1994)</td>
<td>6.4-15</td>
<td>18-54</td>
<td>18-63</td>
<td>24-78</td>
</tr>
<tr>
<td>Brooklyn, South (Lacy et al., 1994)</td>
<td>17.5</td>
<td>70</td>
<td>59</td>
<td>74</td>
</tr>
<tr>
<td>Brooklyn, West (Lacy and Gould, 1985)</td>
<td>2.5-15.2</td>
<td>61</td>
<td>5-56</td>
<td>8-63</td>
</tr>
<tr>
<td>Cedar Creek (Lacy, 1986)</td>
<td>5-10</td>
<td>13-19</td>
<td>9-15</td>
<td>-</td>
</tr>
<tr>
<td>Embarcadero (Clough and Chameau, 1980)</td>
<td>1-5</td>
<td>8-51</td>
<td>8-58</td>
<td>-</td>
</tr>
<tr>
<td>Leningrad (Dalmatov et al., 1967)</td>
<td>2.8</td>
<td>6-11</td>
<td>4.5</td>
<td>8.1</td>
</tr>
<tr>
<td>Lesaka (Picornell and del Monte, 1985)</td>
<td>17.5</td>
<td>250</td>
<td>111</td>
<td>117</td>
</tr>
<tr>
<td>Northbrook pipeline (Linehan, 1992)</td>
<td>2.8</td>
<td>38</td>
<td>37</td>
<td>48</td>
</tr>
<tr>
<td>Tri-beca (Lacy et al., 1994)</td>
<td>2.5-18</td>
<td>38-69</td>
<td>15-112</td>
<td>27-135</td>
</tr>
</tbody>
</table>

2.3 Ground vibration propagation during pile driving

The first commercial application of vibratory pile driving was carried out in Germany, in 1932 (Rodger and Littlejohn, 1980). As the investigations on vibratory driving progresses, Russia started working on impact pile driving mechanisms in 1946. In vibratory pile driving, a set of counter-rotating eccentric masses is used to generate the force driving the pile. The vibrations generated by the pile driver reduce the soil resistance, and accommodates to drive the pile with a smaller surcharge force than that generated during impact driving. Impact pile driving involves dropping a ram mass from a given height, which gives energy necessary for the pile to penetrate into the soil, even in difficult soil conditions. However, due to the high noise and vibration generation, impact driving is less preferred in urban construction activities (Woods, 1997).

Attewell and Farmer (1973) presented a diagram for wave propagation around a pile during driving. They concluded that a driven pile generates different types of elastic waves: compression, shear and surface waves. Compression waves are believed to be
transmitting from the area of pile toe expanding over a spherical wavefront. Vertical shear waves are expected to generate from the friction between pile shaft and soil, which will expand around a conical surface, as shown in Figure 2.4.

Figure 2.4. Possible wavefronts, generated from a driven pile (Attewell and Farmer, 1973).

Woods (1997) suggested an advanced illustration of wave propagation from driven piles in homogeneous soils. According to Woods (1997), the shear stress waves generated from the friction between the pile and soil particles first generate from the upper contact point and propagate out in a conical shape as shown in Figure 2.5. Due to the velocity of the impulse driving the pile, the compression wave velocity of the pile is usually ten times or more than that of the shear wave velocity of the soil. The shear wave front is considered cylindrical in practice. Each impact on the pile creates a volume displacement at the tip of the pile, resulting outward travelling P and S waves. Once these waves reach the ground surface, part of the waves will be transformed to surface waves such as Rayleigh waves and rest of the waves are reflected back into the ground.
(Athanasopoulos and Pelekis, 2000). When the pile is completely embedded, ground vibrations around the pile are dominated by shear waves. Independent of the driven pile depth, compression waves are attenuated and near surface ground vibrations are determined by Rayleigh waves in the far field (Masoumi et al. 2007).

![Figure 2.5. Seismic wave propagation in pile driving (Woods, 1997).](image)

Numerous studies have been carried out investigating free field vibration propagation due to dynamic pile installation methods. Kim and Lee (2000) observed a friction pile driving at a bridge construction site in Pusan, South Korea. Piles were used for piers of the bridge. The piles were hollow steel piles of diameter 0.6 m and a hydraulic hammer was used to drive them. A 0.8 m drop height was used for a hammer weight of 7 t. The ground profile consists of about 15 m gravel fill, 20 m of silty sand
and clay, 5 m sand, 7 m weathered soil and bed rock beyond. Depths of pile driving varied from 16-28 m. The vibrations were captured using 4.5 Hz vertical geophones and two 3D geophones. Figure 2.6 shows the configuration of geophones used at the pile driving site to measure the intensity of ground vibrations.

![Figure 2.6 Site information at bridge site in Pusan, South Korea (Kim and Lee, 2000).](image)

According to Kim and Lee (2000), most of the energy is transmitted by vertical motion at frequencies below 10 Hz apart from transverse motion. It was apparent that the pile driving tries to overpower the friction generated between soil and pile shaft and as a result, the whole soil mass vibrates. Figure 2.7 shows the particle velocities measured by a geophone placed 17 m away from the driven pile and 15 m below the ground surface.

![Figure 2.7. 3D transducer time records for friction pile driving induced vibrations (Kim and Lee, 2000).](image)
In regard to vibration attenuation, Kim and Lee (2000) discussed about geometric damping and material damping. They specified geometric damping as,

\[ w_2 = w_1 \left( \frac{r_1}{r_2} \right)^n e^{-\alpha(r_2 - r_1)} \]  

(2.5)

where \( w_1 \) and \( w_2 \) are vibration amplitudes at distances \( r_1 \) and \( r_2 \), respectively. Geometric damping coefficient was given by \( n \) whilst the material damping coefficient was denoted by \( \alpha \). Geometric damping occurs by diminishing energy density with distance increasing from the source. In determining the geometric damping analytically, Table 2.5, which was originally proposed by Gutowski and Dym (1976) and modified by Kim and Lee (2000), can be used to assess the type of wave, vibration source and the monitoring location.

**Table 2.5 Geometric damping coefficients for various sources (Kim and Lee, 2000).**

<table>
<thead>
<tr>
<th>Physical sources</th>
<th>Type of source</th>
<th>Wave</th>
<th>Location</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highway/Rail line footing array</td>
<td>Line</td>
<td>Surface</td>
<td>Surface</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Body</td>
<td>Surface</td>
<td>1.0</td>
</tr>
<tr>
<td>Car in pothole, Single footing</td>
<td>Point</td>
<td>Rayleigh</td>
<td>Surface</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Body</td>
<td>Surface</td>
<td>2.0</td>
</tr>
<tr>
<td>Tunnel</td>
<td>Buried line</td>
<td>Body</td>
<td>Interior</td>
<td>0.5</td>
</tr>
<tr>
<td>Buried explosion</td>
<td>Buried point</td>
<td>Body</td>
<td>Interior</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Material damping occurs due to the friction and cohesion between soil particles when the waves are propagating. It is affected by frequency of the propagating vibration and the soil type. The material damping, \( \alpha \), can be given by (Kim and Lee, 2000),

\[ \alpha = \frac{\pi f \eta}{c} \]  

(2.6)

where \( f \) is the frequency, \( \eta \) is the loss factor and \( c \) is the velocity of the propagating wave.

Poulos (1994) conducted a study on the effect of pile driving on existing piles. He used theoretical analyses to determine the axial forces and bending moments developed
in existing piles due to the horizontal and vertical movements developed by driven piles in clay. The methodology adopted was first to determine the free-field movements in the soil due to driven piles. Poulos (1994) carried out two separate pile-soil interaction analyses for the existing pile to compute the axial response and the lateral response, imposing the vertical and horizontal free-field movements, respectively. Free-field ground movements were determined from a separate analysis. It was observed that as the pile driving continues, the soil movements change from settlement to heave. The forces at the existing pile are compressive at the beginning but changes to tensile towards the final penetration (Poulos, 1994). It was concluded that the magnitude of the maximum tensile force induced can be significant, up to 25% of the ultimate shaft load capacity, when the spacing between piles is three pile diameters. For the maximum tensile force induced to be dropped below 15% of the ultimate shaft load capacity, pile spacing should exceed more than ten diameters. It is also mentioned that the induced bending moments are relatively small for spacing exceeding five pile diameters.

Poulos (2005) extended his study on effects of pile driving and drilling to existing pile foundations by performing finite element simulations using data available in the literature. Poulos (2005) used two distinct stages in analysing the ground movement effects on piles: (i) estimation of free-field soil movements while the existing pile is not present and (ii) calculation of the response of the existing pile applying computed free-field movements from the first stage, which is similar to the method used by Poulos (1994). He indicated that in both cases there is a potential to cause additional bending moments and axial forces in the nearby existing piles. It was recommended to construct a barrier or a wall to protect existing piles in extreme situations (Poulos, 2005).

Thandavamoorthy (2004) studied the intensity of ground vibrations from a hammer driven pile on the far field in sand by measuring the PPVs near and far from a driven pile. He observed that vertical vibrations generated by a driven pile are impulsive, with high frequency content and significant compared to the horizontal vibrations. Hence, it was concluded that the procedures related to earthquake-generated waves, where horizontal vibrations are significant compared to vertical vibrations, are not applicable in
pile driving induced ground vibrations. Masoumi et al. (2009a) conducted a low strain dynamic test using a hammer with a weight of 5.5 kg on a single pile to study the pile response and the vibration propagation in the free field. There was a reasonable agreement between the measured and predicted responses beyond 4 m from the driven pile. According to Masoumi et al. (2009b), the free field vibrations depend on the material-damping ratio of the soil at high frequencies and at the distance from the driven pile.

A set of full scale field tests were conducted by Lee et al. (2012) to study the characteristics of dynamic behaviour of a vibratory driven sheet pile. They developed a computer programme to predict the penetration rate of a sheet pile. It was validated using the field data extracted for vibratory sheet pile driving. They observed that the effect of assuming the driven pile to be rigid is reasonable by comparing the accelerations measured near the top and the bottom of the tested sheet piles.

Apart from the numerous experimental studies carried out in determining the ground response to vibratory pile driving, several numerical studies have also been carried out on vibratory pile driving. An early study of vibratory pile driving conducted by Rodger and Littlejohn (1980) distinguished vibratory pile driving into two categories: slow and fast vibratory pile driving, which were determined by the initial soil density, pile diameter, displacement amplitude and acceleration of vibration. They concentrated on slow vibratory pile driving and developed a numerical solution to simulate vibratory pile driving. They verified the numerical solution through a full-scale field experiment conducted in dry cohesionless soil.

Masoumi et al. (2007) presented a numerical model based on coupled finite element-boundary element method to predict the free field vibration propagation from vibratory and impact pile driving. For vibratory driving, they categorised the induced ground vibrations as vertical shear waves around the shaft, dilation waves around the pile toe and Rayleigh waves along the surface. They observed that vertically polarised shear waves are dominating vibrations around the driven pile and as vibrations propagate, the body
waves attenuate and the Rayleigh waves dominate near the surface, irrespective of the depth of penetration.

Hügel et al. (2008) presented an analysis of vibratory pile driving, modelled using ABAQUS/Standard and ABAQUS/Explicit. They discussed the use of continuum elements and infinite elements (to represent the far field) in analysing the pile driving problems. Penetration of the piles in homogeneous subsoil was analysed using axisymmetric models. Penetration of piles in areas with existing structures and foundations was analysed using three-dimensional finite element models. Adaptive meshing based Arbitrary Lagrangian Eulerian (ALE) approach was used to overcome the mesh distortion caused by pile penetration, particularly when the wall friction angles are larger than a third of internal friction angle ($\phi/3$) of the soil. A fine mesh around the driven pile overcame the mesh distortion in three-dimensional models. They observed large surface settlements around the pile, due to the strong dynamic shearing between pile and soil. As a result, it reduces the void ratio of the soil, thus densifying the soil around the driven pile.

A three-dimensional finite element model was presented by Lo et al. (2012) to predict free field vibrations from vibratory and impact pile driving. It incorporated scaled boundary finite element method, which could absorb the reflective energy at the boundary of the finite domain. Their study concluded that surface vibrations at a location of five pile diameters away from the driven pile would not be significantly affected by vibratory pile driving. However, the study was limited to a frequency of 20 Hz and amplitude of 1000 kN.

A case study presented by Lidén (2012) on vibratory driving of sheet piles stated the importance of considering vibrations of all orthogonal directions when studying the influence of ground vibration propagation. Lidén (2012) also observed that the soil characteristics influenced the magnitude of vibrations and recommended considering the soil characteristics in investigating vibratory pile driving induced ground vibrations.
Aforementioned research studies confirm the significance of vibratory pile driving on the surrounding environment and the necessity of implementing proper mitigation measures against ground vibration propagation during vibratory pile driving. The next section focuses on mitigation measures currently used in practice against ground vibration propagation.

2.4 Mitigation measures against ground vibration propagation

Different standards and design codes specify different vibration criteria to prevent building damages. Jones and Stokes (2004) summarised the PPV criteria given by the American Association of State Highways and Transportation Officials (AASHTO) in 1990. Table 2.6 shows the limiting PPVs for different types of structures. A research report prepared by Jackson et al. (2007) outlines the PPV criteria given in Swiss standard, SN640312, for different categories of buildings. SN640312 differentiates four building classes and two frequency ranges in each class for machines and traffic generated vibrations, as illustrated in Table 2.7. Eurocode 3 (EN 1993) provides guidelines for acceptable vibration levels generated from any source to avoid structural damages to buildings. As shown in Table 2.8, Eurocode 3 is similar to AASHTO and does not differentiate PPVs based on frequency ranges as in SN640312.

Other standards such as German Standard, DIN4150; British Standards, BS7385 and BS5222-2; and Australian Standard, AS2187.2; also published PPV criteria but the guidelines are developed to use against the vibrations generated due to transient events such as blast loading. Since the occurrences of blast vibrations are transient and not continuous, the PPV values given are much higher than the limiting values given for continuous vibrations such as machine, construction and traffic generated vibrations.
### Table 2.6 AASHTO: Maximum acceptable vibration levels to prevent structural damage (Jones and Stokes, 2004).

<table>
<thead>
<tr>
<th>Type of Situation</th>
<th>PPV in/sec (mm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Historic sites or other critical locations</td>
<td>0.1 (2.54)</td>
</tr>
<tr>
<td>Residential buildings, plastered walls</td>
<td>0.2-0.3 (5.08-7.62)</td>
</tr>
<tr>
<td>Residential buildings in good repair with gypsum board walls</td>
<td>0.4-0.5 (10.16-12.7)</td>
</tr>
<tr>
<td>Engineered structures, without plaster</td>
<td>1.0-1.5 (25.4-38.1)</td>
</tr>
</tbody>
</table>

### Table 2.7. Swiss Standards (SN640312): Maximum acceptable vibration levels to prevent structural damage (Jackson et. al., 2007).

<table>
<thead>
<tr>
<th>Building Class</th>
<th>Frequency Range (Hz)</th>
<th>PPV in/sec (mm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>10-30</td>
<td>0.5 (12.7)</td>
</tr>
<tr>
<td></td>
<td>30-60</td>
<td>0.5-0.7 (12.7-17.78)</td>
</tr>
<tr>
<td>II</td>
<td>10-30</td>
<td>0.3 (7.6)</td>
</tr>
<tr>
<td></td>
<td>30-60</td>
<td>0.3-0.5 (7.6-12.7)</td>
</tr>
<tr>
<td>III</td>
<td>10-30</td>
<td>0.2 (5.08)</td>
</tr>
<tr>
<td></td>
<td>30-60</td>
<td>0.2-0.3 (5.08-7.62)</td>
</tr>
<tr>
<td>IV</td>
<td>10-30</td>
<td>0.12 (3.05)</td>
</tr>
<tr>
<td></td>
<td>30-60</td>
<td>0.12-0.2 (3.05-5.08)</td>
</tr>
</tbody>
</table>

I- Buildings of steel or reinforced concrete, such as factories retaining walls, bridges, steel towers, open channels; underground chambers and tunnels with and without concrete lining
II- Foundation walls and floors in concrete, walls in concrete or masonry; stone masonry retaining walls; underground chambers and tunnels with masonry linings; conduits in loose material.
III- Buildings as previously mentioned but with wooden ceilings and walls in masonry
IV- Construction very sensitive to vibration; objects of historical interest

### Table 2.8. Eurocode 3: Maximum acceptable vibration levels to prevent structural damage (EN 1993).

<table>
<thead>
<tr>
<th>Building type</th>
<th>PPV (mm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Architectural merit</td>
<td>2</td>
</tr>
<tr>
<td>Residential area</td>
<td>5</td>
</tr>
<tr>
<td>Light commercial</td>
<td>10</td>
</tr>
<tr>
<td>Heavy industrial</td>
<td>15</td>
</tr>
<tr>
<td>Buried structures</td>
<td>25</td>
</tr>
</tbody>
</table>
A study conducted by Svinkin (2006) recommended several methods to mitigate the effect of vibratory pile driving on adjacent structures, such as decreasing the vibration level of the source, pre-augering or jetting ahead of the driven pile in the upper soil layers and implementing the pile driving sequence away from the structures. His recommendations are to reduce the intensity of ground vibrations at the vibratory source, however, limiting the propagation of the ground vibrations was not addressed.

Celebi et al. (2009) carried out field experiments on wave propagation and use of trenches to isolate wave propagation. In their research, they used Bentonite, Concrete and Water as fill materials and conducted field experiments. They concluded that the open trenches are the most efficient wave barriers. However, practical applications are limited to shallow depths. Use of a softer in-fill material will enable to use deeper trenches with no supporting for trench walls. Also it was highlighted that the passive isolation, where the wave barrier is near the structure to be isolated from vibration, is more effective than the active isolation, where the wave barrier is near the source of vibration.

Andersen and Nielsen (2005) studied the applicability of a wave barrier against ground vibration propagation using a coupled finite element-boundary element model. They used wave propagation along a railway track to investigate ground vibration propagation. In conclusion, they mentioned that a horizontal force applied in the model gives rise to an amplification of the response at a certain distance from the railway track when open trenches are used to mitigate wave propagation. According to them, this behaviour was related to the depth of trench established along the track. Deep trenches and high frequency excitations will reduce the wave amplification beyond the barrier. In their model, they used a trench filled with rubber chips and concrete, and found that the soft barriers are more efficient than hard wave barriers in reducing ground wave propagation. Their results show about 50% reduction in wave intensity beyond the wave barrier.

A set of three-dimensional finite element analyses was carried out by Ju and Li (2011) investigating the efficiency of the water trenches in attenuating the ground
vibrations. They observed that the water-filled trenches are performing similar or slightly better than the open trenches in attenuating ground vibrations propagating in the form of shear and Rayleigh waves. It was explained that the incident wave, which passed through the water trench and the diffraction wave from the trench bottom had different phases, thus cancelling out much of the wave intensity. However, due to the compressive nature of the dilation wave passing through the water trench, the amplitude of the wave was not reduced significantly by water, making the incident wave much larger than the diffraction wave from the trench bottom, reducing the efficiency of the wave barrier (Ju and Li, 2011, Wang et. al, 2009).

It is clear that different materials are used to fill the wave barriers used in attenuating ground vibrations. However, EPS geofoam offers a number of advantages over other fill materials such as concrete and bentonite because of its light weight, cost effectiveness, energy absorbing characteristics, efficiency in terms of construction time and ease of handling. The next section discusses about EPS geofoam and its use as an in-filled material against ground vibration propagation.

2.5 Characteristics of EPS Geofoam

EPS geofoam has been utilised for geotechnical engineering applications for several decades. A patent was granted in the USA to use EPS geofoam as a pavement insulation material in 1960’s (Hazarika, 2006). Norwegian geotechnical engineers conducted field experiments using EPS geofoam in their construction activities in 1970’s (Aaboe, 1987). EPS geofoam is manufactured using pre-expanded resin beads, which determine the density of the geofoam block based on resin type and the extent of pre-expansion in beads (Negussey, 2007). These beads are exposed to steam and pressure in a confined mould, forming the geofoam block. Due to the heat experienced by the beads, they tend to expand forming polyhedral shapes within the confined mould. The cellular EPS geofoam structure created by beads makes about 95% of its overall volume due to the air entrapped between the cells (Ossa and Romo, 2011). When EPS geofoam is subjected to compression, the cell walls bend and buckle, allowing the entrapped air to escape. Under extreme loading, cell walls may fracture.
EPS geofoam has been receiving the recognition in the geotechnical engineering industry due to its desirable mechanical properties such as light weight, high strength to weight ratio and volume contraction under deviatoric compressive loading, availability, ease of use, low manufacturing costs and environmental friendliness (Horvath, 1994). Gradually EPS geofoam has been receiving the recognition in the civil engineering industry. Many researchers have investigated the behaviour of EPS geofoam, but its full potential in geotechnical applications is yet to be fully discovered.

Horvath discussed the material behaviour of EPS geofoam with a density of 21 kg/m$^3$ using some unpublished data from BASF (BASF SE, Germany) for a short-term strain controlled unconfined compression tests. According to Horvath (1994), the variation of compressive stress with strain presented in Figure 2.8 is qualitatively representative of EPS geofoam with other densities as well. The linear-elastic behaviour occurs between 1-2% strain and the elastic limit increases with increasing density. The yield occurs not at a single point, but over a range, and the post-yield behaviour shows strain-hardening. At larger strains, the behaviour becomes non-linear.
2.5.1 Application of EPS Geofoam in ground vibration attenuation

Itoh et al. (2005) investigated the effect of EPS geofoam barriers in reducing train induced ground vibration propagation. They observed that there was a close relationship between the vibration around the EPS geofoam barrier and the frequency of the dynamic loading. Murillo et al. (2009) discussed about vibration isolation using geofoam barriers. They conducted a centrifuge parametric study to determine the influence of geofoam wave barriers. The efficiency is determined at the beginning using the amplitude reduction ratio, $A_{rr}$, given by,
\[ A_{rr} = \frac{A_l}{A_0} \]  

(2.7)

where \( A_l \) is the amplitude with isolation system and \( A_0 \) is the amplitude without isolation system. Those are expressed in terms of either vertical displacement of ground or spectral density. It was mentioned that the barrier is satisfactorily efficient when \( A_{rr} \leq 0.25 \). Murillo et al. (2009) used a piezoelectric actuator to run centrifuge tests to determine the influential parameters. The tests are carried out at an acceleration of 50\( g \). The schematic configuration of the centrifuge tests is illustrated in Figure 2.9. They concluded that the attenuation is significant with increasing time and frequency. The barrier influence is clearly illustrated in Figure 2.10, which shows the attenuation of the waves with increasing distance from the barrier and frequency. Their tests showed that the efficiency (\( A_{rr} \)) of the deeper barriers (dimensionless barrier depth \( D = d / \lambda_R > 2.0 \)), is 0.2. Here, \( \lambda_R \) is the wavelength of the Rayleigh waves of the system. Results are valid for source barrier distance of 5 cm in model scale (2.5 m in prototype scale).

Note: All dimensions given in this figure are in mm and correspond to the 1:50 scale

**Figure 2.9 Schematic configuration of the centrifuge tests, Murillo et al. (2009).**

For \( D < 1.0 \), \( A_{rr} \) lies between 0.8 and 1.4. According to the test results, farther the barrier is placed from the source, higher the amplitudes recorded behind the barrier. For a
dimensionless width \( W(= w/\lambda_R) > 0.25 \) and for \( r = 5 \) cm, \( A_{RR} \) is approximately 0.2 for all depths. It was also stated that for thinner barriers: \( W < 0.25 \), amplification occurs mostly with shallow barriers. Where \( R(= r/\lambda_R) \) is the dimensionless source-barrier distance, when \( 0.5\lambda_R < R < 1.2\lambda_R \), barrier has an influence on reducing the wave amplitude \( A_{RR} < 1 \).

![Figure 2.10. Attenuation effects of wave barriers, Murillo et al. (2009).](image)

A full scale field experiment was carried out by Alzawi and El-Naggar (2011) to determine the effectiveness of open wave barriers and in-filled wave barriers using EPS geofoam. A mechanical oscillator was used to generate ground vibrations. The effectiveness of the barriers was evaluated based on the reduction in soil particle velocities and the location of the wave barriers. The effectiveness of the open trench barrier depends on the ratio, distance to source/depth. As the ratio increases, the effectiveness of the wave barrier reduces while any significant changes were not identified for the geofoam barrier (Alzawi and El-Naggar, 2011).

In addition to the experimental studies carried out on application of EPS geofoam as a wave barrier, numerical studies have also been carried out.

Wang et al. (2006) carried out a finite element simulation to study the attenuation of stress wave propagation in a concrete medium behind an inclusion of EPS geofoam. They used rectangular EPS geofoam blocks of different dimensions. Based on the results from the numerical analyses, it was concluded that the length of EPS geofoam inclusion
has a significant influence on attenuation of stress waves while the width of the inclusion has a little influence. The model used by them could estimate the minimum length of EPS geofoam inclusion and minimum safe distance from the inclusion.

Finite element modelling will provide useful results only if the constitutive behavior of the geomaterials are accurately captured by the constitutive models representing material behaviour. If the finite model is capable of simulating the boundary value problem, it will be a useful tool in investigating influence of different parameters on the problem. The main advantage is the cost savings achieved by a reliable finite element model in carrying out parametric studies, compared to full scale field tests and centrifuge tests. EPS geofoam is relatively novel construction material. Hence there are not many constitutive models available to simulate the material behaviour of geofoam. Therefore, in the following section, different constitutive models available to describe the mechanical behaviour of EPS geofoam under various loading conditions will be discussed.

2.5.2 Constitutive models for EPS geofoam

Time dependent behaviour of EPS geofoam is discussed by Horvath (1998). Horvath (1998) used Findley equation (Findley and Khosla, 1956) for creep behaviour of polymeric materials. From the basic relationship, \( \varepsilon = \varepsilon_0 + \varepsilon_c \), Findley and Khosla (1956) proposed the following equation,

\[
\varepsilon = \varepsilon_0 + m \left( \frac{t}{t_0} \right)^{n_F}
\]

(2.8)

where \( \varepsilon_0 \) is the immediate strain upon stress application, \( m \) is a dimensionless material parameter, \( n_F \) is a dimensionless Findley material parameter, \( \varepsilon_c \) is the time-dependent (creep) component of strain after time \( t \), \( t \) is time after stress applied with units in hours and \( t_0 \) is one hour. Above equation was then simplified into the following form,

\[
\varepsilon = \varepsilon_0 + mt^{n_F}
\]

(2.9)
Using equation (2.9), Horvath (1998) developed the following equation,

\[ \varepsilon = \varepsilon_0 \sinh \left( \frac{\sigma}{\sigma_{e_0}} \right) + m \sinh \left( \frac{\sigma}{\sigma_{m_0}} \right) t^{n} \]  

(2.10)

where, \( \varepsilon_0 \) and \( m_0 \) are dimensionless Findley material parameters, \( \sigma_{e_0} \) and \( \sigma_{m_0} \) are Findley material parameters with dimensions of stress and \( \sigma \) is the applied stress. In this study, Horvath derived simplified Findley parameters for equation (2.10), given by,

\[ \varepsilon = 0.011 \sinh \left( \frac{\sigma}{54.2} \right) + 0.000305 \sinh \left( \frac{\sigma}{33.0} \right)^{0.20} \]  

(2.11)

The model is only valid for EPS geofoam with density of 20 kg/m\(^3\), under an applied stress magnitude less than or equal to 50 kPa, measuring time more than one hour, and a temperature approximately at 23°C. Due to these limitations, the application of this model to practical problems is limited. Further, the applicability of the model proposed by Horvath (1998) to simulate complex loading patterns such as regular or irregular cyclic loading is not verified.

A series of drained triaxial compression tests for EPS geofoam with different densities varying from 15-30 kg/m\(^3\) was carried out by Chun et al. (2001) to propose a constitutive model to predict the behaviour of EPS geofoam. Their study concluded the elastoplastic behaviour of EPS geofoam. According to them, the stress-strain relationship of EPS geofoam is closely associated with the density and the confining pressure. This study further confirmed that an increase in density and confining pressure increase the axial stress and initial tangent modulus. Based on the experimental results, Chun et al. (2001) proposed the following relationship to illustrate the mechanical behaviour of EPS geofoam,

\[ \sigma_1 = \frac{a \varepsilon_1^b}{c + \varepsilon_1^b} \]  

(2.12)

where \( \sigma_1 \) is axial stress, \( \varepsilon_1 \) is axial strain and \( a, b \) and \( c \) are parameters given by,


\[ a = -60.955 + 9.843d + 0.339\sigma_3 \]  \hspace{1cm} (2.13a)

\[ b = 1.135 + 0.042d - 0.008\sigma_3 \]  \hspace{1cm} (2.13b)

\[ c = -0.437 + 0.102d - 0.002d^2 + 0.011\sigma_3 - 0.389d\sigma_3 \]  \hspace{1cm} (2.13c)

where \( d \) is the density of EPS geofoam (kg/m\(^3\)) and \( \sigma_3 \) is the confining pressure (kPa).

The tangent modulus, \( E_t \), at each stress level is expressed as,

\[ E_t = \frac{abc\varepsilon_i^{b-1}}{\varepsilon_i^{2b} + 2c\varepsilon_i^b + c^2} \]  \hspace{1cm} (2.14)

The relationship between Poisson’s ratio (\( \nu \)), density and the confining pressure is given by,

\[ \nu = 0.0967 + 0.0031d - 0.0023\sigma_3 \]  \hspace{1cm} (2.15)

Further, it was mentioned that the behaviour of EPS geofoam and the gradient of the relationship between the volumetric strain and the axial strain have a close association with the confining pressure and the density. Chun et al. (2004) extended the above discussed model and concluded that the model may not work for complex stress paths such as loading/unloading, dynamic loading and environment factors such as variation of material behaviour due to temperature changes. In addition, the model does not consider the size effects because the samples used for the experiments were of one specific size.

Arellano et al. (2001) compared the existing creep models for EPS geofoam with field measurements. They considered the model developed by Horvath (1998) based on Findley creep model and by Laboratoire Ponts et Chausssess (LCPC) based on general power-law. The general power law derived by LCPC for total strain of EPS blocks is in the following form,

\[ \varepsilon = \left( \frac{\sigma}{E_t} \right) + \left( 0.00209 \left( \frac{\sigma}{\sigma_p} \right)^{2.47} \right) \left( -0.9 \log_{10} \left[ 1 - \left( \frac{\sigma}{\sigma_p} \right) \right] \right) t \]  \hspace{1cm} (2.16)
where $\varepsilon$ is the total strain, $\sigma$ is the applied stress in kPa, $\sigma_p$ is the stress corresponding to the onset of yielding in kPa, $E_i$ is initial tangent modulus in kPa (average slope of stress-strain curve at strains between 0 and 1%) and $t$ is the time in hours after applying the stress. $\sigma_p$ and $E_i$ are given by,

$$\sigma_p = 6.41\rho - 35.2 \quad (2.17a)$$
$$E_i = 479\rho - 2875 \quad (2.17b)$$

where $\rho$ is the density of EPS geofoam in kg/m$^3$. This study discovered that the initial tangent modulus given by equation (2.17b) is higher than the values reported in the literature. Thus, they have used a modified version of equation (2.17b) as shown below.

$$E_i = 479\rho - 3000 \quad (2.17c)$$

Arellano et al. (2001) concluded that both models by Horvath (1998) and LCPC give total strain values similar to or less than the measured values obtained from full size blocks in full scale creep tests, for stresses in the range of 10-55 kPa. According to them, the power-law equation predicts strains smaller than modified Findley equation. Hence they recommended the modified Findley equation to predict total vertical strains in the above stress range. According to their study the stresses ranging from 67-79 kPa, the total strains yield from power-law equation overestimate the test values while the modified Findley equation underestimate them, making both the models invalid for stresses ranging from 67-79 kPa.

A constitutive model to simulate EPS geofoam behaviour under large-strain conditions was proposed by Hazarika (2006). This model is based on the results obtained from uniaxial compression tests, which utilised incremental theory of plasticity and incorporates the size and shape factors together with density. A standard sample size of 50x50x50 mm is used for tests with densities of 16 and 20 kg/m$^3$. Figure 2.11 shows a comparison of compressive strengths for different samples used in unconfined compression tests. The common practice is to use 50 mm cubic samples for compressive
strength testing. However, the Young’s modulus derived from specimens of standard size underestimate the strength of geofoam as illustrated in Figure 2.14. Hazarika concluded that further research is needed to establish a relationship between test results from standard size specimens and full-size blocks manufactured for practical purposes.

![Figure 2.11 Comparison of compressive strengths (Hazarika, 2006).](image)

In deriving the constitutive model, Hazarika (2006) started with a general form of constitutive relation for any material displaying non-linear strain hardening behaviour,

\[
d\sigma = \left[ D^f - \frac{D^c (\partial g / \partial \sigma) \otimes D^c (\partial f / \partial \sigma)}{h + (\partial f / \partial \sigma) \cdot D^c (\partial g / \partial \sigma)} \right] d\varepsilon
\]  

(2.18)

where \( \sigma, \varepsilon, D^f, f, g \) and \( h \) are the stress tensor, the strain tensor, the elasticity matrix, the yield function, the plastic potential function and the normalised hardening modulus, respectively, and \( h' \) is defined as,

\[
h' = \frac{\partial K}{\partial e^p} + \beta \frac{\partial K}{\partial \varepsilon^p}
\]  

(2.19)
where $K$ is the hardening function of the material whilst $e^p$ and $\bar{e}^p$ are the deviatoric and the volumetric components of the plastic strain $\varepsilon^p$. Dilatancy of the material is represented by $\beta$. Since EPS geofoam contains 98% air by volume, the initial volume change is completely due to the expulsion of air from the cells (Hazarika, 2006). Further, it is stated that EPS is a material free from dilatancy and due to the same reason, $e^p$ and $\bar{e}^p$ are uncoupled. Thus, omitting the term $\beta$, the stress increment is given by,

$$d\sigma = \left[ D^e - \frac{D^e m \otimes D^e (\partial f / \partial \varepsilon) - D^e \cdot \partial f / \partial \varepsilon \cdot D^e m}{(\partial K / \partial \varepsilon^p) + (\partial f / \partial \varepsilon) \cdot D^e m} \right] d\varepsilon$$

(2.20)

where $m$ is the unit normal vector in the direction of deviatoric stress.

Finally, assuming that EPS geofoam follows the von Mises yield criteria and isotropic hardening, the yield function is given by,

$$f(\varepsilon, K(e^p)) = s - K$$

(2.21)

where $s = s_\mu \cdot s_\mu$ is the deviatoric stress tensor. It was assumed that the hardening function is given by,

$$K = \frac{e^p}{a + b e^p}$$

(2.22)

where $a$ and $b$ are rheological parameters for EPS geofoam. From equations (2.20)-(2.22), the constitutive model can be represented by,

$$d\sigma = \left[ D^e - \frac{D^e m \otimes D^e s}{(a/(a + b e^p)^2)} + s \cdot D^e m \right] d\varepsilon$$

(2.23)

By comparing the proposed model with few other available models (Preber et al., 1995; Chun et al., 2004), Hazarika (2006) concluded that the model proposed by Chun et al. (2004) and the model by Hazarika (2006) predict respectively the upper bound and lower bound of test results, as illustrated in Figure 2.12.
However, the model proposed by Hazarika (2006) has some limitations because it is only valid for large strain levels and rapid loading applications. His experiments confirmed the significance of the creep behaviour of EPS geofoam but it was not considered in deriving the constitutive model.

An elastoplastic hardening constitutive model for EPS geofoam was presented by Wong and Leo (2006), based on a series of drained triaxial tests carried out by varying confining pressures. The sample used by them had a density of 20 kg/m³, diameter of 50 mm and a height of 50 mm. They selected an aspect ratio of 1:1 to avoid column-type buckling in the sample during loading. The samples were loaded under confining pressures, ranging from 0-60 kPa, and were allowed to be stabilized in volume change. Then a strain controlled axial load was applied.

The proposed model is based on the Drucker-Prager yield criterion, which properly predicts the variation of deviatoric stress and volumetric strain with axial strain. The yield function and the plastic potential function for this constitutive model are given by,

\[
f(\sigma, R) = \sigma_1 - K_p\sigma_3 - R = 0
\]  

(2.24)
where $K_p$ is a rheological constant, $R$ is unconfined compressive strength, which is a rheological parameter, $\sigma_1$ and $\sigma_3$ are the axial stress and the confining pressure applied on the triaxial sample. A detailed discussion of the constitutive model and implementation of the model in ABAQUS/Explicit dynamic finite element modelling programme are presented in Chapter 4.

Verification of the proposed constitutive model was carried out using results from drained triaxial tests. In Figure 2.13 and Figure 2.14, the continuous lines represent the experimental data and the dotted lines represent the results obtained from the proposed constitutive model. The model successfully predicts the behaviour of EPS geofoam under compressive loading, in terms of deviatoric stress vs. axial strain and volume change vs. axial strain. Nevertheless, the model is not intended to address the EPS geofoam behaviour at high temperatures and long term loading situations which incorporates creep. Also, it does not consider the size and the shape factors of the test specimens. The application of this model is limited to normal operating temperatures and strain levels below the sigmoidal strains, where geofoam shows rapid strain hardening.
Leo et al. (2008) performed triaxial tests on EPS geofoam using a true triaxial apparatus to determine the mechanical behaviour of the material. This study showed that the EPS geofoam was softening stiffness-wise under increasing confining pressure. The elastoplastic hardening material showed plastic contractive volume change under compressive loading beyond the onset of yielding. Further, they showed that the EPS geofoam can be modelled as a material following the Drucker Prager yield criteria.

Zarnani and Bathurst (2008) carried out a finite element simulation to replicate a shake table test carried out on a retaining wall with EPS geofoam as a compressible inclusion (Bathurst et al., 2007). To simulate the test results, EPS geofoam was modelled as a linear elastic-plastic material and soil was modelled as an elastic-plastic material following Mohr-Coulomb yield criterion. They showed that numerical results were in agreement with shake table test data although they used an elastic-perfectly plastic model to simulate EPS geofoam behaviour.

The dependency of creep on the density of EPS geofoam was investigated by Vaitkus et al. (2007) using long term uniaxial compression tests. With a total test time of 988 days, they derived a model to predict the creep strains up to 50 years. The creep strains $\varepsilon_c(t)$ are calculated as,

$$\varepsilon_c(t) = \frac{\bar{X}_c(t)}{d_s} \times 100\%$$

(2.26)
where $X_c(t)$ is the average decrease in thickness of specimens over time $t$, $d_s$ is the average thickness of the specimen under preload of 250 Pa. The creep compliance parameter, $J_c(t)$, is determined by,

$$J_c(t) = \frac{\varepsilon_c(t)}{\sigma_c} \quad (2.27)$$

Two assumptions are used to determine the creep (Vaitkus et al., 2007). In the first assumption, creep is assumed as a steady state process leading to,

$$J_c(t) = b_0 t^{b_1} \quad (2.28)$$

where $J_c(t)$ is the average value of creep compliance at time $t$, in MPa$^{-1}$, $b_0$ and $b_1$ are rheological constants related to the material and $t$ is the time given in days.

In the second assumption, creep is assumed as a damped process leading to,

$$J_c(t) = b_0 \left[1 - \exp\left(-b_1 t^{b_2}\right)\right] \quad (2.29)$$

where $b_2$ is a material constant.

Calculating $J_{c(a)}(t = 50)$ and $J_{c(b)}(t = 50)$ from Equations (2.28) and (2.29) respectively,

$$\Delta J_c(t = 50) = \frac{J_{c(a)}(t = 50) - J_{c(b)}(t = 50)}{J_{c(a)}(t = 50)} \times 100\% \quad (2.30)$$

It was proposed that for any given time $t \leq 50$ years, the average creep strains can be represented by,

$$\bar{\varepsilon}_c(t \leq 50) = 166.2 \rho^{-1.98} \sigma_c \overline{m}_{J_c(t_i)} \quad (2.31)$$

where $\rho$ is the density of EPS geofoam, $\sigma_c (=0.35\sigma_{10\%})$ is the fixed compressive strength, $\sigma_{10\%}$ is calculated according to the British Standard EN 826: 1996 E, and $\overline{m}_{J_c(t_i)} = 0.55t^{0.26}$. Above study was extended using test results from different durations.
with compressive strengths at $0.25\sigma_{10\%}$ and $0.35\sigma_{10\%}$ (Gnip et al., 2008 and Gnip et al., 2010). The model can be used to determine the strains in EPS geofoam under constant compressive stress for periods ranging 5-50 years.

Abdelrahman et al. (2008) conducted a series of unconfined compression tests on EPS geofoam to determine the effect of strain rate on the behaviour of EPS geofoam. The tests were carried out on EPS geofoam with two different densities (19.3 and 28.0 kg/m$^3$) under monotonic loading conditions. A maximum axial strain of 15% was applied at vertical strain rates varying between 0.0046-33%/min. It was observed that rate effects on compressive strength of EPS geofoam become significant after the start of large-scale yielding. According to them, at small strains, the effect of strain rate is insignificant. They concluded that the strength of EPS geofoam increases with increasing strain rate. A similar observation was made by Ossa and Romo (2009) in a series of uniaxial and triaxial compression tests carried out to study the micro and macro mechanical behaviour of EPS geofoam under compressive loading. The sample density was varied from 17 to 30 kg/m$^3$ and samples were subjected to different axial strain rates varying from 0.29 to 5.88%/min. Ossa and Romo (2009) concluded that increasing displacement rate caused higher compressive strengths in EPS geofoam, leading to larger initial tangent modulus, yield stress and in many cases, greater plastic modulus. They further observed that the strain rate does not have any influence on the Poisson’s ratio of EPS geofoam.

2.6 Gaps in our understanding

Ground vibrations are hazardous to the structural health of nearby existing substructures and subsequently buildings supported by them. Rectification of damaged substructures and superstructures, and the subsequent delays to the construction activities are highly expensive and extremely inconvenient. Furthermore, disruptions to the services in the vicinity can have an immense impact on the community. Therefore, it is important to implement proper engineering approaches to eliminate, or to minimise, the construction induced ground vibrations. In this thesis, attention is focused on ground vibrations
induced due to vibratory pile driving, their effect on existing nearby structures and mitigation measures utilising EPS geofoam.

Wave barriers are often used to mitigate the construction induced vibration propagation in ground. Open wave barriers are the most effective wave isolators. However, they are vulnerable to localised collapse and numerous maintenance issues can arise when using open wave barriers while construction continues. This can be overcome by introducing a suitable fill material to the wave barrier, which can support the trench walls as well as to reduce the intensity of ground vibrations. Several studies have been carried out investigating capability of EPS geofoam in attenuating ground vibrations. Itoh et al. (2005) conducted centrifuge tests to determine the efficiency of EPS geofoam in attenuating vibrations generated by high-speed railway systems. Murillo et al. (2009) carried out centrifuge tests to determine the capability of EPS geofoam in attenuating traffic induced ground vibrations. Another full scale field study was carried out by Alzawi and El-Naggar (2011) to investigate attenuation of ground vibrations induced by machine foundations. However, the capability of EPS geofoam in attenuating ground vibrations has not been investigated in detail, considering geometry and location of the wave barrier, soil properties, experimentally observed behaviour of EPS geofoam and frequency of the vibratory source. Although finite element studies can be found in the literature simulating EPS geofoam as a fill material in wave barriers, geofoam behaviour has been modelled assuming elastic or elastic perfectly plastic behaviour.

In addition, the few studies which can be found in literature, investigating the applicability of EPS geofoam as a wave barrier, are limited to the attenuation of surface waves. Although, when considering protection of existing foundations, it is necessary to examine the attenuation of both surface waves and body waves since each type can affect nearby existing foundations. Inadequate knowledge in aforementioned areas prevents utilisation of EPS geofoam as a wave barrier in construction industry. Hence, main focus of this research is to address those gaps of knowledge giving special attention to the ground vibrations developed during vibratory pile driving. In the following chapters of
this thesis design charts will be developed which can be used by construction industry to utilise EPS geofoam as a fill material in wave barriers in mitigating ground vibrations.
Chapter 3

Wave transmitting boundaries used in numerical modelling

3.1 Introduction

Wave propagation in an infinite domain is considered in many areas where an object of concern is much smaller compared to the domain or the field it is being held in reality. The infinite domain around the object being analysed should be considered in order to conduct an accurate analysis of the response of the object, which can consume a significant amount of computational time and memory. The common approach would be to truncate the infinite domain around the object by an artificial boundary able to supplant the effect of the domain beyond the boundary on the object. This will make the analysis computationally inexpensive. Early attempts to analyse similar problems by truncating the domain near the object, having reasonably large finite domain to give reliable output and fixing the degrees of freedom along the truncated boundary were satisfactory only if the analysis was a static problem. Dynamic analyses conducted in the same manner were inaccurate as the wave reflection along the fixed boundary caused significant errors in results.

Boundary conditions used in dynamic finite element analysis can be divided into two categories: local and non-local boundaries. Local boundary conditions provide approximate solutions with low accuracy but efficiency is high when compared to non-
Chapter 3  Wave transmitting boundaries used in numerical modelling

local boundary conditions. Non-local boundary conditions provide more accurate solutions to a wave propagation problem but the computational cost of the solution is significant. In local boundary conditions, each degree of freedom at the boundary of the domain is directly coupled with each element of the local boundary. In non-local boundary conditions, all degrees of freedom of the boundary of the domain are coupled with the non-local boundary condition. Generally non-local boundaries are adopted in frequency domain analysis and local boundaries are adopted in time domain analysis. Different boundaries have been developed since late ‘60s and non-reflecting boundaries developed based on the boundary integral method, boundary element method, coupled finite element and boundary element method and infinite elements used in numerical modelling are considered as accurate as they are non-local in space and time (Givoli, 2004). Local wave transmitting boundaries, which consist of a combination of spring/dashpot/mass models, are approximate but they can be conveniently implemented in a nonlinear finite element analysis.

This chapter presents a review of local and non-local wave transmitting boundaries found in the literature. Performance of four of the non-reflecting boundary conditions, which are widely used in finite element modelling, are investigated using axisymmetric and plane-strain wave propagation problems. The analysis is carried out by implementing these wave-transmitting boundaries in ABAQUS/Explicit finite element program. The predictions of the selected boundary conditions are compared with predictions from extended mesh solutions to determine the accuracy of boundary conditions selected for this analysis.

3.2 Non-local wave transmitting boundaries

Smith (1974) proposed a frequency independent superposition boundary condition by defining two boundary value problems: a free boundary using Neumann condition ($\frac{\partial u}{\partial x}=0$, at the boundary) and a fixed boundary using Dirichlet condition ($u=0$, at the boundary). The solutions from the two boundary conditions were added together to eliminate the wave reflections at the boundary. When applying this method, for $n$ number of boundaries to be non-reflective, $2n$ number of equations needs to be solved.
A doubly asymptotic boundary element method was proposed by Underwood and Geers (1981) for two-dimensional plane-strain problems in infinite elastic mediums, which is asymptotically exact at high and low frequency limits. This boundary consists of a stiffness matrix for the low frequency range and a damping matrix for the high frequency range. The results suggest that the doubly asymptotic boundary proposed by Underwood and Geers is reasonably accurate when applied in analysing engineering structures with complex surface geometries. Wolf and Song (1996) proposed doubly asymptotic multi-directional boundary conditions by formulating multi-directional outward plane-wave boundary conditions for interaction forces. Predictions of the Wolf and Song (1996) boundary condition were reasonably close to those obtained using an extended finite element mesh.

Higdon (1986, 1990) developed a low-order multi-dimensional boundary condition for acoustic and elastic mediums. Only second and third order Higdon boundary conditions were considered in practice due to increase in complexity of the boundary conditions at higher orders. Higdon boundary condition of order J can be written as,

$$ H_j : \left[ \prod_{j=1}^{J} \left( \partial_i + C_j \partial_x \right) \right] u = 0 \quad (3.1) $$

where \( x \) is the direction of \( u \), which is the unknown wave field, \( C_j \) is the parameter signifying the phase speeds in the given direction. The boundary condition is exact for waves that propagate with phase speeds equal to \( \prod_{j=1}^{J} C_j \). Givoli and Neta (2003) proposed a new high-order sequence of non-reflecting boundary conditions for time-dependent wave problems, by reformulating the Higdon boundary condition. It does not involve higher derivatives, enabling the boundary condition to be used for any desired order with ease. They noted that the computer cost increased only linearly with increasing order. Dea (2011) presented an improved version of the Higdon boundary condition, incorporating a weighting factor in the finite difference implementation. It was
concluded that the use of weighting factor improved the absorption properties of the Higdon boundary condition significantly.

Liao and Wong (1984) proposed a transmitting boundary applicable to two and three-dimensional wave problems with a time-stepping algorithm. The values at the transmitting boundary nodes are calculated using the interior nodal values from the previous time step by simple interpolation and extrapolation of the values. The accuracy of the boundary condition could be improved by reducing the time increment. Through various numerical examples, they proved that parameter $c\Delta t/\lambda$ is the most important factor determining the accuracy of the extrapolation, where $c$ and $\lambda$ are velocity and Lamé’s first constant, respectively. The proposed method was capable of transmitting elastic waves: without separating shear and dilation components and with increasing order increases accuracy but decreases in computational efficiency. According to Liao and Wong (1984), the efficiency of the boundary condition does not alter with inhomogeneous physical properties of the domain.

A dynamic infinite element that can transmit Rayleigh waves and body waves in homogeneous half space was developed by Rajapakse and Karasudhi (1985) for spherical coordinate systems by using explicit displacement interpolation functions. They proved that the proposed dynamic infinite element is computationally efficient and sufficiently accurate through a series of numerical solutions for wave problems. Burnett (1994) developed a three-dimensional time-harmonic acoustic infinite element based on a multipole expansion for scattered and/or radiated fields. This element was implemented in a finite element program and validated against wave propagation problems. It was successfully used to handle a full range of frequencies and geometric complexities. An extension to this formulation was later presented for ellipsoidal coordinate systems with two angular and one radial coordinates (Burnett, 1998). Astley (1996) proposed a new set of two-dimensional and three-dimensional transient infinite elements based on Fourier transform of the wave envelop approach applied to steady state wave problems. It consists of a set of linear, second order, coupled differential equations. The formulation
produced predictions in wave propagation with a reasonable accuracy when compared to extended mesh solutions.

Liu and Jerry (2003) proposed a finite element approach to analyse wave propagation in an infinitely long plate. They used a set of elements beyond the region of interest with gradually increasing damping properties to diminish the wave reflection. The method requires analysing the problem several times to arrive at a proper boundary condition. Though the proposed boundary condition showed reasonable attenuation of wave propagation, being tedious and the computationally expensive makes it less desirable in complex analyses.

Seo et al. (2007) developed a set of three-dimensional elasto-dynamic infinite elements for soil-structure interaction problems in multi-layered half space. It consists of infinite elements simulating horizontal, horizontal-corner, vertical, vertical-corner and vertical–horizontal-corner elements along the artificial boundary. The predictions made by above infinite elements in various wave propagation problems were in good agreement with the data presented in the literature. They concluded that the proposed infinite element can be effectively used in wave propagation problems such as earthquakes, blasts and machine vibrations.

Falletta and Monegato (2014) proposed an exact non-reflecting boundary condition for two-dimensional time-dependent wave propagation problems using the boundary integral formulations by means of finite element or finite difference methods. The authors concluded that the proposed non-reflecting boundary condition consumes the computational power almost on par with local boundary conditions, although the proposed condition was non-local in time and space. However, they did not prove that the proposed boundary condition is mathematically stable and has the ability to give a solution without convergence issues.

The above discussed wave transmitting boundaries are capable of absorbing the waves travelling towards the boundary without reflecting them back to the finite domain of interest. However, implementation of them in finite element programs are not straight
forward compared to local wave transmitting boundaries discussed in the following section. Therefore, application of non-local wave transmitting boundaries in solving boundary value problems is limited.

### 3.3 Local wave transmitting boundaries

Lysmer and Kuhlemeyer (1969) developed a viscous local wave transmitting boundary, which consists only of a dashpot configuration for both shear and dilation waves. The stresses in the path of the plane waves at a particular point are considered as proportional to the velocities of the waves. The accuracy of the solution diminishes as the boundary moves towards the source. The boundary condition is independent of frequency and can be easily implemented in a time domain finite element analysis. The boundary condition proposed by Lysmer and Kuhlemeyer to replace the infinite domain with an artificial boundary can be expressed as,

\[ \sigma = a \rho V_p \dot{w} \]  \hspace{1cm} (3.2)

\[ \tau = b \rho V_s \dot{u} \]  \hspace{1cm} (3.3)

where \( \sigma \) is the normal stress, \( \tau \) is the shear stress, \( \dot{w} \) is the normal velocity, \( \dot{u} \) is the tangential velocity and \( \rho \) is the material density. \( V_s \) and \( V_p \) are shear and dilation wave velocities respectively and \( a \) and \( b \) are dimensionless parameters.

The local non-reflecting boundary condition developed by Deeks and Randolph (1994) for cylindrical shear and dilation wave propagation is the next advancement in local wave transmitting boundaries. The boundary, based on an approximation of the form of axisymmetric waves, is established by frequency independent models having a linear spring and a linear viscous dashpot for shear wave propagation and an extra mass element attached to the end of dashpot in the case of dilation wave propagation. These boundary conditions are illustrated in Figures 3.1(a) and (b), respectively for shear and dilation waves. The spring \( k \), dashpot \( c \) and mass \( m \) coefficients for these boundary conditions are given in Table 3.1, where \( G, \rho, c_s, c_d \) and \( r_b \) are shear modulus, density, shear wave velocity, dilation wave velocity of the medium and the radial distance to the
boundary, respectively. They concluded that the proposed boundary condition is more accurate than either the viscous boundaries proposed by Lysmer and Kuhlemeyer (1969) or the doubly asymptotic boundaries proposed by Underwood and Geers (1981).

![Diagram](image)

**Figure 3.1** Axisymmetric boundary condition proposed by Deeks and Randolph (1994) for;
(a) Shear Wave and (b) Dilation Wave.

<table>
<thead>
<tr>
<th></th>
<th>Shear</th>
<th>Dilation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>$G/2r_b$</td>
<td>$2G/r_b$</td>
</tr>
<tr>
<td>$c$</td>
<td>$\rho c_s$</td>
<td>$\rho c_d$</td>
</tr>
<tr>
<td>$m$</td>
<td>$\rho r_b$</td>
<td>$2\rho r_b$</td>
</tr>
</tbody>
</table>

Table 3.1 Wave transmitting boundary coefficients for Deeks and Randolph (1994) model.

A local time-domain transmitting boundary with extra complexity was proposed by Du and Zhao (2010) for axisymmetric and plane-strain wave propagation problems. It consisted of a spring and a dashpot, connected to the same node in parallel, representing the current outgoing wave. Another set of dashpot and mass elements connected to each other in series is attached to the same node to take into account the previous outgoing wave. The boundary element arrangement at a given node is illustrated in Figure 3.2.
Figure 3.2 Local time domain transmitting boundary condition proposed by Du and Zhao (2010) for two-dimensional elastic wave propagation.

The dimensionless spring \((k_0)\), dashpot \((c_l)\) and mass \((m_l)\) coefficients, independent of the geometry and the physical properties of the far field, are given as,

\[
K_0 = S_0 k_0 \quad (3.4a)
\]
\[
C_l = \rho c c_l \quad l = 0, \ldots, N \quad (3.4b)
\]
\[
M_l = \rho R m_l \quad l = 1, \ldots, N \quad (3.4c)
\]

where, \(\rho\) is mass density of the far field, \(R\) is the radial distance to the boundary, \(S_0\) is the static stiffness and \(c\) is either shear \((c_s)\) or dilation \((c_d)\) wave velocity, as shown in Table 3.2. The dimensionless coefficients for dilation wave transmitting boundaries with order \(N=3\), which gives the most accurate solution, are given in Table 3.3.

The dimensionless coefficients for horizontal shear wave boundary with order \(N=4\), which gives the most accurate solution, are given in Table 3.3. This artificial boundary has higher order accuracy for axisymmetric elastic wave propagation and can be implemented in a finite element analysis.

Table 3.2 Wave transmitting boundary coefficients for Du and Zhao (2010) model.

<table>
<thead>
<tr>
<th></th>
<th>Shear</th>
<th>Dilation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring constant</td>
<td>((G/2r_b)k_l)</td>
<td>((2G/r_b)k_l)</td>
</tr>
<tr>
<td>Dashpot constant</td>
<td>((\rho c_s)c_l)</td>
<td>((\rho c_d)c_l)</td>
</tr>
<tr>
<td>Mass constant</td>
<td>((\rho r_b)m_l)</td>
<td>((\rho r_b)m_l)</td>
</tr>
</tbody>
</table>
Table 3.3 Coefficients $k_l, c_l$ and $m_l$ for shear ($N = 4$) and dilation ($N = 3$) wave transmitting boundaries.

<table>
<thead>
<tr>
<th>$n$</th>
<th>Spring coefficient ($k_n$)</th>
<th>Dashpot coefficient ($c_n$)</th>
<th>Mass coefficient ($m_n$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shear</td>
<td>Dilation</td>
<td>Shear</td>
</tr>
<tr>
<td>0</td>
<td>0.000</td>
<td>1.000</td>
<td>1.355 $\times 10^{-2}$</td>
</tr>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.000</td>
<td>-1.345 $\times 10^{-4}$</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>0.000</td>
<td>-8.705 $\times 10^{-3}$</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>0.000</td>
<td>-4.531 $\times 10^{-3}$</td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>-9.432 $\times 10^{-4}$</td>
<td></td>
</tr>
</tbody>
</table>

The viscous spring boundary condition discussed above is proposed with $n=0$ for two-dimensional plane-strain wave problems and does not contain mass elements as in the axisymmetric boundary discussed before. Instead, the dimensionless coefficient for spring element ($k_0$) was proposed to be 0.5 and 0.25 for dilation and shear wave boundaries, respectively. The coefficient for dashpot ($c_0$) is 1.0 for both cases. When calculating spring constant ($S_0$), $r_s = \sqrt{2}X$ is used, where $X$ is the distance to the boundary in the $X$ direction. This wave transmitting boundary is approximate but for the two-dimensional plane-strain elastic wave propagation problems it gives a solution of acceptable accuracy. In the following section, performance of four wave transmitting boundaries discussed in this section is investigated for axisymmetric and two-dimensional plane-strain wave propagation problems.

### 3.4 Numerical Examples

Performance of four wave transmitting boundaries discussed above is investigated in this section: (i) viscous boundary by Lysmer and Kuhlemeyer (1969), (ii) axisymmetric wave transmitting boundary by Deeks and Randolph (1994), (iii) axisymmetric and plane-strain wave transmitting boundaries by Du and Zhao (2010) and (iv) wave transmitting boundary based on infinite elements in ABAQUS/Explicit. A model with a large mesh sufficient to extract results at the observation point during the analysis time without interruptions from reflecting waves was simulated to verify the accuracy of the selected boundary conditions.
3.4.1 Axisymmetric wave propagation

An annular disk with a thickness of 0.1 m and an internal radius of 1 m was considered when investigating the axisymmetric elastic wave propagation. A dilation wave was generated in the disc by applying a pressure load over the inner surface of the annular disk and a shear wave was generated by applying a traction load along the inner surface. The density of the disk was considered to be 2000 kg/m$^3$ and it was assigned a shear modulus of 45 MPa and a Poisson’s ratio of 0.45. An artificial boundary truncated the annular disk at 2 m radius. Finite element mesh for the disk was created using eight-node continuum elements with side lengths of 0.1 m.

The efficiency of the wave transmitting boundaries is determined using an extended mesh of the same annular disk with internal radius of 1 m. Based on the properties assigned to the disk, shear wave velocity is 150 m/s and the dilation wave velocity is 497.5 m/s. Therefore, the external radius of the extended mesh was placed at 6 m for shear wave propagation and at 24 m for dilation wave propagation. This will facilitate to extract the response at the observation point, which is at the internal radius, without any disturbance from reflecting waves from the outer radius over a period of 0.08 sec. A schematic diagram of the axisymmetric problem is shown in Figure 3.3 for the shear wave propagation. A triangular pulse load of 500 kN as shown in Figure 3.4 is applied at the inner radius of the annular disk.

![Figure 3.3 Schematic diagram of the axisymmetric finite element model.](image-url)
Chapter 3  
Wave transmitting boundaries used in numerical modelling

Wave transmitting boundaries for shear and dilation wave propagation were investigated separately applying the traction and pressure loads, respectively, for Lysmer and Kuhelemeyer (1969), Deeks and Randolph (1994) and Du and Zhao (2010) wave transmitting boundaries. In addition, the infinite element in ABAQUS/Explicit was investigated. When using the infinite element in ABAQUS/Explicit, the artificial boundary shown in Figure 3.3 was replaced by an infinite element. This element is similar to the infinite element proposed by Zienkiewicz et al. (1983) but for the dynamic analyses, dashpots are attached automatically to the far end nodes of the infinite element similar to Equations 3.2 and 3.3 for the Lysmer and Kuhelemeyer (1969) and Deeks and Randolph (1994). Du and Zhao (2010) boundary condition was implemented with an order of $N=4$ for the shear waves and order of $N=3$ for the dilation waves. Since ABAQUS/Explicit does not allow negative dashpot and mass elements, Du and Zhao boundary is analysed using implicit time integration scheme for dynamic problems available in ABAQUS/Standard. Other three boundaries were analysed using explicit time integration scheme in ABAQUS/Explicit.

Figure 3.5 shows the performance of four wave transmitting boundary conditions considered in the finite element analysis investigating the shear wave propagation. Figure 3.6 shows the results from same four boundaries for the dilation wave propagation.

![Amplitude vs. Time history of the dynamic load.](image)

**Figure 3.4 Amplitude vs. Time history of the dynamic load.**
Displacements at the inner surface of the annular disk from each boundary condition are compared with the displacements extracted from the extended mesh to determine the accuracy.

![Graph showing displacement vs. time history for shear wave propagation.](image)

**Figure 3.5 Displacement vs. time history for shear wave propagation.**

For shear wave propagation, Du and Zhao (2010) boundary condition predicts the displacements at the observation point in agreement with the extended mesh. Deeks and Randolph (1994) boundary condition predicts the maximum displacement accurately but under predicts the displacement beyond the peak displacement due to shear wave propagation. Infinite elements (ABAQUS, 2011) and Lysmer and Kuhlemeyer (1969) boundaries has the ability to predict the maximum displacement but a rigid body motion is apparent beyond the peak displacement.
Displacements during dilation wave propagation predicted by the Du and Zhao (2010) boundary condition agrees well with the displacements from the extended mesh. Deeks and Randolph (1994) wave transmitting boundary also shows good agreement with displacements from the extended mesh during the dilation wave propagation. The viscous boundary condition by Lysmer and Kuhlemeyer (1969) and the infinite elements (ABAQUS, 2011) show similar pattern in their prediction of displacements. Both boundaries under predict the peak displacement and reduce to zero afterwards, deviating from the displacements predicted by the extended mesh.

These results clearly show that for cylindrical shear and dilation wave propagation, Du and Zhao (2010) boundary is the most efficient. However, when applying this model, no extra nodes are required only if the order $N=0$. When $N>0$, extra nodes need to be created in the finite element mesh to add the additional mass and dashpot elements. In some finite element programs, these additional nodes need to be added manually making application of this model with a higher order, such as $N=3$ or $N=4$ for higher order of accuracy, becomes not practical. Therefore, Deeks and Randolph (1994) boundary can be considered as the most efficient wave transmitting boundary for axisymmetric shear and
dilation wave propagations. Predictions from this boundary agree well with the extended mesh at peak although they deviate slightly beyond the peak.

3.4.2 Two-dimensional plane-strain wave propagation

Plane-strain wave propagation is investigated using a mesh with dimensions of \(2 \text{ m} \times 1 \text{ m}\). A dynamic load of 500 kN (as shown in Figure 3.4) was applied at the centre of the mesh to generate waves in the medium. The finite domain was assigned a density of 2000 kg/m\(^3\), a shear modulus of 45 MPa and a Poisson’s ratio of 0.25. Due to the symmetry of the problem, only a half of the model is considered during the analysis, which gives mesh dimensions of \(x = y = 1 \text{ m}\). Eight node continuum elements with offside widths of 0.1 m were used to create the mesh. Along the vertical boundary, shear and dilation wave transmitting boundaries were attached to vertical and horizontal degrees of freedom along the boundary respectively. Along the bottom boundary shear and dilation wave transmitting boundaries were attached to horizontal and vertical degrees of freedom along the boundary respectively. An extended mesh with dimensions of \(x = y = 13 \text{ m}\) was simulated to verify the accuracy of the considered boundary conditions. Figure 3.7 shows the schematic diagram of the plane-strain finite element model used for this analysis. The location of the dynamic load is selected as the observation point to extract the displacement predictions from each analysis. Using this mesh, dilation and shear wave propagation can be investigated over a period of 0.052 s and 0.172 s, respectively, before the reflected wave travels to the observation point. Therefore, displacement predictions at the observation point by various boundary conditions are shown only up to 0.05 s in Figure 3.8. The viscous boundary condition (Lysmer and Kuhlemeyer, 1969) demonstrates a rigid body motion during the analysis. It is closely followed by displacements predicted by the infinite element. Nevertheless, Du and Zhao (2010) boundary condition predicts the displacements very close to the displacements given by the extended mesh. Deeks and Randolph (1994) boundary condition, which is originally derived for axisymmetric wave propagation, does not predict the plane-strain wave propagation accurately. However, it is better than the
infinite element and the viscous boundary in absorbing the waves propagating in the medium.

![Schematic diagram of the plane-strain finite element model](image)

**Figure 3.7** Schematic diagram of the plane-strain finite element model.

![Vertical displacement vs. time history for plane-strain wave propagation](image)

**Figure 3.8** Vertical displacement vs. time history for plane-strain wave propagation.
3.5 Conclusions

A comprehensive review of both local and non-local wave transmitting boundaries found in the literature is presented. Several local and non-local wave transmitting boundaries are then simulated using the finite element method. The displacement predictions are compared with an extended mesh solution, in which the mesh is extended far away from the observation point to avoid the effect of reflected waves on the displacements recorded at the observation point.

Results show that the wave transmitting boundary proposed by Du and Zhao (2010) is the most appropriate one for axisymmetric wave propagation problems. However, extra nodes are necessary to accommodate this boundary condition in a finite element model. Therefore, the axisymmetric wave transmitting boundaries proposed by Deeks and Randolph can be considered as the most practicable boundary which can be applied to get results with a reasonable accuracy for axisymmetric wave propagation problems.

For plane-strain wave propagation problems, Du and Zhao boundary condition outperformed Deeks and Randolph (1994) boundary condition, which is developed only considering axisymmetric wave propagation. Even though the viscous boundary proposed by Lysmer and Kuhlemeyer (1969) is the commonly used boundary condition in finite element analysis of wave propagation problems, it shows a rigid body motion. Infinite element in ABAQUS/Explicit also shows a rigid body motion, compared to the displacements from the case with extended mesh.
Chapter 4

Numerical simulation of EPS geofoam behaviour

4.1 Introduction

Expanded Polystyrene (EPS) geofoam has a special place among traditional geomaterials in modern day construction industry due to its light weight, high strength to weight ratio, volume contraction under deviatoric compressive loading, strain hardening beyond yielding and ease of use. EPS geofoam has been widely used in embankment construction, slope stabilization, retaining walls, bridge approaches and abutments. Nevertheless, the potential of EPS geofoam as an engineering material in geotechnical applications has not been fully realised yet. In this Chapter, the finite element formulation of a constitutive model based on the hardening plasticity, which has the ability to simulate short term behaviour of EPS geofoam, is presented to predict the mechanical behaviour of EPS geofoam and it is implemented in the finite element program ABAQUS. The finite element formulation is verified using triaxial test data found in the literature (Wong and Leo 2006 and Chun et al. 2004) for two varieties of EPS geofoam. Performance of the constitute model is compared with four other models found in the literature to confirm its ability to simulate the short term behaviour of EPS geofoam with sufficient accuracy.
4.2 Constitutive model for EPS geofoam

The constitutive model presented here is developed by Wong and Leo (2006), briefly described in Chapter 2. It incorporates classical plasticity with strain hardening in order to correctly describe the post yield behaviour of EPS geofoam. The model is developed within the framework of the Drucker Prager yield criterion based on the non-associated flow rule.

The yield function, \( f(\sigma, a) \), and the plastic potential function, \( g(\sigma) \), for the constitutive model are given by,

\[
f(\sigma, a) = \frac{1}{1-b}(q - 3bp - a) = 0 \quad (4.1)
\]
\[
g(\sigma) = \frac{1}{1-c}(q - 3cp) = 0 \quad (4.2)
\]

where \( q \) is the deviatoric stress and \( p \) is the mean stress. In a three-dimensional stress space, they are defined as,

\[
q = \frac{1}{\sqrt{2}} \left[ (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\tau_{xy}^2 + 6\tau_{yz}^2 + 6\tau_{zx}^2 \right]^{1/2} \quad (4.3)
\]
\[
p = \frac{\sigma_x + \sigma_y + \sigma_z}{3} \quad (4.4)
\]

where \( \sigma_x \), \( \sigma_y \) and \( \sigma_z \) are respectively the normal stress components in \( x \), \( y \) and \( z \) directions and \( \tau_{xy} \), \( \tau_{yz} \) and \( \tau_{zx} \) are the shear stress components. Parameters \( a, b \) and \( c \) can be extracted from drained triaxial test data and will be discussed in Section 4.5. They are related to three rheological parameters \( R, K_p \) and \( K \) as shown below,

\[
a = \frac{3R}{K_p + 2} \quad (4.5a)
\]
\[
b = \frac{K_p - 1}{K_p + 2} \quad (4.5b)
\]
\[ c = \frac{K - 1}{K + 2} \quad (4.5c) \]

where \( R \) is the hardening variable which increases with the accumulated plastic deviatoric strain. \( K_p \) is a constant and defines the size of the yield surface and \( K \) is a parameter, which depends on the confining pressure, and defines the flow rule and dilatancy behaviour of EPS geofoam. The volumetric strain is negative if \( K > 1 \), implying the dilatancy, whilst the volumetric strain is positive if \( K < 1 \) implying the contractancy of EPS geofoam. \( R \) varies linearly with respect to the equivalent deviatoric plastic strain tensor, \( \varepsilon_q^p \), which is given by,

\[ R(\varepsilon_q^p) = R_0 + \beta \varepsilon_q^p \quad (4.6) \]

\[ \varepsilon_q^p = \int_0^T d\varepsilon_q^p \quad (4.7a) \]

\[ d\varepsilon_q^p = \sqrt{\frac{1}{2} \{d\varepsilon_q^p\}^T \{d\varepsilon_q^p\}} \quad (4.7b) \]

\[ \{d\varepsilon_q^p\} = \{d\varepsilon_q^p\} - \frac{1}{3} d\varepsilon_q^p \{I\} \quad (4.7c) \]

\[ \{d\varepsilon_q^p\} = \begin{bmatrix} d\varepsilon_x^p \\ d\varepsilon_y^p \\ d\varepsilon_z^p \\ d\varepsilon_y^p \\ d\varepsilon_z^p \\ d\varepsilon_z^p \end{bmatrix} \quad (4.7d) \]

\[ \{I\} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \end{bmatrix} \quad (4.7e) \]

where \( \beta \) is the parameter defining rate of hardening, \( R_0 \) is the initial value of \( R \) before material starts yielding, \( T \) in Equation (4.7a) is the time duration of the analysis, and \( t \) in Equation (4.7b) is the transpose of the matrix and \( d\varepsilon_v^p \) is the plastic volumetric strain.
$d\varepsilon^p_x$, $d\varepsilon^p_y$, $d\varepsilon^p_z$, are respectively the plastic normal strain increments in $x$, $y$ and $z$ directions, and $d\varepsilon^p_{xy}$, $d\varepsilon^p_{yz}$, $d\varepsilon^p_{zx}$ are respectively the plastic shear strain increments. $d\varepsilon^p_v$, in Equation (4.7c) is defined as,

$$d\varepsilon_v^p = d\varepsilon_x^p + d\varepsilon_y^p + d\varepsilon_z^p$$  

(4.8)

### 4.3 Finite element formulation

The finite element formulation of the constitutive model discussed in Section 4.2 is outlined in this section.

The total strain matrix for elasto-plastic material, $\{d\varepsilon\}$, is given by,

$$\{d\varepsilon\} = \{d\varepsilon^e\} + \{d\varepsilon^p\}$$  

(4.9)

The stress increment, $d\sigma^e$, can be represented in terms of elastic strain increment as,

$$\{d\sigma\} = [C_e]\{d\varepsilon^e\}$$  

(4.10)

where $[C_e]$, the elastic stiffness matrix in general stress space, is given by,

$$[C_e] = \frac{E}{(1-2\nu)(1+\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$  

(4.11)

where $E$ is the Young’s modulus and $\nu$ is the Poisson’s ratio.

Based on Equations (4.1) and (4.5a), $a$ in Equation (4.1) is a function of $R$. Hence the yield function can be written as,

$$f(\sigma,R) = 0$$  

(4.12)

An incremental change in the yield function is given by,
\[ df = \left\{ \frac{\partial f}{\partial \sigma} \right\}^t \{d\sigma\} + \frac{\partial f}{\partial R} dR = 0 \] (4.13a)

\[ \{d\sigma\} = \begin{pmatrix} d\sigma_x \\ d\sigma_y \\ d\sigma_z \\ d\tau_{xy} \\ d\tau_{yz} \\ d\tau_{zx} \end{pmatrix} \] (4.13b)

where \( d\sigma_x \), \( d\sigma_y \) and \( d\sigma_z \), are respectively the normal stress increments in \( x \), \( y \) and \( z \) directions, and \( d\tau_{xy} \), \( d\tau_{yz} \), \( d\tau_{zx} \) are respectively the shear stress increments.

Therefore, from Equations (4.9), (4.10) and (4.13),

\[ \left\{ \frac{\partial f}{\partial \sigma} \right\}^t \left( [C_e] \{d\epsilon\} - [C_e] \{d\epsilon^p\} \right) + \frac{\partial f}{\partial R} dR = 0 \] (4.14)

The relationship between plastic strain increment, \( \{d\epsilon^p\} \), and the plastic potential, \( g(\sigma) \), is given by,

\[ \{d\epsilon^p\} = \lambda \left( \frac{\partial g}{\partial \sigma} \right) \] (4.15)

where \( \lambda \) is the plastic scaling factor. The term \( \frac{\partial f}{\partial R} dR \) in Equation (4.14) can be rewritten as shown below using Equation (6),

\[ dR = \beta d\epsilon_q^p \] (4.16a)

\[ d\epsilon_q^p = \lambda \frac{\partial G}{\partial q} \] (4.16b)

\[ \frac{\partial f}{\partial R} dR = \frac{\partial f}{\partial R} \beta \lambda \frac{\partial G}{\partial q} \] (4.16c)

By substituting Equations (4.15) and (4.16c) in Equation (4.14), \( \lambda \) becomes,
Combining Equations (4.14), (4.15), (4.16c) and (4.17), the stress increment, \( \{d\sigma\} \), can now be written as,

\[
\{d\sigma\} = \{C_e\} [d\varepsilon] - \lambda \left[ \frac{\partial f}{\partial \sigma} \right]' \left[ \frac{\partial g}{\partial \sigma} \right]' \left[ C_e \right] \left[ C_e \right] [d\varepsilon] - \left[ \frac{\partial f}{\partial \sigma} \right]' \left[ C_e \right] \left[ \frac{\partial g}{\partial \sigma} \right] - \frac{\partial f}{\partial R} \beta \frac{\partial G}{\partial q}
\]

Finally, the elastoplastic tangent matrix, \( [C_{ep}] \), can be expressed as,

\[
[C_{ep}] = \{C_e\} - \lambda \left[ \frac{\partial f}{\partial \sigma} \right]' \left[ \frac{\partial g}{\partial \sigma} \right]' \left[ C_e \right] \left[ C_e \right] - \frac{\partial f}{\partial R} \beta \frac{\partial G}{\partial q}
\]

### 4.4 Implementation of the constitutive model in ABAQUS/Explicit

The finite element formulation presented in Section 4.3 is implemented in ABAQUS/Explicit (ABAQUS, 2012) finite element programme using the user defined material subroutine, VUMAT, which is developed using the explicit integration scheme according to the requirements of the programme. The subroutine will be called for each block of material calculation points (Gauss integration points) during the analysis for which the material is defined. Within the subroutine, the stress increments and the stress state are calculated for Gauss integration points. This is repeated for each time step of the analysis.

The model parameter \( a \) is specified as a solution dependant state variable at the beginning of the first time step. This will be updated after the EPS geofoam reaches yielding, as given in Equations (4.5a), (4.6) and (4.7).
At the beginning of each time step, $t_n$, the final stress of the previous step, $t_{n-1}$, is taken as the initial stress for the current step. Thus,

$$\{\sigma_{n,\text{final}}\} = \{\sigma_{n-1}\}$$  \hspace{1cm} (4.20)

The strain increments corresponding to time step $t_n$ calculated within the main program are transferred to the subroutine. Using these strain increments, stress increments are calculated within the subroutine. Based on the initial stress state at the beginning of the time step, the yield condition is checked to see whether the material is yielding. If,

$$f(\sigma_{n,\text{initial}}) > 0$$  \hspace{1cm} (4.21)

then the material has reached the plastic region as the initial stress state lies outside the yield surface. If the initial stress state lies within the elastic region, the subroutine calculates the stress increment during the time step using the strain increments as shown in Equation (4.10), using the elastic tangent stiffness matrix given in Equation (4.11), and directs the stress state at the end of time $t_n$ back to the main programme.

If the material is yielding at the beginning of time step $t_n$ with current stress state, the consistency parameter, $\lambda$, is calculated using Equation (4.17),

$$\lambda = \frac{\{\partial f / \partial \sigma\} \{C_e \partial \varepsilon_{n,\text{initial}}\}}{\{\partial f / \partial \sigma\} \{C_e \partial \varepsilon_{n,\text{initial}}\} - \partial f / \partial R \beta \partial G / \partial q}$$  \hspace{1cm} (4.22)

In the above equation, $\partial f / \partial R = \partial f / \partial a * \partial a / \partial R$ and by combining Equations (4.1), (4.2) and (4.5) the second term in the denominator can be written as,

$$\frac{\partial f}{\partial R} \beta \frac{\partial G}{\partial q} = -\beta \frac{2 + K}{3}$$  \hspace{1cm} (4.23)

where $\partial f / \partial a = -1 / (1 - b) = \frac{(2 + K_p)}{3}$, $\partial a / \partial R = \frac{3}{2 + K_p}$ and $\partial G / \partial q = \frac{1}{(1 - c)} = \frac{2 + K}{3}$. 


\[
\left\{ \frac{\partial f}{\partial \sigma} \right\} \text{ and } \left\{ \frac{\partial g}{\partial \sigma} \right\} \text{ in Equation (4.22) are two column matrices given by,}
\]

\[
\left\{ \frac{\partial f}{\partial \sigma} \right\} = 
\begin{bmatrix}
\frac{\partial f}{\partial p} + \frac{\partial f}{\partial q} \\
\frac{\partial f}{\partial \sigma_x} \\
\frac{\partial f}{\partial \sigma_y} \\
\frac{\partial f}{\partial \sigma_z} \\
\frac{\partial f}{\partial \tau_{xy}} \\
\frac{\partial f}{\partial \tau_{yz}} \\
\frac{\partial f}{\partial \tau_{zx}} \\
\end{bmatrix}
\]

\[
\left\{ \frac{\partial g}{\partial \sigma} \right\} = 
\begin{bmatrix}
\frac{\partial g}{\partial p} + \frac{\partial g}{\partial q} \\
\frac{\partial g}{\partial \sigma_x} \\
\frac{\partial g}{\partial \sigma_y} \\
\frac{\partial g}{\partial \sigma_z} \\
\frac{\partial g}{\partial \tau_{xy}} \\
\frac{\partial g}{\partial \tau_{yz}} \\
\frac{\partial g}{\partial \tau_{zx}} \\
\end{bmatrix}
\]

\[
\left( \frac{1-K_p}{3} + \frac{2+K_p}{3} \right) \left( 2\sigma_x - \sigma_y - \sigma_z \right) \\
\left( \frac{1-K_p}{3} + \frac{2+K_p}{3} \right) \frac{2q}{3} \\
\left( \frac{1-K_p}{3} + \frac{2+K_p}{3} \right) \left( 2\sigma_y - \sigma_x - \sigma_z \right) \\
\left( \frac{1-K_p}{3} + \frac{2+K_p}{3} \right) \frac{2q}{3} \\
\left( \frac{1-K_p}{3} + \frac{2+K_p}{3} \right) \left( 2\sigma_z - \sigma_x - \sigma_y \right) \\
\left( \frac{1-K_p}{3} + \frac{2+K_p}{3} \right) \frac{2q}{3} \\
\left( \frac{1-K_p}{3} + \frac{2+K_p}{3} \right) \left( 3\tau_{xy} \right) \\
\left( \frac{1-K_p}{3} + \frac{2+K_p}{3} \right) \frac{3q}{3} \\
\left( \frac{1-K_p}{3} + \frac{2+K_p}{3} \right) \left( 3\tau_{yz} \right) \\
\left( \frac{1-K_p}{3} + \frac{2+K_p}{3} \right) \frac{3q}{3} \\
\left( \frac{1-K_p}{3} + \frac{2+K_p}{3} \right) \left( 3\tau_{zx} \right) \\
\left( \frac{1-K_p}{3} + \frac{2+K_p}{3} \right) \frac{3q}{3} \\
\end{bmatrix}
\]

(4.24)

(4.25)

The plastic strain increment, \( \Delta \varepsilon_p^n \), for current step, \( t_n \), can be derived as,

\[
\left\{ \Delta \varepsilon_p^n \right\} = \lambda \left\{ \frac{\partial g}{\partial \sigma} \right\}
\]

(4.26)

Then, the elastic strain increment, \( \Delta \varepsilon_e^n \), can be calculated as,

68
\[ \{ \Delta \varepsilon^e_n \} = \{ \Delta \varepsilon_n \} - \{ \Delta \varepsilon_p^n \} \]  

Then the stress increment in the current step, \( \Delta \sigma_n \), is given by,

\[ \{ \Delta \sigma_n \} = \{ C_n \} \{ \Delta \varepsilon_n \} \]  

Thus, the final stress at the end of the \( n^{th} \) step, \( \sigma^\text{final}_n \), is,

\[ \{ \sigma^\text{final}_n \} = \{ \sigma^\text{final}_{n-1} \} + \{ \Delta \sigma_n \} \]  

The plastic deviatoric strain increment, \( \Delta \varepsilon^p_{q,n} \), is derived as,

\[ \{ \Delta \varepsilon^p_{q,n} \} = \frac{1}{2} \{ \Delta e^p_{n} \} \cdot \{ \Delta e^p_{n} \} \]  

where

\[ \{ \Delta e^p_{n} \} = \{ \Delta e^p_{n} \} - \frac{1}{3} \{ \Delta \varepsilon^e_{y,n} \} \]  

At the end of the time step, parameter \( a \) should be updated. This value of \( a_n \) will be used to calculate \( \lambda \) in time step \( t_{n+1} \). From Equations (4.5a) and (4.6),

\[ a_n = a_{n-1} + \Delta a_n \]  

where

\[ \Delta a_n = \frac{3}{K_p + 2} \beta \{ \Delta e^p_{q,n} \} \]  

Intel Visual Fortran was used to write a user defined subroutine, VUMAT, to implement the constitutive model for EPS geofoam in the finite element programme. Subroutine is given in Appendix A.

### 4.5 Extracting material properties for the constitutive model

Material properties for the model can be extracted using a series of drained triaxial tests carried out on EPS geofoam samples at different confining pressures (at least three
different confining pressures). During elastic loading of the triaxial sample, the change in deviatoric, $d\varepsilon_q$, and volumetric, $d\varepsilon_v$, strains can be related to $dp$ and $dq$ as follows,

$$
\begin{bmatrix}
dp \\
\frac{dq}{d\varepsilon_q}
\end{bmatrix} = \begin{bmatrix}
K_o & 0 \\
0 & 3G
\end{bmatrix} \begin{bmatrix}
d\varepsilon_v \\
d\varepsilon_q
\end{bmatrix}
$$

(4.33)

where $K_o$ is the bulk modulus and $G$ is the shear modulus of the EPS geofoam, which are related to $E$ and $\nu$. During a triaxial test, cell pressure, $\sigma_3$, does not change and only axial stress, $\sigma_1$, changes. Hence, Equation (4.33) can be written as shown below,

$$
\begin{bmatrix}
d\sigma_3 \\
\frac{d\sigma_1}{3}
\end{bmatrix} = \begin{bmatrix}
\frac{E}{3(1-2\nu)} & 0 \\
0 & \frac{3E}{2(1+\nu)}
\end{bmatrix} \begin{bmatrix}
d\varepsilon_1 + 2d\varepsilon_3 \\
\frac{2}{3}(d\varepsilon_1 - d\varepsilon_3)
\end{bmatrix}
$$

(4.34)

By solving Equation (4.34), following two relationships can be obtained,

$$
\frac{d\sigma_3}{d\varepsilon_1} = E
$$

(4.35a)

$$
\frac{d\varepsilon_v}{d\varepsilon_1} = (1-2\nu)
$$

(4.35b)

Using the elastic gradient of graphs of $\sigma_1$ vs. $\varepsilon_1$ and $\varepsilon_v$ vs. $\varepsilon_1$ (similar to initial gradient of Figure 4.2), $E$ and $\nu$ of EPS geofoam can be obtained using Equations (4.35a) and (4.35b).

The two parameters $R_o$ and $K_p$ can be obtained using Yield function given by Equation (4.1). For stress conditions during triaxial tests, the Yield function is given by,

$$
a = q - 3bp = \sigma_1(1-b) - \sigma_3(1+2b)
$$

(4.36)

After substituting $a$ and $b$ given in Equations (4.5a) and (4.5b), Equation (4.36) is given by,

$$
\sigma_{1y} = K_p\sigma_3 + R_o
$$

(4.37)
where \( R_0 \) is the initial value of \( R \) in Equation (4.6), and \( \sigma_{1y} \) is the value of \( \sigma_1 \) when the material starts to yield (the point at which initial gradient changes in the graph of \( \sigma_1 \) vs. \( \varepsilon_1 \)). Equation (4.37) represents a straight line in a graph of \( \sigma_{1y} \) vs. \( \sigma_3 \), where intercept is \( R_0 \) and the gradient is \( K_p \). This graph can be drawn with data from minimum of three triaxial tests carried out at different confining pressures, \( \sigma_3 \).

Now the unknown parameters in the constitutive model are only \( K \) and \( \beta \). When the material starts yielding, the stress strain relationship for the EPS geofoam sample is given by,

\[
\begin{align*}
\begin{bmatrix}
\frac{dp}{dq} \\
\frac{d\varepsilon_y}{d\varepsilon_y}
\end{bmatrix} &= \begin{bmatrix}
C_{ep}
\end{bmatrix}
\begin{bmatrix}
\frac{d\varepsilon_y}{d\varepsilon_y}
\end{bmatrix} \\
&= \begin{bmatrix}
3(G + HA)K_o \\
9bGK_o
\end{bmatrix} + \begin{bmatrix}
2 \varepsilon_1 - 2 \varepsilon_3 \\
2 \varepsilon_1 - 2 \varepsilon_3
\end{bmatrix}
\end{align*}
\]

(4.38)

where \( C_{ep} \) is given by Equation (4.19) and by substituting \( C_{ep} \) in Equation (4.38), stress-strain relationship can be written as shown below,

\[
\begin{align*}
\begin{bmatrix}
\frac{d\sigma_1}{3} \\
\frac{d\sigma_1}{3}
\end{bmatrix} &= \begin{bmatrix}
3(G + HA)K_o \\
9bGK_o
\end{bmatrix} \begin{bmatrix}
2 \varepsilon_1 - 2 \varepsilon_3
\end{bmatrix} + \begin{bmatrix}
2 \varepsilon_1 - 2 \varepsilon_3 \\
2 \varepsilon_1 - 2 \varepsilon_3
\end{bmatrix}
\end{align*}
\]

(4.39)

where \( H \) is the hardening parameter given by Equation (4.23), and \( A \) and \( B \) are given by,

\[
B = \frac{9bK_o + 3G}{A} + H
\]

(4.40a)

\[
A = (1 - b)(1 - c)
\]

(4.40b)

According to Wong and Leo (2006), the Poisson’s ratio for EPS geofoam is nearly zero. Therefore, elastic shear modulus, \( G = E/2 \), and elastic bulk modulus, \( K_o = E/3 \). By solving Equation (4.39), following two equations can be obtained for the elastoplastic gradients for the graphs of \( \sigma_1 \) vs. \( \varepsilon_1 \) and \( \varepsilon_\nu \) vs. \( \varepsilon_1 \).
\[
\frac{d\sigma_1}{d\varepsilon_1} = \frac{EH}{H + E} \quad (4.41)
\]

\[
\frac{d\varepsilon_v}{d\varepsilon_1} = \frac{H + E(1-K)}{H + E} \quad (4.42)
\]

Using Equations (4.41) and (4.42), \( K \) and \( \beta \) can be evaluated for the EPS geofoam sample.

Table 4.1 shows the rheological parameters derived by Wong and Leo (2006) based on experimental results for an Australian manufactured EPS geofoam with a density of 20 kg/m\(^3\). Assigning the values from Table 4.1 on Equations (4.5a), (4.5b) and (4.5c), parameters \( a, b \) and \( c \) can be evaluated as 160.67 kPa, \(-0.64\) and \(-0.71\), respectively, at the beginning of the analysis. Since parameter \( a \) depends on the plastic deviatoric strain, value of \( a \) changes as the model deforms plastically.

**Table 4.1 EPS geofoam properties extracted for finite element modelling of triaxial tests by Wong and Leo (2006).**

<table>
<thead>
<tr>
<th>( E ) (kPa)</th>
<th>( \nu )</th>
<th>( R_0 ) (kPa)</th>
<th>( K_p )</th>
<th>( \beta ) (kPa)</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3950</td>
<td>0</td>
<td>98</td>
<td>-0.17</td>
<td>225</td>
<td>-0.25</td>
</tr>
</tbody>
</table>

### 4.6 Verification of finite element formulation

The finite element formulation developed to simulate the mechanical behaviour of EPS geofoam is verified using the finite element programme with the user defined material subroutine, VUMAT. In this section triaxial test data published by Wong and Leo (2006) and Chun et al. (2004) have been used for the verification.

#### 4.6.1 Triaxial tests by Wong and Leo (2006)

During the laboratory triaxial tests, samples with an aspect ratio of 2:1 showed some buckling as discussed by Wong and Leo (2006). Therefore they used samples with aspect ratio of 1:1 for triaxial tests, which is less likely to show column-type buckling during loading. The ends of the samples were sand papered to reduce the end friction. Therefore,
finite element simulations are carried out using an EPS geofoam sample with a diameter of 50 mm and an aspect ratio of 1:1. The ends of the triaxial sample are restrained only in the vertical direction to simulate a frictionless contact condition at the top and bottom of the sample. The finite element mesh is created using axisymmetric four-node elements with reduced integration and an element size of 5 mm. Figure 4.1 shows the sketch of the axisymmetric finite element model created to simulate the laboratory test.

Figure 4.1. Axisymmetric finite element mesh used to simulate EPS geofoam block with an aspect ratio of 1:1 (triaxial tests by Wong and Leo 2006).

The sample is subjected to different confining pressures: 0, 20, 40 and 60 kPa. The corresponding pressure for the test is applied to the cylindrical surface of the sample. The strain is applied as a displacement boundary condition at the top of the sample.

Figure 4.2 illustrates the variation of deviatoric stress with axial strain at different confining pressures. The proposed finite element formulation predicts the mechanical behaviour of EPS geofoam exhibiting good agreement with the experimental results presented by Wong and Leo (2006). However, as the confining pressure increases, a slight decrease in the accuracy of the prediction can be observed.
Wong and Leo (2006) derived an improved $K$ for each confining pressure to improve the volumetric strain prediction because the value used previously for $K$ (-0.25) gave similar volume change for all confining pressures. Variation of $K$ with respect to the confining pressure is illustrated in Figure 4.3, which was derived from experimental data. The new values are used in the constitutive model to calculate the change in volume for each confining pressure in consideration. Figure 4.4 shows the variation of volume change with axial strain.

Figure 4.2. Variation of deviatoric stress with axial strain for different confining pressures.
These results confirm that the finite element model has the ability to predict the volume change of EPS geofoam with increasing axial strain. The agreement between experimental and numerical results increases with decreasing confining pressure.

Figure 4.3 Variation of model parameter $K$ with confining pressure (Wong and Leo, 2006).

Figure 4.4 Variation of volume change with axial strain for different confining pressures.
4.6.2 Triaxial tests by Chun et al. (2004)

Triaxial tests by Chun et al. (2004) were carried out on EPS geofoam samples with aspect ratio of 2:1. Similar to triaxial tests described in the previous section, specimens were tested in dry condition. They tested samples with four different densities: 15, 20, 25 and 30 kg/m$^3$. The confining pressures applied to the samples were 0, 20, 40 and 60 kPa.

The model parameters necessary for the finite element simulations were derived based on the variation of volumetric strain and major principle stress with major principal strain as described in Section 4.5. The derived model parameters are listed in Table 4.2. Figure 4.5 shows the major principal stress and strain variation measured during the tests carried out on samples with density of 20 kg/m$^3$ at four different confining pressures and results from the finite element simulations. Results given by Chun et al. (2004) show non-zero principal strains at the beginning of loading. Hence they are adjusted to show zero major principal strain at the beginning of the triaxial test because in the finite element simulations, strains are recorded when the sample starts to deform due to applied axial strain.

Table 2. EPS geofoam properties extracted for finite element modelling of triaxial tests by Chun et al. (2004).

<table>
<thead>
<tr>
<th>$E$ (kPa)</th>
<th>$\nu$</th>
<th>$R_0$ (kPa)</th>
<th>$K_p$</th>
<th>$\beta$ (kPa)</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6287</td>
<td>0</td>
<td>121</td>
<td>0.35</td>
<td>204</td>
<td>0.25</td>
</tr>
</tbody>
</table>

These results further confirm that the proposed finite element formulation has the ability to simulate EPS geofoam behaviour if the triaxial test data are available to derive model parameters as described in Section 4.5.
4.7 Deformation of EPS geofoam sample during loading

During the triaxial testing of soils, bulging of the soil samples are normally observed with increasing axial strain. The triaxial tests carried out by Wong and Leo (2006) clearly showed volumetric contraction of EPS geofoam samples as shown in Figure 4.6. The reason for this volumetric contraction is the near zero Poisson’s ratio of EPS geofoam. According to Wong and Leo (2006), during the initial stages of loading when the strains within the sample are very small, Poisson’s ratio is a small positive value. However, with the progression of loading, strains start to increase reducing the Poisson’s ratio to a zero or small negative value.

In order to investigate the volumetric contraction observed during experiments, a set of simulations were carried out on EPS geofoam samples with aspect ratio 1:1,
introducing a frictional contact at the top and bottom platen-geofoam interfaces. Figure 4.7 shows the deformation patterns of samples under different confining pressures. At the end of 30% and 50% axial strain, deformed shapes of all samples demonstrate a deformation comparable to experimentally observed deformed shapes shown in Figure 4.6.

Figure 4.6 Laboratory observed volumetric contraction of EPS geofoam at different levels of axial compression.

Figure 4.7 Deformed shape of samples from finite element formulation at axial strain 18, 30 and 50% under confining pressures 0, 20, 40 and 60 kPa for EPS geofoam samples (Frictional contact).
The concave shape due to volumetric contraction is due to the frictional contact introduced at the top and bottom boundaries of the triaxial specimen. Therefore this study shows that the platen-geofoam interface was not perfectly smooth during the experiments. The experimental results are extracted for strain levels up to 18% (Figure 4.2 and 4.4). At lower strain levels, the sample has not shown the concave shape. Therefore, the end friction has not contributed significantly on the data used for model formulation and validation.

These results show that the formulation is capable of simulating EPS geofoam behaviour. Although this formulation is proposed for ABAQUS/Explicit finite element program, it can be incorporated into any finite element code to solve boundary value problems involving EPS geofoam. In its current form, it does not take into account the behaviour of EPS geofoam under high temperatures or creep behaviour under long-term loading. However, the proposed finite element formulation of the constitutive model is a valuable tool in solving problems involving normal operating temperatures (less than 30°C), strain levels below sigmoidal strains where rapid strain hardening occurs (Wong and Leo, 2006) and for short term loading where creep effects are not significant.

4.8 Comparison of Wong and Leo (2006) model with other constitutive models

In this section, performance of the constitutive model used in this study developed by Wong and Leo (2006) is compared with four other constitutive models found in the literature: (i) Findley model (Findley and Khosla, 1956), (ii) LCPC model (Magnan and Serratrice, 1989), (iii) Bilinear model (Preber et al., 1995) and (iv) Hyperbolic model (Chun et al., 2004).

4.8.1 Findley model

Findley and Khosla (1956) developed a power-law model to describe the time dependent behaviour of EPS geofoam. The model is in the form of,
\[ \varepsilon = \varepsilon_i + m \left( \frac{t}{t_0} \right)^{n_F} \]  
(4.43)

\[ \varepsilon_i = \varepsilon_{if} \sinh \left( \frac{\sigma}{\sigma_{if}} \right) \]  
(4.44)

\[ m = m_F \sinh \left( \frac{\sigma}{\sigma_{mf}} \right) \]  
(4.45)

where \( \varepsilon \) is the total strain; \( \varepsilon_i \) is the immediate strain; \( t \) is the time after application of stress (units in hours); \( t_0 \) is one hour; \( \varepsilon_{if}, m_F \) and \( n_F \) are dimensionless Findley material parameters; \( \sigma \) is the applied stress; \( \sigma_{if} \) and \( \sigma_{mf} \) are Findley material parameters with dimensions of stress. The Findley material parameters were dependent on environment factors such as temperature and water content.

### 4.8.2 LCPC model

LCPC model (Magnan and Serratrice, 1989) is a modified version of the power-law model developed by Findley and Khosla (1956). This model is derived to predict the total strain of EPS geofoam blocks in the form of,

\[ \varepsilon = \varepsilon_i + at^n \]  
(4.46)

\[ \varepsilon_i = \left( \frac{\sigma}{E_t} \right) \]  
(4.47)

\[ a = 0.00209 \left( \frac{\sigma}{\sigma_y} \right)^{2.47} \]  
(4.48)

\[ n = -0.9 \log_{10} \left[ 1 - \left( \frac{\sigma}{\sigma_y} \right) \right] \]  
(4.49)

where \( \varepsilon \) is the total strain, \( \varepsilon_i \) is the immediate strain, \( \sigma \) is the applied stress in kPa, \( \sigma_y \) is the stress corresponding to the onset of yielding developed in EPS in kPa, \( E_t \) is the
initial tangent modulus in kPa (average slope of stress-strain curve at strains between 0 and 1%) and \( t \) is the time in hours after applying the stress. \( \sigma_y \) and \( E_i \) are given by,

\[
\sigma_y = 6.41\rho - 35.2 \tag{4.50}
\]

\[
E_i = 479\rho - 2875 \tag{4.51}
\]

where \( \rho \) is the density of EPS geofoam in kg/m\(^3\). This model does not take into account the effect of confining pressure on geofoam behaviour.

### 4.8.3 Bilinear model

Preber et al. (1995) proposed a time-independent constitutive model to predict the behaviour of EPS geofoam. This model takes into account the influence of confining pressure and density on the stress-strain behaviour of geofoam. The relationship between stress and strain is represented by,

\[
\sigma = (I + E_p\varepsilon) \left[ 1 - \exp\left(-C \varepsilon^2 - \frac{E_i\varepsilon}{I}\right) \right] \tag{4.52}
\]

Parameters in the above equation are defined as,

\[
C = - \frac{E_i}{IX_0} - \frac{1}{X_0^2} \ln \left( 1 - \frac{Y_0}{I + E_pX_0} \right) \tag{4.53}
\]

\[
I = (-107 + 8.9\rho) + (0.63 - 0.062\rho)\sigma_3 \tag{4.54}
\]

\[
E_p = (104 + 4.3\rho) + (-3.6 + 1.5\rho)\sigma_3 \tag{4.55}
\]

\[
E_i = (-4180 + 382.2\rho) + (-6.2 - 0.52\rho)\sigma_3 \tag{4.56}
\]

\[
Y_0 = (1.4 + 8.7\rho) + (-1.1 + 0.04\rho)\sigma_3 \tag{4.57}
\]

where \( \sigma \) is the axial stress, \( \varepsilon \) is the axial strain, \( X_0 \) is the axial strain at which EPS geofoam starts plastic deformation, \( \rho \) is the density of EPS geofoam and \( \sigma_3 \) is the confining pressure.
4.8.4 Hyperbolic model

Chun et al. (2004) proposed a hyperbolic model incorporating the effect of density and confining pressure on the geofoam behaviour. The stress-strain relationship in the hyperbolic model is given by,

\[
\sigma_1 = \frac{a \varepsilon_1^b}{c + \varepsilon_1^b}
\]  

(4.58)

where \( \sigma_1 \) is the axial stress, \( \varepsilon_1 \) is the axial strain and \( a \), \( b \) and \( c \) are parameters given by

\[
a = -60.955 + 9.843 \rho + 0.339 \sigma_3
\]  

(4.59)

\[
b = 1.135 + 0.042 \rho - 0.008 \sigma_3
\]  

(4.60)

\[
c = -0.437 + 0.102 \rho - 0.002 \rho^2 + 0.011 \sigma_3 - 0.00039 \rho \sigma_3
\]  

(4.61)

where \( \rho \) is the density of EPS geofoam (kg/m\(^3\)) and \( \sigma_3 \) is the confining pressure (kPa).

Figure 4.8 shows a comparison between the predictions from above described models and the model proposed by Wong and Leo (2006). Accuracy of the Wong and Leo (2006) model and the Chun et al. (2004) model surpass the accuracy of the Findley model, LCPC model and Bilinear model by Preber et al. (1995). LCPC model displays significant deviations from the experimental data during inelastic loading but elastic loading path agrees well with the experimental results. While Findley model and Bilinear model by Preber et al. (1995) under predict the stress state during both elastic and elastoplastic regions, the ultimate strength prediction agrees well with the experimental data.
Figure 4.8 Comparison of finite element results based on the Wong and Leo (2006) model with previous constitutive models.
4.9 Conclusion

Finite element formulation of a constitutive model developed to simulate mechanical behaviour of EPS geofoam based on the Drucker Prager yield criterion and utilisation of the formulation in analysing a boundary value problem are presented. The formulation is carried out based on the explicit integration scheme. The mechanical behaviour of EPS geofoam under varying confining pressures is then investigated by implementing the developed finite element formulation in ABAQUS/Explicit. The finite element formulation is verified with data available from triaxial tests conducted on two varieties of EPS geofoam manufactured in Australia and Korea. A comparison of results with four other constitutive models found in the literature suggests that the model used in the finite element formulation has the ability to simulate EPS geofoam behaviour. Hence, it can be concluded that the finite element formulation presented here can be successfully applied to solve boundary value problems in geotechnical engineering, which employ EPS geofoam under normal operating temperatures and strain levels where rapid strain hardening does not occur. The application of EPS geofoam for vibration isolation will be further discussed in later chapters of the thesis.
Chapter 5

Free-field ground vibration propagation during vibratory pile driving

5.1 Introduction

Rapid population growth in last few decades has increased the demand for high rise buildings in urban areas. Due to lack of space, developers seek to maximise the use of valuable land by constructing taller and more ambitious structures. Hence deployment of pile foundations to transmit increasingly larger loads from structure to firmer soil or rock at a greater depth has also increased significantly over the last few decades. Vibratory pile driving has become the preferred pile installation method in urban construction due to its low cost and less noise pollution compared to impact driving. In addition, vibratory pile driving is assumed to be less damaging to the nearby existing structures. However, so far no research has been carried out quantifying the intensity of ground vibration propagation during vibratory pile driving and its impact on different types of structures. Hence, this chapter concentrates on the propagation of ground vibrations due to vibratory and resonance driving of closed-ended piles using axisymmetric finite element models. The numerical modelling technique adopted for the analysis takes into account the large
soil deformations around the pile during driving and is based on the Arbitrary Lagrangian Eulerian technique. It has the ability to drive a pile few pile diameters below the initial position without mesh distortions. The intensity of ground vibrations was measured using the peak particle velocity (PPV), which is calculated as the peak of the true vector sum of the particle velocities in three orthogonal directions. Then, the effect of ground vibration propagation on existing structures was investigated by varying the loading configurations, where amplitude and frequency of the loading were changed within the operating ranges for commercially available vibratory pile driving equipment. In addition, the influence of rigidity index, which is the ratio between shear modulus and undrained shear strength of the soil, and material damping on the propagation of ground vibrations were investigated. The impact of vibratory pile driving on nearby structures is discussed comparing the peak particle velocity distributions with the specifications given by the American Association of State Highways and Transportation Officials (AASHTO), Swiss Standard SN640312 and Eurocode 3. The upper and lower bounds for the peak particle velocity distributions around a vibratory driven closed-ended pile are presented to determine the influence zones for different types of nearby structures. Finally attenuation relationships, and upper and lower bounds for the peak particle velocity distributions around a driven closed-ended pile are presented to determine the influence zones for different types of nearby structures.

5.2 Numerical model simulating ground vibration propagation

The dynamic finite element model used for the analysis of vibratory pile driving was developed using the ABAQUS/Explicit finite element program (ABAQUS, 2011), where dynamic analysis is performed using an explicit central difference time integration. The integration scheme satisfies the dynamic equilibrium equations at the beginning of the time step, \( t \). Then the accelerations calculated at time \( t \) are used to obtain velocities at time \( (t + \Delta t) \) and the displacements at time \( (t + \Delta t) \) as shown below,

\[
\ddot{u}_{N}^{N}\left(t, \frac{\Delta t}{2}\right) = \ddot{u}_{N}^{N}\left(t + \frac{\Delta t}{2}\right) + \ddot{u}_{i}^{N} \Delta t
\]  

(5.1)
\[ u_{(i+\Delta t)}^N = u_i^N + \Delta t \left[ \frac{\Delta u_i}{2} \right] \] (5.2)

where \( u_i^N \) is the \( N^{th} \) displacement or rotation degree of freedom and \( \Delta t \) is the time increment. Computational efficiency of the system is achieved by the use of diagonal lumped mass matrices because the inversion of the lumped mass matrix is simple to obtain the accelerations at the beginning of the time step as shown below,

\[ \ddot{u}_i^N = (M)^{-1}(P_i - I_i) \] (5.3)

where \( P_i \) is the applied load vector and \( I_i \) is the internal force vector (ABAQUS, 2011).

Since pile driving is an axisymmetric problem, an axisymmetric finite element model was developed using a technique similar to Henke and Grabe (2006) to simulate pile installation from the ground surface to few diameters below the ground surface avoiding mesh distortions.

In this model, along the axis of symmetry, a tube of 1.0 mm radius, which was in frictionless contact with the soil and the pile, was modelled allowing the driven pile to slide over the tube. Both pile and the tube are modelled as analytical rigid surfaces and each surface has a single reference point to apply loads and the relevant boundary conditions. When the pile was penetrating the soil, it separated soil from the tube and generated the pile-soil contact. The pile was laterally restrained during installation, while the tube was restrained in all directions throughout the analysis. The tip of the pile was modelled as a conical surface. As shown in Figure 5.1, the tip was embedded in the soil at the beginning of the pile driving process to avoid excessive deformation of soil elements around the pile tip. Pile installation is simulated using the Arbitrary Lagrangian Eulerian (ALE) method in which the quality of the mesh is preserved during the pile driving. Soil around the pile is modelled using four-node axisymmetric elements with reduced integration (CAX4R) because the ALE method in ABAQUS/Explicit is applicable only for linear elements.
The soil domain of the axisymmetric model used to simulate pile jacking is extended about 30D in both radial and vertical directions away from the pile, where D is the diameter of the pile. For vibratory pile driving, the soil domain is extended 40D away from the pile in the radial direction and 20D in the vertical direction. Figure 5.2 shows the finite element mesh used to simulate pile installation with finer elements with minimum dimension of 0.05 m towards the driven pile and coarser elements with maximum dimension of 0.2 m towards the boundary.

The ALE method available in ABAQUS uses adaptive meshing where the same number of elements in the initial mesh is preserved throughout the analysis but elements are readjusted to avoid the distortions otherwise inherent in small strain finite element procedures. The ALE method performs two steps: (1) generation of a new mesh and (2) remapping solution variables from the old mesh to the new mesh. The efficiency of an analysis is governed by the way these two tasks are performed for a particular problem.

The user can specify the frequency at which a new mesh is generated during an analysis and the adaptive meshing region where the highly distorted elements are likely to evolve during an analysis. New mesh is created by performing a number of mesh sweeps, which can be specified by the user. During each mesh sweep, nodes in the domain are relocated only a fraction of the size of surrounding elements to reduce the
element distortions. For the results presented in this chapter, ALE adaptive meshing is defined only for the finer mesh region closer to the driven pile shown in Figure 5.2, which extends up to one pile diameter away from the centre line of the mesh. A new mesh is generated for the adaptive mesh region at the end of each time increment using ten mesh sweeps.

**Figure 5.2 Axisymmetric model for pile installation.**

In ABAQUS the frequency of mapping solution variables from an old mesh to a new mesh is determined automatically and the user cannot override it. If the movement of each node is greater than 50% of the size of any adjacent elements, then the solution variables are mapped from the old mesh or last intermediate mesh to the new intermediate mesh. The solution mapping will continue until the number of mesh sweeps reaches the specified number. Instead of mapping solution variables from the old mesh to
the new mesh in one step, these intermediate remapping steps are performed to maintain the stability of the numerical procedure.

The ALE adaptive meshing technique changes the mesh during the analysis irrespective of the material movement. This avoids possible mesh distortions but does not allow tracking the material movement using nodal displacements. However, ABAQUS provides the option of tracing the material movement by defining tracer particles. Thus, a set of tracer particles were defined along the mesh in the radial direction at the level of embedded pile tip to track the material movement during pile driving. The soil was modelled as a homogeneous continuum.

The contact between the pile and the soil is modelled using the master-slave formulation in ABAQUS/Explicit. The rigid pile is defined as the master surface and the soil surface in contact with the pile is defined as the slave surface. The contact formulation will not allow slave surface to penetrate into the master surface but the master surface can penetrate the slave surface. However, this can be minimized by refining the mesh of the slave surface. The friction between two surfaces is modelled using a friction coefficient of 0.2 and the limiting shear stress transmitted between the two surfaces is computed by multiplying the normal force across the interface by the friction coefficient between the surfaces.

ABAQUS/Explicit introduces damping to dynamic models in order to improve the modelling of high-speed dynamic events (ABAQUS, 2011). It is known as bulk viscosity; a type of damping associated with volumetric straining, which is further categorised into two: linear and quadratic. Linear bulk viscosity is introduced as a bulk viscosity pressure, to damp the “ringing effect” in the highest element frequency by default in every ABAQUS/Explicit analysis. Linear bulk viscosity pressure, \( p_{bv1} \), is defined as,

\[
p_{bv1} = b_1 \rho c_d L_c \dot{\varepsilon}_{vol}
\]  

\[ (5.4) \]
where $b_1$ is the linear bulk viscosity damping coefficient, $\rho$ is the material density, $c_d$ is the current dilation wave speed, $L_c$ is the characteristic length of an element and $\dot{e}_{vol}$ is the volumetric strain rate. The default value assigned by ABAQUS/Explicit for $b_1$ is 0.06. Quadratic bulk viscosity is introduced to prevent elements from collapsing under extremely high velocity gradients when the volumetric strain rate is compressive and is found only in solid continuum elements. The quadratic bulk viscosity pressure, $p_{bv2}$, introduces a resisting pressure to the elements, preventing the collapse. It is defined as,

$$
p_{bv2} = \rho \left( b_2 L_c \dot{e}_{vol} \right)^2
$$

where $b_2$ is the quadratic bulk viscosity damping coefficient and the rest of the parameters are as defined for linear bulk viscosity. The default value introduced by ABAQUS/Explicit for quadratic bulk viscosity is 1.2. In the current study, default values for linear and quadratic bulk viscosity coefficients were used.

### 5.3 Verification of the numerical model

The numerical model described in the previous section is verified using the field data published by Cooke et al. (1979) for pile jacking. These data were selected instead of a vibratory pile driving case due to the availability of soil properties for this site. The data consists of field tests conducted in North London and the soil at the site is London clay, which has a density of 1947 kg/m$^3$, Young’s modulus of 50 MPa and a Poisson’s ratio of 0.49. The undrained shear strength of the soil varies with the depth from 35 kPa at the ground surface to 78 kPa at 4.6 m below the surface level. Three hollow closed ended steel piles of 0.168 m diameter and approximately 5 m length were jacked to the soil to a depth of 4.6 m from the surface. They recorded the vertical ground movements about 0.5 m below the ground surface in the radial direction up to 3 m away from the pile.

The finite element mesh used for this analysis is similar to Figure 5.2, where the conical pile tip is embedded in the ground at the beginning of the analysis to avoid mesh distortions around the tip at the beginning of the penetration. The mesh is truncated at the
bottom by restraining both radial and vertical movements and at the radial direction by restraining only the radial movements. Then the pile is pushed 4.6 m to the ground applying a displacement-controlled boundary at the pile head.

Figure 5.3 shows the comparison between the field data from Cooke et al. (1979) and the results from the finite element model, when the pile is jacked 4.6 m into the ground. The finite element predictions of the ground settlement at 0.5 m below the surface level are in good agreement with the field data. The maximum heave predicted by the finite element model is 8.5 mm and the measured maximum heave in the field is 10.1 mm. The difference of 1.6 mm is acceptable considering the non-homogeneity of the soil domain at the field.

![Figure 5.3 Ground settlement at 0.5 m below the surface level after penetrating 4.6 m.](image)

Figure 5.4 shows the deformed mesh at the end of 4.6 m penetration. It is clear that the proposed method has the ability to avoid mesh distortions. Hence these results confirm the ability of the proposed model in simulating pile installation involving large deformations.
The axisymmetric model verified in earlier section was used to simulate the different pile installation methods. The soil properties published by Hwang et al. (2001), extracted at a large scale pilot pile test project site located in Taiwan, were used in this study. The soil at the site had a density of 1947.5 kg/m$^3$, a Young’s modulus of 20 MPa, a Poisson’s ratio of 0.4 and undrained shear strength ($s_u$) of 25 kN/m$^2$. The soil was modelled as a homogeneous elastic, perfectly-plastic material with von Mises yield criterion. The specifications published by ABI Group (ABI, 2011), manufacturers of vibrator rigs for pile driving, were used to determine the driving force and the frequency range for vibratory pile driving. As an extension to vibratory pile driving, resonant pile driving was modelled using the specifications provided by Resonance Technology International (Resonance, 2012) for high frequency vibrator rigs.

A pile with a diameter (D) of 0.5 m was modelled as an analytical rigid surface as described in Section 5.2. For vibratory pile driving and resonant pile driving, analyses were carried out when the driven pile tip was near the surface and 4 m below the surface level for duration of 0.5 seconds to investigate the influence zones by means of the PPV.

The wave transmitting boundaries proposed by Deeks and Randolph (1994) described in Chapter 3 are selected in truncating the finite element mesh beyond the region of interest, when analysing the dynamic pile installation methods in this study.
5.4.1 Vibratory pile driving

ABI vibrators MRZV_12S and MRZV_30S were selected to model vibratory pile driving. Each vibrator has a driving force of 600 kN and 1500 kN, respectively. The operating frequencies were selected as 28 and 40 Hz for all the cases as those were the minimum and maximum operating frequencies for the selected vibrators (ABI, 2011). Vibratory driving was continued for a period of 0.5 seconds for each case. Pile driving below the surface level was modelled by embedding the pile 4 m below the surface level at the beginning of the analysis to save the computational time required to drive the pile to that depth from the surface. Since the pile tip was modelled as a cone, the tip of the pile was at about 5.5 m below the surface level at the beginning of the analysis. The forces with sinusoidal amplitude, as shown in Figure 5.4, were applied to the reference point of the analytical rigid surface representing the pile.

Figure 5.5 shows the variation of particle velocities through the medium when the pile tip is 4 m below the ground surface, for vibrator MRZV_12S operating at a driving frequency of 28 Hz. The particle velocities attenuate with the propagation of wave fronts in radial and vertical directions. Several wave fronts generated during the analysis can be seen extending towards the boundary of the domain and the velocities distribute spherically around the pile tip confirming the model proposed by Woods (1997), as shown in Figure 2.5 in Chapter 2.

![Figure 5.4 Dimensionless amplitude curve for vibrators operating at 40 Hz.](image-url)
The PPVs were extracted at selected locations along the surface and at each 1 m increment in depth up to 6 m depth. The particle velocities were calculated using the total vector sum of vertical and radial components of particle velocities and were plotted against r/D where r is the radial distance from the centre of the pile. Comparisons were made between the PPVs for different configurations of vibratory pile driving.

Figures 5.6 (a) and (b) show that the attenuation of PPVs when the pile is driving closer to the surface and 4 m below the surface. According to this figure, when the pile is driving closer to the surface (pile tip is 0.5 m below the surface), the highest PPVs are observed at the surface but when the pile is driving 4 m below the surface, the highest PPVs are observed closer to the level of pile tip. Also these figures show that the PPVs decay rapidly when the pile tip is 4 m below the surface when compared to the driving near the ground surface. Hence it can be concluded that the ground vibrations generated...
due to vibratory pile driving is more critical when the pile is driving closer to the surface than when driving few meters below the ground surface.

Figures 5.7 (a) and (b) compare the peak particle velocities with respect to the driving frequency and the magnitude of the driving force, when the pile tip is near the surface and 4 m below the surface, respectively. It is clear that the magnitude of the driving force has a significant influence on the PPV only closer to the pile wall. Away from the pile wall, magnitude or frequency does not have a significant influence on PPVs.

Considering the criterion published by AASHTO, SN640312 and Eurocode 3, as shown in Tables 2.6, 2.7 and 2.8 in Chapter 2 respectively, typical residential buildings can withstand PPV of 5 mm/s and the minimum PPV limit for buildings with historical value or architectural merit is 2 mm/s. Only Eurocode 3 provides PPV limits for buried structures, 25 mm/s. Therefore, in the following sections, influence zones for different types of structures are identified using the PPV limits given in Eurocode 3.

According to Figures 5.7 (a) and (b), for the selected vibrator rigs, the radial extent of the influence zone is about 5D less for driving below the surface compared to driving at or near the surface level. Therefore the influence zone can be determined considering the pile driving near surface based on the types of structures given in Eurocode 3. For buildings with architectural merit or historical significance, an influence zone could not be determined with a finite element mesh with a reasonable size due to the extremely low PPV limit assigned for this category (2 mm/s). Therefore, we can conclude that the vibratory piling near buildings of architectural merit and historical significance is not suitable. This will be further discussed in the following section.
(a) Pile is driving near the surface (pile tip 0.5 m below surface).

(b) Pile is driving 4 m below the surface (pile tip 4 m below surface).

Figure 5.6 PPV variation with depth for Vibrator MRZV_12S operating at 28 Hz.
5.4.2 Resonant pile driving

Resonant pile driving is an advanced form of typical vibratory driving. This technology is used where the adjacent structures are highly vibration sensitive, preventing the use of traditional vibratory pile rigs. Resonant driving rigs use higher operating frequencies with reduced forces compared to conventional vibratory driving rigs. In this study, specifications provided by Resonance Technology International (Resonance, 2012) were used to simulate resonant pile driving. Selected vibratory rigs were RD140 and RD260 with maximum driving forces of 180 kN and 280 kN respectively. Operational frequency
range for both rigs was 80-150 Hz. Similar to previous section, all the configurations were simulated having the driven pile tip 0.5 m below the surface and 4 m below the surface level. Sinusoidal vibratory forces were applied at the pile head to determine the radial extend of the influence zone based on the PPV limits given in Eurocode 3. Figures 5.8 (a) and (b) illustrate the PPV variations in the soil medium for above configurations.

![Graph showing PPV variations](image)

**Figure 5.8 PPV variations for a resonant vibrator driven pile.**

These figures clearly show that there is a significant difference in the extent of the influence zone when the frequency increases but the effect of amplitude is less
significant for the influence zone. There is not much difference in derived influence zones when the piles are driven at the surface or below the surface.

When the frequency of the driving force increases from 80 Hz to 150 Hz, the influence zones reduce more than 50%. Therefore, resonant pile driving is more suitable for urban construction where new pile installations are carried out near existing buildings. Also this driving method is suitable when piles need to be installed near buildings with architectural merit or historical value.

5.5 Effect of soil rigidity index

Stiffness of the soil varies with location and can significantly affect the propagation of ground vibrations. In order to determine the effect of soil stiffness on ground vibration propagation, a parametric study was conducted by varying the rigidity index, \( \frac{G}{s_u} \), of the soil, where \( G \) is the shear modulus of the soil and \( s_u \) is the undrained shear strength of the soil. Rigidity index was varied from 285 to 1200 by changing the Young’s modulus of the soil, while keeping the Poisson’s ratio and the undrained shear strength constant. The wave transmitting boundaries applied to each model were modified according to the respective shear modulus used for the model.

According to the results presented in Section 5.4.1, for vibratory driving, the extent of the influence zone is large when the pile is driving near the ground surface. For resonant pile driving, this zone is nearly the same at both near and below ground surface. Therefore in the following sections extent of the influence zone is investigated considering pile driving near the surface.

For the vibratory pile driving (Section 5.4.1), the influence zone is determined considering the ABI vibrator, MRZV_12S, operating at 40 Hz. Therefore in this section the effect of soil rigidity index is investigated considering the same vibrator. For resonant pile driving, vibrator RD260 operating at 80 Hz and 150 Hz was selected.

Figure 5.9 shows the variation of PPVs along the surface for ABI vibrator MRZV_12S (40 Hz) whilst Figure 5.10 (a) and (b) show the variation of PPV along the
surface for vibrator RD260 operating at 80 Hz and 150 Hz, respectively. When the vibratory pile driving is considered, it is apparent that the PPVs are highest near the driven pile when the rigidity index of the soil is high. With the increasing distance from the driven pile, the rate of attenuation of PPVs is high for soil types with higher rigidity index. Soil types with lower rigidity index displays lower PPVs near the driven pile and experience a lower rate of attenuation of ground vibrations with increasing distance from the driven pile.

![Graph](image)

**Figure 5.9 PPV variation for vibratory driving with varying soil stiffness.**

For resonant pile driving, there is not any clear pattern between radial influence zone for different building categories and the soil rigidity indexes as shown in Figures 5.10 (a) and (b). According to these figures, at lower frequencies, radial influence zone is governed by the lower rigidity indexes giving larger influence zones but for higher frequencies, radial influence zone is governed by PPVs extracted for higher rigidity indexes. Also the PPV variation along the radial direction with rigidity index is confined to a narrow band for higher driving frequencies. The influence zones for both vibratory and resonant pile driving will be further discussed in following sections.
Chapter 5  Free-field ground vibration propagation during vibratory pile driving

5.6 Effect of material damping of soil

In this section, Rayleigh damping is applied to the soil domain in order to introduce material damping, as described in ABAQUS (2011). Rayleigh damping consists of both stiffness and mass proportional damping to damp higher and lower frequency range behaviour, respectively. In mass proportional damping, damping forces are generated by velocities of the model and simulates the model moving through a still viscous fluid. Hence motion at points within the model causes damping. In stiffness proportional
damping, damping forces are generated by strain rates. Hence the damping is associated with the material rather than the motion of the model. Therefore, in the finite element model, only stiffness proportional damping is introduced. It is applied as damping stress, $\sigma_d$, which is added to the stresses caused by constitutive response at each integration point when the dynamic equilibrium equations are generated. $\sigma_d$ is defined as,

$$\sigma_d = \beta R D^e \dot{\varepsilon}$$

(5.6)

where $\beta_R$ defines the viscous material damping in ABAQUS, $D^e$ is the material’s current elastic stiffness in ABAQUS (2011) and $\dot{\varepsilon}$ is the strain rate. For ground vibration propagation due to earthquake loading, the material damping considered to be in the order of 20% of critical damping (Liyanapathirana and Poulos, 2002). Thus, the material damping properties introduced in this parametric study are adjusted to provide 2-20% fraction of critical damping for the first natural mode of vibration. According to ABAQUS (2011) the parameter $\beta_R$ can be calculated as,

$$\beta_R = \frac{2\xi}{\omega_1}$$

(5.7)

where $\xi$ is the fraction of critical damping and $\omega_1$ is the first natural mode.

Hashash and Park (2002) discussed application of damping in non-linear site response analysis. They showed that in addition to the first natural mode, higher modes also have an important contribution to the material damping. However, for shorter soil columns with height of 50 m, traditional method, which uses the first natural mode, gave similar results to the proposed formulation with full Rayleigh damping. Therefore, only the first natural mode is used in this analysis when computing $\beta_R$.

A natural frequency extraction analysis was carried out for the axisymmetric pile driving model using ABAQUS/Standard in order to extract the eigen frequencies. The first eigen frequency value extracted for the model was unrealistically small, presumably due to uncompleted stabilization procedure at the time of extraction. Thus, the second eigen frequency was considered for each model, which was 3.14 rad/sec for models.
having pile tip near the surface. The $\beta_R$ was then calculated varying $\xi$ from 2-20%. A force of 600 kN operating at 40 Hz was applied to the driven pile for vibratory pile driving and a force of 280 kN operating at 80 Hz and 150 Hz was applied to the driven pile for resonant driving.

Variation of PPV along the soil domain at different $\xi$ values for vibratory pile driving is shown in Figure 5.11. PPV variations for resonant driving at different frequencies are shown in Figures 5.12 (a) and (b). In both Figures 5.11 and 5.12, results are for the case with $G/s_u = 285$. Understandably, as the $\xi$ value increases, PPVs attenuate rapidly within the domain surrounding the driven pile. The decrease in PPV variation when a little amount of damping is introduced to the system is much significant than the decrease in PPV variation when the $\xi$ value is increased.

**Figure 5.11** PPV variation with material damping for vibratory driving ($G/s_u = 285$).

In addition to the finite element results, Figure 5.12 shows PPV variation with radial distance given by Kim and Lee (2000) and Athanasopoulos and Pelekis (2000) based on PPV measurements recorded during pile driving operations. The PPV variation for Athanasopoulos and Pelekis (2000) in Figure 5.11 is based on the attenuation equation
derived by them, considering all measured PPVs in the radial direction during sheet pile driving, as shown below.

\[ PPV = 32r^{-1.5} \]  

(5.8)

Figure 5.12 PPV variation with material damping for resonant driving \((G/s_u = 285)\).
Both sets of field measurements suggest that PPVs calculated without material damping is much higher compared to the field measurements. According to this figure, PPV variation in the radial direction measured during sheet pile driving given by Athanasopoulos and Pelekis (2000) agrees well with the PPV distributions obtained from the finite element analysis when material damping is about 2-5% of critical damping for the first mode of vibration. PPV distribution based on Kim and Lee’s data is higher than finite element results including damping and PPV measurements by Athanasopoulos and Pelekis (2000). Kim and Lee’s PPV distribution is available only for a region 10 m away from the driven pile and the distribution is based on only few data points. Therefore we can consider that the PPV distribution given by Athanasopoulos and Pelekis (2000) is reliable and 2% damping is sufficient to take into account the material damping within the finite element model.

Also we can use Bornitz equation (Athanasopoulos et al. 2000) to validate whether the 2% material damping is a reasonable amount. Bornitz equation includes both radiation and material damping effects as shown below,

\[ w_2 = w_1 \left( \frac{r_1}{r_2} \right)^n e^{-a(r_2 - r_1)} \]  \hspace{1cm} (5.9)

where \( w_1 \) and \( w_2 \) are amplitudes of vibration at distances \( r_1 \) and \( r_2 \) from the source, respectively, \( n \) is attenuation due to radiation damping which depends on the wave type, type of vibration source and the position of the vibration source, and \( a \) is the attenuation coefficient due to material damping. \( a \) can be calculated by applying Equation (5.9) to two points selected from Figure 5.11, for the case with 2% damping,. Based on (6,6.63) and (30,0.82) for \((r/D, \text{PPV})\), \( a \) is 0.11, if \( n \) is equal to 0.5 (surface wave due to a point shaped vibration source). According to Woods (1997), \( a \) should be between 0.1 \( \text{–} \) 0.3 for weak or soft soils when the frequency of vibration is 50 Hz. For the pile driving considered here the frequency of vibration is 40 Hz and \( G/s_u = 285 \) represent a soft soil. Hence, 2% of critical damping (based on the first mode of vibration) in Figure 5.11 is realistic.
If the PPV variation during pile driving is given in terms of $(r/D)$ instead of the radial distance $r$, Equation (5.8) can be modified based on the results presented in Figure 5.11 for a 0.5 m diameter pile as shown below.

$$PPV = 90\left(\frac{r}{D}\right)^{-1.5}$$  \hspace{1cm} (5.10)

In Figures 5.12 (a) and (b), for resonant pile driving with frequencies 80 and 150 Hz, respectively, Equation (5.8) by Athanasopoulos and Pelekis (2000) is also plotted. However, it is clear that the equation for pile driving over predicts the PPV variation in the radial direction for resonant pile driving in the high frequency range. Therefore two equations are proposed for resonant pile driving as shown below.

$$PPV = 34\left(\frac{r}{D}\right)^{-1.5} \quad \text{Frequency} = 80 \text{ Hz}$$  \hspace{1cm} (5.11a)

$$PPV = 25\left(\frac{r}{D}\right)^{-1.5} \quad \text{Frequency} = 150 \text{ Hz}$$  \hspace{1cm} (5.11b)

Based on these results it is clear that the PPV variation in the radial direction depends on the frequency of vibration, rigidity index of soil and material damping. Although Equations (5.11a) and (5.11b) can be used as a guide to determine the radial influence zone for different types of structures, it is clear that it is more appropriate to give an upper and lower bound regions for the radial influence zone considering different rigidity indices for soil.

Figures 5.13 and 5.14 show the upper and lower bounds for vibratory and resonant pile driving excluding and including material damping, which is 2% of the critical damping based on the first mode of vibration. According to Woods (1997), amount of material damping should change with the frequency of vibration source and the soil type. However, for resonant pile driving with frequencies in the range of 80 – 150 Hz, there is no clear guide available to select the material damping parameter, $a$. Therefore, material damping equivalent to 2% or critical damping based on the first mode of vibration is applied for resonant pile driving, similar to vibratory pile driving. Also Figures 5.13,
5.14 (a) and 5.14 (b) show the safe PPV limits for different types of structures. These figures are useful in assessing radial influence zone when there is a pile driving operation adjacent to an existing structure.

![Figure 5.13 Upper and lower bounds of PPV variation for vibratory pile driving.](image)

Figure 5.13 Upper and lower bounds of PPV variation for vibratory pile driving.
Figure 5.14 Upper and lower bounds of PPV variation for resonant pile driving.
5.7 Conclusion

The effect of vibratory and resonant pile driving around a driven pile is investigated using a finite element model developed in ABAQUS/Explicit incorporating the Arbitrary Lagrangian Eulerian technique. Excessive mesh distortions around the driving pile are avoided by implementing the adaptive meshing technique, which carry out remeshing without changing the topology of the mesh. The proposed model is verified with field data available for pile jacking. The model was then used to simulate vibratory and resonant pile driving, truncating the finite element mesh with wave transmitting boundaries for both shear and dilation waves propagating in the soil domain. A parametric study was carried out by varying the amplitude and the frequency of the vibratory forces pertinent for commercially available driving rigs, soil rigidity index and soil material damping. The influence zones for different structures are derived by considering the PPV limits given in Eurocode 3. Results show that for vibratory pile driving, the influence zone does not significantly change with the amplitude or frequency of the driving force, and the depth of driving (driving near surface or below surface). For resonant pile driving, the influence zone changes with the frequency of the driving force but not with the depth of driving or the amplitude of the driving force. Also these results indicate that if pile driving needs to be carried out near buildings with historical value or architectural merit, resonant pile driving is more suitable than vibratory pile driving.

When the rigidity index of the soil is changed, for vibratory pile driving, PPVs are higher near the pile wall for higher rigidity indexes. The rate of attenuation of PPV near the pile wall increases with increasing rigidity index. For all rigidity indexes, rate of PPV attenuation decreases with increasing radius. For resonant pile driving, PPV near the pile wall increases with decreasing rigidity index, in contrast to the vibratory pile driving. Also the rate of attenuation of PPV near the pile wall is low for soils with higher rigidity indexes. Away from the driven pile, PPV variation and rigidity index does not show any clear pattern. This may be due to the method used to compute the PPV for a particular location. It is the maximum value of true vector sum of radial and vertical components of velocities recorded during the analysis.
The parametric study carried out by varying the material damping, which is a fraction of the critical damping based on the first mode of vibration. The comparison of finite element results with field measurements for vibratory pile driving show that 2% of critical damping is appropriate to simulate the field conditions. Also a back analysis is carried out using the Bornitz equation confirm that the 2% of critical damping is an appropriate value. Finally attenuation relationships, and upper and lower bounds for the peak particle velocity distributions around vibratory and resonant driven closed-ended piles are presented to determine the influence zones for different types of nearby structures.
Chapter 6

Attenuation of ground vibrations using in-filled wave barriers

6.1 Introduction

Ground vibrations generated by construction activities can adversely affect the structural health of adjacent buildings and foundations supporting them. Therefore propagation and rate of attenuation of construction induced ground vibrations is important during construction activities, particularly in urban areas where constructions are carried out in the vicinity of existing structures. In practice wave barriers are installed in the ground to mitigate the ground vibration propagation and hence to minimise the effect of ground vibrations on surrounding structures. Different types of fill materials such as bentonite, EPS geofoam and concrete are used in constructing wave barriers. In this study, the effectiveness of EPS geofoam and water filled wave barriers against ground vibration propagation is studied conducting a parametric study varying the physical properties of the wave barriers as well as the location and the frequency of the vibration source. A three-dimensional finite element model is developed to study the efficiency of different fill materials in attenuating ground vibrations. The model is first verified using data from full scale field experiments, where EPS geofoam has been used as a fill material in wave barriers. Then the model is used to conduct a parametric study to determine the efficiency
of EPS geofoam and water as fill materials for wave barriers, by varying the geometric properties of the wave barriers: depth, length and the width; the location of the barrier and frequency of the vibration source.

6.2 Numerical model for ground vibration propagation

A three-dimensional dynamic finite element model was developed using the ABAQUS/Explicit (ABAQUS, 2011) finite element programme to numerically simulate the attenuation of ground vibrations using open and in-filled wave barriers. ABAQUS/Explicit performs dynamic analyses using explicit central difference time integration. The dynamic equilibrium equations are satisfied at the beginning of the time step, \( t \), and the integration scheme computes velocities at time \( t + \Delta t \) and displacements at time \( (t + \Delta t) \), using the accelerations calculated at time \( t \), as discussed in Chapter 5.

The finite soil domain was modelled using eight-node linear brick continuum elements with reduced integration and hourglass control (C3D8R). Instead of a higher order element with twenty-nodes, eight-node elements were selected due to restrictions in the finite element programme. It allows only linear continuum elements with eight nodes when adaptive meshing is used in the analysis. The soil domain was truncated at 30 m in the vertical direction and a fixed boundary condition was applied to simulate the bedrock. The region beyond the interest in lateral direction was truncated using an artificial boundary condition in order to save computational cost of the numerical simulations.

When analysing dynamic three-dimensional finite element models, it is necessary to select a wave transmitting boundary that can be easily implemented and have a reasonable accuracy. Previously investigated (Chapter 3) boundary conditions with high accuracy (Deeks and Randolph, 1994; Du and Zhao, 2010) are not applicable to three-dimensional finite element models because those boundaries are developed considering axisymmetric wave propagation. Since it is not in the scope of this research to develop a boundary condition to simulate three-dimensional wave propagation, the artificial boundary was modelled using the eight-node linear continuum infinite elements (CIN3D8) available in ABAQUS/Explicit. Infinite element displays a rigid body motion.
Chapter 6  
Attenuation of ground vibrations using in-filled wave barriers

when used in wave propagation problems towards the residual deformations as shown in Chapter 3. However, this research investigates the peak particle velocities. These peak values can be achieved with reasonable accuracy using infinite elements.

The maximum element size was determined based on the minimum Rayleigh wave length ($\lambda_{R_{\text{min}}}$) for the ground, in order to avoid filtering of higher frequencies by large elements. Zerwer et al., (2002) recommended that the maximum element dimension ( $L_{\text{max}}$) in a finite element model should be,

$$L_{\text{max}} \leq \frac{1}{4} \lambda_{R_{\text{min}}}
$$

(6.4)

The minimum wavelength ($\lambda_{\text{min}}$), which can occur during an analysis, is calculated as

$$\lambda_{\text{min}} = \frac{V_R}{f_{\text{max}}}
$$

(6.5)

where $V_R$ is the Rayleigh wave velocity and $f_{\text{max}}$ is the highest frequency. Rayleigh wave velocity ($V_R$) is given by,

$$V_R \approx \left[ \frac{0.72 - \left( \frac{V_S}{V_D} \right)^2}{0.75 - \left( \frac{V_S}{V_D} \right)^2} \right] \times V_S
$$

(6.6)

The domain is symmetrical along the line connecting the source and the locations of PPV extraction, as there were no structures present in the field to obstruct the ground vibration propagation. Hence, only half of the domain was modelled in order to save computational time. Figure 6.1 illustrates the finite element mesh with an open trench used for the analysis.
Finite element program introduces damping to dynamic models in order to improve the modelling of high-speed dynamic events as described in Chapter 5.

6.3 Verification of the numerical model

The model was verified using a set of field tests conducted by Alzawi and El Naggar (2011). The field tests were conducted in a flat area located in Ponoka, Alberta, Canada. The excitation source used was a vertical harmonic load which has the shape of a sine wave. Tests were carried out varying the excitation frequency between 15 and 58.84 Hz but for the model verification, tests carried out with excitation frequencies of 40 and 50 Hz were selected. For the model verification three cases were considered: (i) free field ground motions without a wave barrier, (ii) open wave barrier and (iii) geofoam in-filled wave barrier. In latter cases, a wave barrier with a length of 20 m, a width of 0.25 m and a depth of 3.0 m is placed 2.5 m away from the source of excitation. During the field tests, geophones were placed at 2.5 m interval, in a line perpendicular to the wave barrier and measured the particle velocities in the vertical direction.
Bedrock at the site was located at 30 m below the ground surface. The shear wave velocity of the soil increased from 225 - 456 m/s along the depth from surface to 30 m, and the density increased from 1812.5 - 1955.3 kg/m$^3$. The variation of material properties along the depth cannot be assigned to the infinite elements. Therefore, the material properties were applied to the infinite domain in ten layers with thickness of 3 m and each layer was assigned an average value of Young’s modulus and density. The Poisson’s ratio of the soil deposit was considered as 0.4. Variation of the shear wave velocity and the density were considered to be linear during the numerical simulations. For the geofoam in-filled trenches, a light weight polyurethane material known as URETEK expanded polymer has been used. This material has a shear wave velocity of 330 m/s, density of 61 kg/m$^3$ and Poisson’s ratio of zero. Since only these properties are available for the geofoam, it is simulated as an elastic material in the finite element model. This is a reasonable assumption when the magnitude of the excitation is not high enough to cause plastic deformations.

According to Equation 6.6, Rayleigh wave velocity near the surface is 214.8 m/s. The maximum frequency used in this study is 50 Hz, which gives a $L_{\text{max}}$ of 1.074 m. However, in order to have a finer mesh, maximum element size of 0.5 m is used in the numerical model, which is smaller than the maximum allowable element size.

A natural frequency extraction analysis was carried out for the three-dimensional finite element model using ABAQUS/Standard in order to extract the eigen frequencies. The first three eigen frequency values were zero and the next three values were unrealistically small, which is likely to occur due to uncompleted stabilization procedure at the time of extraction. Hence, the seventh eigen frequency was considered for each model, which was 17.825 rad/sec. The damping factors, $\alpha_R$ and $\beta_R$, were then calculated as described in Chapter 5, considering 5% of critical damping. A vertical point load of 11.75 kN operating at 40 Hz as a sine wave was applied on the ground 2.5 m away from the centre of the wave barrier. Analyses were repeated after increasing the frequency of the vertical point load to 50 Hz.
The total analysis time for each model was 0.5 s, which was sufficient for waves to propagate over the half-space. The results were extracted along the surface at intervals of 0.5 ms, totalling 1000 extraction points for each model. Figures 6.2 and 6.3 show the finite element results of PPVs of the ground in the vertical direction for wave propagation in the free field, for the case with an open trench and for the case with geofoam in-filled trench. The PPVs are normalised by the highest velocity extracted at 0.5 m away from the source of vibration. $PPV^*$ shown in figures denote the normalised PPVs. The vertical solid line shows the location of the wave barrier.

![Figure 6.2](image-url) Normalised vertical peak particle velocities with vibratory source operating at (a) 40 Hz and (b) 50 Hz.
There are small discrepancies in the finite element results and field measurements. This may be due to the non-homogeneous nature of the soil at the Ponoka site. Overall, the results are in good agreement with field data. Hence, these results confirm that the finite element model developed in this study, including the wave absorbing boundary, is capable of simulating ground vibration propagation and attenuation of ground vibrations in the presence of a wave barrier.

### 6.4 Material models for in-filled wave barriers

The verified finite element model was then used to simulate the effect of different fill materials on wave attenuation. In the current study, EPS geofoam and water were selected as fill materials. In the previous section, geofoam was simulated assuming it behaves elastically because only shear modulus, density and Poisson’s ratio were available for the geofoam used for the field tests. However, geofoam shows hardening plasticity during loading according to the published literature. Hence, the parametric study described in following sections modelled EPS geofoam using a constitutive model based on the hardening plasticity developed by Wong and Leo (2006) for a variety of geofoam commonly used in Australia.
6.4.1 EPS geofoam

A detailed description of this material model and implementing the model in ABAQUS/Explicit are given in Chapter 4. Table 4.1 gives material properties for the geofoam variety used in this chapter.

6.4.2 Water

The effect of a water filled wave barrier in attenuating ground vibrations was investigated by modelling water with the Mie-Grüneisen equation of state (ABAQUS, 2011; Mahamadi et al., 2004). The Mie-Grüneisen equation defines pressure for the compressed material as shown below,

\[ p = p_H \left(1 - \frac{\Gamma_0 \eta}{2}\right) + \Gamma_0 \rho_0 E_m \]  

(6.19)

where \( p_H \) is the Hugoniot pressure, \( \Gamma_0 \) is the Grüneisen ratio, \( \eta \) is the nominal volumetric compressive strain, \( \rho_0 \) is the initial density of the medium, which is water in this case, and \( E_m \) is the internal energy per unit mass. \( \eta \) can be expressed in terms of density as shown below,

\[ \eta = 1 - \frac{\rho_0}{\rho} \]  

(6.20)

where \( \rho \) is the current density. The Hugoniot pressure can be expressed as,

\[ p_H = \frac{\rho_0 c_0^2 \eta}{(1 - s \eta)^2} \]  

(6.21)

where \( c_0 \) is the compression or \( p \) wave velocity in water and \( s \) is the gradient of the linear relationship between shock velocity \( U_s \) and the particle velocity \( U_p \), which is given by,

\[ U_s = s U_p + c_0 \]  

(6.22)

From Eq. (17) and (20), the Mie-Grüneisen linear \( U_s - U_p \) equation is written as,
Chapter 6  
*Attenuation of ground vibrations using in-filled wave barriers*

\[ p = \frac{\rho_0 c_0^2 \eta}{(1 - s \eta)^2} \left( 1 - \frac{\Gamma_0 \eta}{2} \right) + \Gamma_0 \rho_0 E_m \]  
(6.23)

The water filled wave barrier is modelled using a density of 1000 kg/m\(^3\). A viscosity of \(1 \times 10^{-3}\) Ns/m\(^2\) was applied to water which has a compression wave velocity \((c_0)\) of 1490 m/s. During the analysis, \(s\) and \(\Gamma_0\) were assigned 1.79 and 1.65, respectively, based on the properties for the model given by Otsuka et al., (2004).

6.5 **Comparison of free field ground vibrations with open and in-filled wave barriers**

Figure 6.4 shows the attenuation of PPVs in the radial direction away from the source of vibration. Results are presented in the form of \(PPV^*\) vs normalised distance, \(d/\lambda_R\), for an EPS geofoam filled wave barrier, water filled wave barrier and open trench. Also in the same figure \(PPV^*\) distribution is given when there is no wave barrier is present.

It can be seen that both water filled wave barrier and EPS geofoam wave barrier display a considerable attenuation of the ground vibration propagation for both frequencies considered for the analysis, 40 Hz and 50 Hz. When \(d/\lambda_R\) is 3 and frequency of excitation is 40 Hz, water in-filled trench attenuated the \(PPV^*\) by 50% and EPS in-filled trench attenuated the \(PPV^*\) by 60%. When \(d/\lambda_R\) is 3 and frequency of excitation is 50 Hz, water in-filled trench attenuated the \(PPV^*\) by 45% and EPS in-filled trench attenuated the \(PPV^*\) by 70%. Normalised distance, \(d/\lambda_R\), is computed by dividing the actual distance, \(d\), by the Rayleigh wave length, \(\lambda_R\). At both frequencies, open trench outperforms water or geofoam in-filled trench but in an open trench, wall stability could be an issue. During wave propagation, unsupported walls of an open trench may collapse into the excavation. These results confirm that both EPS geofoam and water are effective as fill materials for wave barriers. Overall, EPS geofoam in-filled barriers outperform water in-filled barriers.
In the following sections, the efficiency of different configurations of EPS geofoam and water in-filled wave barriers are investigated varying the geometric properties and the location of the wave barriers, and the frequency of the ground vibration source.

6.6 Influence of in-filled wave barrier size on wave attenuation

In this section, a parametric study was carried out varying the depth, length, width and location of the wave barrier, and the operating frequency of the ground vibration source. During the parametric study, the depth of the wave barrier was varied from 2 – 9 m and the length of the barrier was varied from 4 – 24 m. The width of the barrier was varied from 0.25 – 2 m. The frequency of the vibratory source was varied from 25 – 55 Hz. The location of the vibratory source was varied by increasing the distance between barrier and the source from 2.5 – 10 m. PPV* distributions are plotted against the normalised distance, \( d/\lambda_R \), measured from the ground vibration source. The soil properties used in this section are same as those described in Section 3. The computed \( V_R \) at the surface based on these soil properties is 214.8 m/s, which gives a \( \lambda_R \) of 5.37 m when the frequency of the vibratory source is 40 Hz. For clarity, the legends of the graphs are not given in terms of \( \lambda_R \).
6.6.1 EPS geofoam in-filled wave barriers

The effect of varying the depth of the EPS geofoam wave barrier was investigated by varying the depth from 2 – 9 m (0.37 – 1.68λR), while the length and the width of the wave barrier remained at 20 m and 0.25 m, respectively. A vertical point load of 11.75 kN operating at 40 Hz as a sine wave was applied on the ground 2.5 m away from the centre of the wave barrier and PPV was measured at points along a line perpendicular to the wave barrier. Figure 6.5 shows the variation of $PPV^*$ along the ground surface for different depths of the wave barrier.

According to Figure 6.5, wave attenuation is at the lowest when the barrier depth is 2 m (0.37λR). The highest wave attenuation is observed when the barrier depth is 7 to 8 m (1.30 – 1.49λR). The deepest barrier, which has a depth of 9 m, does not show any improvement in wave attenuation, compared to the 8 m deep wave barrier. Behind the wave barrier, towards the vibration source, $PPV^*$ is the same for all depths. However, a general pattern could be observed in the attenuation of $PPV^*$s beyond the wave barrier:
increase in depth of wave barrier increases the wave attenuation. Figure 6.5 shows the upper and lower bounds for the $PPV^*$ distributions given by,

Upper bound: $PPV^* = \frac{0.03}{d/\lambda_R}$ \hspace{1cm} (6.24a)

Lower bound: $PPV^* = \frac{0.006}{d/\lambda_R}$ \hspace{1cm} (6.24b)

Figure 6.6 shows the $PPV^*$s along the surface of the soil domain for different wave barrier lengths. In this case, the length of the wave barrier was varied from 4 – 24 m (0.74 – 4.47$\lambda_R$) while maintaining a depth of 3 m and a width of 0.25 m. The amplitude, frequency and the location of the vibratory source are as discussed earlier in this section. The $PPV^*$ distributions show that the increase in barrier length does not have a significant influence on the wave attenuation because the $PPV^*$ distributions varied within a narrow range.

Figure 6.7 shows the $PPV^*$ distribution with the width of the wave barrier. A clear pattern of ground vibration attenuation could be observed for this case. Here, the width of the wave barrier was varied from 0.25 – 2.0 m (0.05 – 0.37$\lambda_R$) while maintaining a length of 20 m and a depth of 3 m. The same vibratory source is used as in the previous cases. This figure shows that the wave attenuation increases marginally with increasing width of the wave barrier, displaying lowest attenuation for the barrier with a width of 0.25 m (0.05$\lambda_R$) and highest attenuation for the barrier with a width of 2.0 m (0.37$\lambda_R$).
Chapter 6

Attenuation of ground vibrations using in-filled wave barriers

Figure 6.6 Variation of normalised peak particle velocities with length of EPS geofoam wave barrier.

Figure 6.7 Variation of normalised peak particle velocities with width of EPS geofoam wave barrier.
The effect of the location of the wave barrier on its efficiency was observed by varying the distance between the source and the wave barrier from 2.5 m to 10.0 m (0.47 – 1.86λR) at 2.5 m intervals. The length, width and depth of the wave barrier were 20 m, 0.25 m and 3.0 m, respectively. Figure 6.8 illustrates the $PPV^*$ variation along the surface with the location of the wave barrier. The locations of the wave barrier are also shown in Figure 6.8 using the same line type used for the $PPV^*$ variation curves corresponding to the wave barrier location.

![Figure 6.8 Variation of normalised peak particle velocities with location of EPS geofoam wave barrier.](image)

If Figure 6.8 is examined carefully, wave barriers located 7.5 m and 10.0 m away from the source are more efficient in ground vibration attenuation than those located 2.5 m and 5.0 m away from the vibration source. If active and passive ground vibration isolation are considered, it can be concluded that the ground vibration attenuation using passive isolation, where the wave barrier is placed away from the ground vibration source towards the structure to be protected, is more effective than the active isolation, where the wave barrier is placed closer to the vibration source. Although the stiffness of the wave barrier is less than the adjacent ground, part of the waves propagating towards
the barrier will reflect back towards the vibration source. As a result $PPV^*$ immediately before the barrier location is higher than the $PPV^*$ at the same location for free field ground vibration propagation.

Figure 6.9 illustrates the variation of $PPV^*$ along the surface of the soil domain with the operating frequency of the ground vibration source. These results were extracted for different frequencies of the vertical vibratory source with a magnitude of 11.75 kN, varying from 25 – 55 Hz. This changes the Rayleigh wave length of the soil medium from 8.59 – 3.91 m during the parametric study. The dimensions of the wave barrier remained the same as in the previous case.

![Figure 6.9](image)

**Figure 6.9 Variation of normalised peak particle velocities with frequencies of vibratory source.**

The results indicate that the efficiency of the EPS geofoam wave barrier increases with increasing frequency of the ground vibration source. Increasing frequency of the vibration source decreases the wave length of the ground vibrations and hence the rate of wave attenuation increases with decreasing wave length. The upper and lower bounds for the $PPV^*$ distributions shown in Figure 6.9 are given by,
Upper bound: \[ PPV^* = \frac{0.25}{d} \] (6.25a)

Lower bound: \[ PPV^* = \frac{0.04}{d} \] (6.25b)

### 6.6.2 Water filled wave barriers

A series of three-dimensional finite element analyses, similar to those described in previous section, were used to conduct a parametric study on water in-filled wave barriers. The base case used in the parametric study is an in-filled wave barrier with a length of 20 m, a width of 0.25 m and a depth of 3.0 m. The vibratory source was a point load of 11.75 kN with a frequency of 40 Hz and was applied at 2.5 m away from the wave barrier.

The effect of varying the depth of the water filled wave barrier from 2 – 9 m (0.37 – 1.68\( \lambda_R \)) on the ground vibration propagation is shown in Figure 6.10.

![Figure 6.10 Variation of normalised peak particle velocities with depth of water filled wave barrier.](image)

The lowest attenuation of ground vibrations is observed at the shallowest depth of 2 m (0.37\( \lambda_R \)). The highest attenuation of PPV* adjacent to the barrier location was observed.
for barrier with a width of 9 m (1.68\(\lambda_R\)). As the normalised distance increases, barrier with a depth of 7 m (1.30\(\lambda_R\)) displays the highest attenuation of \(\text{PPV}^*\). However, the attenuation of \(\text{PPV}^*\) exists within a narrow range, defined by,

Upper bound: \(\text{PPV}^* = \frac{0.03}{d/\lambda_R} \) \hspace{1cm} (6.26a)

Lower bound: \(\text{PPV}^* = \frac{0.02}{d/\lambda_R} \) \hspace{1cm} (6.26b)

Variations of \(\text{PPV}^*\) with the length of the water filled wave barrier are given in Figure 6.11. Increasing the length of the water filled wave barrier from 4 – 24 m (0.74 – 4.47\(\lambda_R\)) does not show a significant attenuation of ground vibrations.

**Figure 6.11 Variation of normalised peak particle velocities with length of water filled wave barrier.**

Figure 6.12 shows the variation of \(\text{PPV}^*\) along the surface of the soil domain for varying the width of the water filled wave barrier. Here the width of the water filled wave
barrier is increased from 0.25 – 2.00 m (0.05 – 0.37\(\lambda_R\)) while keeping the rest of the parameters at the base case described earlier in this section.

Figure 6.12 Variation of normalised peak particle velocities with width of water filled wave barrier.

Results show that the efficiency of the water filled wave barrier can be slightly increased by increasing its width. The \(\text{PPV}^*\) curves closer to barrier location shows similar attenuation efficiency. As the distance from the wave barrier increases, the variation increases displaying highest attenuation from the widest barrier (0.37\(\lambda_R\)) and the lowest attenuation from the narrowest barrier (0.05\(\lambda_R\)). The upper and lower bounds for the \(\text{PPV}^*\) can be represented by,

\[
\text{Upper bound: } \text{PPV}^* = \frac{0.03}{\left(\frac{d}{\lambda_R}\right)} \tag{6.27a}
\]

\[
\text{Lower bound: } \text{PPV}^* = \frac{0.025}{\left(\frac{d}{\lambda_R}\right)^2} \tag{6.27b}
\]
Influence of the location of water filled wave barrier on the attenuation of ground vibrations was also observed similar to EPS geofoam wave barriers. Variation of $PPV^*$ attenuation at different barrier locations is shown in Figure 6.13. The dimensions of the wave barrier were kept at the initial configuration and the location was changed by increasing the distance between the source and the barrier from 2.5 – 10.0 m ($0.47 – 1.86\lambda_R$) at 2.5 m intervals. For water in-filled wave barriers both active and passive isolation approaches are efficient unlike for the EPS geofoam in-filled wave barriers.

![Figure 6.13 Variation of normalised peak particle velocities with location of water filled wave barrier.](image)

Figure 6.14 shows the variation of the $PPV^*$ along the ground surface when the frequency of the vibratory source varied from 25 – 55 Hz ($\lambda_R = 8.59 – 3.91$ m). The highest attenuation is observed for the highest frequency (55 Hz) while the lowest attenuation is recorded for the lowest frequency, 25 Hz. However, it can be seen that the $PPV^*$ variation for frequency of 30 Hz shows significant attenuation when compared to the rest of the lower frequency $PPV^*$ variation curves. It can be concluded that the higher operational frequencies improve the efficiency of the water filled wave barrier in
attenuating ground vibrations. The upper and the lower bounds of the $PPV^*$ attenuation due to change in source frequency are given by,

Upper bound: $PPV^* = \frac{0.4}{d^{1.2}}$

Lower bound: $PPV^* = \frac{0.2}{d^{1.2}}$

![Figure 6.14 Variation of normalised peak particle velocities with different frequencies of vibratory source.](image)

Based on the results of this parametric study, it can be concluded that the physical parameters of the wave barriers are effective in controlling the efficiency of ground vibration attenuation. Figures 6.15 to 6.18 compare the range of attenuation in ground vibrations by different wave barriers. According to Figure 6.15, attenuation achieved by varying the depth of the water trench is smaller compared to the attenuation achieved by the EPS geofoam wave barrier. Higher attenuation of ground vibrations can be achieved by using EPS geofoam as the fill material in wave barriers. A shallower EPS geofoam wave barrier (depth of 4 m) can be used to achieve the same ground vibration attenuation achieved by a deeper water filled wave barrier (depth of about 9 m).
Figure 6.15 Effect of depth of the wave barrier in attenuating ground vibrations.

Attenuation ranges for varying barrier length for water and geofoam in-filled wave barriers are shown in Figure 6.16. EPS geofoam wave barrier shows higher attenuation of ground vibrations closer to the vibratory source, when compared to the water in-filled wave barrier. However, as the distance from the source increases, attenuation from the water-filled wave barrier increases and both wave barriers have attenuation ranges closer to each other.

Figure 6.17 shows the attenuation ranges for in-filled wave barriers when the width of the wave barrier increases. EPS geofoam wave barrier shows wider attenuation range closer to the vibratory source, while water-filled wave barrier has a narrow range. As the distance from the vibratory source increases, the attenuation range for EPS geofoam wave barrier narrows down and the attenuation range for water in-filled wave barrier widens. Water-filled wave barrier outperforms the EPS geofoam wave barrier as the distance from the vibratory source increases.
Chapter 6  
Attenuation of ground vibrations using in-filled wave barriers

Figure 6.16 Effect of length of the wave barrier in attenuating ground vibrations.

Figure 6.17 Effect of width of the wave barrier in attenuating ground vibrations.
The effect of frequency of vibratory source in the efficiency of the in-filled wave barriers is shown in Figure 6.18. For lower frequencies, efficiency of the EPS geofoam wave barriers is closer to the efficiency of the water in-filled wave barriers. However, as the frequency increases, the EPS geofoam wave barrier outperforms the water in-filled wave barrier, resulting in wider range of attenuation. These figures can be used as a guide in determining the geometry of the wave barrier and fill material to be used in attenuating ground vibrations, when it is necessary to mitigate the damages to existing nearby structures during construction activities which generate ground vibrations.

![Figure 6.18 Effect of frequency of the vibratory source in attenuating ground vibrations.](image)

6.7 Conclusion

The effect of a water and EPS geofoam in-filled wave barriers is investigated using a three-dimensional finite element model developed using the finite element program ABAQUS/Explicit. The proposed model is first verified with field data available for attenuation of ground vibrations in free field and at the presence of an EPS geofoam wave barrier. Then the model was used to conduct a parametric study varying the physical properties of the wave barrier: depth, length, width and location, as well as the
frequency of the vibratory source for different fill materials: EPS geofoam and water. It is concluded that EPS geofoam is the most efficient fill material in attenuating ground vibrations.

Efficiency of EPS geofoam wave barriers increase significantly when the depth of the barrier is increased. However, an increase in width or length of EPS geofoam wave barriers slightly improves the efficiency of the barrier. When EPS geofoam wave barrier is used in active isolation, vibration attenuation is less than that obtained from passive isolation, concluding that the barrier efficiency can be improved by adopting passive isolation. A considerable improvement of ground vibration attenuation can be achieved by increasing the frequency of the vibratory source. In addition, the efficiency of EPS geofoam wave barriers are superior to water in-filled wave barriers in attenuating Rayleigh waves with shorter wave lengths.

The efficiency of the water in-filled wave barrier can be improved by increasing the depth of the wave barrier, though the influence is less compared to an EPS geofoam barrier. The length of the wave barrier is not a significant factor in improving the efficiency of water in-filled wave barriers, similar to EPS geofoam wave barriers. However, a considerable improvement of the efficiency can be achieved by increasing the width of the wave barrier. Frequency of the vibratory source shows a moderate influence on the efficiency of the wave barrier, increasing the attenuation of ground vibrations when the frequency is increased. The location of the water in-filled wave barrier does not have any significant influence on the attenuation of ground waves beyond the wave barrier.

When the performance of EPS geofoam and the water-filled wave barriers are compared, EPS geofoam wave barrier outperforms the water-filled wave barrier as the depth of the wave barrier increases. When the length of the wave barrier is varied, EPS geofoam wave barrier performs slightly better than water in-filled wave barrier in attenuating ground vibrations closer to the vibratory source. However, both barriers perform similarly as the distance from the vibratory source increases. When the width of the wave barrier widens, water filled wave barrier outperforms EPS geofoam wave
barrier in attenuating ground vibrations as the distance from the vibratory source increases. However, closer to the vibratory source, EPS geofoam barrier is more efficient than the water filled barrier. Frequency of the vibratory source has a significant influence on the performance of wave barriers. At lower frequencies, efficiency of both EPS geofoam and water-filled wave barriers are similar. However, as the frequency of the vibratory source increases, EPS geofoam wave barriers outperform the water in-filled wave barriers. These results are useful in determining the geometry and fill material of wave barriers depending on the frequency of source of ground vibrations and the safe limit of peak particle velocity required to avoid damages to the existing nearby structures.
Chapter 7

Application of EPS geofoam in attenuating ground vibrations during vibratory pile driving

7.1 Introduction

Propagation of ground vibrations during urban construction activities has become a major concern due to severe damages to adjacent structures caused by vibrations. Vertical wave barriers are installed in the ground to minimise the detrimental effects of ground vibrations on surrounding structures. Different types of fill materials are used in constructing wave barriers. This chapter examines the efficiency of Expanded Polystyrene (EPS) geofoam in-filled wave barriers on ground vibration attenuation during vibratory pile driving in different soil types. A three-dimensional dynamic finite element model developed based on the Arbitrary-Lagrangian-Eulerian approach has been adopted to simulate the deep penetration of the pile from the ground surface. The mechanical behaviour of EPS geofoam is simulated using a constitutive model developed based on the hardening plasticity, which is an extension of the Drucker Prager yield criteria, described in Chapter 4. The effect of wave barrier on attenuating the ground vibration propagation is quantified by evaluating the reduction in bending moments
generated on an existing nearby pile. A parametric study was conducted varying the geometric parameters of the EPS geofoam wave barrier: length, depth, width and the location; the soil properties: density and Poisson’s ratio; and the soil type: Ariake clay, Bangkok clay and Singapore marine clay as well as a range of rigidity indexes for soils in general.

7.2 Numerical model for vibratory pile driving

The effect of EPS geofoam as a wave barrier in isolating ground vibration propagation is analysed using a dynamic finite element model developed to simulate vibratory pile driving using ABAQUS/Explicit finite element programme (ABAQUS, 2011). The technique adopted to simulate vibratory pile driving was based on the method developed by Henke and Grabe (2006). The finite element model consists of an analytically rigid surface modelling a tube of 1.0 mm radius placed along the axis of pile penetration. The tube is in frictionless contact with the finite element mesh representing the soil domain at the beginning of the analysis and the contact between the driven pile and tube is established during the deep penetration of the pile. The driven pile is a closed ended pile with a conical tip and it is also modelled as an analytical rigid surface. The tip of the pile is initially embedded in soil as illustrated in Figure 7.1, to avoid excessive element deformations, which can occur in the soil domain around the pile tip if the pile penetration is started at the ground surface. As the pile penetrates into the soil, it slides over the tube separating soil from the tube and generating pile-soil contact. This technique enables the finite element model to simulate penetration of a closed ended pile few diameters below the surface level, avoiding large mesh distortions, which are common for deep penetration problems (Ekanayake et al., 2012b).

Explicit central difference method is used for the time integration of the dynamic analyses in ABAQUS/Explicit. This integration scheme satisfies the dynamic equilibrium equations at the beginning of the time step, \( t \), and then computes velocities at time \( \left( t + \frac{\Delta t}{2} \right) \) and displacements at time \( (t + \Delta t) \), using the accelerations calculated at time \( t \), as discussed in Chapter 5.
Chapter 7  

Application of EPS geofoam in attenuating ground vibrations during vibratory pile driving

![Diagram](image)

**Figure 7.1 Conical tip of the pile embedded in soil.**

The soil domain was modelled using eight-node linear brick continuum elements with reduced integration and hourglass control (C3D8R). The soil domain was truncated beyond the region of interest using an artificial boundary in order to reduce the computational cost of numerical simulations. Artificial boundary was modelled using eight-node linear continuum infinite elements (CIN3D8). Detailed discussion of this boundary condition is presented in Chapter 3. Maximum element dimensions are determined considering the minimum wavelength, which can occur during an analysis. Damping is introduced to improve the modelling of high-speed dynamic events. A comprehensive description of selecting maximum element dimension and damping in dynamic models is given in Chapter 5.

The elastoplastic hardening constitutive model developed by Wong and Leo (2006) is used to simulate the mechanical behaviour of EPS geofoam wave barrier. Chapter 4 presents a detailed description of the constitutive model and implementation of the constitutive model in the finite element programme. Material properties used in this study are given in Table 4.1.
7.3 Numerical simulation of vibratory pile driving

A three-dimensional finite element model was developed as explained in Section 7.2 to simulate vibratory pile driving. The efficiency of EPS geofoam wave barrier in reducing vibration intensity on a nearby pile is investigated during vibratory pile driving. The diameter of the driven pile was 0.3 m and it was modelled as an analytical rigid surface to be in frictionless contact with the surrounding soil. A single vertical sinusoidal force of 600 kN operating at 30 Hz was applied to the driven pile at the reference point. Amplitude curve of the sinusoidal force is similar to the amplitude curve given in Figure 5.4. The soil domain was extended 20 m in radial direction and 15 m in vertical direction and region beyond that was truncated using infinite elements. Soil domain was modelled as an elastic-perfectly plastic material with von Mises yield criterion.

Three soil types were selected to conduct the parametric study investigating the efficiency of EPS geofoam in attenuating ground vibrations. The soil properties were extracted from a site investigation carried out by Tanaka et al. (2001) on Ariake clay, Bangkok clay and Singapore marine clay, as given in Table 7.1. The variation of material properties along the depth cannot be assigned to the infinite elements because this facility is not available in the finite element program. Therefore, the material properties were applied to the infinite domain in five layers with thickness of 3 m and each layer was assigned an average value of Young’s modulus and density. Based on the soil properties, the smallest Rayleigh wave velocity \( V_R \) is calculated for Ariake clay, which was 29.21 m/s near the surface of the soil domain. Hence, \( L_{\text{max}} \) was selected as 0.24 m.

A natural frequency extraction analysis was carried out for the three-dimensional finite element model to extract the eigen frequencies. For each soil type, the first six eigen frequency values were unrealistically small, possibly due to uncompleted stabilisation procedures at the time of frequency extraction. Therefore, the seventh eigen frequency was considered for each soil type, which were 5.27, 9.82 and 13.55 rad/s for Ariake, Bangkok and Singapore marine clay, respectively. The damping factors, \( \alpha_R \) and \( \beta_R \), were then calculated considering 2% of critical damping, as discussed in Chapter 5.
Table 7.1 Soil properties (Tanaka et al., 2001).

<table>
<thead>
<tr>
<th></th>
<th>Ariake clay</th>
<th>Bangkok clay</th>
<th>Singapore marine clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ (MPa)</td>
<td>6.8-12.2</td>
<td>29.0-46.4</td>
<td>65.0</td>
</tr>
<tr>
<td>(depth)</td>
<td>(5-10 m)</td>
<td>(7.5-15 m)</td>
<td>(15 m)</td>
</tr>
<tr>
<td>$s_u$ (kPa)</td>
<td>12-17</td>
<td>28-48</td>
<td>35</td>
</tr>
<tr>
<td>(depth)</td>
<td>(5-10 m)</td>
<td>(7.5-15 m)</td>
<td>(15 m)</td>
</tr>
<tr>
<td>Density (kg/m$^3$)</td>
<td>2600-2630</td>
<td>2720-2750</td>
<td>2760</td>
</tr>
<tr>
<td>(depth)</td>
<td>(3-12 m)</td>
<td>(5-12 m)</td>
<td>(15 m)</td>
</tr>
<tr>
<td>Poissons ratio</td>
<td>0.35</td>
<td>0.45</td>
<td>0.45</td>
</tr>
</tbody>
</table>

The existing pile is modelled as a concrete pile with a fixed connection at the head, in order to model a pile connected to a superstructure. The pile was assigned a diameter of 0.3 m and a length of 5 m. The pile is located 10 m away from the driven pile. A density of 2400 kg/m$^3$ and a Young’s modulus of 30 GPa were assigned to concrete. The Poisson’s ratio of concrete was assumed to be 0.2. A friction coefficient of 0.2 was assigned to the interface between the soil and the concrete pile and the pile was modelled using eight-node solid continuum elements with reduced integration.

An EPS geofoam wave barrier was installed between the existing pile and the driven pile, 5 m (center-to-center) away from the center of the existing pile. In the initial configuration, the barrier was given a length of 5 m, a depth of 5 m and a width of 0.4 m. The constitutive model and finite element formulation presented in Chapter 4 was used to simulate the mechanical behaviour of EPS geofoam using the user subroutine VUMAT. Figure 7.2 shows a plane section through the existing pile and the driven pile.

Only a quarter of the soil domain within which the existing pile and the EPS geofoam trench were located was modelled in order to reduce the computational cost of the analysis. Figure 7.3 shows the three-dimensional finite element model used for the analysis.
Chapter 7

Application of EPS geofoam in attenuating ground vibrations during vibratory pile driving

Figure 7.2 Plane section of the 3D finite element model through the plane of the driven pile and the existing pile.

Figure 7.3 Three-dimensional finite element model used for vibratory pile driving.
7.4 Performance of EPS geofoam wave barrier in attenuating ground vibrations

An initial study is conducted varying the dimensions of the EPS geofoam wave barrier to investigate the performance of the EPS geofoam wave barrier in different soil types. Then, the study is extended to examine the influence of the location of the wave barrier, and soil properties: stiffness, density and Poisson’s ratio. Waves were allowed to propagate in the soil over 0.5 s and the results were extracted at 0.5 ms, totalling 1000 extraction points for each model. Bending moments developed across a given cross section of the existing pile were recorded when the new pile is driving below the surface level. They were used to quantify the effect of ground vibrations induced due to vibratory pile driving on the existing nearby concrete pile and the efficiency of EPS geofoam trench in attenuating the ground vibrations.

The results plotted in this section are the highest bending moments recorded at each depth during the analysis. The highest bending moment at a given depth in the existing pile does not necessarily occur at the same instance as another section within the existing pile. That is the reason for irregularities observed in the bending moment distributions presented in this section.

7.4.1 Effect of geometry of the wave barrier in different soil types

A parametric study is conducted to determine the effect of the geometric properties of the barrier: length, depth and the width; in attenuating the ground vibrations. Basic configuration of the wave barrier was considered to have a length of 5 m, a depth of 5 m and a width of 0.4 m. The length and depth of the barrier were varied from 3 to 7 m while the width of the barrier was varied from 0.2 to 1.0 m. Figures 7.4, 7.5 and 7.6 show the maximum bending moments developed over cross sections of the existing pile at different depths, when the new pile is driving below the surface level in Ariake clay, Bangkok clay and Singapore marine clay, respectively.
Figure 7.4 Variation of maximum bending moment developed along the existing pile in Ariake clay for varying (i) depth (ii) length and (iii) width of wave barrier.
Figure 7.5 Variation of maximum bending moment developed along the existing pile in Bangkok clay for varying (i) depth (ii) length and (iii) width of wave barrier.
Figure 7.6 Variation of maximum bending moment developed along the existing pile in Singapore marine clay for varying (i) depth (ii) length and (iii) width of wave barrier.
Chapter 7

Application of EPS geofoam in attenuating ground vibrations during vibratory pile driving

Variation of maximum bending moment of the existing pile in Ariake Clay is shown in Figure 7.4. When the depth of the wave barrier is varied, greater attenuation of wave propagation can be observed at wave barrier depths 3 m (3.09λ_R) and 6 – 7 m (6.18 – 7.21λ_R). Rest of the wave barriers also attenuates the intensity of ground vibrations, but to a lesser extent. When the length of the wave barrier is varied, the barrier with a length of 3 m (3.09λ_R) shows the highest attenuation of wave propagation, reducing the maximum bending moment transferred across the pile. Wave barriers with length varying 4 – 7 m (4.12 – 7.21λ_R) attenuate the ground vibrations; however, they show attenuation efficiency closer to each other. Variation of width of the wave barrier 0.2 – 0.6 m (0.20 – 1.03λ_R) does not improve the wave attenuation significantly. Barrier with a width of 0.4 m (0.41λ_R) displays the highest attenuation of ground vibrations while the barrier with a width of 0.6 m (0.62λ_R) attenuates ground vibrations at some of the depths. Barrier with a width of 0.2 m (0.20 – 1.03λ_R) attenuates ground vibrations at 0.5 m below the surface and amplifies vibrations beyond that. Wave intensity is amplified with rest of the barrier configurations.

Figure 7.5 shows the variation of the maximum bending moment across the existing pile, in Bangkok clay. Young’s modulus of Bangkok clay is about four times higher than that of Ariake clay. A significant improvement in wave attenuation can be seen when the depth of the wave barrier is varied. The barrier with a depth of 7 m (3.63λ_R) shows the highest attenuation of ground vibrations while barrier with a depth of 5 m (2.59λ_R) shows the least attenuation. Nevertheless, mixed results are observed for the rest of the configurations with varying depth. When the length of the wave barrier is varied, highest attenuation can be observed at a length of 3 m (1.55λ_R), and lowest at 5 m and 7 m (2.59λ_R and 3.63λ_R). Attenuation data show mixed pattern closer to the pile head, however, towards the pile toe, shorter barriers shows highest attenuation. When the width of the wave barrier is varied, the barrier with a width of 0.2 m (0.10λ_R) shows the lowest attenuation while barriers with a width of 0.6 m and 0.8 m (0.31λ_R and 0.41λ_R) show the highest attenuation.
Attenuation data for different wave barrier configurations in Singapore marine clay are plotted in Figure 7.6. Singapore marine clay has the highest Young’s modulus, which is about 1.5 times higher than that of the Bangkok clay at 15 m depth. Considering the varying depth of the wave barrier, the ground vibrations are attenuated by the each configuration of the wave barrier. Highest attenuation is observed when the barrier depth is 3 m (1.05λ_R) and the lowest attenuation is observed when the barrier depth is 6 m (2.09λ_R). When the length of the wave barrier is varied, wave barrier with a length of 7 m (2.44λ_R) shows highest attenuation of ground vibrations. Barriers with lengths of 3 m and 6 m (1.05λ_R and 2.09λ_R) show the least attenuation of ground vibrations. It can be concluded that in Singapore marine clay, higher attenuation of ground vibrations can be achieved by increasing the length of the EPS geofoam wave barrier. The variation of width of the wave barrier in Singapore marine clay could alter the attenuation data, as shown in Figure 7.6(iii). The barrier with a width of 0.6 m (0.21λ_R) displays the highest attenuation and mixed results are observed for lowest attenuation when the width of the wave barrier is varied.

### 7.4.2 Effect of varying wave barrier location

Figure 7.7 shows the variation of maximum bending moment transferred across the existing pile when the location of the wave barrier was varied from 3 – 9 m from the driven pile at 2 m intervals. The distance between the driven pile and the existing pile is 10 m. The soil type used in these analyses was Singapore marine clay. It is apparent that when the wave barrier is closer to the driven pile (3 m) or the existing pile (9 m), the attenuation of ground vibrations is at the lowest. If the wave barrier is in the middle of the existing and driving piles or 7 m from the driven pile, the highest attenuation of ground vibrations is observed. Out of the wave barriers with lowest attenuation, wave barrier closer to the existing pile (9 m from the driven pile, i.e. 1 m to the existing pile) shows the lowest attenuation of ground vibrations.

In a free field wave propagation problem, discussed in the previous chapter, results showed that the barrier is most effective when it is closer to the structure. In that case, the order of materials between the source of vibration and the assumed location of the
structure is soil, geofoam and soil. When the incident wave transmits into geofoam, amplitude of the wave increases but when it enters the soil again at the opposite end, magnitude decreases due to higher density of soil than the geofoam. If the distance to the geofoam barrier increases, then the amplitude of the incident wave at the barrier decreases due to wave attenuation with distance from the source of vibration. Therefore the PPV recorded at the assumed location of the structure shows the highest attenuation when the barrier is closer to the structure. However in this case, when the pile is physically modelled in the finite element mesh, there are three mediums with different densities: soil, geofoam and pile. When the propagating wave exits the geofoam wave barrier, amplitude is decreased. It is further decreased when entering the pile with density higher than the soil. The reflected wave at the pile has a higher magnitude than the wave transmitted to the pile but reversed in direction. This will develop a wave scattering effect between the pile and the geofoam barrier. When the barrier moves closer to the pile this effect is intensified and as a result, bending moments developed in the pile become highly irregular and reaches the maximum when the barrier moves closer to the pile.

![Graph showing variation of maximum bending moment](image-url)

**Figure 7.7** Variation of maximum bending moment developed along the existing pile at varying barrier locations.
7.4.3 Effect of varying soil density
The effect of varying soil density (considering Singapore marine clay) in attenuating ground vibrations using an EPS geofoam wave barrier is shown in Figure 7.8. In this section densities were selected because wave velocities depend on the soil density. The bending moment distributions for densities of 2000 and 2250 kg/m$^3$ appear to be the dominant. Bending moment distributions for densities of 2770 and 3000 kg/m$^3$ show the lowest values for maximum bending moment transferred across the existing pile. With increase in soil density, velocity of both shear and compression waves will decrease. When the waves reach the geofoam wave barrier, due to low density of geofoam compared to soil, the amplitude of the wave transmitted to geofoam is higher than the amplitude of the incident wave. With increasing density of soil, the amplitude of wave transmitting into the geofoam increases. At the same time, when the wave passes from geofoam to the opposite side, the increase in density of soil will decrease the amplitude of the transmitted wave into to the soil. As a result with increasing soil density, bending moments developed over the existing pile decreases.

![Figure 7.8 Variation of maximum bending moment developed along the existing pile with varying soil density.](image)

7.4.4 Effect of varying Poisson’s ratio
Poisson’s ratio of Singapore marine clay was varied from 0.25 – 0.49 to determine the effect of Poisson’s ratio on ground vibration propagation. The variation of bending
moment across the existing pile in Singapore marine clay for each Poisson’s ratio considered is shown in Figure 7.9. As expected, the lower values for maximum bending moment transferred across the existing pile are recorded at the lower values of Poisson’s ratio. The highest bending moments were recorded for the highest Poisson’s ratio of 0.49 while the lowest was recorded for Poisson’s ratio of 0.25. By definition, Poisson’s ratio is the negative ratio of transverse to axial strain. When the Poisson’s ratio of soil increases, for the same axial strain, transverse strain developed in the soil increases. As a result lateral deformation of the pile will also increase. Therefore, an increase in pile bending moment can be observed with increasing Poisson’s ratio. Also with increasing Poisson’s ratio, shear wave velocity of soil decreases but the dilation or compression wave velocity increases. As a result, an increase in Poisson’s ratio will reduce the particle velocities in the vertical direction but increases the particle velocities in the radial direction. Hence pile experiences higher bending moments with increasing Poisson’s ratio.

![Figure 7.9 Variation of maximum bending moment developed along the existing pile with varying Poisson’s ratio.](image)

### 7.4.5 Effect of rigidity index of soil

In order to determine the effect of stiffness of soil in attenuating ground vibrations using EPS geofoam wave barriers, a parametric study was conducted by varying the rigidity
index, $G/s_u$, of Singapore marine clay. Here, $G$ is the shear modulus of soil and $s_u$ is the undrained shear strength of the soil. Stiffness of the soil is varied by changing the Young’s modulus of Singapore marine clay while keeping undrained shear strength as a constant. Rigidity index is varied from 300-1000 and the maximum bending moment transferred across the existing pile is given in Figure 7.10. Soil type with highest rigidity index carries the maximum impact on the existing pile while the soil type with lowest rigidity index carries the lowest impact on the existing pile. In this analysis, rigidity index of the soil is increased by increasing the shear modulus of soil. Hence with increasing rigidity index, both shear and dilation wave velocities in soil increases.

![Figure 7.10 Variation of maximum bending moment developed along the existing pile with varying rigidity index (RI).](image)

### 7.5 Conclusion

The effect of EPS geofoam wave barriers in attenuating ground vibrations during vibratory pile driving is investigated using a three-dimensional finite element model. The three-dimensional finite element model was developed adopting a novel technique, which has the ability to simulate the driving of closed ended piles, few diameters below the surface level, avoiding large mesh distortions common for deep penetration problems. An EPS geofoam wave barrier is installed between the driven pile and the existing pile and the effect of the EPS geofoam wave barrier in attenuating ground vibrations was
observed. A parametric study was conducted varying the depth, length and width of the
EPS geofoam wave barrier to evaluate the effect of the geometric parameters of the wave
barrier in attenuating ground vibrations in different soil types. The influence of the
barrier location together with the density, the Poisson’s ratio and the rigidity index of the
soil on the efficiency of the EPS geofoam wave barrier is also investigated.

The results presented in this chapter do not show a uniform trend with the parameters
varied during the finite element analysis. This may be due to the bending moments
recorded at different locations of the pile are maximum bending moments and they do
not occur at those sections at the same time. In addition, due to three different materials
present in the model (soil, geofoam and concrete), wave velocities are not uniform over
the finite domain. Due to the restraining effect of the surrounding soil at the vicinity of
the pile toe, which is modelled using solid continuum elements, recorded maximum
bending moments are high at pile toe for some cases although the existing pile used is a
floating pile.

In all soil types investigated, all three physical parameters: depth, length and width;
are crucial on the degree of efficiency of the wave barrier. While changes made to the
depth and length of the wave barrier can attenuate ground vibrations, changes to the
width of the wave barrier can either attenuate or amplify the ground vibrations. Highest
ground vibration attenuation in Ariake clay could be achieved at wave barrier depths
3.09λ_R and 6.18 – 7.21λ_R and at wave barrier length of 3.09λ_R. While wave barrier with a
width of 0.41λ_R attenuates ground vibrations at all depths, other wave barrier
configurations amplify the wave intensities. For Bangkok clay, highest attenuations of
ground vibrations were observed at a wave barrier depth of 3.63λ_R and at a wave barrier
length of 1.55λ_R. When the width of the wave barrier in Bangkok clay is varied, barriers
with widths of 0.31 λ_R and 0.41λ_R showed highest attenuation of ground vibrations, while
barrier with a width of 0.10λ_R showing the lowest attenuation of ground vibrations.
Attenuation of ground vibration in Singapore marine clay was at highest level when the
wave barrier depth is 1.05λ_R and the length is 2.44λ_R. When the width of the wave barrier
is varied, barrier with a width of 0.21λ_R displays the highest attenuation.
Varying the barrier location between the existing and the driven pile proved that extreme active isolation (vibration isolation closer to the source of vibration) and extreme passive isolation (vibration isolation closer to the existing structure) are not effective in attenuating ground vibrations. As the propagating wave enters the pile (which has a higher density), it is partially reflected back to soil medium, developing a wave scattering effect between the pile and the geofoam barrier. Wave scattering intensifies as the barrier moves closer to the pile. As a result, bending moments developed in the pile become highly irregular and reach the maximum when the barrier moves closer to the pile.

Variation in soil density affects the ground vibration propagation with higher densities resulting in lower bending moments in existing nearby pile. Increase in soil density result a decrease in wave velocities in soil medium. Further, with increasing density of soil, the amplitude of wave transmitting into the geofoam increases. However, when the wave passes from geofoam to the opposite side, the increase in density of soil will decrease the amplitude of the transmitted wave in to the soil. Hence, increase in soil density will decrease the bending moments developed across the existing pile.

The Poisson’s ratio shows a substantial influence on ground vibration attenuation, higher Poisson’s ratios resulting in lower vibration attenuation. When the Poisson’s ratio of soil increases, for the same axial strain, transverse strain developed in the soil increases. This will increase the lateral deformations experienced by existing pile, increasing the bending moments along the pile. Further, with increasing Poisson’s ratio, shear wave velocity decreases marginally but dilation wave velocity increases substantially. Consequently, particle velocities in vertical direction decrease and particle velocities in radial direction increase, resulting in higher bending moments in existing pile with increasing Poisson’s ratio.

Rigidity index also shows a significant influence in ground vibration propagation. Rigidity index is increased by increasing shear modulus of soil. Hence, when the rigidity index is increased, both shear and dilation wave velocities in soil increase. This intensifies ground vibrations experienced by existing pile, resulting in higher bending moments developed across the pile when the rigidity index of the soil is increased.
Chapter 8

Summary and Conclusion

8.1 Summary

Vibratory pile driving is often preferred in urban construction activities due to low noise and less ground disturbances during installation when compared to other pile installation methods. However, vibratory pile driving induced ground vibrations may cause severe damages to structures in the vicinity, depending on the proximity and the sensitivity of the structures. In-filled wave barriers can be used to diminish the ground vibration propagation induced by vibratory pile driving. There are number of fill materials used in practice such as concrete, bentonite, water or expanded polystyrene (EPS) geofoam. EPS geofoam offers number of advantages over other fill materials due to its light weight, cost effectiveness, energy absorbing characteristics and ease of handling. However, the construction industry lacks the understanding of full potential of EPS geofoam as a wave barrier.

This thesis presented a comprehensive numerical study of ground vibration propagation by vibratory pile driving and the efficiency of EPS geofoam in attenuating vibratory pile driving induced ground vibrations. A finite element formulation of a constitutive model to simulate the mechanical behaviour of EPS geofoam is first developed and verified based on the experimented data. Then, a set of two-dimensional axisymmetric finite element models were developed to investigate the free field ground vibration propagation induced by vibratory pile driving. The effect of ground vibration
propagation on existing structures was investigated varying the loading configurations and soil properties. Influence zones for different structures were derived based on the PPV limits given in design standards. Next, a three dimensional finite element model is developed to investigate the efficiency of water and EPS geofoam in-filled wave barriers. Knowledge gathered from investigation proved that EPS geofoam is the most efficient fill material in attenuating ground vibrations. Finally, the effect of EPS geofoam wave barriers in attenuating vibratory pile driving induced ground vibrations was studied. A three-dimensional dynamic finite element model is developed to simulate vibratory pile driving, which can drive a pile deep into soil medium avoiding mesh distortions. The efficiency of EPS geofoam wave barrier in attenuating vibratory pile driving induced ground vibrations was then investigated in detail by varying the barrier dimensions, location of the barrier, soil types and soil parameters. The outcomes of this research are useful to increase the knowledge on the applicability of EPS geofoam as wave barriers in attenuating ground vibration propagation, especially ground vibrations due to vibratory pile driving, and also the outcomes of this thesis will contribute for the development of new design guidelines for the use of EPS geofoam as wave barriers.

8.2 Conclusion

Major findings from the comprehensive numerical study carried out in this research are summarised below.

- Wave transmitting boundaries proposed by Du and Zhao (2010) for shear and dilation wave propagation are the most appropriate boundaries for axisymmetric wave propagation problems. However, extra nodes necessary to accommodate this boundary condition in a finite element model makes it less preferred in finite element modelling. Hence, the axisymmetric wave transmitting boundaries for shear and dilation wave propagation proposed by Deeks and Randolph (1994) can be considered as the most practicable boundary which can be applied to get results with a reasonable accuracy for axisymmetric wave propagation problems.
• Du and Zhao boundary condition outperformed Deeks and Randolph (1994) boundary condition, which is developed considering only cylindrical wave propagation, for plane-strain wave propagation problems. The viscous boundary proposed by Lysmer and Kuhlemeyer (1969) and infinite element in ABAQUS/Explicit show a rigid body motion, compared to the displacements from the case with extended mesh.

• Three-dimensional wave propagation can be simulated using infinite elements in ABAQUS/Explicit. Infinite element displays a rigid body motion, however, when considering peak displacements or particle velocities generated within the finite domain, infinite element can be used to simulate three-dimensional wave propagation problems with a reasonable accuracy.

• Finite element formulation is developed for the constitutive model developed by Wong and Leo (2006) to simulate mechanical behaviour of EPS geofoam extending the Drucker Prager yield criterion. Comparison of results with triaxial test data shows that this model has the ability to simulate EPS geofoam behaviour. It can be successfully applied to solve boundary value problems in geotechnical engineering, which employ EPS geofoam under normal operating temperatures and strain levels where rapid strain hardening does not occur.

• For vibratory pile driving, the influence zone does not significantly change with the amplitude or frequency of the driving force, and the depth of driving (driving near surface or below surface).

• For resonant pile driving, the influence zone changes with the frequency of the driving force but not with the depth of driving or the amplitude of the driving force.

• If pile driving needs to be carried out near buildings with historical value or architectural merit, resonant pile driving is more suitable than vibratory pile driving.

• When the rigidity index of the soil is varied for vibratory pile driving, PPVs are higher near the pile wall for higher rigidity indexes. The rate of attenuation of
PPV near the pile wall increases with increasing rigidity index. For all rigidity indexes, rate of PPV attenuation decreases with increasing radius.

- For resonant pile driving, PPV near the pile wall increases with decreasing rigidity index, in contrast to the vibratory pile driving. The rate of attenuation of PPV near the pile wall is low for soils with higher rigidity indexes. Away from the driven pile, PPV variation and rigidity index does not show any clear pattern.

- 2% of critical damping is appropriate to numerically simulate the field conditions in ground vibration propagation. A back analysis carried out using the Bornitz equation confirms that the 2% of critical damping is an appropriate value.

- Efficiency of EPS geofoam wave barriers increase significantly when the depth of the barrier is increased. Increase in width or length of EPS geofoam wave barriers slightly improves the efficiency of the barrier. The efficiency of EPS geofoam wave barrier can be improved by adopting passive isolation, where barrier is placed away from the vibratory source, closer to the point of interest.

- A considerable improvement of ground vibration attenuation can be achieved by increasing the frequency of the vibratory source. The efficiency of EPS geofoam wave barriers are superior to water in-filled wave barriers in attenuating Rayleigh waves with shorter wave lengths.

- The efficiency of the water in-filled wave barrier can be improved by increasing the depth of the wave barrier. The length of the wave barrier is not a significant factor in improving the efficiency of water in-filled wave barriers. A considerable improvement of the efficiency can be achieved by increasing the width of the water-filled wave barrier.

- Frequency of the vibratory source shows a moderate influence on the efficiency of the water-filled wave barrier, increasing the attenuation of ground vibrations when the frequency is increased. The location of the water in-filled wave barrier does not have any significant influence on the attenuation of ground waves beyond the wave barrier.
• EPS geofoam wave barrier outperforms the water-filled wave barrier as the depth of the wave barrier increases. When the length of the wave barrier is varied, EPS geofoam wave barrier performs slightly better than water in-filled wave barrier in attenuating ground vibrations closer to the vibratory source. Both wave barriers perform similarly as the distance from the vibratory source increases. When the width of the wave barrier widens, water filled wave barrier outperforms EPS geofoam wave barrier in attenuating ground vibrations as the distance from the vibratory source increases.

• Closer to the vibratory source, EPS geofoam barrier is more efficient than the water filled barrier. At lower frequencies, efficiency of both EPS geofoam and water-filled wave barriers are similar. As the frequency of the vibratory source increases, EPS geofoam wave barriers outperform the water in-filled wave barriers.

• All three physical parameters of EPS geofoam wave barriers: depth, length and width; are crucial in designing the barrier configuration. While increase in depth and decrease in length of the wave barrier can improve the attenuation of ground vibrations, changes to the width of the wave barrier can either attenuate or amplify the ground vibrations.

• Varying the barrier location between the existing and the driven pile proves that extreme active isolation (vibration isolation closer to the source of vibration) and extreme passive isolation (vibration isolation closer to the existing structure) are not effective in attenuating ground vibrations.

• Variation in density influences the ground vibration propagation with higher densities resulting in lower intensities in ground vibrations. The Poisson’s ratio shows a substantial influence on ground vibration attenuation, higher Poisson’s ratios result lower vibration attenuation. Rigidity index can also affect the ground vibration propagation significantly. Higher rigidity indices amplify the amplitude of ground vibrations propagating in ground.
8.3 Recommendations for future research

Findings of this research are important to understand the applicability of EPS geofoam as a wave barrier in attenuating ground vibrations induced by vibratory pile driving. The design charts developed are useful for practicing geotechnical engineers to use EPS geofoam as a wave barrier. However, more research in this area is necessary to enhance the understanding and current design practices of EPS geofoam as a wave barrier. Hence, following recommendations are made to be considered in future studies.

1. Findings of this research can be added to the knowledge base of the influence of vibratory driving of close-ended piles and its influence on existing structures. However, the research can be extended by investigating the influence of different types of piles used in urban construction activities: open ended piles and sheet piles, physical dimensions of the pile; diameter and cross section, and shape of the pile.

2. This research is conducted using properties extracted from a variety of EPS geofoam manufactured in Australia. However, manufacturing process and quality control may vary from one production plant to another, and hence the properties of EPS geofoam vary among different manufactures. The effectiveness of attenuating vibratory pile driving induced ground vibrations using another EPS geofoam variety as wave barriers can therefore vary from what has been observed in this research. This should be taken into account when developing design guidelines to use EPS geofoam wave barriers against ground vibration propagation.

3. Attenuation of influence of vibratory pile driving induced ground vibrations using EPS geofoam wave barriers was investigated by evaluating the reduction in bending moment in a nearby pile. The existing pile simulated here is a floating pile fixed at the pile head. It is assumed that it simulates a friction pile attached to a superstructure. In future research studies, the single configuration of the friction pile used in this research needs to be extended including the geometric properties
of the existing pile: length and diameter and material properties. Also different configurations of pile groups should be considered.

4. As an extension to this research, influence of vibratory pile driving on end bearing piles also needs to be investigated. Apart from the factors recommended to investigate on friction piles, profile of the bedrock should also be considered as it can also affect the ground vibration propagation during vibratory pile driving. Also an assessment should be made about the effect of ground vibrations on overall bearing capacity of the nearby existing pile.

5. Research can be further strengthened by considering the influence of water table and excess pore pressure generation during ground vibration propagation. In addition, presence of water in soil domain can influence the wave velocities in the soil domain. Predictions made by conducting ground vibration propagation analyses for a soil domain with no water table will not be valid in such situations. However, this stage will be challenging to perform because currently ABAQUS/Explicit does not have facilities to incorporate pore pressure effects into the finite element analysis.

6. Robust analysis methods should be developed to compute the influence of different pile installation methods such as impact pile driving and for different pile types (sheet piles and open-ended piles) on existing foundations. The numerical methods developed in this thesis are able to simulate only driving of closed ended piles. This will enable practicing geotechnical engineers to determine whether the existing foundations need extra protection. Design guidelines should also be developed to calculate the degree of protection provided by EPS geofoam against different ground vibration sources such as drilling, blasting and traffic induced ground vibrations in addition to the vibrations generated due to pile driving. These design guidelines are useful for practicing geotechnical engineers, where remedial measures are required to protect existing foundations or structures supported by them.
References

Aaboe, R. (1987). ‘13 years of experience with expanded polystyrene as a lightweight fill material in road embankments’, Norwegian Road Research Laboratory.

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References


References


Appendix A

VUMAT Subroutine

Intel Visual Fortran code to implement the VUMAT subroutine of EPS geofoam constitutive model (Wong and Leo, 2006) is presented here.

******************************************************************************
** VUMAT, FOR ABAQUS/EXPLICIT INCORPORATING ELASTIC-PLASTIC LINEAR, **
** NONASSOCIATIVE FLOW. LARGE DEFORMATION FORMULATION FOR THREE  **
** DIMENSIONAL ELEMENTS. EXPLICIT INTEGRATION IS USED **
******************************************************************************

*USER SUBROUTINE

SUBROUTINE VUMAT(

C Read only -
1 NBLOCK, NDIR, NSHR, NSTATEV, NFIELDV, NPROPS, LANNEAL,
2 STEPTIME, TOTALTIME, DT, CMNAME, COORDMP, CHARLENGTH,
3 PROPS, DENSITY, STRAININC, RELSPININC,
4 TEMPOLD, STRETCHOLD, DEFGRADOLD, FIELDOLD,
5 STRESSOLD, STATEOLD, ENERINTERNOLD, ENERINELASOLD,
6 TEMPNEW, STRETCHNEW, DEFGRADNEW, FIELDNEW,

C Write only -
7 STRESSNEW, STATENEW, ENERINTERNNEW, ENERINELASNEW)

C

INCLUDE 'VABA_PARAM.INC'

C

DIMENSION PROPS(NPROPS), DENSITY(NBLOCK), COORDMP(NBLOCK),
1 CHARLENGTH(NBLOCK), STRAININC(NBLOCK, NDIR+NSHR),
2 RELSPININC(NBLOCK, NSHR), TEMPOLD(NBLOCK),
3 STRETCHOLD(NBLOCK, NDIR+NSHR),DEFGRADOLD(NBLOCK,NDIR+NSHR+NSHR),
4 FIELDOLD(NBLOCK, NFIELDV), STRESSOLD(NBLOCK, NDIR+NSHR),
5 STATEOLD(NBLOCK, NSTATEV), ENERINTERNOLD(NBLOCK),
6 ENERINELASOLD(NBLOCK), TEMPNEW(NBLOCK),
7 STRETCHNEW(NBLOCK, NDIR+NSHR),DEFGRADNEW(NBLOCK,NDIR+NSHR+NSHR),

Appendix A

8 FIELDNEW(NBLOCK, NFIELDV), STRESSNEW(NBLOCK,NDIR+NSHR),
9 STATENEW(NBLOCK, NSTATEV), ENERINTERNNEW(NBLOCK),
1 ENERINELASNEW(NBLOCK)

C

CHARACTER*80 CMNAME

C

PARAMETER (ZERO=0.D0,ONE=1.D0, TWO=2.D0, THREE=3.D0)

C

DIMENSION DSTRAN(NBLOCK, 6), STROLD(NBLOCK, 6), STRNEW(NBLOCK, 6)
DOUBLE PRECISION EBULK3, EG2, EG, ELAM, TRACE, PRESS,
1 DSTR1, DSTR2, DSTR3, DSTR4, DSTR5, DSTR6, PJ, x, a, ZY,
2 F1, F2, F3, F4, F5, F6, G1, G2, G3, G4, G5, G6,
3 DVA1, DVA2, DVA3, DVA4, DVA5, DVA6, DVB1, DVB2, DVB3, DVB4,
4 DVB5, DVB6, TERM2, TERM3, LAM, DPSTRAN1, DPSTRAN2, DPSTRAN3,
5 DPSTRAN4, DPSTRAN5, DPSTRAN6, DESTRAN1, DESTRAN2, DESTRAN3,
6 DESTRAN4, DESTRAN5, DESTRAN6, DSTRESS1, DSTRESS2, DSTRESS3,
7 DSTRESS4, DSTRESS5, DSTRESS6, DEP1, DEP2, DEP3, DEP4, DEP5,
8 DEP6, T, DEQ, DLAMBDA

C

SPECIFY MATERIAL PROPERTIES

C

E = 3950.0       ! given in kN/m2
XNUE = 0.0
b = -0.6393
C = -0.714
h = 113.67

C

EBULK3 = E/(ONE-TWO*XNUE) ! BULK MODULUS (3K)
EG2 = E/(ONE+XNUE)       ! SHEAR MODULUS (2G)
EG = EG2/TWO             ! SHEAR MODULUS (G)
ELAM = (EBULK3-EG2)/THREE ! LAME'S 1ST PARAMETER (3K-2G)/3

C

DO I=1, NBLOCK

C

DSTRAN(I,1)=-STRAININC(I,1)
DSTRAN(I,2)=-STRAININC(I,2)
DSTRAN(I,3)= -STRAININC(I,3)
DSTRAN(I,4)= -STRAININC(I,4)
DSTRAN(I,5)= -STRAININC(I,5)
DSTRAN(I,6)= -STRAININC(I,6)
STROLD(I,1)= -STRESSOLD(I,1)
STROLD(I,2)= -STRESSOLD(I,2)
STROLD(I,3)= -STRESSOLD(I,3)
STROLD(I,4)= -STRESSOLD(I,4)
STROLD(I,5)= -STRESSOLD(I,5)
STROLD(I,6)= -STRESSOLD(I,6)

C
END DO

DO I=1, NBLOCK

C
TRACE = DSTRAN(I,1) + DSTRAN(I,2) + DSTRAN(I,3)
STRNEW(I,1) = STROLD(I,1)
  * + EG2 * DSTRAN(I,1) + ELAM * TRACE
STRNEW(I,2) = STROLD(I,2)
  * + EG2 * DSTRAN(I,2) + ELAM * TRACE
STRNEW(I,3) = STROLD(I,3)
  * + EG2 * DSTRAN(I,3) + ELAM * TRACE
STRNEW(I,4)=STROLD(I,4) + EG * DSTRAN(I,4)
STRNEW(I,5)=STROLD(I,5) + EG * DSTRAN(I,5)
STRNEW(I,6)=STROLD(I,6) + EG * DSTRAN(I,6)

C
END DO

C

IF (totalTIME.EQ.ZERO) THEN
  STATENEW(I,1)=ZERO
END IF

C

DO I=1, NBLOCK

C
CALCULATE THE AVERAGE NORMAL STRESS (P)

C
PRESS = (STROLD(I,1)+STROLD(I,2)+STROLD(I,3))/THREE
CALCULATE THE DEVIATORIC STRESS (Q)

\[
\begin{align*}
  \text{DSTR1} &= \text{STROLD}(I,1) - \text{PRESS} \\
  \text{DSTR2} &= \text{STROLD}(I,2) - \text{PRESS} \\
  \text{DSTR3} &= \text{STROLD}(I,3) - \text{PRESS} \\
  \text{DSTR4} &= \text{STROLD}(I,4) \\
  \text{DSTR5} &= \text{STROLD}(I,5) \\
  \text{DSTR6} &= \text{STROLD}(I,6)
\end{align*}
\]

CALCULATE DEVIATORIC STRESS (Q)

\[
\text{PJ} = \sqrt{\frac{3}{2} \left( \text{DSTR1}^2 + \text{DSTR2}^2 + \text{DSTR3}^2 + 2 \text{DSTR4}^2 \right)/2}
\]

RECOVER a, FROM PREVIOUS TIME STEP

\[
\begin{align*}
  \text{x} &= \text{STATEOLD}(I,1) \\
  \text{IF} \ (\text{x.LT.1}) \ \text{THEN} \\
  \quad \text{a} &= 160.6560000 \\
  \text{ELSE} \\
  \quad \text{a} &= \text{STATEOLD}(I,2) \\
  \text{END IF}
\end{align*}
\]

CHECK THE YIELD CONDITION

\[
\begin{align*}
  \text{ZY} &= \frac{(\text{PJ} - (a+b*\text{THREE*PRESS}))/\text{ONE}-b)}{\text{ONE}-b} \\
  \text{DLAMBDA} &= 0. \\
  \text{IF} \ (\text{ZY.GT.0.}) \ \text{THEN} \\
  \text{GRADIENT OF THE YIELD FUNCTION } "F", (d.f/d.sigma)
\end{align*}
\]

\[
\begin{align*}
  \text{F1} &= \frac{((\text{THREE}/\text{TWO}/\text{PJ})*\text{DSTR1}-b)/\text{ONE}}{\text{ONE}-b} \\
  \text{F2} &= \frac{((\text{THREE}/\text{TWO}/\text{PJ})*\text{DSTR2}-b)/\text{ONE}}{\text{ONE}-b} \\
  \text{F3} &= \frac{((\text{THREE}/\text{TWO}/\text{PJ})*\text{DSTR3}-b)/\text{ONE}}{\text{ONE}-b}
\end{align*}
\]


Appendix A

\[
F_4 = \frac{3}{P_J} \cdot \text{DSTR}_4 / (1 - b)
\]
\[
F_5 = \frac{3}{P_J} \cdot \text{DSTR}_5 / (1 - b)
\]
\[
F_6 = \frac{3}{P_J} \cdot \text{DSTR}_6 / (1 - b)
\]

CALCULATE THE PLASTIC FLOW DIRECTION "G", (d.g/d.sigma)

\[
G_1 = \frac{3}{2/P_J} \cdot \text{DSTR}_1 - c / (1 - c)
\]
\[
G_2 = \frac{3}{2/P_J} \cdot \text{DSTR}_2 - c / (1 - c)
\]
\[
G_3 = \frac{3}{2/P_J} \cdot \text{DSTR}_3 - c / (1 - c)
\]
\[
G_4 = \frac{3}{P_J} \cdot \text{DSTR}_4 / (1 - c)
\]
\[
G_5 = \frac{3}{P_J} \cdot \text{DSTR}_5 / (1 - c)
\]
\[
G_6 = \frac{3}{P_J} \cdot \text{DSTR}_6 / (1 - c)
\]

DETERMINE THE PLASTIC MULTIPLIER [D]*DSTRAIN

\[
DVA_1 = (E + \text{LAM}) \cdot \text{DSTRAN}(I,1)
\]
\[
\quad + \text{LAM} \cdot (\text{DSTRAN}(I,2) + \text{DSTRAN}(I,3))
\]
\[
DVA_2 = (E + \text{LAM}) \cdot \text{DSTRAN}(I,2)
\]
\[
\quad + \text{LAM} \cdot (\text{DSTRAN}(I,1) + \text{DSTRAN}(I,3))
\]
\[
DVA_3 = (E + \text{LAM}) \cdot \text{DSTRAN}(I,3)
\]
\[
\quad + \text{LAM} \cdot (\text{DSTRAN}(I,1) + \text{DSTRAN}(I,2))
\]
\[
DVA_4 = E \cdot \text{DSTRAN}(I,4)
\]
\[
DVA_5 = E \cdot \text{DSTRAN}(I,5)
\]
\[
DVA_6 = E \cdot \text{DSTRAN}(I,6)
\]

NUMERATOR OF LAMBDA

\[
\text{TERM}_1 = (DVA_1 \cdot F_1) + (DVA_2 \cdot F_2) + (DVA_3 \cdot F_3) + (DVA_4 \cdot F_4)
\]
\[
\quad + (DVA_5 \cdot F_5) + (DVA_6 \cdot F_6)
\]

FIRST TERM IN DENOMINATOR

\[
DVB_1 = (E + \text{LAM}) \cdot G_1 + \text{LAM} \cdot (G_2 + G_3)
\]
\[
DVB_2 = (E + \text{LAM}) \cdot G_2 + \text{LAM} \cdot (G_1 + G_3)
\]
\[
DVB_3 = (E + \text{LAM}) \cdot G_3 + \text{LAM} \cdot (G_1 + G_2)
\]
\[
DVB_4 = E \cdot G_4
\]
DVB5 = EG*G5
DVB6 = EG*G6

C
TERM2 = (DVB1*F1)+(DVB2*F2)+(DVB3*F3)+(DVB4*F4)
1 + (DVB5*F5)+(DVB6*F6)

C
HARDENING PARAMETER IN THE DENOMINATOR

C
TERM3 = h

C
LAM = TERM1/(TERM2+TERM3)

C
DLAM = DABS(LAM)

C
CALCULATE PLASTIC STRAIN INCREMENT

C
DPSTRAN1 = DLAM*G1
DPSTRAN2 = DLAM*G2
DPSTRAN3 = DLAM*G3
DPSTRAN4 = DLAM*G4
DPSTRAN5 = DLAM*G5
DPSTRAN6 = DLAM*G6

C
DETERMINE ELASTIC STRAIN INCREMENT

C
DESTRAN1 = DSTRAN(I,1)-DPSTRAN1
DESTRAN2 = DSTRAN(I,2)-DPSTRAN2
DESTRAN3 = DSTRAN(I,3)-DPSTRAN3
DESTRAN4 = DSTRAN(I,4)-DPSTRAN4
DESTRAN5 = DSTRAN(I,5)-DPSTRAN5
DESTRAN6 = DSTRAN(I,6)-DPSTRAN6

C
DETERMINE STRESS INCREMENT

C
DSTRESS1 = (EG2+ELAM)*DESTRAN1 + ELAM*(DESTRAN2+DESTRAN3)
DSTRESS2 = (EG2+ELAM)*DESTRAN2 + ELAM*(DESTRAN1+DESTRAN3)
DSTRESS3 = (EG2+ELAM) * DESTRAN3 + ELAM * (DESTRAN1+DESTRAN2)
DSTRESS4 = EG * DESTRAN4
DSTRESS5 = EG * DESTRAN5
DSTRESS6 = EG * DESTRAN6

C
C UPDATE ALL QUANTITIES USING EXPLICIT INTEGRATION
C

STRNEW(I,1) = STROLD(I,1) + DSTRESS1
STRNEW(I,2) = STROLD(I,2) + DSTRESS2
STRNEW(I,3) = STROLD(I,3) + DSTRESS3
STRNEW(I,4) = STROLD(I,4) + DSTRESS4
STRNEW(I,5) = STROLD(I,5) + DSTRESS5
STRNEW(I,6) = STROLD(I,6) + DSTRESS6

C

STRAIN = DPSTRAN1 + DPSTRAN2 + DPSTRAN3

C

DEP1 = DPSTRAN1 - STRAIN / THREE
DEP2 = DPSTRAN2 - STRAIN / THREE
DEP3 = DPSTRAN3 - STRAIN / THREE
DEP4 = DPSTRAN4
DEP5 = DPSTRAN5
DEP6 = DPSTRAN6

C

DEVIATORIC PLASTIC STRAIN
C

T = DEP1**2 + DEP2**2 + DEP3**2 + DEP4**2 + DEP5**2 + DEP6**2
C

DEQ = SQRT(T/TWO) !DEVIATORIC PLASTIC STRAIN
DEQ = ABS(DEQ)
C

STATENEW(I,3) = STATEOLD(I,3) + DEQ
a = 160.656 + 368.85 * STATENEW(I,3)
C

STORE UPDATED STATE VARIABLES
C

STATENEW(I,1) = ONE
STATENEW(I,2)=a
C
END IF
C
END DO
C
DO I=1, NBLOCK
STRESSNEW(I,1)=-STRNEW(I,1)
STRESSNEW(I,2)=-STRNEW(I,2)
STRESSNEW(I,3)=-STRNEW(I,3)
STRESSNEW(I,4)=-STRNEW(I,4)
STRESSNEW(I,5)=-STRNEW(I,5)
STRESSNEW(I,6)=-STRNEW(I,6)
END DO
C
RETURN
END