AREA: THE BIG COVER-UP

DIANE MCPHAIL

DOCTOR OF PHILOSOPHY

UNIVERSITY OF WESTERN SYDNEY

2007
ACKNOWLEDGMENTS

I would like to thank my three supervisors, Dr Kay Owens, Associate Professor Bob Perry and Dr Allan White. Kay inspired me to commence my research journey and supported me through the data collection phase. Bob and Allan patiently guided me through the analysis and the final documentation. I am indebted to their constant encouragement and wise counseling.

My friend and mentor, Dr Lynne Outhred, provided guidance and a critical ear.

This research was made possible by the teachers and children who trialed the area lessons and the professional learning models. My sincerest thanks go to the teachers who shared their programs, thoughts, and classes with me. Together we explored the many facets of how to help children to describe and measure area.

I would also like to acknowledge the assistance provided by my employer, the New South Wales Department of Education and Training, and the support and patience extended by colleagues.
STATEMENT OF AUTHENTICATION

The work presented in this thesis is, to the best of my knowledge and belief, original except as acknowledged in the text. I hereby declare that I have not submitted this material, either in full or in part, for a degree at this or any other institution.
TABLE OF CONTENTS

LIST OF TABLES ........................................................................................................................................... vii
LIST OF FIGURES .......................................................................................................................................... ix
GLOSSARY .................................................................................................................................................. x
ABSTRACT .................................................................................................................................................. xii
CHAPTER 1 - INTRODUCTION ....................................................................................................................... 1

1.1 Experiences and Concerns .................................................................................................................... 2
1.2 Measurement and Area within the Mathematics Curriculum ............................................................... 6
  1.2.1 Measurement in the Curriculum ........................................................................................................ 6
  1.2.2 The Area Substrand in the New South Wales Curriculum ................................................................. 6
    1.2.2.1 The New South Wales 1989 syllabus, Mathematics K-6 ................................................................. 6
    1.2.2.2 The New South Wales 2002 Syllabus ............................................................................................ 8
1.3 Teacher Professional Learning ............................................................................................................... 9
1.4 Questions to be addressed by this research ......................................................................................... 12
  1.4.1 Phase 1 Aim and Research Question ................................................................................................ 12
  1.4.2 Phase 2 Aim and Research Questions ............................................................................................... 13
1.5 Designing the Study as a Research Project ......................................................................................... 13
1.6 Plan of the Thesis .................................................................................................................................. 14
1.7 Conclusion ............................................................................................................................................. 15

CHAPTER 2 - REVIEW OF THE LITERATURE ............................................................................................ 16

2.1 Teaching and Learning about Measuring the Area of Rectangles ......................................................... 16
  2.1.1 Understanding the Attribute of Area .................................................................................................. 20
  2.1.2 The Use of Units to Measure ............................................................................................................... 21
  2.1.3 Measuring Area with Informal Units .................................................................................................. 23
  2.1.4 Counting the Number of Informal Units Used to Measure Area ...................................................... 27
  2.1.5 Estimating Measurements ................................................................................................................ 29
  2.1.6 Implications for the Design and Implementation of an Area Teaching Program .............................. 30
2.2 Teacher Professional Learning ............................................................................................................. 33
  2.2.1 What is Teacher Professional Learning? ............................................................................................. 33
  2.2.2 The Principles of Effective Professional Learning .............................................................................. 35
  2.2.3 Implementing Professional Learning Programs ................................................................................. 39
  2.2.4 Contributing Factors to Successful Teacher Professional Learning .................................................. 41
    2.2.4.1 Attitudes and values that teachers bring to their profession .......................................................... 42
    2.2.4.2 Teachers’ commitment to achieving positive student learning outcomes ................................... 43
    2.2.4.3 Teacher collaboration .................................................................................................................... 44
    2.2.4.4 Teacher reflection .......................................................................................................................... 45
    2.2.4.5 Making time for professional learning ........................................................................................... 46
  2.2.5 Teacher Professional Learning and the Teaching of Mathematics .................................................. 47
    2.2.5.1 Professional learning and classroom questioning strategies ......................................................... 49
    2.2.5.2 Teacher knowledge and skills to teach the measurement of area ................................................ 50
  2.2.6 The Role of Teacher Leadership in Curriculum Implementation .................................................... 51
2.3 Designing the Methodology of the Research Program .......................................................................... 54
  2.3.1 Phase 1: Development and trial of area lessons .................................................................................. 55
  2.3.2 Phase 2: Trial of Teacher Professional Learning Models ................................................................... 56
    2.3.2.1 Action research projects ............................................................................................................... 58
  2.3.3 Research Methods to Evaluate the Lessons and the Professional Learning Models .......................... 60
    2.3.3.1 Quantitative research methods ..................................................................................................... 61
    2.3.3.2 Qualitative research methods ....................................................................................................... 61
CHAPTER 3 - FIRST STEPS ........................................................................................ 64

First Steps Part 1: Teachers’ Knowledge and Practices in Teaching Area..................64

3.1 Part 1: Design..................................................................................................65
3.1.1 Aims of the Investigation ...........................................................................65
3.1.1.1 Research questions .............................................................................65
3.1.2 Research Context .....................................................................................66
3.1.4 Ethical Issues ...........................................................................................67

3.2 Part 1: Method ................................................................................................67
3.2.1 Interviews ..................................................................................................67
3.2.2 Analysis ....................................................................................................68

3.3 Part 1: Findings ................................................................................................69
3.3.1 Teachers’ Knowledge and Practices in the Teaching of Area .......................69
3.3.1.1 How is area being taught? .....................................................................69
3.3.1.2 Is area integrated with other mathematics concepts? .........................70
3.3.1.3 What resources do teachers use when planning area activities? .............71

Teachers were asked what skills and understandings about area they thought that children
needed to develop during Year 1 or Year 2. Teachers’ responses were reasonably
similar for area skills, but some teachers mentioned other skills as well (Table 3.4). ....71
3.3.1.6 Summary of research questions .............................................................73
3.3.2.1 Aim: To establish familiarity with planning and conducting interviews. ....74
3.3.2.2 Aim: To analyse interview and observational data ................................74

3.4 Part 1: Implications for the Major Study ......................................................75

First Steps Part 2: Trial of a Sequence of Area Lessons .............................................75

3.5 Part 2: Design ..................................................................................................76
3.5.1 Aims ..........................................................................................................76
3.5.1.1 Research questions .............................................................................76
3.5.2 Research Context .....................................................................................77

3.6 Part 2: Method ..................................................................................................77
3.6.1 Design of the Four-Lesson Program ..........................................................78
3.6.2 Sequence of Tasks ..................................................................................79
3.6.2.1 Lesson 1 ..........................................................................................79
3.6.2.3 Lesson 3 ..........................................................................................80
3.6.2.4 Lesson 4 ..........................................................................................80
3.6.3 Data Collection .........................................................................................80
3.6.3.1 Interviews with the classroom teacher ...............................................81
3.6.3.2 Interviews with a group of children ...................................................81
3.6.3.3 Field notes .......................................................................................83
3.6.3.4 Video recordings ...............................................................................84
3.6.4 Data Analysis ...........................................................................................84

3.7 Part 2: Findings ................................................................................................85
3.7.1 Analysis of the Lessons .............................................................................85
3.7.1.1 Lesson 1 ..........................................................................................85
3.7.1.2 Lesson 2 ..........................................................................................86
3.7.1.3 Lesson 3 ..........................................................................................88
3.7.1.4 Lesson 4 ..........................................................................................89
3.7.2 Research Question 1 ................................................................................90
3.7.2.1 Covering the area of a shape with repeated, tessellating units .............90
3.7.2.2 Calculating the number of units in multiples (using rows and columns) ....90
3.7.2.3 Identifying the pattern of covering units as a grid constructed from continuous lines...90
3.7.3 Research Question 2 ................................................................................91
CHAPTER 6 – RESULTS OF IMPLEMENTATION OF AREA LESSONS

6.1 Outline of Data Collection ................................................................. 112

6.1.1 Data Collected from Interviews with Teachers and Researcher’s Field Notes ................................................................. 112
6.1.2 Data Collected from Children ................................................................................................................................. 113

6.2 Findings ........................................................................................... 113

6.2.1 Results from Interviews with Teachers and Researcher’s Field Notes ................................................................. 113
6.2.1.1 Labelling of comments ................................................................................................................................. 113
6.2.1.2 Coding scheme .............................................................................................................................................. 114
6.2.1.3 Analysis of interviews and field notes comments .............................................................................................. 117
6.2.1.4 Presentation of data for each lesson and the groups of lessons .................................................................................. 117
6.2.1.5 Area syllabus lesson 1 ...................................................................................................................................... 118
6.2.1.6 Area syllabus lesson 2 ...................................................................................................................................... 122
6.2.1.7 Area syllabus lesson 3 ...................................................................................................................................... 125
6.2.1.8 Area syllabus lesson 4 ...................................................................................................................................... 129
6.2.1.9 Area syllabus lessons final teacher interview .................................................................................................................. 132

6.2.2 Analysis of Data .................................................................................. 109

6.2.3 Student assessment task .................................................................... 108

6.2.4 Interviews with teachers .................................................................... 106

6.2.5 Collection of Data ............................................................................... 105

6.2.6 Labelling of comments ........................................................................ 113

6.2.7 Coding of student responses, drawing the tiles ...................................... 109

CHAPTER 5 - IMPLEMENTATION OF A SEQUENCE OF AREA LESSONS .... 100

5.1 Design ................................................................................................. 100

5.1.1 Research Context ............................................................................. 100

5.2 Method ................................................................................................. 101

5.2.1 Designing the Lesson Sequences ......................................................... 101
5.2.1.1 Area syllabus lesson sequence ............................................................... 101
5.2.1.2 Area research lesson sequence ............................................................ 102
5.2.2 Implementation of lessons ................................................................... 104
5.2.3 Collection of Data ............................................................................... 105
5.2.4 Student assessment task ..................................................................... 108
5.2.5 Analysis of Data ................................................................................... 109
5.2.5.1 Analysis of teacher interviews ............................................................... 109
5.2.5.2 Coding of student responses, drawing the tiles ...................................... 109

5.3 Conclusion ............................................................................................ 110

CHAPTER 4 – DESIGN OF THE STUDY ......................................................... 94

4.1 Phase 1: Implementation of a Sequence of Area Lessons .......................... 94

4.1.1 Aim .................................................................................................. 96
4.1.1.1 96
4.1.2 Outline of Data Collection ................................................................. 96
4.1.2.1 Student activity ................................................................................. 96
4.1.2.2 Interviews with teachers ................................................................. 97
4.1.2.3 Field notes ..................................................................................... 97

4.2 Phase 2: Implementation of Teacher Professional Learning Models .......... 97

4.2.1 Aim .................................................................................................. 98
4.2.1.2 Research questions .......................................................................... 98
4.2.2 Outline of Data Collection ................................................................. 99
4.2.2.1 Interviews with teachers ................................................................. 99
4.2.2.2 Student pre-test and post-test assessment task ................................. 99

4.3 Conclusion ............................................................................................ 99

CHAPTER 3 – OUTLINE OF DATA COLLECTION ....................................... 82

3.1 Interview .............................................................................................. 83

3.2 Student activity ..................................................................................... 83

3.3 Interview with teachers .......................................................................... 84

3.4 Coding of student responses, drawing the tiles ........................................ 84

3.5 Labelling of comments ........................................................................... 85

3.6 Coding scheme ..................................................................................... 87

3.7 Preparation of data for each lesson and the groups of lessons ................... 88

3.8 Results .................................................................................................. 89

3.9 Conclusion ............................................................................................ 90

CHAPTER 2 – IMPLEMENTATION OF RESEARCH CONTEXT ....................... 63

2.1 Research Context .................................................................................. 63

2.2 Data Collected ....................................................................................... 63

2.3 Student activity ..................................................................................... 64

2.4 Interview with teachers ......................................................................... 65

2.5 Coding of student responses, drawing the tiles ........................................ 66

2.6 Labelling of comments ........................................................................... 67

2.7 Coding scheme ..................................................................................... 70

2.8 Preparation of data for each lesson and the groups of lessons ................... 71

2.9 Results .................................................................................................. 72

2.10 Conclusion ............................................................................................. 73

CHAPTER 1 – RESEARCH QUESTIONS ....................................................... 48

1.1 Research questions ................................................................................ 48

1.2 Interview with teachers ......................................................................... 49

1.3 Student activity ..................................................................................... 49

1.4 Interview with teachers ......................................................................... 50

1.5 Coding of student responses, drawing the tiles ........................................ 50

1.6 Labelling of comments ........................................................................... 51

1.7 Coding scheme ..................................................................................... 54

1.8 Preparation of data for each lesson and the groups of lessons ................... 55

1.9 Results .................................................................................................. 56

1.10 Conclusion ............................................................................................. 57
8.2 Findings ............................................................................................................................................................................. 181
8.2.1 Results of Teacher Initial Interviews .......................................................................................................................... 181
8.2.1.1 Teachers' practices in the planning and teaching of area measurement ................................................................. 182
8.2.1.2 Frequency of area lessons ........................................................................................................................................... 183
8.2.1.3 Teachers' rationale in planning and teaching the measurement of area ................................................................. 183
8.2.2 Results of Teacher Final Interviews .............................................................................................................................. 184
8.2.2.1 Changes that teachers had already made to teaching mathematics ........................................................................... 185
8.2.2.2 Intended future changes to be made to teaching mathematics .................................................................................. 187
8.2.2.3 What teachers learnt during the area project .............................................................................................................. 188
8.2.2.4 Teachers' achievement of planned professional learning goals ..................................................................................... 189
8.2.2.5 Teachers' perceptions of successful professional learning strategies ........................................................................ 192
8.2.3 Results of School-based Team Meetings ......................................................................................................................... 194
8.2.3.1 Teachers' descriptions of children’s learning .................................................................................................................. 195
8.2.3.2 Teachers' surprise at student knowledge ........................................................................................................................ 195
8.2.3.3 Teachers gave and accepted advice from colleagues .................................................................................................. 196
8.2.3.4 Teachers described their own learning .......................................................................................................................... 196
8.2.3.5 Teachers discussed integrating area with other mathematics ....................................................................................... 197
8.2.3.6 Teachers sustained a focus on area concepts throughout the lessons ......................................................................... 197
8.2.4 Modelling the Professional Learning ............................................................................................................................. 198
8.2.5 Results from Children's Pre- and Post-Assessment Tasks ............................................................................................... 201
8.2.6 Results of Teacher Follow-up Interviews ......................................................................................................................... 205
8.2.6.1 The number of teachers who had taught the lessons again .............................................................................................. 207
8.2.6.2 Effects of participation on teaching ............................................................................................................................ 207
8.2.6.3 Teachers' perceptions of the professional learning strategies .......................................................................................... 208
8.2.6.6 Evidence of teamwork at each school site ..................................................................................................................... 208
8.2.6.5 Assistance given by the principal .................................................................................................................................. 209
8.3 Research Questions ............................................................................................................................................................... 209
8.3.1 How do teachers currently plan and implement learning experiences in area? .......................................................... 209
8.3.2 Which of the three models implemented was the most successful for the professional learning of the teachers who participated in this study? .................................................................................. 210
8.3.3 What factors were important to the success of the professional learning models? .......................................................... 211
8.4 Conclusion ................................................................................................................................................................................. 213

CHAPTER 9 - CONCLUSIONS AND RECOMMENDATIONS ...................................................................................................... 214

9.1 The Outcomes of Phase 1 and Implications for the Design of Area Teaching Programs ................................................................. 214
9.1.1 Phase 1 Research Question and Findings .......................................................................................................................... 215
9.1.2 Implications for the Design of Area Teaching Programs .................................................................................................. 216
9.1.2.1 Establishing the attribute of area .................................................................................................................................. 216
9.1.2.2 Focusing on the array structure of repeated, tessellating units .................................................................................... 217
9.1.2.3 Integrating area measurement concepts with counting strategies .................................................................................. 217
9.1.3 Further Development of Lessons ....................................................................................................................................... 217

9.2 The Outcomes of Phase 2 and Implications for the design of Teacher Professional Learning Programs ................................................................. 218
9.2.1 Phase 2 Research Questions and Findings .......................................................................................................................... 219
9.2.2 Implications for the design of Professional Learning Programs .......................................................................................... 220
9.2.2.1 Providing Teaching Resources to Implement a New Program ..................................................................................... 220
9.2.2.2 The Role Played by Teacher Leaders ........................................................................................................................... 221
9.2.2.3 Promoting collegial sharing and discussion .................................................................................................................. 222

9.3 Conclusion ................................................................................................................................................................................. 223

LIST OF APPENDICES ............................................................................................................................................................... 225

Appendix 1: Teacher interview schedule for First Steps Part 1 ................................................................................................. 226
Appendix 2: Refined area lesson 1 .................................................................................................................................................. 227
Appendix 3: Refined area lesson 2 ............................................................... 229
Appendix 4: Refined area lesson 3 ............................................................... 231
Appendix 5: Refined area lesson 4 ............................................................... 232
Appendix 6: Refined area lesson 5 ............................................................... 234
Appendix 7: Refined area lesson 6 ............................................................... 236
Appendix 8: Suggested student interview, lesson 1 ..................................... 237
Appendix 9: Suggested student interview, lesson 2 ..................................... 238
Appendix 10: Suggested student interview, lesson 3 ................................... 239
Appendix 11: Suggested student interview, lesson 4 ................................... 240
Appendix 12: Suggested student interview, lesson 5 ................................... 241
Appendix 13: Suggested student interview, lesson 6 ................................... 242
Appendix 14: Cutting template for 5 cm tiles ............................................. 243

REFERENCES .................................................................................. 244
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Best Practice in Professional learning</td>
<td>35</td>
</tr>
<tr>
<td>3.1</td>
<td>Features of the three schools represented in the study</td>
<td>66</td>
</tr>
<tr>
<td>3.2</td>
<td>Details of teachers interviewed</td>
<td>67</td>
</tr>
<tr>
<td>3.3</td>
<td>Frequency of area units of work each term</td>
<td>69</td>
</tr>
<tr>
<td>3.4</td>
<td>Skills that will assist children to measure area in later years</td>
<td>72</td>
</tr>
<tr>
<td>3.5</td>
<td>Understandings that assist children to measure area in later years</td>
<td>73</td>
</tr>
<tr>
<td>3.6</td>
<td>Lesson 1 comments from children, teacher and researcher</td>
<td>86</td>
</tr>
<tr>
<td>4.1</td>
<td>Implementation of lesson sequences</td>
<td>95</td>
</tr>
<tr>
<td>5.1</td>
<td>Comparison of two sequences of lessons</td>
<td>103</td>
</tr>
<tr>
<td>5.2</td>
<td>Implementation of lessons and collection of data</td>
<td>105</td>
</tr>
<tr>
<td>6.1</td>
<td>Number of children who were involved</td>
<td>113</td>
</tr>
<tr>
<td>6.2</td>
<td>Labelling of teachers’ and researcher’s comments</td>
<td>114</td>
</tr>
<tr>
<td>6.3</td>
<td>Coding scheme</td>
<td>116</td>
</tr>
<tr>
<td>6.4</td>
<td>Interview comments area syllabus lesson 1</td>
<td>118</td>
</tr>
<tr>
<td>6.5</td>
<td>Field notes area syllabus lesson 1</td>
<td>119</td>
</tr>
<tr>
<td>6.6</td>
<td>Analysis of area syllabus lesson 1</td>
<td>121</td>
</tr>
<tr>
<td>6.7</td>
<td>Interview comments area syllabus lesson 2: Superimpose shapes</td>
<td>122</td>
</tr>
<tr>
<td>6.8</td>
<td>Field notes area syllabus lesson 2: Superimpose shapes</td>
<td>123</td>
</tr>
<tr>
<td>6.9</td>
<td>Analysis of area syllabus lesson 2</td>
<td>124</td>
</tr>
<tr>
<td>6.10</td>
<td>Interview comments area syllabus lesson 3</td>
<td>125</td>
</tr>
<tr>
<td>6.11</td>
<td>Field notes area syllabus lesson 3</td>
<td>127</td>
</tr>
<tr>
<td>6.12</td>
<td>Analysis of area syllabus lesson 3</td>
<td>128</td>
</tr>
<tr>
<td>6.13</td>
<td>Interview comments area syllabus lesson 4</td>
<td>129</td>
</tr>
<tr>
<td>6.14</td>
<td>Field notes area syllabus lesson 4</td>
<td>130</td>
</tr>
<tr>
<td>6.15</td>
<td>Analysis of area syllabus lesson 4</td>
<td>131</td>
</tr>
<tr>
<td>6.16</td>
<td>Teachers’ final interview comments: Area syllabus lessons</td>
<td>132</td>
</tr>
<tr>
<td>6.17</td>
<td>Analysis of final interview comments: Area syllabus lessons</td>
<td>134</td>
</tr>
<tr>
<td>6.18</td>
<td>Interview comments area research lesson 1</td>
<td>135</td>
</tr>
<tr>
<td>6.19</td>
<td>Field notes area research lesson 1</td>
<td>137</td>
</tr>
<tr>
<td>6.20</td>
<td>Analysis of area research lesson 1</td>
<td>139</td>
</tr>
<tr>
<td>6.21</td>
<td>Interview comments area research lesson 2</td>
<td>140</td>
</tr>
<tr>
<td>6.22</td>
<td>Field notes area research lesson 2</td>
<td>142</td>
</tr>
<tr>
<td>6.23</td>
<td>Analysis of area research lesson 2</td>
<td>143</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>6.24</td>
<td>Interview comments area research lesson 3</td>
<td>144</td>
</tr>
<tr>
<td>6.25</td>
<td>Field notes area research lesson 3</td>
<td>145</td>
</tr>
<tr>
<td>6.26</td>
<td>Analysis of area research lesson 3</td>
<td>147</td>
</tr>
<tr>
<td>6.27</td>
<td>Interview comments area research lesson 4</td>
<td>148</td>
</tr>
<tr>
<td>6.28</td>
<td>Field notes area research lesson 4</td>
<td>150</td>
</tr>
<tr>
<td>6.29</td>
<td>Analysis of area research lesson 4</td>
<td>151</td>
</tr>
<tr>
<td>6.30</td>
<td>Teachers’ final interview comments: Area research lessons</td>
<td>152</td>
</tr>
<tr>
<td>6.31</td>
<td>Analysis of final interview comments: Area research lessons</td>
<td>154</td>
</tr>
<tr>
<td>6.32</td>
<td>Area refined lessons</td>
<td>164</td>
</tr>
<tr>
<td>7.1</td>
<td>Participating teachers’ experience and class taught</td>
<td>167</td>
</tr>
<tr>
<td>7.2</td>
<td>Schools represented by participating teachers</td>
<td>168</td>
</tr>
<tr>
<td>7.3</td>
<td>Children who completed the pre-test and post-test activity</td>
<td>169</td>
</tr>
<tr>
<td>7.4</td>
<td>Three models of teacher professional learning support</td>
<td>170</td>
</tr>
<tr>
<td>8.1</td>
<td>Frequency of responses to changes already made</td>
<td>185</td>
</tr>
<tr>
<td>8.2</td>
<td>Frequency of responses to intended future changes</td>
<td>187</td>
</tr>
<tr>
<td>8.3</td>
<td>Frequency of responses to teacher learning</td>
<td>189</td>
</tr>
<tr>
<td>8.4</td>
<td>Teachers’ achievement of planned goals in high support schools</td>
<td>190</td>
</tr>
<tr>
<td>8.5</td>
<td>Teachers’ achievement of planned goals in medium support schools</td>
<td>191</td>
</tr>
<tr>
<td>8.6</td>
<td>Teachers’ achievement of planned goals in low support schools</td>
<td>191</td>
</tr>
<tr>
<td>8.7</td>
<td>Frequency of responses to successful professional learning strategies</td>
<td>193</td>
</tr>
<tr>
<td>8.8</td>
<td>Students’ improvement between pre-test and post-test</td>
<td>205</td>
</tr>
<tr>
<td>8.9</td>
<td>Results of high support teacher follow-up interviews</td>
<td>206</td>
</tr>
<tr>
<td>8.10</td>
<td>Results of medium support teacher follow-up interviews</td>
<td>206</td>
</tr>
<tr>
<td>8.11</td>
<td>Results of low support teacher follow-up interviews</td>
<td>207</td>
</tr>
<tr>
<td>8.12</td>
<td>Schools with an active team or teacher leader</td>
<td>209</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

| Figure 5.1 | Coding of children’s responses to tiling assessment task ............. 110 |
| Figure 6.1 | Results at completion of research lessons and syllabus lessons..... 159 |
| Figure 6.2 | Post- syllabus and post- research results in schools 3 and 4 ........ 159 |
| Figure 6.3 | Post- syllabus lessons and post- research lessons, Year 1 child..... 160 |
| Figure 8.1 | Projected model of teacher professional learning .......................... 198 |
| Figure 8.2 | Tentative model of teacher professional learning .......................... 199 |
| Figure 8.3 | Action research model of teacher professional learning ............... 200 |
| Figure 8.4 | Avenue school results of pre-test and post-test............................. 202 |
| Figure 8.5 | Mall school results of pre-test and post-test................................ 203 |
| Figure 8.6 | Vista school results of pre-test and post-test............................. 203 |
| Figure 8.7 | Hills school results of pre-test and post-test ............................. 203 |
| Figure 8.8 | Plains school results of pre-test and post-test .......................... 204 |
| Figure 8.9 | Forest school results of pre-test and post-test .......................... 204 |
| Figure 8.10 | Station school results of pre-test and post-test....................... 204 |
GLOSSARY

Area
Part of a two-dimensional surface (plane or curved) enclosed within a specified boundary or geometric figure. The measure or extent of such a surface, or part of it (measured in square units). The lesson activities in this study refer to the measurement of rectangular area, or the area of rectangles.

Area lessons
- **Area syllabus lessons**: Sequence of four lessons taken from the Sample Activities listed in Area units 2, 3 and 4 from the syllabus document *Mathematics K-6* (New South Wales Department of Education, 1989), and taught by the researcher in Year 1 and Year 2 classes.
- **Area research lessons**: Sequence of four lessons designed and taught by the researcher in Year 1 and Year 2 classes, to assist in developing an understanding of the attribute of area, and the measurement of rectangular area using tessellating, congruent, informal units.
- **Area refined lessons**: Sequence of six lessons that combined components of both the area research lessons and the area syllabus lessons in phase 1 of this study. The lessons were implemented by teachers during a trial of teacher professional learning models in phase 2 of this study.

Array
A set of elements arranged in a pattern of rows and columns.

Attribute
Property or characteristic.

Composite units
A collection or unit group of set items which are treated spatially or numerically as a unit.

Congruent
Having the same shape and size.

Conservation
Understanding that an amount of material (continuous or discontinuous) remains constant, despite being organised into several smaller quantities or into a wider or taller arrangement.

Grid
A pattern of horizontal and vertical lines that cross each other to make a uniform set of squares or rectangles.
<table>
<thead>
<tr>
<th><strong>Iteration</strong></th>
<th>The act of doing something repeatedly, repeated application of a procedure.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Partitioning</strong></td>
<td>The division of a quantity into a number of smaller equal units of a chosen size in preparation for calculating the total value of the quantity in terms of the chosen unit.</td>
</tr>
<tr>
<td><strong>Perimeter</strong></td>
<td>The sum of the lengths of the segments that form the sides of a polygon, or the total length of any closed curve, such as the circumference of a circle.</td>
</tr>
<tr>
<td><strong>Row or column</strong></td>
<td>Numbers or objects arranged in a straight line (the convention for an array or grid is that rows are horizontal and columns are vertical).</td>
</tr>
<tr>
<td><strong>Tessellation</strong></td>
<td>A covering of a surface (a paving) with congruent (equal in all aspects) shapes. The shapes fit together without gaps or overlap and can be extended infinitely in any direction.</td>
</tr>
<tr>
<td><strong>Transitivity</strong></td>
<td>The relationship between three quantities such that if it holds between the first and second and it also holds between the second and third it must necessarily hold between the first and third.</td>
</tr>
</tbody>
</table>
ABSTRACT

The research problem reported in this thesis is an investigation of the teaching and learning of area measurement in the early years of school. Research indicates that children confuse the measurement of area and perimeter and also the use of linear and two-dimensional units of measure. The first phase of the study investigated the knowledge and skills which underpin an understanding of the L x B formula for calculating rectangular area. Those factors were used to plan a teaching program of four lessons for Year 1 and Year 2 children, focusing on: establishing the attribute of area; making, describing and drawing the spatial structure of arrays of repeated informal units to measure areas; and methods of counting to determine the total number of units. The effectiveness of the program was evaluated by implementing the lessons in four classes. Lessons planned from the then current syllabus were implemented in another four classes. Comparison of student learning outcomes from all classes indicated that the research lessons were more effective in assisting children to develop an understanding of a grid pattern or array of repeated informal units.

The second phase of this study described the researcher’s investigation, design and trial of teacher professional learning strategies which would assist teachers to adopt successful methods of teaching young children to measure area. Seventeen volunteer teachers in seven school teams participated in one of three models of professional learning, based on varying levels of consultancy support. The models were based on the provision of lesson notes and teaching materials, facilitation of team meetings to discuss the implementation, and the provision of additional time to interview individual children following each lesson.

Participation in the project assisted all of the participating teachers to develop their content knowledge and to modify their teacher-centred teaching practices. The key strategies and factors which contributed to this success included ongoing school-based professional dialogue and support, the provision of a teaching program which emphasised students’ conceptual development within a sequence of activities, the role played by teacher leaders within each team, opportunities to develop questioning techniques and the motivation and disposition of the participating teachers.
CHAPTER 1 - INTRODUCTION

The study of mathematics has been a central component of knowledge-seeking and knowledge-building from early civilisations to current times. An understanding and knowledge of mathematics gives access to practical as well as theoretical tools. In a technological world permeated by mathematics, opportunities to work and study are governed by a facility with the principles of mathematics (Australian Association of Mathematics Teachers, 1996, 1997; Australian Education Council, 1991; Department of Employment Education Training and Youth Affairs, 1997; Ministerial Council on Education Employment Training and Youth Affairs, 1999).

The mathematics education encountered by generations of children in western education systems has frequently relied on rote learning of facts and memorisation of rules and procedures (Malloy, 1999; Raghavan, Sartoris, & Glaser, 1998). The pressures of contemporary living and working dictate that children participate in an education that focuses on an understanding of mathematical principles and applications, rather than an acceptance of memorised processes (Australian Education Council, 1991; S. Cohen, 2004; Lappan, 1999; National Council of Teachers of Mathematics, 2000). Children need to be able to think about, use and discuss the deductive and reasoning powers of mathematics (Ball, 1990). Children also need to interpret and analyse a range of mathematical representations such as graphs, tables and data, make accurate and effective mental computations and estimate the measurement of quantities (Pfannkuch & Watson, 2004; Willis, 1990).

The research reported in this thesis focuses on the teaching and learning of area, an everyday application of mathematics. The problems that children have with calculating area and perimeter in mathematics lessons in primary school and high school have been well-documented (M. A. Clements & Ellerton, 1995; Ferrer, Hunter, Irwin, & Shelden, 2001; Hart, 1981a; Strutchens, Martin, & Kenney, 2003). Mathematics educators have suggested that children’s difficulty in understanding area and the measurement of rectangular area can be traced back to the teaching methods in early and middle primary school years (Hart, 1981b; Nitabach & Lehrer, 1996; Outhred, 1993; Stephan & Clements, 2003).
Phase 1 of this study implemented and evaluated two sequences of teaching activities in Year 1 and Year 2 classrooms. One sequence of lessons, the area syllabus lessons, was planned from the current curriculum document *Mathematics K-6* (New South Wales Department of Education, 1989). The second sequence, titled the area research lessons, was designed to assist children to develop the measurement, spatial and counting concepts underpinning the calculation of the area of rectangles. The two sequences of lessons were evaluated and compared by studying the children’s learning outcomes and class teachers’ and the researcher’s observations and comments. A revised sequence of lessons, area refined lessons, was designed to incorporate selected activities from each of the previous sequences. In Phase 2 of the study, the area refined lessons were implemented by 17 volunteer teachers who chose to participate in one of three models of professional learning. Data collected from teachers and students was used to evaluate and compare the three models.

This chapter describes the context of the study in terms of the researcher’s experiences of teaching in New South Wales (Australia) primary schools, and the issues surrounding the teaching of measurement and area within this state, as well as nationally and internationally. The design of the study is described in broad terms, together with the principal research questions that were examined.

**1.1 Experiences and Concerns**

The decision to undertake this study arose from the researcher’s personal frustration with the approach taken to the teaching of area in the mathematics program in many Year 1 and Year 2 classrooms in primary schools in New South Wales (Australia). In New South Wales, children attend primary school from Kindergarten to Year 6. The teaching and learning content from these years is taken from the K-6 syllabus documents. Children continue to high school to complete the years 7 to 12, typically completing Year 12 by the age of 17 to 18 years.

Most Year 1 and Year 2 teachers, including the researcher, used a combination of the then current syllabus *Mathematics K-6* (New South Wales Department of
Education, 1989), and textbook activities to introduce 6 and 7 year-old children to the concept of area and the measurement of rectangular area with informal units including a variety of found and created shapes or tiles. Early whole-class discussions included descriptions of the play area, reading area and the wet area. When asked to measure the area of the surface an object, children were frequently confused by the use of the term area in a mathematical context. When measuring area, children were encouraged to use non-standard shapes as informal units of measure. These units included non-tessellating shapes such as circles, handprints and leaf prints.

The researcher’s concern was that children and teachers were directed to look at some aspects of area that were not helpful in establishing that “Area is the measure of the amount of surface” (New South Wales Department of Education, 1989, p. 126). Lessons included the use of informal and non-tessellating units that frequently did not cover the whole area. The researcher and her colleagues concentrated on hands-on, busy activities with limited reference to an explanation of the attribute of area, or mathematical terminology that would help children to explain their actions and results. Children became adept at covering and many could remember a ‘no gaps, no overlaps’ rule, but there was limited systematic order or patterning to the covering. The researcher felt that children were not being assisted to develop the broader concepts of how to describe and measure area.

The researcher knew that when many children were introduced to the formula for calculating rectangular area, in Year 4 or Year 5 mathematics lessons, they had limited understanding of the attribute of area, and the mathematical concepts behind the formula. These children were able to apply the formula for calculating rectangular area when completing a series of exercises on an activity sheet or textbook page, but had difficulty in generalising to a single episode of how to calculate area in a problem-solving situation.

During the preliminary phase of this study, the researcher found that many teachers were unaware of how the covering exercises in Years 1 and 2 related to the use of the area formula in the primary years. A teacher who was interviewed during the pilot study of this research explained:
When they get into primary in Year 3, they expect that they suddenly know this concept and they’re going to able to measure the area. Well, if they haven’t got the basic concepts then they’re still not going to know what’s going on. I’m sure that children, once they get to high school, just go and learn the formula and have no idea of what they’re actually doing. Somewhere there has to be a transition from here, what we’re doing with the hands on, to when you get into primary and suddenly, what is the area of this, you know, shape, 3 cm by 2 cm (Cathy, Initial interview).

The researcher was also concerned that children appeared to have difficulty in recognizing which formula to use when calculating perimeter and area, in lessons in Years 4, 5 and 6. Children did not understand the units of measure they had used and were confused as to whether their calculation should be expressed in linear units such as centimetres and metres, or units of area measure such as square centimetres, square metres and hectares.

Mathematics educators have noted the difficulties that children appear to have in applying area and perimeter formulas, from the primary school years when these are introduced, to early high school, and even to adulthood (Baturo & Nason, 1996; Malloy, 1999; Raghavan et al., 1998). The reasons for the difficulties may be varied, including a lack of basic skills such as using a ruler correctly when measuring lengths, counting the number of units of measurement, or multiplying two numbers correctly. Other children may be unable to visualise geometric shapes within a composite two-dimensional figure. Some children may not understand why the formula is used, and what the resulting units of measurement will represent (Murdoch, 1998).

As far back as 1981, Hart warned that premature use of instruments or formula leaves children without the understanding necessary for solving measurement problems.

Once the child has been taught the formula he is unfortunately likely to remember it in forms other than that which his teacher presented, e.g. L + B instead of L x B or perimeter (2L + 2B) instead of area (Hart, 1981b, p.10).

Many primary and secondary school teachers, frustrated with children’s confusion between area and perimeter, and an assumption that a larger perimeter will have a larger area, have sought to assist children to connect conceptual understanding to the processes or procedures, through problem solving tasks that encourage students to
investigate and discuss the relationships (Brindle, 1994; Chappell & Thompson, 1999; Ferrer et al., 2001; Moyer, 2001; Stone, 1994).

The researcher’s goals in commencing this study were twofold. Firstly, she wanted to identify how children acquire understandings of the attribute and the measurement of rectangular area. She also wanted to investigate the planning and implementation of activities that assist children to develop these concepts. As a District Mathematics Consultant and a former member of school executive teams, the researcher understood the opportunity to encourage colleagues to make changes to their current teaching practices, by sharing and critically discussing the results of lesson trials and programs. The second goal of this study was to explore the factors that assist teachers to make effective changes to teaching practices when participating in professional learning programs.

The researcher was employed as a District Mathematics Consultant by the New South Wales Department of Education and Training (NSW DET) in a metropolitan area. The NSW DET provides free, secular education to approximately 750 000 students from Kindergarten to Year 12, representing two-thirds of the children in New South Wales. The Department currently employs 65 000 teachers to staff approximately 2200 primary schools, high schools, and schools for specific purposes and is the largest single organisation, public or private, in Australia (New South Wales Department of Education and Training, 2005). During the research phase of this study, the state was divided into 40 Districts, each with a District Superintendent and Literacy, Mathematics, Student Equity and Student Welfare consultants based in a District Office. The researcher was responsible for coordinating and presenting teacher professional learning strategies to the 55 schools in her District. At the conclusion of the data-collection phase of this study, the researcher was promoted to the role of statewide numeracy coordinator within the New South Wales Department of Education and Training.
1.2 Measurement and Area within the Mathematics Curriculum

1.2.1 Measurement in the Curriculum

Measurement has an important place in the mathematics curriculum, not only because of the obvious connections to everyday life, but also because of the possibilities measurement presents for the development and application of number concepts, spatial understanding and statistical operations (National Council of Teachers of Mathematics, 2000; Shuard, 1986). The necessity for an understanding of how to measure and calculate with quantities was noted in *A national statement on mathematics for Australian schools* (Australian Education Council, 1991) which describes how number and quantification are used for daily living, in civic life, at work, and as a part of the culture of all Australians. The necessity of measurement understandings and skills to employment was described by the Cockcroft Report:

A very great deal of the mathematics which is used in employment is concerned with measurement in one or other of a wide variety of forms, by no means all of which are directly concerned with the use of measuring instruments. Measurements are specified in a variety of ways, for instance in terms of number of items or total of money; of length, weight or volume; or ratio, percentage or rate. (Great Britain Committee of Inquiry into the Teaching of Mathematics in Schools, 1982, p. 23)

Measurement skills and concepts also have a broad application across other areas within the school curriculum, including science, social studies, art and physical education (D. Clements, 2003; National Council of Teachers of Mathematics, 2000). The study of measurement and the acquisition of measurement skills have significance both within the school curriculum and also in the successful application of mathematical understandings in other subject areas, everyday activities and the workplace.

1.2.2 The Area Substrand in the New South Wales Curriculum

1.2.2.1 The New South Wales 1989 syllabus, Mathematics K-6

The mathematics curriculum for primary children in New South Wales that was current as this study was commenced, was *Mathematics K-6* (New South Wales Department of Education, 1989). The curriculum was presented in three strands: Space, Measurement and Number. Each strand was divided into a number of substrands and Area was one of the six substrands in Measurement (Length, Area,
Volume, Mass, Temperature and Time). The content and suggested teaching activities for each substrand were described in numbered units of work. The substrand Area had ten teaching units from Kindergarten to Year 6. In Year 1 and Year 2, teachers planned four or five mathematics lessons each week and had to adequately cover the three strands and seventeen substrands during the school year. Each syllabus unit included content, the main idea, objectives and activities. In Years 1 and 2, teachers typically planned, documented and presented classroom lessons based on the material in Area Units 1 to 5.

Area Unit 1: Area is the measure of the amount of surface (Awareness of surface)
Area Unit 2: Area is the measure of the amount of surface (Awareness of the attribute of area)
Area Unit 3: Areas can be compared (Comparison of two areas)
Area Unit 4: Informal units can be used to measure area (Measurement with informal area units)
Area Unit 5: Three or more areas can be ordered (Comparison and ordering of area)


In 1998, the Board of Studies New South Wales (the statutory authority responsible for setting school syllabuses in New South Wales) published Mathematics K-6 Outcomes and Indicators to accompany the syllabus document Mathematics K-6. When documenting lessons and assessment tasks, teachers were encouraged to reference all relevant outcomes from this document (Appendix 2). The Area outcome within Measurement Stage 1 was:

M1.3: Estimates, compares and orders the areas of shapes using informal units (Board of Studies New South Wales, 1998, p. 48).

The sequence of area teaching units in Mathematics K-6 suggested that when children had an understanding of the attribute of area, and were able to compare the areas of two or more given surfaces by superimposing, they were ready to measure areas with a wide variety of informal units, progressing to using the formal units of the square metre and then the square centimetre. The initial activities with square centimetres involved student-devised counting strategies for large numbers of units, typically on grid paper, but no formal recognition of the grid pattern formed by these units. Later still, the children would be introduced to the formula for calculating area.
This learning sequence, with an emphasis on busy “covering” activities and the absence of any focus of grid patterns that could lead seamlessly into an understanding of the area formula, were the researcher’s motivators for undertaking this study.

1.2.2.2 The New South Wales 2002 Syllabus

The Board of Studies New South Wales published a new *K-6 Mathematics Syllabus* in late 2002 (Board of Studies New South Wales, 2002). The NSW Department of Education and Training notified principals of government schools that the Measurement, Data, and Patterns and Algebra strands were to be implemented in 2004, with the remaining strands due for progressive implementation in 2005 (New South Wales Department of Education and Training, 2003b). The new syllabus emphasised a continuum of learning, with Stage outcomes and key ideas presented in a format to encourage teachers to look beyond the material relevant to their children’s current stage of working. The units of work with suggested activities and mathematical language in the previous syllabus were replaced by lists of Knowledge and Skills (content). The new syllabus was in its first year of full implementation as this study was being documented. The teachers who were involved in the research were working to the earlier syllabus.

The Key Ideas and Knowledge and Skills in the Area substrand of the 2002 syllabus emphasised children’s understanding of the attribute of area, prior to the introduction of activities involving measurement of area with tessellating units in a grid pattern.

When children understand why tessellating units are important, they should be encouraged to make, draw and describe the spatial structure (grid). Children should develop procedures for counting the tile or grid units so that no units are missed or counted twice. Children should also be encouraged to identify and use efficient strategies for counting e.g using repeated addition or rhythmic counting (Board of Studies New South Wales, 2002, p.97).

The sequence of area outcomes and background information for teachers was guided by the success of the teacher professional development program *Count Me Into Measurement*, conducted by the New South Wales Department of Education and Training (Outhred & McPhail, 2000; Outhred, Mitchelmore, McPhail, & Gould, 2003). As a component of her employment with the NSW Department of Education
and Training, the researcher worked with Dr Lynne Outhred to develop the program and coordinate its implementation across NSW. The program culminated in the publication of two teacher resource books, *Teaching Measurement: Early Stage 1 and Stage 1* and *Teaching Measurement: Stage 2 and Stage 3* (New South Wales Department of Education and Training, 2003c, 2004). The area lessons, documentation and teacher professional learning strategies that the researcher developed in the early part of this study were incorporated into the Departmental project *Count Me Into Measurement*. The outcomes of the research that is described in this thesis have already had a significant impact on the teaching and learning of measurement in New South Wales schools. Over 200 government schools participated in the initial trial of the *Count Me Into Measurement* project. The frameworks and activities which focused on establishing the attribute of area and the identification of the pattern of repeated, tessellating units used to measure rectangular area were then incorporated into the 2002 Mathematics syllabus (Board of Studies New South Wales, 2002), which now has mandatory implementation in all schools in New South Wales.

1.2.3 Planning and Teaching Measurement

The implementation of the main idea, objectives and content of each of the 1989 Mathematics K-6 teaching units were mandatory in all government schools, but implementation was planned and achieved in a variety of ways. Some schools prepared their own scope and sequence charts that designated the content to be taught each year, or even each term. In other schools, teachers chose the appropriate content for their classes according to children’s needs and children’s achievement of relevant outcomes. These teachers frequently based their teaching program on the suggested scope and sequence chart in the syllabus, and the staged outcomes.

1.3 Teacher Professional Learning

The researcher’s lengthy teaching career spanned primary school teaching in two Australian states and the Australian Capital Territory and included the roles of classroom teacher, school administrator, mathematics consultant and statewide coordinator of numeracy programs in government schools. However, in her primary
role of a committed, effective educator her overriding concern was that the research study should have practical implications and positive effects on teaching in schools. The researcher therefore determined that the study should also include a teacher professional learning component that would assist teachers to both understand how children acquire area concepts as well as guiding them in the design and implementation of suitable programs.

The challenge of this study was to give teachers a purpose, based on an understanding of how children learn about area. This understanding would make explicit the concepts and understandings behind area:

- Understanding of the attribute of area, to enable children to progress to using the area and volume formulas in later years;
- The thinking skills required to measure and make sense of area.

The emphasis on a knowledge nation and a numerate society has encouraged teachers to plan beyond teaching for memorisation of mathematical formulas and procedures (Australian Association of Mathematics Teachers, 1996; Department of Employment Education Training and Youth Affairs, 1997). The teacher’s role encompasses assisting children to develop and practise mathematical skills and to discuss the thinking behind their mathematical processes that will be used confidently in everyday and employment situations (Driscoll & Lord, 1990). Additionally, there is an expectation that all children are entitled to the achievement of basic curriculum standards (Ministerial Council on Education Employment Training and Youth Affairs, 1999).

The researcher considered that teachers required some understanding of the ‘big picture’ of the development of measurement concepts to enable them to think ahead, to plan more effective activities for children who have not understood the basic concepts, and to be able to adapt learning activities for a wide range of children’s skills and interests. In addition, teachers also needed to have a clear understanding of content and concepts, to enable them to identify children’s achievement of planned goals.
Personal experiences and beliefs influenced the initial focus of the study. The researcher considered that giving teachers the opportunities and shared language to communicate with colleagues about successful or challenging programs would assist in encouraging reflective thinking. As Jaworski (1994) suggested, and as the researcher’s experiences confirmed, teachers who discuss their thinking with other teachers, are frequently able to clarify objectives and teaching practices in their own minds.

As a District Mathematics Consultant with responsibility for providing programs and support to teachers in 55 schools, the researcher was also interested in investigating alternative formats for the delivery of teacher professional learning. Her work with the early numeracy project *Count Me In Too*, developed by the New South Wales Department of Education and Training, had illustrated the positive results of intensive, extended training and in-class support for teachers (Bobis, 1996, 1997, 1999). The *Count Me In Too* project focused on the number strand of the mathematics curriculum, and it was unlikely that District Mathematics Consultants and schools would have the time or resources to dedicate similar, intensive teacher training and support for a measurement program.

The researcher designed a professional learning program which translated to three models that differed by the amount of consultancy support provided to each school. The models were based on the proven tenets of professional learning programs, as described in recent research, but their initiation and ongoing implementation were designed to be achievable within the researcher’s district workload. Two of the three models were suitable for implementation in remote area schools, with ongoing support provided by telephone conversations with a consultant or mentor.

A communiqué developed by participants at the Australian Association of Mathematics Teachers Seventeenth Biennial Conference, 1999, summarised the combination of resources and professional development that may contribute towards the provision of high-quality mathematics education for all children. The communiqué included the following conditions:
Sufficient provision for continuing professional development to ensure that all teachers are competent and confident in the teaching of mathematics that they undertake;

Sufficient time for mathematics learning on school timetables and for teachers of mathematics to plan and reflect (Australian Association of Mathematics Teachers, 1999, p. 6).

The issue of designing teacher professional learning raised the question of what do teachers identify as effective professional learning. If the impact of professional learning was change in teacher practice, this study would also need to identify the factors that will assist such change. As Nisbet and Warren (2000) noted, the interaction between teachers’ beliefs and their classroom practice can work both ways, so that practice influences beliefs and beliefs influence practice. Given that many primary teachers agree that they lack confidence in teaching mathematics (M. Stephens, 2000; Zevenbergen, 2005), this study had the potential to look beyond the attitudes and practices, to examine whether any other teacher characteristics or outcomes are instrumental in achieving effective professional development.

1.4 Questions to be addressed by this research

The overall aim of this study was to improve student learning outcomes in the measurement of rectangular area, in the early years of primary school. It was intended to achieve this through a two-phase intervention study involving student learning and teacher professional learning.

The two major points addressed by the study determined the organisation of the research questions.

1.4.1 Phase 1 Aim and Research Question

The first phase of this research aimed to design and trial a sequence of lessons to assist Year 1 and Year 2 students to identify, describe, compare and measure rectangular area using repeated units. Two sequences of learning activities (area syllabus lessons and area research lessons) were planned, implemented and evaluated.

- How effective were the area research lessons compared to the area syllabus lessons in developing:
  - an understanding of the attribute of area; and
o an understanding of a grid pattern or array of repeated, informal units to measure area?

1.4.2 Phase 2 Aim and Research Questions

The aim of the phase 2 research was to implement, evaluate and compare three professional learning models which assisted teachers to change their current practices in the teaching of area.

• How do teachers currently plan and implement learning experiences in area?
• Which of the three models implemented was the most successful for the professional learning of the teachers who participated in this research?
• What factors were important to the success of the professional learning models?

1.5 Designing the Study as a Research Project

The results from this study had the potential to influence the design and implementation of state-wide numeracy programs in government schools. The study was intentionally very practical in its conception, design and execution. However, the quality of the program as research and the validity of the results were dependent upon careful design and execution within each step, to ensure that the study was relevant, sound, and could be generalised to teaching other mathematics programs and teacher professional learning in general. As Pirie (1998b) noted when discussing qualitative research design in mathematics education, the methodology and the methods need to be tailored to the questions that are explored by the research, instead of assuming that a particular approach will be suitable. This includes a consideration of methodological strategies appropriate to the research aims as well as decisions of how to collect data that are relevant to the task.

At the commencement of the study the author was an inexperienced researcher. Chapter 3 of this thesis, titled First Steps, describes the researcher’s initial research questions, teacher interview schedules and resultant data collection. The results from this trial process highlighted teachers’ willingness to share their skills and knowledge of teaching mathematics, specifically area. The researcher’s experiences of interviewing teachers, analysing the transcriptions of interviews and reporting on teachers’ beliefs about teaching area, influenced the design of the next stage of the
study, involving the development and trial of the research area lessons. During this stage, the researcher taught the lessons herself, in four schools, with supervisory assistance from the class teachers. The researcher also taught a sequence of lessons planned from the *Mathematics K-6* syllabus. At the conclusion of each lesson the teachers were interviewed to discuss the intended and achieved outcomes, as well as possible modifications. In addition, the children in each class completed pre-and post-assessment tasks. By comparing the children’s growth in learning at the conclusion of each lesson sequence, the researcher and teachers were able to identify the most helpful aspects of each lesson sequence.

The final stage of the study was conducted in seven metropolitan primary schools, located in a broad range of communities with varying socio-economic characteristics. The 17 participating teachers were all volunteers, and represented career stages from beginning teachers to several with 30 years experience. The student assessment task that was used again in the final stage had proven to be effective in providing quantitative data about individual learning. The researcher was also much more experienced in interviewing teachers, and encouraging them to share their observations and beliefs. This information was gathered before and after the professional learning program, and was examined by collating responses to each interview question. In addition, school-based teams of teachers from two of the three groups of schools met with the researcher each fortnight to discuss the progress of the area lessons. The transcriptions from these discussions were studied to develop a model of professional learning.

### 1.6 Plan of the Thesis

This thesis is organised into nine chapters.

*Chapter 1*

The researcher’s focus has been introduced, together with a broad outline of the problem, and a statement of the data to be collected.

*Chapter 2*

The research literature that concerns the development of area concepts and measurement, and knowledge of rectangular arrays is reviewed. As well, the literature dealing with models of teacher professional learning is reviewed.
Chapter 3
The researcher’s First Steps as a mathematics education researcher are described, with the results of the study of teachers’ understanding of teaching about area and an initial trial of a sequence of four area lessons.

Chapter 4
The design of the research study is described, with a brief outline of each of the two phases.

Chapter 5
The methodology of phase 1, the implementation and evaluation of the teaching sequence of area research lessons, is described.

Chapter 6
The results of the area research lessons in four schools are reported.

Chapter 7
The methodology of phase 2, the implementation and evaluation of three models of teaching professional learning, is described.

Chapter 8
The results of the three teacher professional learning models are described.

Chapter 9
The findings of the study and implications for teachers, schools, and school systems are presented.

1.7 Conclusion

This research had the potential to provide a sound basis for a revision of classroom practices in the teaching of area in primary schools: both the concept of area, and the measurement of rectangular area. The study aimed to define how children learn about area and its measurement, and how teachers can assist and enhance that learning in an integration of curriculum development, classroom teaching, and mathematics educational research (D. Clements, Battista, & Sarama, 1998). An understanding of how and what children are learning should support teachers to plan, implement, evaluate and modify teaching activities that will assist children to develop as competent mathematicians.

This chapter has described the genesis and the scope of the study. The next chapter reports the results of an investigation of the research literature that underpins the design of both the teaching intervention and the teacher professional learning program.
CHAPTER 2 - REVIEW OF THE LITERATURE

This chapter reviews the research literature from the two focus areas of the study: how children learn to measure area, and how Year 1 and Year 2 teachers can be assisted to plan, implement and evaluate an effective mathematics teaching program, specifically in the measurement of area. During the current study, a sequence of teaching and learning experiences was developed and refined before implementation as the focus of a professional learning program for the participating teachers. The theoretical design and the implementation of the study were based on a review of literature, to address the focus areas. This review of literature draws largely upon American, European and Australian studies, with other findings included wherever possible. It concentrates upon research in the domains of mathematics education and teacher professional learning.

The first section of this chapter investigates the research on how children develop concepts of the attribute of area and measurement of the area of rectangles. In the second section of this chapter, models of teacher professional learning were sourced that would encourage teachers to adapt basic ideas to their own classroom needs and to make changes to classroom practices.

2.1 Teaching and Learning about Measuring the Area of Rectangles

This study focused on designing and implementing classroom activities that would assist children in Year 1 and Year 2 (typically six to eight years of age) to develop and practise understandings of how to measure rectangular area using informal measurement units. The children’s current mathematics programs were planned from tasks suggested by the mandatory New South Wales syllabus Mathematics K-6 (New South Wales Department of Education, 1989). The syllabus activities included identifying closed and open shapes, superimposing shapes to make direct comparisons of areas, covering the surfaces of objects with informal measurement units such as books, counters or leaves, and comparing areas by covering each with informal measurement units, including books or tickets. As the children progressed to Years 3 and 4 they would be introduced to formal units, square metres and square centimetres. Area lessons would then progress to
experimenting with and using the formula Area = Length x Breadth to calculate the area of rectangular shapes.

The researcher’s intent was to plan a sequence of tasks that would establish an understanding of the attribute of area and a conceptual understanding of how and why the formula Length x Breadth is used when rectangular area is calculated. The researcher’s extensive teaching experience and initial literature review (Hart, 1981a; Hirstein, 1981; F. Kidman, 1997; McDonough, 2001; Outhred, 1993; Outhred & Mitchelmore, 2000) indicated that many children did not understand why the formula worked. Children confused area with perimeter, or did not understand the difference between units of square measure and units of linear measure (Baturo & Nason, 1996). Children’s difficulties with the topic included a failure to connect the measurement processes with the underlying concepts and problems with choosing which dimensions to measure. These difficulties appeared to stem from a teaching approach, rather than arithmetical skills (Baturo & Nason, 1996; G. Kidman & Cooper, 1996a, 1996b; Zacharos, 2006).

When Gholam (1994) analysed the results of the CSMS assessments in British secondary schools, she concluded that the poor results in the area tasks were due to the application of rules which children had devised or modified and applied with little understanding. The children’s formulae were invariably incorrect and also inappropriately applied. Analysis of similar testing of middle school and high school students in North America, from the second National Assessment of Educational Performance (Hirstein, 1981, p. 706) indicated several misconceptions about measurement and a confusion between area and perimeter:

The results of NAEP area exercises show that many 17-year-olds do not use the correct numerical manipulation. It is of no practical use if our students correctly calculate using an inappropriate operation when a simple conceptual question is asked.

Year 5 students in New South Wales demonstrated a similar confusion in a multiple choice question in the 2000 statewide Basic Skills Test. Only 32% of students were able to select the correct answer for the area of a pool, from a diagram with the measurements 50 m by 20 m. Thirty-five percent of students selected 140 m², indicating the application of the perimeter formula (New South Wales Department of Education and Training, 2000, Numeracy p. 6).
Stephan and Clements (2003), in describing children’s development of measurement concepts, noted that an understanding of measurement does not keep pace with other mathematical ideas, and this is probably due to instructional methods which emphasise procedures for calculating measurements, rather than conceptual understandings of how measurements are made. They described four foundational concepts in learning to measure area as: (1) partitioning, (2) unit iteration, (3) conservation, and (4) structuring an array, with the suggestion that teachers who don’t understand the complexity of these concepts, are at risk of making incorrect assumptions about children’s level of comprehension.

The children who participated in the researcher’s study were concurrently studying spatial concepts within the Space strand of the syllabus (New South Wales Department of Education, 1989). Activities planned from the Space 2D substrand included identifying and manipulating tessellating shapes, and making symmetrical patterns from shapes by flipping (reflecting), sliding (translating) and turning (rotating). The children were covering the area of a given surface by experimenting with tessellating shapes, but the syllabus notes for teachers did not indicate the possible links between the Space and Measurement Strands. The syllabus consisted of a large number of discrete units for each substrand and strand with accompanying teaching notes. This format did not encourage teachers to identify and emphasise conceptual links between and within the strands of Number, Space and Measurement.

This study sought to identify the key factors in planning a successful mathematics teaching program for developing an understanding of the measurement of rectangular area. A focus on a practical understanding of how area is compared and measured would align with the emphases suggested by mathematical education research including learning with understanding and developing the essential knowledge for the concepts introduced in later years (Australian Association of Mathematics Teachers, 1996, 1999; Australian Education Council, 1991; National Council of Teachers of Mathematics, 2000).

Learning how to measure a quantity entails more than the procedures of using a measuring instrument and counting and recording the units of measure. The process
of measuring also involves identifying the attribute to be measured, selecting and using appropriate units and fundamental mathematical ideas such as comparison, conservation and transitivity (Piaget, Inhelder, & Szeminska, 1960; Stephan & Petty, 1999). As Nitabach and Lehrer (1996, p. 473) suggested, measurement skills and knowledge enable children to explore and quantify the space around them and to answer questions such as “How can shapes look different yet cover the same amount of space?”

Piaget et al.’s (1960) theory of developmental stages and conservation of area guided the early research on area learning. However, their emphasis on one-dimensional quantitative aspects was questioned by a second body of research, commencing in approximately 1975 (G. Kidman & Cooper, 1996a). This approach, as investigated by Anderson and Cuneo (1978) was based on children’s conceptions and misconceptions of area and the use of judgmental processes. Researchers emphasised students’ misunderstanding and teaching experiments (Hart, 1981a; Hirstein, 1981), rather than the developmental stages theory of centration, as propounded by Piaget et al. (1960). Anderson and Cuneo (1978) developed assessment tasks and methods of analysis that enabled children’s area problem solving strategies to be identified as an additive model (the length + width rule) or a multiplicative model (the length x width rule). Anderson and Cuneo’s research was conducted across a variety of experimental manipulations in eight experiments. The findings, that young children (5 years) use an additive model and older children, from 11 years, use the multiplicative model, was confirmed by other researchers including Silverman and Paskewitz (1988) and Lautrey, Mullet and Paques (1989). Anderson and Cuneo (1978) suggested that the height + width model confused area with perimeter, therefore an understanding of the multiplicative model was the key to understanding the calculation of area. The lessons described in this research focussed on establishing the concept and use of a multiplicative or array model of tessellating informal units to measure rectangular area.

This review will commence with children’s ability to define and describe the area which is being measured, followed by the use of appropriate informal units to measure and compare areas, the identification of the array pattern formed by
repeated, congruent, tessellating units, and the use of composite units to assist in counting the total used to express the measurement of a rectangular area.

2.1.1 Understanding the Attribute of Area

Children may have difficulty in describing or measuring the area of a surface if they do not have an understanding of the attribute of area:

The first stage of a child’s development will be to perceive area as an amount of surface without necessarily measuring it. It would appear that the child’s understanding of area comes later than that of some of the more readily recognised attributes such as length or mass (Primary Mathematics Project Team Curriculum and Development, 1989, p. 58).

An understanding of the attribute of area includes the two dimensional nature of a surface, and identification of the limits of the surface to be measured. Zacharos (2006) quoted comments from Simon and Blume (1992) in noting that a study of area involves an understanding of area as a quantity and then an evaluation of that quantity.

Children should understand how to cover a surface, because this is basic to the measurement of area (Reeves, Crosse, & Green, 1981). Some young children focus on the boundedness criterion of a given shape, as opposed to the space filling criterion which will measure area. Nitabach and Lehrer (1996) asked children to cover a square with pennies, and found that the pennies were placed carefully on the borders so that none were outside the square, but the children were not concerned about the gaps between the pennies.

The Measure Up project in Hawai’i (Dougherty & Zilliox, 2003) focused on establishing mathematical concepts with young children, in preparation for the introduction of complex topics in the Middle Grades. The Russian mathematics curriculum (Davydov, Gorbov, Mauulina, Savelyeva, & Tabachnikova, 1999) was used as a guide. This project encouraged children to explore number through a comparison of quantities. Instead of a mathematics program that commenced with the counting of objects, children started by describing and defining attributes of objects and then comparing the objects using attributes such as length, area, volume and mass. When direct comparison was no longer possible, due to the location or size of quantities, children used informal units to measure each object, and were
introduced to the notion of number. The children understood that the size of the units had to remain constant for the quantities to be compared.

In the New South Wales syllabus, *Mathematics K-6* (New South Wales Department of Education, 1989), activities described in the first unit of the Area substrand emphasised the identification of surfaces and an ability to cover surfaces. In Area 2, the syllabus objective stated that children should:

use and understand the term “area” when applied to parts of the school or classroom (New South Wales Department of Education, 1989, p. 126).

This statement encouraged teachers to refer to the reading area, canteen area, or playing area, using the term *area* to describe large spaces with loosely-defined boundaries. Researchers have suggested that these kinds of activities lead to confusion when children measure smaller and very specific areas (Outhred, 1993). Reynolds and Wheatley (1997) drew on the work of Acredolo (1981), Berthelot and Salin (1994) to propose that space is experienced and measured in three different forms: micro space (manipulated small objects), meso space (within the immediate vicinity) and macro spaces (urban, rural). As Brindle (1994) noted, the teacher’s understanding of the term *area* may not be shared by her students, requiring development and negotiation through meaningful experiences and discussions.

### 2.1.2 The Use of Units to Measure

Primary mathematics teaching which emphasises children’s understanding of concepts, relationships and patterns, needs to focus on sense-making rather than procedural competence and this has a special significance when using measuring devices and units of measure (Nitabach & Lehrer, 1996). Piaget et al. (1960) stated that measurement required more than an application of skills or steps in a procedure, and that a true understanding of measurement was dependent on concepts of subdivision or displacement. Stephan and Petty (1999) described these concepts as:

understanding space or the length of an object as being partitionable (subdivision), and partitioning off a unit from an object and iterating that unit without overlap or empty intervals (p. 3).

Units are used in all aspects of measurement. When measuring a quantity or comparing two or more quantities, each quantity is subdivided into a number of
identical units which are repeated and then counted. Clements and Stephan (2004) commented that children are ready to begin measuring after they have compared two quantities and identified the equality or inequality of the objects. By measuring, children are “assigning a number to continuous quantities” (D. Clements & Stephan, 2004, p. 300). The repetition of measurement units is the basis for constructing tools such as rulers, which can then be aligned with the objects to be measured (Barrett, Jones, Thornton, & Dickson, 2003).

To measure a quantity, students must select a suitable unit and then iterate the unit appropriately, without gaps or overlaps. Grant and Kline (2003) noted that iteration of measurement units requires both procedural knowledge (how to align and count the units) and conceptual understanding (how the measurement is calculated and why the unit is iterated). Mathematics teaching programs commonly recommend learning experiences using informal units of measurement before formal units are introduced, to assist in establishing the concepts of measurement (Hirstein, 1981; National Council of Teachers of Mathematics, 2000; New South Wales Department of Education, 1989; Owens & Outhred, 2006). Reynolds and Wheatley (1997) advised that students do not easily and automatically develop the concepts necessary to measuring space, therefore early experiences should involve the use of informal units in a variety of meaningful situations which encourage comparing and measuring. These researchers studied children’s responses to problem-solving situations in which the children compared the lengths of jumps by constructing the space of each jump as a linear unit. In another activity, pairs of children used rhythmic counting to measure the time taken to complete an obstacle course. Reynolds and Wheatley (1997) observed that when several pairs of children practised the rhythmic counting to ensure fairness, they recognised that a standard unit of measure was necessary.

These early measuring experiences may involve the use of familiar objects such as pencils and straws for length, tiles or shapes for area, cups or small containers for volume, and blocks or marbles for mass. The informal units are usually items that are familiar and able to be easily manipulated by young children, and provided from classroom resources. The importance of using measurement units and tools which are appropriate to the physical qualities of the quantity being measured was
demonstrated by Zacharos (2006) in a teaching experiment based on the earlier work of Nunes, Light and Mason (1993). Zacharos (2006, p. 234) advised to measure: “length with linear units, areas with two-dimensional units, volume with three-dimensional units and fluid capacity with appropriate vessel tools.” Nitabach and Lehrer (1996, p. 273) stated that “Units of measure should be adapted to the objects of measure” with the example that children who attempt to measure area by repeatedly drawing lines across a rectangle, have assumed that length has space-filling properties. Children may also assume that the units of measurement must bear a physical similarity to the area being measured, so that squares can only be measured by smaller squares and rectangles must be covered by smaller rectangles (Lehrer, Jaslow, & Curtis, 2003).

2.1.3 Measuring Area with Informal Units

The need to measure area is prompted by the necessity to quantify an area or to compare two or more areas. In some instances, a comparison can be achieved by directly comparing two areas, as when one shape is placed on top of another, or by a visual comparison when one area is distinctly larger than another. When two surfaces cannot be readily superimposed, or different shapes make a comparison very difficult, each area must be measured. The process of selecting and repeatedly using a smaller two-dimensional unit to measure an area uses the ideas of unitization and iteration. These concepts also underpin the measurement of length (Barrett et al., 2003). Researchers have focused on children’s difficulties with using units in spatial measurement of length, area and volume, where units of measure must be congruent and spatially aligned within a unit structure, as opposed to mass, temperature and time where spatial structure is not important (Outhred et al., 2003; Owens & Outhred, 2006). Educators suggest that a good understanding of the measurement of length is a prerequisite for a conceptual understanding of area (D. Clements & Stephan, 2004).

Bragg and Outhred (Bragg & Outhred, 2000a; 2000b) found that many 6 to 10-year old students had difficulty with understanding the measurement of length, and suggested that students were following procedural strategies. Students showed their lack of understanding of a subdivision of a linear quantity by counting marks instead
of spaces, and an inability to apply skills to practical problems. Bragg and Outhred recommended that students be exposed to activities involving the partition of quantities rather than simply counting prescribed units. Bragg and Outhred (2004) continued their earlier research by assessing Year 6 students and found that in an ‘offset-ruler’ task, 24% of the students were still reading the number at the end of the ruler as the length, without checking that the object had been aligned with zero. Kamii and Kysh (2006) quoted Martin and Strutchens’ (2000) analysis of a similar task in the Seventh NAEP, in which only 22% of the fourth grade children and 63% of the eighth grade children gave the correct answer. Owens and Outhred (2006) in reviewing students’ difficulties with iterated units, quoted previous studies by Cannon (1992) and Stephan and Cobb (1998), indicating that students counted the number of marks or paces instead of the number of units or amount of space, when measuring length.

Given that many students have difficulty with understanding the use of units to measure length, the measurement of area is likely to cause additional problems. The subdivision of area into congruent, tessellating units requires an understanding of square units, multiplication, and the spatial pattern of a two-dimensional pattern or array (Lehrer, 2003; Lehrer et al., 2003). In a three year study of 37 children in primary grades, Lehrer et al. (1998) found that students could recall the standard formulas for finding the areas of squares and rectangles, but fewer than 20% in their study knew that area measurement required identical units and fewer than half could use the known area measures to find the areas of unknown figures. Lehrer et al. (1998) also found that younger children were also likely to use resemblance when selecting informal units to measure area, so that square units were chosen for square areas and rectangular units were used to measure rectangles.

Kamii and Kysh (2006) assessed 38 children in grade 4 with a task involving two “chocolate bars” of different shapes and sizes on geoboards. When asked to count the number of squares in each bar, only 16% of the children counted squares, and 68% counted pegs. The results demonstrated that fourth grade children do not see a square as a unit of measurement. This result was also shown in a study by Hirstein, Lamb and Osborne (1978) who found that children in grades 3 to 6 considered unit squares to be discrete units which could not be partitioned
Nunes, Light, Mason and Allerton (1994), in investigating the impact of representations on children’s problem solving skills, found that children who were given bricks as informal units were more successful in measuring area than students who used rulers. The research showed that children were more likely to discover and understand the multiplicative relationship of the number in each row times the number of rows, when using appropriate resources. Zacharos (2006) also investigated the use of appropriate tools and found that upper-primary children in an experimental group who measured the area of rectangles with a square unit and marked (but not numbered) ruler were more successful than children in a control group who used a ruler. The results indicated that children who used a ruler and the length x width formula were able to calculate an answer more easily, but did not understand what they were doing or what their answer represented.

Researchers have found that children’s ability to structure an array is the key to understanding that the area formula Length x Breadth is an expression of the number of units in each row multiplied by the number of rows (Battista, Clements, Arnoff, Battista, & Borrow, 1998; Outhred, 1993; Outhred & Mitchelmore, 1992, 2000). Battista et al. (1998) interviewed 12 second grade students for forty-five minute sessions, asking the children to firstly predict how many small square tiles would cover a rectangular area and then to draw where the tiles would be located on the rectangle. The researchers found that drawing the array structure was difficult for the children, as they needed to disembed the composite units of rows from columns, in order to repeat these as successive rows. Battista et al. (1998) commented that the row-by-column structure of rectangular arrays was not automatically comprehended by all children, but must be personally constructed by each individual.

Outhred and Mitchelmore (2000) interviewed and observed 115 children from grades one to four, as the children completed tasks involving drawing, counting and measurement. Results from the study showed that more than a quarter of third graders and about two thirds of fourth graders successfully calculated the number of square tiles needed to cover the area of a rectangle. Outhred and Mitchelmore (2000) commented that many more primary school children should be able to learn to use the area formula, than is currently the case. The researchers also noted that an unintentional result of the study was the emergence of drawing as a teaching and
learning tool. The experience of drawing the covering tiles resulted in new insights for several children, as they experimented with different ways of reproducing the horizontal and vertical lines. Outhred and Mitchelmore (2000) observed that the manipulation of tiles did not provide the children with the same opportunities to reproduce the pattern of lines as an array.

Clements and Stephan (2004) described the levels in the development of an understanding of the array structure as:

a) little or no ability to organise, coordinate, and structure two-dimensional space (cannot represent covering a rectangle with tiles without overlaps or gaps);

b) complete covering, but counting incorrectly (cannot keep track of which units were counted; e.g. counts around the border and then unsystematically counts internal units);

c) covering and counting but again with no row or column structuring;

d) the local, incomplete use of rows or columns (e.g. counts some, but not all, rows as a unit)

e) structuring the rectangle as a set of rows;

f) iterating those rows (e.g. counting each row of 5, 10, 15…);

g) iterating the rows in coordination with the number of squares in a column (e.g. count by 5)

h) Understanding that the rectangle’s dimensions provide the number of squares in rows and columns and thus meaningfully calculating the area from these dimensions (p. 309).

As Battista (2003) explained, to measure area and volume with standard units, the quantity is first subdivided into square or cube units. An understanding of the structure of the array of units is necessary for counting. The four mental processes described by Battista to count arrays of squares and cubes were: “forming and using mental models, spatial structuring, units-locating, and organizing-by-composites” (p. 122). Battista explained that composite units may be comprised of the units in a row, and then the composite unit may be iterated to generate an array. The importance of children’s ability to structure arrays as mental models and to consistently iterate the composite unit when calculating area will be mentioned again in 2.1.4.

Other basic tenets of measuring area include using identical units to measure and compare areas, finding the total area of a figure by adding the area of its subregions, and knowing that measuring area is based on space filling, as opposed to leaving
gaps or only attending to covering the boundaries (Grant & Kline, 2003; Nitabach & Lehrer, 1996). Mathematics educators have also highlighted the importance of children’s understanding of the inverse relationship between the size of the unit and the number of units (D. Clements & Stephan, 2004; Outhred et al., 2003). Grant and Kline (2003) used the transcript of a lesson to demonstrate how young children observed, commented and explained the variation in results when a line on the floor was measured in terms of different children’s feet. Grant and Kline’s (2003) observation was that the teacher’s questioning strategies and an encouraging classroom environment provided opportunities for the students to engage in a mathematical discussion.

Mulligan, Prescott, Mitchelmore and Outhred (2005) found that young children may not have the skills necessary for subdividing area, including grouping, partitioning and unitising and may not recognise the array structure of repeated units. These researchers emphasised the importance of structure to an understanding of mathematics in general, after assessing the skills of 103 Grade 1 children with tasks of time, pattern, triangles, area and length. The results indicated that:

Young children’s perception and representation of mathematical structure generalises across a wide variety of mathematical tasks.

Early school mathematics achievement is strongly correlated with the child’s development and perception of mathematical structure (J. Mulligan et al., 2005, p. 400).

2.1.4 Counting the Number of Informal Units Used to Measure Area

When measuring area, identification and understanding of the pattern formed by the repetition of tessellating units is crucial to an understanding of how the formula for rectangular area works (D. Clements et al., 1998; Outhred, 1993). Stephan and Clements (2003) warned that when children simply counted units to calculate area, and then progressed to using a formula, they did not develop the conceptual basis of the measurement of area. Children need to:

a) construct the idea of measurement units (including measurement sense for standard units);

b) have many experiences covering quantities with appropriate measurement units and counting those units;

c) structure spatially the object they are to measure; and
d) construct the inverse relationship between the size of a unit and the number of units used in a measurement (p.14).

Outhred and Mitchelmore (1992; 1996) investigated the strategies used by young children to count the informal units used to measure an area. The majority of children counted individual units or used repeated addition. Outhred and Mitchelmore (2000; 2004) identified a relationship between the pattern of children’s drawings and their counting strategies. Students who used rows and columns to record the pattern of units were more likely to count by groups or to use multiplication. These researchers suggested that an emphasis on counting based on the structure of the array would assist both multiplication and area understandings. This finding concurs with Battista (Battista, 2003) who described the relationship between constructing mental models, spatial structuring, units-locating and organizing-by-composites. These understandings enable children to count the rows of units as composite units and then to coordinate the number in each row with the number of rows.

An emphasis on the spatial structure of area measurement and the need for children to experience structured and practical spatial activities such as geoboards and dot paper were also suggested by Kidman (1997; 1999), following her investigation of the strategies used to calculate rectangular area. Kidman (1999) found that children in Grades 4, 6 and 8 used an additive (perimeter) or multiplicative (area) judgment rule, with around 50% of the children from all grades using an additive rule to calculate area. Kidman also suggested that the introduction of formulae for the calculation of area and perimeter be delayed.

Hart (1984) tested the ability of 12 to 15 year-old children to measure length, area, and volume. Hart found that children misunderstood the concept of using units to measure and placed greater emphasis on the number of units used, rather than the comparative size of different units used to measure two lengths. Hart suggested that children looked for a number as a simple answer, at the expense of all other considerations. This result was further demonstrated by children’s misunderstanding when 1 cm square tiles used to tile a rectangle were reduced to $\frac{1}{2}$ cm x $\frac{1}{2}$ cm. Hart found that 60% of the group simply doubled the number of tiles. Hart also noted that when presented with shapes on a background grid of squares, many children used
counting to calculate the area, even though a formula to calculate area would have been taught in mathematics classes.

Reynolds and Wheatley (1996) reported that fourth grade children, who were asked how many 3x5 cards would be needed to cover a 12x30 rectangle, were able to do this task if they recognised individual cards as iterable units. Children who used the smaller card as a repeating unit, establishing a pattern of lines or grid, were able to successfully partition the larger card and calculate the number of iterable units. The unsuccessful student tried to visualise the smaller card as a single, repeating unit. She had no multiplication structure, to determine the number of cards needed and was not able to construct the relationship between the card and the larger rectangle. The three successful children were able to use one unit of measure consistently, and also use the 3x5 card as an iterable unit, which became part of a pattern, to be repeated and counted. Reynolds and Wheatley (1996)argue, “Unitising is a fundamental component of children’s construction of a concept of area” (p. 564).

During a research and development project funded by the Education Department of Western Australia, Willis (2005) found that children between five and eight years were able to use rods to count the total needed to make a line across a table, but were unable to utilise this measurement or any other measuring skills to predict whether the table would fit through a doorway. Willis argued that young children who are asked to measure a quantity using given units, learn that they are expected to count the units, but may not understand that the activity is aimed at measuring a quantity. Consequently, the children may be unable to link this counting task with using an understanding of measurement units to complete practical tasks. Willis (2005) noted that discussion of measurement skills should include reference to the outcomes of measuring which leaves gaps or overlaps as children may be able to repeat the mantra of 'no gaps, no overlaps' without considering why this rule is important to accurate measurement.

2.1.5 Estimating Measurements

Estimation skills are an important component of learning how to make and use measurements (New South Wales Department of Education and Training, 2003c). The confident use of estimation assists children to interpret measurements, to apply
numerical skills to everyday situations and to recognise when a measurement is reasonable (Adams & Harrell, 2003). Scott (2002) described a fifth class investigation of estimating and measuring a dog, stimulated by the children’s book *Measuring Penny* (Leedy, 1997). Children estimated then measured the dog’s height, width of paws and mass, using informal then standard units. Scott (2002) argued that estimation is an important mathematical skill which needs to be developed through practical activities rather than pencil and paper calculations.

2.1.6 Implications for the Design and Implementation of an Area Teaching Program

An understanding of how rectangular area is measured involves numerical and spatial, as well as measurement concepts. The need for integration of these concepts, in a way that makes sense to children, is imperative for learning and understanding (Lott & Souhrada, 2000; Willoughby, 2000) and includes the recognition of structures and patterns (Mulligan, Prescott, & Mitchelmore, 2004) and an understanding of the nature of multiplication with its two-dimensional coordination of horizontal and vertical, and the commutative law of multiplication (Rozek & Urbanska, 1999).

Learning experiences that assist children to recognise and work with the properties of tessellating shapes will assist with the spatial aspect of an understanding of area.

A plane tessellation is a pattern made up of one or more shapes, completely covering a surface without any gaps or overlaps. Tessellations can be extended in the plane infinitely in every direction (Seymour & Britton, 1989, p. 3).

*Mathematics K-6* (NSW Department of School Education, 1989) suggested that tessellations be introduced to children in mid- to late-Year 2, but did not suggest to teachers that they should emphasise or discuss the links between tessellations and the measurement of area. The underpinning skills of aligning shapes, angles and equal sides and the links to measuring area by using repeated, congruent, tessellating shapes have not been recognised by many primary teachers (McPhail, 2004; Outhred & McPhail, 2000).
Beyond an understanding of the attribute of area, children must also understand what happens mathematically when two dimensions are multiplied to calculate the area of a rectangular shape. The success of links made by children between multiplication and the calculation of area may be simplified by a teaching program that emphasises the use of rectangular arrays (Haylock & Cockburn, 1997). Rozek and Urbanska (1999) referred to this structure as RCAF (row-column arrangement of figures) and noted that this two-dimensional distribution of elements can also be extended to diagonal arrangements. These researchers warned that young children who do not have an understanding of structures will fail in learning some aspects of mathematics. Malloy (1999) suggested that appropriate tasks accompanied by an enquiry-based pedagogy would assist children to identify the relationships in mathematics. The relationships could be further investigated in other areas of mathematics. Malloy noted that children who have opportunities to develop concepts of area and perimeter, would be able to describe the attributes and derive and use the formula for simple shapes such as rectangles and triangles. By comparison, children who are taught by rote may be able to solve problems, but not understand the concepts behind the words.

Wilson and Rowland (1993) expanded on the need for children to construct meaning for themselves by adding that when given time for reflection and discussion, children will develop abstract ideas about units, perimeter, area and rate. Wilson and Rowland (1993) referred to Piaget et al.’s research, that found that children developed spatial concepts at two levels: perceptual and representational. Wilson and Rowland (1993) acknowledged that recent research suggested that children may develop spatial concepts much earlier than stated in Piaget et al.’s findings, but they maintained that the basic premise of the need to move from perceptual to representational still held true, and the further need to develop abstract ideas cannot be achieved within a measurement teaching program that focuses on memorised facts and processes.

If children construct ideas for themselves instead of just memorizing properties, they will be able to discuss these ideas and move beyond physical reality to abstract representations and relationships between properties. They will be able to operate with their new ideas (Wilson & Rowland, 1993, p. 171).
Malloy (1999) suggested that appropriate tasks accompanied by an enquiry-based pedagogy will assist students to identify the relationships in mathematics and then these relationships can be further investigated in other areas of mathematics. Malloy maintained that children who have opportunities to develop concepts of area and perimeter will be able to describe the attributes and derive and use the formula for simple shapes such as rectangles and triangles. Similarly, Kidman (1999) after finding that around 50% of students from Grades 4, 6, and 8 used a perimeter rule to determine area, suggested that children need to experience structured classroom activities, focusing on the spatial foundations of area and perimeter. Specific tasks included the use of geoboards and dot paper.

McDonough (2002) found that children’s beliefs about measurement were “idiosyncratic”, and that teacher’s and children’s understanding of the relationship between measurement and mathematics, and the use of informal units and measurement, frequently do not align (p. 455). McDonough’s advice to teachers was to identify and model mathematical language through discussions, and to assist children to make the links between measuring and mathematics.

This literature review has included many studies of children’s misconceptions about the measurement of area and perimeter (Baturo & Nason, 1996; Hart, 1981b; Malloy, 1999; Raghavan et al., 1998; Strutchens et al., 2003). However, few examples of relevant teaching programs applicable to whole class implementation in the early years of schooling have been identified. Some researchers and teachers have worked with a small group or individual children (Outhred, 1993; Reynolds & Wheatley, 1996) and others have implemented a restricted number of lessons (Copley, Glass, Nix, Faseler, & De Jesus, 2004; Nitabach & Lehrer, 1996; Reynolds & Wheatley, 1997; Scott, 2002; Trafton & Hartman, 1997).

This review has outlined the skills and knowledge identified by researchers as critical to children’s early understandings of how to compare and measure rectangular areas. The essential factors include an understanding of the attribute of area, the use of informal and formal units to measure quantities, knowledge of array structures, the need to understand the mathematical conceptual and spatial links between multiplication and area, and an overarching understanding of and
familiarity with the processes and uses of measurement. Combining these factors into a cohesive, sequential teaching program was the challenge undertaken by the author of this thesis.

### 2.2 Teacher Professional Learning

The aim of this study was to assist teachers to make changes to the ways in which they taught the measurement of rectangular area. The achievement of such changes would also encourage teachers to apply the principles of the area lesson program, including sequential activities and an emphasis on mathematical and spatial structure, to other aspects of their mathematics teaching. To achieve this ambitious aim, appropriate teacher professional learning models and strategies were reviewed, including definitions, models of development, and strategies of implementation and consideration of what teachers need to know to be able to teach mathematics.

This discussion will begin with a general description of professional learning, followed by an examination of the principles of professional learning and models of successful implementation, the components of professional learning, an exploration of teachers’ mathematical pedagogical and content knowledge that is pivotal to successful teaching in primary mathematics, and teacher leadership in curriculum innovation.

#### 2.2.1 What is Teacher Professional Learning?

Teacher professional learning, with an emphasis on the word *professional*, conveys a sense of commitment to learning, as a prerequisite of belonging to a profession. The range of specific examples of both formal and informal teachers’ professional learning is as broad as the immediate, long term, and ongoing needs of teachers and their students, and may be described as:

> the opportunities offered to educators to develop new knowledge, skills, approaches, and dispositions to improve their effectiveness in their classrooms and organisations (Loucks-Horsley, Hewson, Love, & Stiles, 1998, p. xiv).
This description of teacher programs focuses on the achievement of effective and positive change in teaching practice, leading to enhanced student learning outcomes (Australian Association for Mathematics Teachers, 2002). An explicit link to student outcomes was not acknowledged by some of the early models of in-service in which student learning was incidental. Hargreaves (1995, p. 149) was critical of the “one-shot deal” inservices which raised teachers’ awareness but neglected follow-up support or opportunities for teachers to integrate new knowledge with existing strategies. Participation in successful and relevant professional learning programs encourages teachers to update their pedagogy, to become familiar with curriculum changes and to critically reflect on their craft and their essential role in society as the educators of the next generation. However, even though a significant quantity of educational literature has explored the planning, implementation and problems of teacher professional learning, there is very little systematic research on the effects of professional development on improvements in teaching or on student outcomes (Garet, Porter, Desimone, Birman, & Yoon, 2001).

Educational researchers and educational administrators have advocated a dual emphasis on improving teacher knowledge and skills, as well as teaching content (Ernest, 1989). Cohen (2004) suggested that professional learning plans should define what is useful in helping teachers to deepen their knowledge of subject matter, as well as the teaching strategies which will assist their students. Most approaches to professional learning focus on providing teacher training that results in improved teaching and ultimately enhanced student learning outcomes. McRae, Ainsworth, Groves, Rowland and Zbar (2001) suggested a reverse approach that commenced with defining the desired student outcomes. Appropriate teacher training targeted to the project aims, was then planned and implemented. McRae, et al. argued that this approach utilised natural learning opportunities for teachers, was as close as possible to authentic contexts and had “an explicitly defined, achievement-oriented purpose” (p. 18). The emphasis on the achievement of student outcomes presupposed that significant and goal-oriented student learning could and would be realized.
Studies have demonstrated that changes to the curriculum have not proven to be productive, without an accompanying focus on the pedagogy that will deliver those changes to children in classrooms (Lovitt, Stephens, Clarke, & Romberg, 1990).

There is little evidence that changing the curriculum will improve the level of student outcomes unless there are significant attempts to change what teachers do. It may change them, but not improve them (Ramsay, 2001, p. 37).

Professional learning may traditionally have involved attendance at a training course, usually outside of school hours, and led by an “expert” or consultant (Loucks-Horsley et al., 1998). The focus of teacher change has moved from something that is done to teachers, to “change as a complex process which involves learning” (Clarke & Hollingsworth, 2002, p. 950)). Current literature has suggested a broad range of professional learning activities, mostly during school hours and based on the school site (Garet et al., 2001). These successful practices have acknowledged that teacher learning takes place over a longer length of time than a one-day inservice course, emphasise collaborative planning with colleagues, focus on the explicit goals of improving student learning outcomes, and are based on curriculum, pedagogy and classroom practice. School-based initiatives have provided opportunities to focus on deepening teachers’ knowledge of subject matter which is grounded in student work and classroom practice (Hiebert, 1999).

2.2.2 The Principles of Effective Professional Learning

Louden (1994, p. 32) constructed a simple, but accessible list of factors which combine to provide a guideline for planning, facilitating, implementing and applying professional learning programs (Table 2.1).

<table>
<thead>
<tr>
<th>Planning</th>
<th>Is founded on teacher and staff needs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Is relevant to teachers’ interests and experience</td>
</tr>
<tr>
<td></td>
<td>Provides enough flexibility to accommodate the needs of individual teachers</td>
</tr>
<tr>
<td></td>
<td>Clearly defines goals, processes and outcomes</td>
</tr>
<tr>
<td></td>
<td>Involves committed participants</td>
</tr>
<tr>
<td></td>
<td>Offers equal access to all teachers regardless of their geographical location or type of school</td>
</tr>
<tr>
<td></td>
<td>Brings together teams of classroom teachers</td>
</tr>
<tr>
<td></td>
<td>Requires preparation by participants</td>
</tr>
<tr>
<td></td>
<td>Addresses various levels of need, (e.g. teacher, school and system)</td>
</tr>
</tbody>
</table>
Facilitation

- Involves leaders with expert knowledge and practical know-how
- Locates activities in pleasant and comfortable surroundings
- Involves teachers drawn from both similar and diverse professional settings
- Provides high-quality ‘user friendly’ materials and resources
- Has the support of the school executive

Implementation

- Demonstrates a clear relationship between theory, research and practice
- Provides opportunities for active engagement
- Contains both structured and unstructured time, with participants able to reflect upon implications for their professional practice
- Makes optimal use of the time available
- Involves modelling of exemplary practice
- Sequences and spaces activities over time
- Builds knowledge and ownership through action research
- Uses a variety of presentation styles

Application

- Involves planned follow-up
- Creates a feeling of excitement, empowerment and ownership
- Translates into practice knowledge and skills gained in professional learning
- Demonstrates accountability for student outcomes
- Supports teachers’ accountability for student outcomes
- Rewards participation and achievements through academic credit, employer recognition, career advancement opportunities, or remuneration for time spent
- Encourages transferability of learning across school sectors and subjects

Garet et al. (2001) used data collected as a part of a national evaluation of the Eisenhower Professional Development Program. The researchers surveyed 1027 mathematics and science teachers to compare the core features described in professional development literature. The core features were evaluated by applying the teachers’ self-reported responses to activities. The researchers identified the kinds of activities that had a positive effect on teacher learning, both subject matter and teaching practices. These were:

(a) focus on content knowledge;

(b) opportunities for active learning;

(c) coherence with other learning activities.

The structural features which implemented the core features and which had the most impact on teacher learning and satisfaction were:

(a) the form of the activity (e.g., workshop vs. study group);

(b) collective participation of teachers from the same school, grade, or subject, and
Results indicated that school-based activities, as opposed to external activities, provided longer time periods for teams of teachers with common needs to meet, discuss and learn. The impact of this sustained professional learning over a length of time was reported as more successful teacher learning and positive changes to teaching practices. Teachers also reported that effective professional learning that is coherent and directly related to their needs, had resulted in improved teaching practices.

The focus on providing for teachers’ needs was also noted by Bobis (2004) who identified the five most helpful aspects of training provided to 95 teachers implementing the early numeracy project *Count Me In Too* in New South Wales. The teachers nominated practical features which assisted both classroom implementation and teacher professional learning: “the practical resources and activities, the assessment process, classroom support, the influence of significant people and the opportunity to share ideas” (p. 147).

The *Australian Mathematics Curriculum and Teaching Project* (Lovitt et al., 1990) charted the successful processes which primary and secondary teachers implemented to improve classroom practices. The features essential to successful professional learning programs included:

- address issues of concern recognized by the teachers themselves;
- be as close as possible to the teacher’s working environment;
- take place over an extended period of time;
- have the support of both teachers and the school administration;
- provide opportunities for reflection and feedback;
- enable participating teachers to feel a substantial degree of ownership;
- involve a conscious commitment on the part of the teacher
- involve groups of teachers rather than individuals from a school;
- use the services of a consultant or critical friend (p. 234).
Clarke (1994, p. 38) developed ten key principles from research for the professional development of mathematics teachers, elaborating each principle with support from the literature and his own experiences:

- Address issues of concern and interest, largely (but not exclusively) identified by the teachers themselves, and involve a degree of choice for participants.

- Involve groups of teachers rather than individuals from a number of schools, and enlist the support of the school and district administration, students, parents and the broader school community.

- Recognise and address the many impediments to teachers’ growth at the individual, school and district level.

- Using teachers as participants in classroom activities or students in real situations, model desired classroom approaches during inservice sessions to project a clearer vision of the proposed changes.

- Solicit teachers’ conscious commitment to participate actively in the professional development sessions and to undertake required readings and classroom tasks, appropriately adapted for their own classroom.

- Recognise that changes in teachers’ beliefs about teaching and learning are derived largely from classroom practice; as a result, such changes will follow the opportunity to validate, through observing positive student learning, information supplied by professional development programs.

- Allow time and opportunities for planning, reflection, and feedback in order to report successes and failures to the group, to share “the wisdom of practice,” and to discuss problems and solutions regarding individual students and new teaching approaches.

- Enable participating teachers to gain a substantial degree of ownership by their involvement in decision-making and by being regarded as true partners in the change process.

- Recognise that change is a gradual, difficult and often painful process, and afford opportunities for ongoing support from peers and critical friends.

- Encourage teachers to set further goals for their professional growth.

Clarke was concerned that the implementation of planned reforms to mathematics teaching was not possible without providing support for teachers. His ten principles embody a practical and realistic summation of the factors proposed by other researchers, including addressing teachers’ stated needs and acknowledging the strength of classroom applications as suggested by Louden (1994) and Lovitt et al. (1990). All of the researchers cited commented on the need to establish teams of teachers for collective participation and the necessity of allowing considerable time for teacher professional learning to be effective. However, Clarke’s principles also describe the need to begin by identifying impediments to teacher learning. He
described the importance of gaining teacher commitment to actively participate in the planned program by completing all tasks and ensuring that activities are modified appropriately before implementation in individual classrooms. Another aspect of professional learning noted by Clarke was the need for teachers to continually examine and update their current teaching practices, and to plan ahead for future growth, rather than participating in programs which happen to be available.

2.2.3 Implementing Professional Learning Programs

The goal of the author’s research was to identify and trial practical and effective professional learning programs suitable for schools of different sizes and teachers with a variety of teaching expertise. Models were sought which were economically sustainable for limited school budgets, involving a minimal financial outlay for teacher release, travel, or consultancy visits. Mathematics teacher educators (Lee, 2005; Tytler, Smith, Grover, & Brown, 1999) noted that the results of a wide range of teacher professional learning programs indicate that no single model or approach was guaranteed of success. Rather, the goal of achieved and continuing change in teaching practices may be reached through establishing a supportive environment and providing appropriate materials (Lovitt et al., 1990).

The five models of professional development listed by Sparks and Loucks-Horsley (1989) affirmed the principles outlined by Louden (1994) and Garet et al. (2001) and were also practical and versatile:

**Individually Guided Development**: The teacher designs his or her learning activities.

**Observation and Assessment**: A colleague or other person observes a teacher's classroom and provides feedback.

**Involvement in a Development or Improvement Process**: This might include developing curricula, designing programs, or changing classroom practice through reading, discussion, observation, training, and experimentation.

**Training**: A training design includes an expert presenter who selects the objectives, learning activities, and outcomes.

**Inquiry**: Teachers formulate questions about their own practice and pursue answers to those questions.
As Lee (2005) commented, many organisations have selected several of these strategies to be simultaneously implemented in successful teacher professional learning programs.

Tytler, Smith, Grover and Brown (1999) implemented two professional learning programs in Victoria, Australia, in mathematics and science. They compared the advantages and limitations of a whole-school model with a workshop model. Each model was based on an extensive and affirmative research background. Joyce and Showers (1995) were quoted as arguing strongly for school-based professional learning. Their research showed that teachers who were organised into teams provided collegial and social support, and a collective sense of purpose, generating cultural change within the team or school. Tytler, Smith, Grover and Brown (1999) quoted Borko and Putnam (1995) in the case for workshop-based programs. These researchers referred to the effectiveness of the CGI (Cognitively Guided Instruction) program as a strong indicator of the success of teacher workshops, but added that the greatest differences to instructional methods were generated by teachers listening to their own students solve problems. Tytler, Smith, Grover and Brown (1999) found that both of the programs were successful in achieving the planned outcomes and both catered to the existing knowledge base of participants. The science (workshop) program was able to focus on basic knowledge and skills whereas the mathematics (school-based) program built on and elaborated existing school cultures, team supports and programs. The researchers noted that for both programs, the trialling of sequences of activities, followed by group discussion, was a productive strategy.

Teachers from five small primary schools in western suburbs of Melbourne, Australia, participated in a professional learning project with the objectives to develop skills of appropriate questioning, to motivate teachers and provide ongoing, mutual support and to raise student numeracy skills and the ability to select and use problem solving strategies (Vale, 2003). The professional development model was designed to provide collaboration and support and included a curriculum day for all teachers, network meetings for teachers of similar grade levels and school-based planning meetings. The teachers were also supported by a curriculum consultant and the academic researcher as critical friend.
The project team collected data from an initial questionnaire from 42 teachers, and a final questionnaire from 19 of the participants. The data collection was based on teachers’ perceptions of their questioning strategies at the beginning and the end of the year. The study found a significant difference in two areas: a wide variety of question types when interacting with students and a wide variety of question types in classroom activities (Vale, 2003).

Putt and McLean (2002) surveyed 341 primary teachers in northern Queensland to gauge teachers' perceptions of their preparedness to use calculators in the teaching of mathematics. Approximately 40% of the teachers considered themselves as poorly prepared for calculator use in their teaching. The teachers were also asked which professional development strategy they preferred to assist them in using calculators. The results indicated that teachers preferred a face-to-face session with an "expert" facilitator to assist with implementation of calculator activities in classrooms. This strategy was seen as more useful than a book of student activities, a book on how to teach with calculators, a video on the use of calculators, or a book on how to develop personal calculator skills.

The results from these cases and the professional learning literature indicate that many teachers prefer some initial assistance from an expert or facilitator, but the majority of the teacher development has occurred during classroom implementation of teaching materials and interactions with colleagues and students. There are no generic models of teacher professional learning that can be applied to all situations, but the broad outlines suggested by Sparks and Loucks-Horsley (1989) provide an accessible guide. Another important consideration is that teachers need the opportunity to choose professional learning programs which align with their needs, available time, preferred methods of learning, and which will be helpful to them, rather than confronting or focussed on an approach which highlights the teacher's inadequacies (Irwin & Britt, 1999).

2.2.4 Contributing Factors to Successful Teacher Professional Learning

The success of any professional learning implementation is dependent, to a great extent, on the attitudes, knowledge, beliefs, culture and organisation which school communities and individual teachers bring to the venture. Issues include the
establishment and maintenance of teachers’ positive attitudes towards mathematics and teaching mathematics, their own learning, and the existence of reflective practices, collaborative dialogue and collegial support.

2.2.4.1 Attitudes and values that teachers bring to their profession

Teachers face high expectations from their students, parents of students, colleagues, administrators and employers. These elevated expectations encompass standards of general knowledge, academic skills and pedagogical knowledge as well as aspects of conduct, demeanour and moral responsibilities (Hilferty, 2000; Noddings, 1992). The Australian Senate Committee of Inquiry into the Status of Teachers (Australian Senate Employment Education and Training Reference Committee, 1998) referred to the UNESCO (1966) statement of the professional status of teaching:

Teaching should be regarded as a profession: it is a form of public service which requires of teachers expert knowledge and specialised skills, acquired and maintained through rigorous and continuing study; it also calls for a sense of personal and corporate responsibility for the education and welfare of the pupils in their charge (Chapter 3).

Building a culture of enduring professionalism is dependent on leadership and opportunities to develop, as well as the attitudes, values and skills that teachers bring to their workplace. There is also a need to examine the professional and emotional support that may be necessary to assist teachers to make changes to their beliefs and practices. These principles were emphasised by Robinson’s (1989) philosophy that change in teaching practices cannot be managed, but rather can be encouraged by giving people:

- the opportunity to be fully themselves;
- the vision to become aware of the choices around them;
- the support to venture down whatever road they choose (p. 280).

In a study of fifteen secondary schools in England and Wales facing social, cultural and unemployment problems, a focus on educational values enabled teachers to review “a strong sense of corporate purpose and professional identity” (Nixon, 1995, p. 221). Nixon argued that teacher’s values cannot be mandated, but are embedded within professional practice. Teachers need to focus on what they plan to achieve through their practice, and to debate issues and approaches with colleagues.
within the context of their school and children. Teaching becomes more than the application of theoretical knowledge and skills of how children learn, or a series of “technical operations”. Teaching is based on the values that define it as a profession.

The prime task for teachers as professionals, therefore, is to work out their educational values, not in isolation and abstraction but in collaboration with colleagues and amid the complexities of school life (p. 220).

2.2.4.2 Teachers’ commitment to achieving positive student learning outcomes

In her study of 1,213 teachers in 78 schools in Tennessee, Rosenholtz (1991) found that teachers’ commitment to student learning is determined by the three workplace conditions of empowerment (autonomy and discretion), learning opportunities that enable continuous growth and mastery of their environment, and rewards that ensure continuous contributions to the school. Rosenholtz found that the absence of these conditions may result in negative consequences for teachers, including disaffection, alienation, frequent absences or a desire to leave the school. Furthermore, these attitudes translated to unhelpful outcomes for children:

Teachers’ lack of motivation and commitment, as manifest in a lack of future planning and in complacency with the present, is visited upon children through their diminished opportunity to learn basic skills (p. 7).

Rosenholtz (1991) evaluated the effect of teacher commitment on student learning outcomes by examining variables of teachers’ commitment, school SES, teachers’ mean undergraduate status, their mean educational attainment and their mean years of experience. When the children’s second grade mathematics results were held constant, Rosenholtz found that teacher commitment was the only significant predictor of student achievement on fourth grade mathematics scores (p. 163). Rosenholtz further commented that committed teachers described academic plans in their goals and forward thinking and were prepared to break the rules of imposed policies if these conflicted with their personal plans, goals and projects. Conversely, “moderately stuck” teachers did not describe future or professional plans, or goals that were aligned with student learning. These teachers passively accepted rules and conditions even when imposed policies contradicted their professional judgment.
Bobis (2004) surveyed 95 Year 3 and Year 4 teachers in New South Wales to determine the positive and negative factors of their implementation of the early numeracy program *Count Me In Too*. A high proportion of the teachers recorded significant issues with their implementation of the program. However, 30% stated they would continue with the program despite the difficulties, because they could see the benefits for their students. Despite their stated issues of time, resources, class management and information overload, these teachers were willing to persist. Bobis concluded:

Hence, the factor that seems to emerge as assuming greater significance than concerns surrounding issues of time, resources and the like, is teachers’ inherent perception of the program’s worth for children (p. 149).

2.2.4.3 Teacher collaboration

Many models of teacher development focus on the establishment of a professional team that works to support the growth of individual members and the effective implementation of the chosen strategy or policy (Garet et al., 2001; Hiebert, 1999). Teams meet formally or informally and typically discuss the needs and progress of their students and their professional practice. Collaborative discussions provide an opportunity for teachers to share and compare the decisions they have made about student learning, to challenge their practices and develop new understandings (Jaworski, 1999). These may be based on observations, conversations with children, and children’s responses to questioning or the analysis of work samples. Collegial professional conversations facilitate the sharing of teachers’ assessments of their children’s work and also encourage the growth of these skills (Bryant & Driscoll, 1998).

Effective teamwork and collegial support have the potential to change the isolated conditions and “secret spaces” (Grubb, 2000, p. 696) of many classrooms. By establishing communication and shared development of programs, activities and assessment tasks, teachers have the opportunity to develop professionally. As Kilpatrick and Silver (2000) noted, collaboration with peers can have a positive influence on teachers’ confidence and competence in their teaching practices. Collaboration can also provide a safe environment by establishing “a climate in
which risk taking and growth are encouraged” (Widee, Mayer-Smith, & Moon, 1996, p. 201).

Rogers (2002) found that collaborative teaming and decision making helped to develop a shared decision making culture in the school and a greater feeling of control over pressures on the school from the government, from public expectations, from the media – “such collegial behaviour was clearly affecting their perception and level of stress, their coping ability and their professional empowerment in a positive way” (p. 9). Noddings (1992) commented on the social benefits of working collaboratively:

Teachers who have opportunities to plan together, observe each other, and diagnose and evaluate children together are apparently happier with teaching as a profession than those who do not have such opportunities (p. 204).

Teachers need solutions to the issues of professional learning that are practical and effective, and also encourage reflection and the ability to respond to the identified needs of students. Collaborative, workplace-oriented strategies have provided teachers with the space and support to discuss, evaluate and implement improved teaching practices and programs. However, Fullan (2001) warned that strong communities can make the situation worse if they emphasise or reinforce bad habits.

Collaborative cultures, which by definition have close relationships, are indeed powerful, but unless they are focusing on the right things they may end up being powerfully wrong. Moral purpose good ideas, focusing on results, and obtaining the views of dissenters are essential, because they mean that the organisation is focusing on the right things (pp.67-68).

2.2.4.4 Teacher reflection

Stoll and Fink (1996) referred to two different ways described by Elliot (1991) in which teachers reflected on and developed their practice through research. By researching and reflecting on a problem, teachers developed new understanding which lead to a change in practice. The reverse happened when a teacher made a change prompted by a practical problem and then self-monitored the effect of the change, leading to a new understanding of the situation. In addition, Stoll and Fink (1996) added a third stimulus, that was identified during the Halton Effective Schools Project. As teachers became aware of the results of the research, they used
the findings as an impetus to use the research in their classrooms to assist in identifying areas of need.

2.2.4.5 Making time for professional learning

One of the major challenges facing teachers, school executive and system administrators is finding or making quality time for teachers to engage in professional learning (Loucks-Horsley et al., 1998). In addition to their daily focus on the routines and procedures of classroom activities, teachers must also be accorded the time to investigate and trial changes to their teaching practices (Fullan, 1991).

Campbell (1985) noted four kinds of time for professional learning - group time (collaborative planning), snatched time (hurried consultations), personal time (out of school time for reading or attending courses) and other contact time (preparation or release time). Campbell noted that release time was more frequently used for preparation or marking than collaborative planning.

Stevenson and Stigler (1992), argued that time needs to be given on a daily basis for teachers to mark children’s work, to meet to tutor individual children and to attend staff meetings. When comparing time available to North American and Asian teachers for routine tasks and professional learning, they found that:

When we informed the Chinese teachers that American elementary school teachers are responsible for their classes all day long, with only an hour or less outside the classroom each day, they looked incredulous. How could any teacher be expected to do a good job when there is no time outside of class to prepare and correct lessons, work with individual children, consult with other teachers, and attend to all the matters that arise in a typical day at school! (Stevenson & Stigler, 1992, p. 163).

Stevenson and Stigler (1992) reported that the Asian classes are larger to make more non-teaching time for the teachers. By making larger classes with the same number of teachers, fewer teachers are in the classroom at any time. Additionally, Japanese and Chinese teachers spend longer at school each day.

Bobis (2004) asked teachers to nominate the challenges or barriers to a continuing implementation of *Count Me In Too* in their classrooms, and 45.3% of the teachers referred to “time”.

Teachers considered there to be a lack of time “to meet” with other teachers “to gain new ideas”, to “complete the testing”, “to make the resources”, “to think of different ways to
2.2.5 Teacher Professional Learning and the Teaching of Mathematics

The Australian Association of Mathematics Teachers described the “specialised professional work of teaching mathematics” (2002, p.1) as being in addition to the general characteristics and attributes of teachers. The paper *Standards for Excellence in Teaching Mathematics in Australian Schools* (Australian Association of Mathematics Teachers, 2002) outlines three separate fields within the domain of professional knowledge. These are described as knowledge of children, knowledge of mathematics and knowledge of children’s learning of mathematics. These fields reflect the findings of Putnam, Heaton, Prawat and Remillard (1992). This acknowledgement of the specialised knowledge that is required of teachers of mathematics has implications for the provision of effective professional learning.

Many teachers are ill-prepared to implement teaching programs that assist children to build conceptual understanding of mathematics (Garet et al., 2001). These teachers’ own mathematical learning and skill acquisition was based on memorising facts and procedures, an approach that does not fit well with an emphasis on classroom dialogue and investigation of mathematical relationships. The current emphasis in mathematics education on problem solving and the need for learners to construct meaning has challenged teachers to review their beliefs about learning classroom strategies (Tytler et al., 1999).

Loucks-Horsely et al. (1998) in discussing professional learning for science and mathematics teachers, explained that generic development activities did not address fully the needs of these teachers, and that pedagogical content knowledge for specific concepts was crucial. They also noted that:

Principles that guide the reform of student learning should also guide professional learning for educators. People teach as they are taught. Therefore, engaging in active learning, focusing on fewer ideas more deeply, and learning collaboratively are all principles that must characterise learning opportunities for adults (Loucks-Horsley et al., 1998, p. xix).

Franke, Carpenter, Fennema, Ansell, and Behrend (1998) argued that teacher professional development needed to extend beyond the acquisition of new skills or procedures, to encompass the ability to continue learning, through *self-sustaining*
generative change (p.67). Self-sustaining, generative change was essential if teachers were to continue to modify and adapt their practices to suit the needs of successive classes of students, over an extended length of time. Franke et al. (1998) described self-sustaining generative change as the result of teacher enquiry and reflection which has focused on the thinking of the teacher and her students, and the meanings constructed by the teacher. The results of a study of three teachers involved in Cognitively Guided Instruction (CGI) indicate that teachers’ ability to achieve self-sustaining, generative change is determined by engagement with the principles of the program. The teacher who achieved the greatest ongoing change had focused on her students’ thinking and explanations, analysed the responses and then used these to write programs which extended her students’ ability to solve number problems. This research indicated the enhanced results of a professional development program which actively encouraged teachers to construct and refine their pedagogical knowledge and practices through an examination of children’s mathematical thinking. The results suggest that opportunities and encouragement to focus on children’s thinking and explanations and to make this the basis of reflective practice, will enhance teachers’ ability to continue to learn and to grow.

Ball (1990) argued that the goal of mathematical teaching is for children to develop mathematical understanding. Understanding presupposes mathematical procedures and relationships as well as mathematical ways of knowing. When an enquiry-based teaching approach replaces a traditional text book approach with a focus on practising mathematics facts and procedures, teachers require mathematical understanding which will allow them to initiate discussion, share mathematical ideas and guide students’ thinking (Schifter, 1998). Ramsay (2001, p. 36) spoke strongly about the mathematical knowledge of Australian primary teachers:

A major reason often overlooked for poor mathematics learning in secondary school is the limited levels of mathematics knowledge and skill most primary teachers have. It is a delight to see a primary class being taught mathematics by a teacher who has some depth of knowledge, who can stretch the high achiever and yet be clear and focused in developing concepts with a slower child.

Bryant and Driscoll (1998) noted that professional learning needs to be based on achieving change in student understanding, rather than the implementation of teaching strategies. Programs of teacher change that have focused on techniques such
as grouping or inquiry learning have emphasised discrete teacher behaviour, rather than children’s learning. Bryant and Driscoll (1998) found that “Challenging teachers’ belief systems – about mathematics and about how children effectively learn mathematics – is rarely done” (p. 4).

Mewborn (2003) found that a “multitude of studies have been undertaken in the past 40 years in an attempt to determine which teacher characteristics affect student achievement, and the results largely have been inconclusive” (p. 45). The teacher characteristics investigated by Mewborn included years of teaching, academic records, mathematics qualifications and completion of inservice courses. By contrast, Mewborn cited the results from Cognitively Guided Instruction program (Fennema et al., 1996) and the adoption of the California Mathematics Framework (Gearhart et al., 1999) as positive evidence that students’ learning is influenced by what teachers do in the classroom.

2.2.5.1 Professional learning and classroom questioning strategies

Mathematics educators have argued that the teacher’s use of open-ended questions and tasks will encourage higher-order thinking and problem solving skills and allow students to explore concepts in a way which closed tasks do not (Sullivan, Warren, & White, 2000). These types of questions encourage students to think about relationships, seek patterns and explore mathematical ideas (Vale, 2003). Students develop their language skills and articulation skills and boost their self confidence when they contribute to a class discussion (D. S. Mewborn & Huberty, 1999).

In a year-long study with teachers from five Melbourne schools, Vale (2003) reported that teachers were changing their questioning strategies by the end of the year, and the early career teachers were using probe questions that required explanations from children. Vale’s comment was that the teachers’ questioning strategies needed to move beyond explanation to a classroom model of discussion, justification, argumentation and generalisation as described by Wood (Wood, 2002). Vale’s (2003) suggestions of mentoring or visiting a colleague’s classroom to observe and reflect on questioning strategies were not accepted by the teachers. Vale’s study indicated both the need for professional learning and specific strategies that may be helpful.
2.2.5.2 Teacher knowledge and skills to teach the measurement of area

Researchers have identified the potential for teachers’ confusion when teaching the measurement of rectangular area (D McPhail, 2004; Murphy, 2005). Simon and Blume (1992) found that prospective elementary teachers were able to use multiplication to calculate area, but the choice of operation was more likely to be from a knowledge of the Length x Breadth formula, then from an understanding of multiplication and the link to area. Twenty-six prospective teachers measured the length and width of their tables with a small cardboard rectangle, but were unable to answer why they multiplied the two numbers, giving answers such as “Cause that’s the way we’ve been taught”, and “it’s a mathematical law” (Simon & Blume, 1992, p. 13). The researchers commented that the rectangular area must be envisioned as an array of units. The prospective teachers appeared to concentrate on the linear aspects of the formula without attention to the array structure of rows or columns.

Baturo and Nason (1996) examined student teachers’ substantive knowledge of area measurement, and also looked at the students’ knowledge of mathematics in general and their disposition towards mathematics. They found that student teachers regarded mathematics as the application of many rules and formulas, with poor ability to calculate accurately, predict and check the answer, and little connectedness between the use of a formula to calculate area, and concrete examples of how the formula works.

Reinke (1997) implemented an area and perimeter assessment task with 76 preservice elementary teachers. The participants were asked to explain how to calculate the area and perimeter of a rectangular shape which had a semicircle removed from one end. No dimensions were given and the students were allowed to work as long as necessary. Only 11.8% of the preservice teachers gave a correct response for the perimeter problem and 52.7% were able to correctly answer the area problem. Although a number of strategies were described for the perimeter problem, approximately one-fourth of the preservice teachers ignored the semicircle and simply focused on the sides of the rectangle. Approximately 22% of the subjects applied an area calculation to the perimeter problem. Again, 21.1% of the preservice teachers ignored the semicircle and focused on finding the area of a rectangle.
Reinke also reported that three preservice teachers used degrees to find perimeter or area, and had obviously confused degree measure with linear measure.

When Berenson, Van Der Valk, Oldham, Bunesson, Moreira and Brockman (1997) investigated the content knowledge of 25 prospective teachers from five countries (USA, Netherlands, Ireland, Sweden and Portugal), the researchers found similar results across the countries. The prospective teachers prepared an area lesson for students of eleven to thirteen years of age and were interviewed about their lesson plans. Analysis of mathematical content knowledge was made by judging the lesson plans and interviews within a continuum of a concept-centred approach (the relationship of ideas) to a procedure-centred approach (knowledge of algorithms and formulae). The prospective teachers’ procedure-centred approaches were based on counting unit squares, using a formula and learning an area definition. In the final grouping of the prospective teachers the researchers found that some had a procedure-centred approach, but their aim of conceptual understanding would be compromised by a lack of suitable representations. Another group had limited conceptual knowledge and “some of them may not have learnt all aspects of the mathematics curriculum they will teach” (Berenson et al., 1997, p. 147).

2.2.6 The Role of Teacher Leadership in Curriculum Implementation

In 2006 the Australian Education Union (AEU) released a discussion paper Educational Leadership and Teaching for the twenty-first century: A desirable scenario. The AEU represents teachers and allied educators in schools, colleges and early childhood centres throughout Australia, and has promoted dialogue about the roles of teachers and leaders in the future of public education. The discussion paper argued that leadership within a school is crucial to high quality student learning. However, providing leadership should not be the exclusive domain of school executive members.

Crowther (1997) was invited to evaluate the outstanding and surprising success of a small number of Queensland schools in the Australian Commonwealth Government’s Special Program Schools Scheme. He was given the names of 15 educators who were identified as having led their schools to striking achievement, despite economic and social disadvantage. Thirteen of these educators were teachers.
and two were paraprofessionals. Crowther (1997, p. 13) found that these teacher leaders had five common traits:

- The participants were able to articulate clear views of what they regarded as a better world;
- The participants enjoyed remarkable levels of trust in their communities;
- The participants confronted structural barriers in their schools and communities with confidence;
- The participants seemed to communicate proactively, confidently, and assertively and to use their interactive talents in a number of ways to develop networks of support;
- All were characterised by enthusiasm and a sense of optimism.

Crowther’s (1997) argument that school leadership dialogue and research has focused on the positional authority of principals and school executive and has not recognised the leadership dimensions or potential of teacher leaders, especially within the wider school community, has been voiced by educators who see the need to alter the hierarchical nature of schools through empowering teachers (Boles & Troen, 1996; Fullan, 2001; Heller & Firestone, 1995; Smylie, 1995). Schools and teachers need to understand that teacher leaders do not have to undertake administrative or other formal roles. As the AEU (2006) and Moller and Katzenmeyer (1996) noted, teachers need the confidence to acknowledge themselves as leaders and their school needs the capacity to recognise and encourage teacher leadership.

Urbanski and Nickolaou (1997) described the difference between teacher leadership and administrative leadership as collegiality versus management, with the advice that leadership and management need not be mutually exclusive. Crowther, Kaagan, Feguson and Hann (2002) suggested that Parallel Leadership allowed teacher leaders and administrative leaders to work together in collective action to build school capacity. These researchers described the positive outcomes of teacher leadership as facilitating learning communities, nurturing a culture of success, and the provision of opportunities for every teacher to be a leader. Urbanski and Nickolaou (1997) found that teacher leaders had assumed leadership roles to ensure curriculum implementation and teacher and student learning. Teacher leaders were
mentoring new teachers, coaching each other, visiting colleagues’ classrooms and conducting peer evaluations. The AEU (2006) described teacher leadership as:

Leadership involves motivating others and creating a sense of common purpose, shared values and direction while also bringing out the best in others through consultation and involvement in decision-making (p. 37).

Ackerman and McKenzie (2006) described the leadership role played by teacher leaders in implementing classroom reforms and ensuring the provision of appropriate experiences for all students. Teacher leaders were described as having a strong philosophical and knowledge base, combined with confidence in their own expertise. These attributes were used to strive for improved learning conditions and innovation. Teacher leaders continually questioned their own practices and facilitated collaborative discussions with colleagues. They were prepared to leave the confines and isolation of their classrooms to share expertise and mentor colleagues. They also voiced their concerns about school policies, routines and curriculum.

However, Ackerman and McKenzie (2006) reported that teacher leadership involved risk taking and left some teachers vulnerable and open to critical questioning by disaffected colleagues. A teacher leader who criticised existing school practices was labelled as being “rude”. Teacher leaders needed to develop and use effective communication and negotiation skills to gently persuade other teachers to introduce reforms and to enable the full potential of their enthusiasm and skills, and the potential leadership of colleagues to be realised.

2.2.7 Implications for the Design and Implementation of Professional Learning

The literature surveyed in this review has indicated that effective and ongoing teacher professional learning was critical to the implementation of updated curriculum programs and the achievement of positive student learning outcomes. The results from repeated studies of prospective teachers or student teachers indicated that many primary teachers’ general understanding of mathematical concepts may be wanting (Ball, 1990; Baturo & Nason, 1996; Berenson et al., 1997; Simon & Blume, 1992).
Grant and Kline (2003) reported that a conceptual understanding of mathematical relationships, as opposed to a procedural knowledge of algorithms and formulae, would assist teachers to encourage and lead classroom investigations and discussions of mathematical applications. This view was shared by Ball (2001, p. 12) who questioned the impact of reforms to mathematics teaching in the United States, and found that skill practice and teacher explanation were the reality in many classrooms:

Most evidence suggests that mathematics teaching that focuses on understanding as well as skill, that involves children in significant mathematical reasoning, and that strives for high standards of progress and accomplishment, is not commonplace (p. 12).

Ball discussed the difficult role of teachers in successfully managing a classroom where a range of solutions and representations can be shared and evaluated, with significant learning for all.

The professional learning literature has provided practical and effective guidelines for planning and implementing successful programs (Louden, 1994; Lovitt et al., 1990), and these can be supplemented with strategies to ensure teacher collaboration, reflection and the provision of time or resources. However, as Rosenholtz and Bobis (Bobis, 2004; Rosenholtz, 1991) reported, in some cases teacher commitment to improving student learning outcomes was the final predictor of success. Programs which encouraged and harnessed teacher commitment appeared to encourage ongoing planning, implementation and use of personal professional judgement. Similarly, the emergence of teacher leaders who used informal networks to mentor colleagues and facilitate joint problem solving, ensured rich professional learning for teachers and guaranteed the success of curriculum innovations.

The literature has indicated that the author’s study has the potential to identify successful strategies and processes which assist in the development of teachers’ learning communities and to make a valuable contribution to current research.

2.3 **Designing the Methodology of the Research Program**

The aims of this research were to develop and evaluate a sequence of area measurement lessons in phase 1 (2.3.1), and then to implement the lessons in a trial of teacher professional learning models in phase 2 (2.3.2). The methodology selected was a quasi-experimental design for phase 1 and an action research approach for
phase 2. To evaluate the lesson sequence and the teacher professional learning models, a combination of both qualitative and quantitative research methods was used and these are described in 2.3.3.

2.3.1 Phase 1: Development and trial of area lessons

Cohen, Manion and Morrison (2000) described experimental design as the only research design which can establish causality, by eliminating other factors.

The essential feature of experimental research is that investigators manipulate the conditions which determine the events in which they are interested (p. 211).

Cohen et al. (2000) in listing the steps to designing a research methodology, described defining the research problem, formulating a hypothesis, selecting levels at which independent variables will be tested, deciding sample size in relation to the need to generalise results, choosing instruments and appropriate methods of analysis, and conducting the data collection and observations in meticulous detail before embarking upon the processes of analysing and reporting.

In the evaluation of the area lesson sequences, the sample size and sampling methods were determined by the need to generalise results, in addition to factors of practicality in the management of the research and the collection of data. The sample size of four classes for each of the two lesson sequences allowed the researcher to teach all of the lessons herself, ensuring uniformity of implementation. However, because it was not possible to match whole classes and students within such a teaching project, a quasi-experimental design was appropriate. As Kerlinger and Lee (2000) noted, a sample of children taken from a range of schools would have allowed a generalisation of the results across government schools in a metropolitan area.

Cohen et al. (2000) refer to the longitudinal study conducted by Mason, Mason and Quayle from 1984 to 1992 as an example of a quasi-experimental design. The experimental classes experienced explicit teaching of linguistic features. The experimental and control groups were not randomly allocated, because whole class groups were used. The students in the experimental classes demonstrated substantial gains in the public examinations. However, Cohen et al. question whether the
students’ success was due solely to their participation in the project, given the researchers’ difficulties with controlling all variables.

In phase 1 of this research, two sequences of area lessons were trialled and evaluated. The four classes in the control group were taught the *area syllabus lessons*. The independent variable was introduced in the form of the *area research lessons*, based on students’ achievement of a conceptual understanding of the measurement of area, using square informal units within identifiable array structures, and a focus on using rows and columns to count the number of tiles necessary to measure the area. To measure the effect of the area research lessons, all students completed an assessment task at the conclusion of the implementations. This experimental plan was a post-test only control group design (L. Cohen et al., 2000). Students in the control group were assessed again at the conclusion of a second sequence of lessons, to enable further evaluation of learning outcomes.

### 2.3.2 Phase 2: Trial of Teacher Professional Learning Models

In selecting appropriate research methodology for phase 2 of the study, the desired final outcomes were considered, including achieving effective teacher professional learning through establishing collaborative communities, building on teacher knowledge, encouraging professional dialogue and supporting teachers to trial and modify existing programs. Previous studies (Atweh & Heirdsfield, 2003; Kemmis & Wilkinson, 1998; Sax & Fisher, 2001) have indicated that these results were successfully integrated through a variety of action research models. In each case, the use of action research methodology assisted in achieving a positive change in teaching practices and enabled the collection and analysis of data that identified the crucial change processes.

Johnson (2005) compared the outcomes of teachers’ participation in action research projects with traditional teacher inservices and commented that many after-school workshops do not relate to teachers’ needs, interests, classroom situations or teaching styles. Additionally,

> Action research is also very economical in terms of the time and money invested and the returns garnered in the form of increased learning by students and improved practice by teachers (Johnson, 2005, p. 44).
Parsons and Brown (2002) listed the positive outcomes of action research in terms of professional learning as improving teachers’ problem solving skills and attitudes towards change together with increasing confidence and professional self-esteem. As Mertler (2006) noted:

Action research affords teachers opportunities to connect theory with practice, to become more reflective in their practice, and to become empowered risk takers. All of these opportunities enable the inservice classroom teacher to grow professionally and ultimately to realize growth in student understanding (p. 17).

The methodology used in this study of teacher professional learning sought to draw on the advantages of involving motivated teachers in action research. A simple explanation of action research is:

Action research involves learning in and through action and reflection, and it is conducted in a variety of contexts, including the social and caring sciences, education, organization and administration studies, and management (McNiff & Whitehead, 2002, p.15).

Kemmis and Wilkinson (1998, p. 21) described the process of action research as a spiral of self-reflective cycles, rather than a prescribed sequence of steps:

- planning a change
- acting and observing the process and consequences of the change
- reflecting on these processes and consequences, and then
- replanning, and so forth.

Kemmis and Wilkinson (1998) argued that the success of an action research project should be gauged in terms of the participants’ development and understandings, rather than their adherence to the execution of the steps of action research. This view has been repeated by other researchers, such as McNiff and Whitehead (2002, p. 52):

Practitioners need to see these models for what they are: guidelines for how we hope things will eventually fall out. To propose that action research models can be imposed on practice is to turn action research into a technology, an oppressive instrument which can potentially distort other people’s creative practice.

McNiff and Whitehead (2005) also explored the notion of teachers reviewing their beliefs and values and then testing current teaching practices and outcomes against identified personal principles. An action research cycle which may be appropriate in this kind of investigation involves:

- We review our current practice;
- identify an aspect that we want to improve;
• imagine a way forward
• try it out
• and take stock of what happens.
• We modify our plan in the light of what we have found, and continue with the ‘action’;
• evaluate the modified action;
• and reconsider the position in light of the evaluation (McNiff & Whitehead, 2005, p. 29).

The social aspect of action research, as proposed by Kemmis and Wilkinson (1998), emphasised the study and restructure of social practices through collaborative participation, rather than an individual’s isolated application focusing on self reflection. This view of action research indicates its ideal application to educational processes, assisting teachers to study and analyse their practices and to learn together within the physical and social environment which is specific to those teachers. The argument that action research involves a group with an agreed agenda, as opposed to an individual’s project, has been debated by researchers. Stringer (2004) argued that practitioner research described the observation, assessment, and reflective analysis which teachers may employ in their classrooms. Stringer’s definition of action research involves the study which a group of teachers may undertake to solve an educational problem, by:

seeking to engage teachers in reflective processes illuminating significant features of their classroom practice and enabling them to understand the experience and perspective of other participants in classroom contexts (Stringer, 2004, p. 5).

2.3.2.1 Action research projects

Atweh and Heirdsfield (2003), in planning an induction program for three early career female teachers in schools in isolated areas of Australia, implemented the project through an action research network. The project focused on the teachers’ need to make mathematics more inclusive for all students. The outcomes were described in terms of the teachers’ learning and professionalisation. The researchers’ choice of action research was guided by the opportunities this methodology provided for teachers to both generate and practise knowledge. Atweh and Heirdsfield cited the Crawford and Adler (1996) argument that an action research model allows participating teachers to use their prior experiences and collegial support to reflect upon and negotiate new understandings, as in a constructivist perspective.
In their use of action research to plan and then evaluate the three teachers’ learning, Atweh and Heirdsfield (2003) broadened a conventional view that action research is restricted to recurring cycles of planning, action and reflections, by including five additional characteristics identified by Kemmis and Wilkinson (1998). These were participatory (involving the participants in the process), collaborative (participants from within and also the outside working together), social (describing the practice within the social context), critical (presenting alternative ways of thinking and doing) and emancipatory (the potential for empowerment of all participants to improve practice and knowledge). The researchers analysed the data collected from meetings, emails and journals to identify the teachers’ learning. The results indicated that managing an inclusive curriculum and developing self-confidence in teaching mathematics were key outcomes for the participating teachers.

Dawe (1998) worked with a group of Year 4 teachers in five different schools across western Sydney. He invited the teachers to use action research to find out how learning was taking place in their classrooms. The teachers agreed on a central question - How could we as teachers facilitate children’s application of mathematical knowledge in problem solving? Data were collected through children’s journals, teacher observations, conversations with children and teachers’ reflective diaries. Dawe reported that at the follow-up meeting, the teachers had found it “extremely difficult” to maintain their diaries, but were very willing to talk about their experiences and case study reports. The teachers reported that their teaching strategies and planning had changed as a result of their observations, particularly those involving learning styles, cultural and gender differences.

Sax and Fisher (2001) investigated the effects of participation in an action learning project by a disparate group of 21 teachers in a school district in southeast California. The teachers met over a two-year period to study for a master’s program and devised their own projects based on an initial investigation of “What is a good student?” This simple question raised many issues, particularly when teachers compared their own ideas with their students’ responses. During the next phase of the program, all 21 teachers examined classroom management strategies and 19 of the teachers modified the reward and intrinsic motivation strategies in their
classrooms. Sax and Fisher monitored the teachers’ increased use of reflection and shared discussion to describe and solve the problematic situations in their classrooms and analysed the changes on three levels – personal, professional and political. At the personal level, the sense of community and relationships which were established assisted teachers to approach their work with increased confidence and deeper understanding. Professional knowledge was enhanced through the action research strategies of discussing, sharing and making decisions. At the political level, teachers were more aware of how to take an active role in school improvement issues. Engagement in the action research cycle encouraged teachers to make and implement informed decisions, based on their reflections and careful analysis of all factors. The project also resulted in the formation of an effective learning community, built on trust, critical discussions and shared understandings.

2.3.3 Research Methods to Evaluate the Lessons and the Professional Learning Models

As noted by Pirie (1998) and Kuhn (1970), researchers’ choices of methodology and methods are governed by the questions which underpin the research, the researcher’s aim and the situation. The two phases in this study presented research questions and aims which required proof and comparison through a number of methodological strategies and data collections.

The essential data in phase 1 of the study included teacher’s statements of planning strategies and beliefs of how to teach area in their mathematics programs, scores to indicate the growth of children’s learning, and the teachers’ and researcher’s professional judgements of the efficacy of individual lessons and the lesson sequence within each of the classroom settings. In phase 2 of the study, the data focused on children’s learning as indicated by scores on pre- and post-tests, and analyses of the interviews and meetings which described the experiences, comments and reflections from each teacher and the interactions and shared dialogue of teams of teachers. A survey of both qualitative and quantitative methods was made in preparation for selecting the methods and data which would address the needs of the specific study.
As Schoenfeld (2002) noted, mathematics education research is a relatively young discipline, having developed largely in the last third of the twentieth century, and resulting in the current situation of a wide variety of perspectives and methods. The ongoing debate about the conduct of mathematics education research has included the validity, generality and importance of results as well as the analysis of data and the interpretation of analyses. Taylor and Wallace (2007) and Pirie (1998a) commented that the current research in mathematics and science education is exciting and refreshingly new, but the questions of validity and even what constitutes research are still confusing and confronting.

2.3.3.1 Quantitative research methods

The use of quantitative research in education was a development of the quantitative research used in natural science (Carr & Kemmis, 1986) for two main reasons – because the purposes and methods were applicable to educational research, and because quantitative methods provided a statement of truth. This latter point was elaborated by Smith (1983) who found that quantitative research was quarantined from the researcher’s subjective viewpoints. Neutral scientific language would be used to explain statistical truth, without bias from the researcher’s value judgements. Popkewitz (1984) identified the advantages of quantitative research as being the emergence of a theory that was both universal and value-free. Other researchers identified the absence of the researcher’s viewpoint as being a weakness of quantitative methods, particularly in studies where participants’ experiences, interpretations and emotions contributed to the outcomes of the research.

2.3.3.2 Qualitative research methods

Denzin and Lincoln (2005, p. 3) defined qualitative research as:

a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible…qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them.

Qualitative research is a term for the investigative methodologies including ethnographic, naturalistic, anthropological, field, and participant observer research.
Qualitative research in education focuses on the variables which exist in natural settings and accepts the researcher’s viewpoint as being a crucial and valid to the research. As Carr and Kemmis noted, the researcher is “an insider to the research” (1986, p. 8). This approach enabled researchers to more accurately describe the variables of interpersonal, behavioural and cultural aspects of education through the collection of a wide variety of data including field notes, interviews and observations. Researchers found that the focus on site-based specific interactions and social behaviours resulted in a disadvantage of qualitative research - the difficulty of generalising findings from a unique situation, across many settings (Eisner & Peshkin, 1990).

2.3.3.3 Combining quantitative and qualitative research methods

Qualitative and quantitative research methods have developed from very different philosophical bases and some researchers may argue that the two methods are incompatible. However, as Glesne and Peshkin (1992) noted, different approaches to research allow examinations of different aspects of situations and phenomenon. The review of qualitative and quantitative paradigms indicated that a combination of both approaches was appropriate for the author’s study. A quantitative approach using predetermined and structured methods was suitable for displaying and comparing the children’s learning through an analysis of pre- and post- test scores. By comparison, qualitative methods were appropriate for identifying, describing and hypothesising the factors which influenced teacher learning and student learning within each classroom and school team.

2.4 Conclusion

The literature review described in this chapter commenced with a consideration of children’s early understandings that assist with a smooth transition to using and understanding the area formula (Length x Breadth) in later years. Successful mathematics programs were researched and analysed and the key factors identified, to assist with developing an appropriate sequence of classroom activities that were implemented as a major component of this research.
The review also considered the factors, strategies and organisation of effective professional learning models that guided the design of the teacher professional learning models used in the implementation of the area research lessons. The next chapter describes the author’s *First Steps* in the execution of the research questions and research program.
CHAPTER 3 - FIRST STEPS

This chapter describes the results of two pilot investigations and the researcher’s *first steps* as a mathematics education researcher. These pilot investigations guided the development of the research into teacher professional learning strategies and a sequence of lesson plans to teach early area concepts. The investigations also provided essential background and research experiences for the researcher. Both investigations were conducted within the researcher’s school district, where she held the position of District Mathematics Consultant.

The first investigation considered teachers’ knowledge and practices in teaching area. Interviews were conducted with seven Year 1 and Year 2 teachers and the resulting data were validated by comparison with teachers’ documented programs and student work samples. An analysis of all data enabled generalisations to be made of the teachers’ approach to planning and teaching area lessons.

The second investigation trialled a sequence of four area lessons in a Year 1 class. The lessons were taught by the researcher with supervisory assistance from the class teacher. To evaluate the effectiveness of the teaching program, the teacher and four children were interviewed immediately following each of the four lessons. The collected data also included student work samples and videotapes of two classroom lessons and two student interviews. The intended lesson outcomes were compared with the teacher’s comments, analysis of student work samples and observations of children’s responses to the interview tasks. This evaluation enabled the researcher to modify and extend the activities in preparation for the major study. The investigation indicated that the children could be encouraged to focus on the pattern of repeated tessellating units, and to use this structure in their measurement of rectangular area.

**First Steps Part 1: Teachers’ Knowledge and Practices in Teaching Area**

This investigation had the twofold purpose of establishing background information about teachers’ practices in planning and implementing area lessons as a component of the mathematics program, as well as providing an opportunity for the author to trial and develop research procedures.
3.1 Part 1: Design

3.1.1 Aims of the Investigation

The study aimed to provide relevant and current information about how Year 1 and Year 2 teachers planned and taught area lessons and activities. It was intended that this information would be used as the initial basis for planning teacher professional learning in the major study.

This study was the researcher’s first project involving the use of qualitative research methods. The study therefore had process aims including:

- To establish familiarity with planning and conducting interviews;
- To analyse interview and observational data leading to the identification of trends.

The findings include a brief analysis of the researcher’s learning experiences during the study, as identified in reflections of interviews with teachers, interpretations of teachers’ responses and the collation of data.

3.1.1.1 Research questions

The research questions explored the factors that contribute to teachers’ planning and implementation of area lessons, such as the frequency and design of the activities, teaching resources used, the teachers’ choices of materials used in implementation and the skills and understandings which teachers believed would assist children to measure area in later years.

- How is area being taught? (frequency of lessons, materials used)
- Is area integrated with other mathematics concepts?
- What resources do teachers use when planning area activities?
- What skills do teachers consider will assist children to measure area in later years?
- What do teachers believe are the understandings that are prerequisite to area concepts in later years?
3.1.2 Research Context

The study was based on the teaching of area concepts and skills to children in Year 1 and Year 2, in New South Wales Department of Education and Training schools. Three principals were approached to discuss the study and their permission was requested to interview teachers. Principals nominated teachers whom they considered to be confident and experienced professionals who would not feel threatened by an interview with a researcher.

The three schools were similar in size and within three kilometres of each other in a metropolitan area, but had quite different teaching programs due to student needs, staff skills and parental influences. Because of these differences in culture and clientele, the three schools formed an interesting and varied sample of urban, government primary schools.

Table 3.1 Features of the three schools represented in the study

<table>
<thead>
<tr>
<th>School</th>
<th>Number of students</th>
<th>LBOTE*</th>
<th>Teaching focus</th>
<th>Special features</th>
</tr>
</thead>
<tbody>
<tr>
<td>School 1</td>
<td>480</td>
<td>55%</td>
<td>Whole-grade groupings for mathematics and literacy sessions, daily.</td>
<td>School situated in a low socio-economic area.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>High student mobility.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>No dominant cultural groups.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Limited parent involvement in schooling.</td>
</tr>
<tr>
<td>School 2</td>
<td>440</td>
<td>94%</td>
<td>Emphasis on need to teach basic language functions to ensure understanding.</td>
<td>School situated in a low socio-economic area.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>High student mobility.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Main cultural groups Indian and Chinese.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Parents favour traditional teaching and textbooks.</td>
</tr>
<tr>
<td>School 3</td>
<td>470</td>
<td>68%</td>
<td>Emphasis on an intensive reading program; children learn strategies, rather than content.</td>
<td>School situated in a low to middle socio-economic area.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Stable student mobility.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Main cultural groups Vietnamese, Korean and Chinese.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Parents very supportive and request homework from Kindergarten to Year 6.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Extracurricular tuition popular.</td>
</tr>
</tbody>
</table>

*LBOTE = Language Background Other Than English
3.1.3 Sample

The group of seven participant teachers had between 5 years and 30 years teaching experience. All teachers were female and one teacher was in an executive position (Table 3.2). All teachers willingly and confidently shared their experiences, beliefs and mathematics programs with the researcher.

<table>
<thead>
<tr>
<th>Teacher (Pseudonym)</th>
<th>School</th>
<th>Position in the school</th>
<th>Class taught</th>
<th>Years of experience on this class</th>
<th>Total Years of teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helen</td>
<td>School 1</td>
<td>Teacher</td>
<td>Year 1</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Cathy</td>
<td>School 1</td>
<td>Teacher</td>
<td>Year 1</td>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>Jenny</td>
<td>School 2</td>
<td>Teacher</td>
<td>Year 1</td>
<td>5</td>
<td>18</td>
</tr>
<tr>
<td>Elizabeth</td>
<td>School 2</td>
<td>Executive Teacher</td>
<td>Year 1</td>
<td>9</td>
<td>30</td>
</tr>
<tr>
<td>Anne</td>
<td>School 1</td>
<td>Teacher</td>
<td>Year 2</td>
<td>6</td>
<td>25</td>
</tr>
<tr>
<td>Linda</td>
<td>School 3</td>
<td>Teacher</td>
<td>Year 2</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Prue</td>
<td>School 3</td>
<td>Teacher</td>
<td>Year 2</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

3.1.4 Ethical Issues

The researcher had already established credibility with participating principals and teachers as a professional colleague and focused on improving mathematics teaching and learning. The purpose of the study and use of audiotapes and transcript material were clearly explained to all participants.

Interviewees were assured that at all stages of their participation in the research, (preliminary meeting, interview, post-interview feedback, publication of results) the content of their interactions would remain confidential. All procedures adhered to the guidelines stated in the project’s University Ethics Approval.

3.2 Part 1: Method

3.2.1 Interviews

The seven interviews were conducted in the teachers’ classrooms, either before or after school or during the lunch break. Each interview took 20 to 25 minutes. The audiotapes of interviews were transcribed by the researcher and a copy of the relevant transcript was given to each teacher the following week for checking, feedback and
further comments. The main questions from the interview schedule (Appendix 1) were frequently followed by additional questions to encourage the interviewee to expand on answers.

Four of the teachers pointed to student work samples on display in the classroom as they described area activities. These were noted and documented, together with copies of teachers’ programs for the teaching of area (an outline of the mathematics program for Term 2 in School 2 and the area units for Terms 1 and 2 in Schools 1 and 3).

3.2.2 Analysis

Data analysis is the process of systematically searching and arranging the interview transcripts, field notes, and other materials that you accumulate to increase your own understanding of them and to enable you to present what you have discovered to others (Bogdan & Biklen, 1992, p.145).

The advice from Bogdan and Biklen to search and arrange collected data, provided a starting point for the analysis. The key themes and repeated responses in the transcriptions of interviews were identified. Teachers’ answers to several questions were collated in tables, to enable the different versions of the same basic theme to be studied (Table 3.3 and Table 3.4). Responses to other questions were tabulated, using the key words from teachers’ comments. The transcriptions were read again, and some categories were merged, to simplify collated data that would contribute to forming generalisations. Teachers’ responses to questions about children’s skills and understandings were reasonably similar, and this material was organised into a matrix, similar to those described by Miles and Huberman (1984) to allow a ready comparison of comments (Table 3.5 and Table 3.6).

The researcher triangulated the data for this study by collecting and comparing material from several sources. These included interviews with the teachers, copies of area lessons from the teachers’ programs and the researcher’s observations of children’s work samples on display in the classrooms or stored in children’s individual folders. A comparison was made between the lessons listed in the teachers’ programs and the individual teacher’s beliefs about appropriate tasks for the teaching of area. All teachers showed a consistency between the lessons they listed as
being implemented that year, their views of good teaching practice, documented programs and children’s work samples.

### 3.3 Part 1: Findings

The findings of the study are presented in two parts:

1. Teachers’ knowledge and practices in the teaching of area.
2. The researcher’s learning experiences as a beginning qualitative researcher.

#### 3.3.1 Teachers’ Knowledge and Practices in the Teaching of Area

Each statement is prefaced by the appropriate research question and a short description of the analysis of data that led to this generalisation.

##### 3.3.1.1 How is area being taught?

Teachers were asked how often they presented area lessons, and whether those were single lessons or part of a unit. Responses (Table 3.3) indicated that two or three area lessons were presented each term.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Frequency of area units</th>
<th>Duration of area units</th>
<th>Total lessons each term</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helen</td>
<td>three/term</td>
<td>one lesson</td>
<td>three</td>
</tr>
<tr>
<td>Cathy</td>
<td>one/term</td>
<td>two or three lessons</td>
<td>two/three</td>
</tr>
<tr>
<td>Jenny</td>
<td>two/term</td>
<td>one lesson</td>
<td>two</td>
</tr>
<tr>
<td>Elizabeth</td>
<td>two/term</td>
<td>one lesson</td>
<td>two</td>
</tr>
<tr>
<td>Anne</td>
<td>one/term</td>
<td>two or three lessons</td>
<td>two/three</td>
</tr>
<tr>
<td>Linda</td>
<td>one/term</td>
<td>two or three lessons</td>
<td>two/three</td>
</tr>
<tr>
<td>Prue</td>
<td>one/term</td>
<td>two or three lessons</td>
<td>two/three</td>
</tr>
</tbody>
</table>

Teachers’ responses to the opening question in the interview “Tell me about the area lessons that you have taught this year” were categorised and coded. The categories that related to the research question “How is area being taught?” were:

- The children work in practical, hands-on activities with concrete materials
- Children work in groups or pairs for area activities
- Teacher chooses a variety of items for covering objects
- Teacher chooses a variety of objects for children to cover.
All teachers mentioned covering everyday objects with a variety of units. The measured objects included odd and irregular shapes, desks, the reading corner, book covers, areas bounded by skipping ropes, and children’s own choices. The small items that teachers cited for measuring or covering the objects varied from coloured paper circles, books, counters, squares of paper, coins, paddle pop sticks, to “a variety of leaves and other paraphernalia”.

Five of the seven teachers specifically mentioned that children worked in groups or pairs during area activities, with comments such as:

- Some they do with a partner and some they do with a group.
- They worked in groups they didn’t do it individually.

All of the teachers described the physical aspects of handling materials during area lessons, including comments like:

- Informal, fun and problem solving, open-ended type questions so they can work it out for themselves.
- They’ve had to measure things around the house.

The comments about group work, items, objects and activities, indicate that the teachers implement syllabus activities using hands-on, small group activities that feature covering objects with a wide variety of items.

3.3.1.2 Is area integrated with other mathematics concepts?

Teachers were asked if they taught skills or knowledge from other mathematics substrands or Key Learning Areas during their area lessons. Five of the seven teachers made specific mention of other mathematics concepts that are integrated when teaching area. These included two dimensional space, length, number, position, and tessellations. In one particular lesson, the teacher suggested that the combination occurred through good luck:

- It worked out that 2D went in together with the area.

However, most teachers demonstrated an awareness that area is taught with other mathematical concepts:

- Obviously using number because you’re counting.
The comments indicate that most of the teachers saw area as a part of the whole mathematics curriculum and were able to make some connections between substrands and strands.

3.3.1.3 What resources do teachers use when planning area activities?

Teachers were asked how they planned their area lessons - from the syllabus, textbooks, or favourite activities from previous years. Teachers responded that they initially planned from the syllabus, and five of the seven teachers used additional ideas from textbooks. Several teachers shared their considerable files of photocopied activity sheets with the researcher, and all were happy to display the relevant pages from their class programs.

These results showed that teachers used *Mathematics K-6* (New South Wales Department of Education, 1989) as their primary source for planning, and most teachers used textbooks for supplementary activities.

3.3.1.4 What skills do teachers consider will assist children to measure area in later years?

Teachers were asked what skills and understandings about area they thought that children needed to develop during Year 1 or Year 2. Teachers’ responses were reasonably similar for area skills, but some teachers mentioned other skills as well (Table 3.4).

The specific skills that teachers thought children should acquire, to prepare them for future work with measuring area were covering or filling a given shape and counting the number of items used. Three teachers included shapes in the specific skills, such as distinguishing shapes, recognising shapes and drawing a shape. Some teachers also considered that estimation, risk-taking or problem solving were important mathematical skills used in area activities.
### Table 3.4  
*Skills that will assist children to measure area in later years*

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Area skills</th>
<th>Skills from other mathematics substrands</th>
<th>General mathematical skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helen</td>
<td>Put on blocks without overlapping</td>
<td>Count large or small items</td>
<td>None mentioned</td>
</tr>
<tr>
<td>Cathy</td>
<td>Cover a shape with materials and fill the space</td>
<td>Know the difference between regular and irregular shapes</td>
<td>None mentioned</td>
</tr>
<tr>
<td>Jenny</td>
<td>Completely cover a space without leaving gaps, Say whether something overlaps</td>
<td>Count items</td>
<td>None mentioned</td>
</tr>
<tr>
<td>Elizabeth</td>
<td>Fit shapes onto a shape to cover it</td>
<td>Recognition of shapes</td>
<td>Report on what they have done</td>
</tr>
<tr>
<td>Anne</td>
<td>Fill in a shape themselves</td>
<td>Draw a shape – triangle or rectangle, Counting - count by 1’s</td>
<td>Solve open-ended questions</td>
</tr>
<tr>
<td>Linda</td>
<td>Completely cover an object</td>
<td>Need to be able to count.</td>
<td>Learn to take risks; Estimation</td>
</tr>
<tr>
<td>Prue</td>
<td>Look at the whole area</td>
<td>Count by 1’s or 5’s or 10’s</td>
<td>Risk taking</td>
</tr>
</tbody>
</table>

### 3.3.1.5 What do teachers believe are the understandings that are prerequisite to area concepts in later years?

As in the question about skills, teachers’ responses were fairly similar (Table 3.5). Six of the seven teachers mentioned an understanding of covering in their response. The seventh teacher (Helen) was more concerned about the concept of measuring, particularly following her experiences with a Year 2 class the previous year. Helen had ascertained that her children had a poor concept of area and she was making a concerted effort to plan many activities involving counting, tessellating shapes and verbal reporting.

Four of the seven teachers emphasised the need to work with uniform items that fitted together (tessellated) when covering an area. These teachers stated that an understanding of tessellating items was necessary to the measurement of area. Four of the seven teachers expected children to be able to solve measuring problems by choosing items and experimenting. Most teachers thought that children should understand that an area needs to be covered in order for it to be measured. Four of the
seven teachers encouraged children to choose, experiment with, and make decisions on the suitability of a range of items to completely cover an area (Table 3.5).

<table>
<thead>
<tr>
<th>Teacher</th>
<th>The need to cover the whole shape</th>
<th>Choosing suitable items for covering</th>
<th>Practical understandings/problem solving</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helen</td>
<td>None mentioned</td>
<td>Some shapes are better than others.</td>
<td>Work with the resources they have available.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Shapes the same size are easier to work with.</td>
<td></td>
</tr>
<tr>
<td>Cathy</td>
<td>Take a regular or irregular shape and completely cover it.</td>
<td>None mentioned</td>
<td>None mentioned</td>
</tr>
<tr>
<td>Jenny</td>
<td>Completely covering without spaces.</td>
<td>Using something uniform that fits together.</td>
<td>By touching, looking, doing it, counting and test and retest.</td>
</tr>
<tr>
<td>Elizabeth</td>
<td>Cover the whole area.</td>
<td>What shapes cover an area better than other shapes.</td>
<td>None mentioned</td>
</tr>
<tr>
<td>Anne</td>
<td>Completely covered, within a closed shape.</td>
<td>None mentioned</td>
<td>Open-ended type questions so they can work it out for themselves.</td>
</tr>
<tr>
<td>Linda</td>
<td>Mostly cover the area.</td>
<td>Use shapes the correct size.</td>
<td>None mentioned</td>
</tr>
<tr>
<td>Prue</td>
<td>Look at the whole area as it is</td>
<td>Find something that will tessellate, Find something that won’t leave any gaps</td>
<td>Real-life situations like ordering playground sizes</td>
</tr>
</tbody>
</table>

3.3.1.6 Summary of research questions

The interviews with teachers indicated that area lessons were scheduled at least twice each term, with initial planning guided by *Mathematics K-6* (New South Wales Department of Education, 1989). Teachers described how area could be integrated with other mathematics topics with specific mention of counting and recognising the shape of objects used as informal units. The greatest emphasis within area activities was on covering a surface area, with no gaps and no overlaps. Four of the seven teachers noted that the items used to measure area should tessellate and that children need to identify the tessellations.
3.3.2 The Researcher’s Learning Experiences

The successful completion of this study exposed the researcher to some of the intricacies, rewards and disciplines of qualitative research, including the collection and documentation of data.

3.3.2.1 Aim: To establish familiarity with planning and conducting interviews

The interview process had presented several challenges. The advice to be mindful of “the closed question, the unclear or vague question and the complex question” (Maykut & Morehouse, 1994, p. 93) was excellent, especially when writing the interview schedule. During the interviews the researcher experienced difficulty in maintaining the interviewer’s role - constantly wanting to discuss thoughts when teachers raised interesting issues, instead of simply listening and asking further questions.

The interview and reporting-back process demonstrated the teachers’ enjoyment of discussing both their craft and their observations of students’ responses to learning experiences. Teachers also reacted very generously to the researcher’s request for assistance. The teachers were treated as “collaborators” in the research process (Maykut & Morehouse, 1994, p. 98) and they willingly provided copies of programs and work samples when the triangulation process was explained.

3.3.2.2 Aim: To analyse interview and observational data

The study provided a timely opportunity to collect and analyse data from a variety of sources. The researcher was able to identify the themes in the teachers’ responses to the interview questions, code the transcriptions and then ask university colleagues to verify her analysis. The importance of triangulation of data was very apparent and advice from Miles and Huberman (1984) was noted:

Perhaps triangulation is a state of mind. If you self-consciously set out to collect and double-check findings, using multiple sources and modes of evidence, the verification process will largely be built into the data-gathering process, and little more need be done than to report on one’s procedures (p. 235).
The researcher determined that during the next phase of the research she would be more aware of how to document, store and collate data including field notes, work samples and documented school programs. Adequate time would also be scheduled for reflecting and writing field notes immediately after interactions and classroom lessons with teachers and children.

3.4 Part 1: Implications for the Major Study

The findings from this study demonstrated homogeneity of planning and teaching processes amongst the group of seven interviewed teachers. However, only one of the seven teachers explained what she thought the children would be doing in area lessons in Years 3 and 4 (Cathy, Interview 4). This result suggested that the topic should be made an explicit question in further interview schedules, to enable an exploration of:

- Do teachers know what children will be doing in area in Years 3 and 4?
- How do teachers think their current program prepares children for later learning experiences in area?

This study enabled some preliminary generalisations to be made of how Year 1 and Year 2 teachers plan and teach area activities. Teachers described a wide variety of interesting resources and encouraged children to participate in active, enjoyable lessons. However, research indicates that the concept of “covering”, as described by most of the teachers, may not be the only understanding that children need to develop. Research suggests that if children are to make a successful transition to using area and perimeter formulas in Years 4, 5 and 6, they also need a sound basis for understanding that calculating rectangular area is based on the structure and pattern of repeated units (McPhail, 2003; Outhred, 1993; Reynolds & Wheatley, 1996).

First Steps Part 2: Trial of a Sequence of Area Lessons

The lesson trial investigated Year 1 children’s understanding of the measurement of area with informal, tessellating units and evaluated a sequence of four area lessons which were planned to assist the development of area concepts. The lessons were planned and then taught by the researcher in a Year 1 class with assistance
from the class teacher. Student learning outcomes and the effectiveness of the
teaching program were evaluated through analysis of data including interviews with
the teacher and children, student work samples, videotapes of the activities and the
researcher’s field notes.

3.5 Part 2: Design

3.5.1 Aims

The aims of the lesson trial were to explore Year 1 children’s understanding of
the measurement of rectangular area and to investigate whether this understanding
could be extended beyond the concept of covering a surface indiscriminately with
units, to the identification of the structured pattern of repeated, tessellating units. The
study also aimed to evaluate the implementation of a sequence of four classroom
lessons that would:

• assist children to develop the concepts of measuring area with informal units;
• be suitable to a range of student abilities and interest levels;
• be appropriate for whole-class management;
• utilise inexpensive and readily-obtainable materials.

3.5.1.1 Research questions

The trial of area lessons was planned and implemented to investigate two issues:

• Can Year 1 children demonstrate an understanding of the measurement of
  rectangular area by:
    o Covering the area of a shape with repeated, tessellating units;
    o Calculating the number of units in multiples (using rows or columns);
    o Identifying the pattern of covering units as a grid constructed from
      continuous lines.
• What are the major elements that need to be included in a sequence of practical
  and relevant area lessons for Year 1 children, so that children will be assisted to
  identify and use the pattern of repeated, tessellating units to measure
  rectangular area?
3.5.2 Research Context

The trial study was conducted in a Year 1 class in a small government primary school (180 children) in metropolitan Sydney. The school community was drawn from a range of socio-economic levels, from those living in public housing to the families in an expensive estate of large, freestanding homes. Members of the teaching staff had close professional bonds, were given whole-school responsibilities, and were encouraged to make autonomous decisions.

The study provided an initial trial of a teaching sequence of activities and an investigation of children’s area concepts. This preliminary program had the potential to inform the researcher’s major study that was planned to involve Year 1 and Year 2 children. The researcher chose to work with a Year 1 class to ensure that the activities to be implemented in the major study would be suitable for Year 1 children, but could also be extended for Year 2 children.

The teacher and the researcher had a good working relationship, established during a numeracy program that featured in-class support from the researcher in her role as the District Mathematics Consultant. The researcher knew that the teacher was keen to discuss effective teaching and learning strategies. The lesson times, classroom materials that the class teacher would provide and a suitable space for the student interviews were negotiated with the teacher.

In previous encounters with the children, the researcher had found them to be articulate, noisy workers, ready to think and to give answers, and very task-oriented. The children were not always easy to manage, but the researcher felt that their responses to the tasks as learning opportunities would be more easily observed and measured than some less responsive groups of children.

3.6 Part 2: Method

A sequence of four, 45-minute lessons was designed and documented. Each lesson included whole-class discussion and revision, small group or individual activities and whole-class reporting back and further discussion. Documented lesson plans included lists of outcomes, materials, organisation of groupings, the sequence of activities, and suggested questioning to initiate whole-class discussions. The
activities were designed to be appropriate for children aged from 5 to 8 years, and of mixed ability, skills and understandings. The lessons were taught by the researcher with supervisory assistance from the class teacher.

3.6.1 Design of the Four-Lesson Program

The program was designed to assist children to consolidate an understanding of the attribute of area and then to investigate the measurement of area by covering with tessellating units and using the emerging grid patterns to cover the area and also count the number of units used. The lessons were planned with an enquiry-based pedagogy which would assist children to construct meaning for themselves (Wilson & Rowland, 1993), and to identify and generalise relationships, encouraging the later development of formulae for measuring the area of simple shapes, rather than rote learning procedures with little understanding (Malloy, 1999).

The first two lessons in the program revised earlier classroom tasks to establish an understanding of the term “area” by identifying the surfaces of objects and defining the area of a shape enclosed by a closed line. Reeves et al. (1981) noted that a perception of area included the concepts of surfaces, boundaries and regions and therefore children needed to be aware of different applications and regions in their environment before commencing any comparisons of area. These lessons and further lessons were designed to encourage children to examine the entire area, rather than focusing on the boundary, as Nitabach and Lehrer (1996) reported.

The Lesson 2 activity was designed to highlight the spatial pattern and alignment of repeating, tessellating tiles that were manipulated to cover a given area. This emphasis on spatial alignment was suggested in earlier work by Outhred et al. (2003) and Owens and Outhred (2006). The purpose of the activity was to stress the measurement of area as essentially a space-filling operation with no gaps or overlaps (Grant & Kline, 2003; Nitabach & Lehrer, 1996).

In Lesson 3, the children were asked to make larger squares from their collected tiles, firstly in small groups and secondly as a whole class. These activities emphasised the array structure through a different perspective, of making larger and larger areas by adding whole rows or columns, rather than creating rows and columns
within a given area. The use of the array structure was the focus of the lesson, assisting to cover the area and also to count the units efficiently (Clements & Stephan, 2004). This activity was also designed to demonstrate the multiplicative relationship of the number of rows and the number in each row, as suggested by Nunes et al. (1994) and Zachoros (2006). The focus on arrays would assist with establishing Battista’s (2003) four mental processes of “forming and using mental models, spatial structuring, units-locating, and organizing-by-composites” (p. 122).

Lesson 4 emphasised the grid patterns of tiles arranged to measure area, and was designed to encourage children to experiment with organising a variety of grid patterns (McPhail, 2003; Outhred, 1993; Reynolds & Wheatley, 1996). The activities also revisited the skills of making and identifying arrays and using multiplicative relationships to count the total number of tiles, as described in Lesson 3. To encourage children to identify the pattern of tessellating squares and equilateral triangles, the tiles had a wide (0.5 cm) black line drawn on the perimeter, so that when placed side-by-side, the grid pattern of repeated tiles was clearly demonstrated (Appendix 14).

The tasks used teaching resources that were common to most classrooms, or materials that were economical to produce. The smallest tiles with the black borders were photocopied light card, but all other measuring tiles were cut from breakfast cereal boxes.

### 3.6.2 Sequence of Tasks

#### 3.6.2.1 Lesson 1

Children worked in groups of three or four to:

- Make a shape with a piece of braid 2 metres long and cover it with square tiles;
- Cover the surface of desks using square and equilateral triangle tiles with sides of 5 cm or 10 cm, commenting on gaps, overlaps, corners and edges.

#### 3.6.2.2 Lesson 2

Children worked with a partner to:
• Cover given triangle and square shapes with a variety of decorative patterns made by arranging and pasting small coloured paper squares and triangles;
• Discuss what was inside and what was outside the edges of the triangle and the square.

3.6.2.3 Lesson 3

Children worked in groups of four to:
• Cut out the decorated squares and triangles from the previous lesson and place these together to make a large square and large triangle.

The whole class worked together on the floor at the front of the classroom to:
• Make a large square, using decorated square tiles collected from all of the children. This activity commenced by making a 2x2 square, then a 3x3 square, then a 4x4 square, until a 10x10 square had been made;
• Calculate the number of tiles used as each large square was made, by counting rows then columns of tiles.

3.6.2.4 Lesson 4

Children worked with a partner to:
• Make squares by arranging 4 kebab sticks (cut to a length of 15 cm) as the perimeter and measure the area inside the square shape with 5 cm square tiles;
• Discuss how to count the number of tiles used;
• Use an additional 2 kebab sticks to make a rectangle. Measured the area in 5 cm square tiles;
• Use 12 square tiles to find which shapes had an area of 12 squares.

3.6.3 Data Collection

Data to evaluate the effectiveness of the teaching sequence and student learning were collected in three major forms:
• An interview with the class teacher, following each area lesson;
• An interview with a group of four children immediately following each area lesson;
Field notes written by the researcher.

In addition, lesson activities and the student interview were videotaped in Lessons 2 and 4, and work samples were collected from all children in Lesson 4.

3.6.3.1 Interviews with the classroom teacher

An interview with the classroom teacher was conducted after each lesson. The interview questions invited the teacher to state what she thought were the main concepts covered in the lesson, how the children responded to those concepts, what worked best, and why. The teacher was also asked if she would normally plan different tasks to address those concepts.

The interviews provided material for the final evaluation of the trial program, and also served as ongoing evaluation of how the lessons were stimulating the children’s thinking. The teacher made suggestions for improving content and classroom management and several of these strategies were incorporated as changes in the lesson plans as the trial progressed. Audiotapes of the interviews were transcribed by the researcher, and a copy given to the teacher for checking before the final data analysis commenced.

3.6.3.2 Interviews with a group of children

An interview was conducted with a group of four children immediately following each of the four lessons, to ascertain what the children’s perceptions of the activities, what they enjoyed, and whether they understood the main concepts presented in the lesson. Interviews were conducted in a small room adjacent to the classroom. The four children were selected by the teacher to represent a range of abilities across the class. The researcher chose to conduct the interviews with the group rather than as individuals, because of her time constraints, the necessity for minimal disruption to the class program, and to give the children an opportunity to discuss the activities in a group situation (Kress & Shoffner, 2007; Krueger & Casey, 2000).

A preliminary analysis of the transcription of lesson 1 interview was disappointing. The children did not appear to understand all of the questions:

Researcher: What did I ask you to find out?
Kevin: Ah......We.....Find out....... (Student interview, lesson 1).

and

Researcher: Did you find out anything that you didn’t know before?

Kevin: Mmmmmm. This is a tricky bit (Student interview, lesson 1).

Additionally, children sometimes gave contradictory answers. Children listed what they had done during the lesson (making shapes and covering shapes) but for an effective evaluation of the researcher’s lessons, she needed to probe the thinking behind their actions and reactions to tasks.

After reviewing relevant literature, and the aims of this study, it became apparent that the interview questions for lessons 2, 3 and 4 would require some modifications. The goal was to ascertain what the children understood as a result of the learning experiences, to identify the meanings that children were applying to mathematical ideas and to enable judgment of the appropriateness of activities in terms of stimulating learning (Lawson, 1997; Webb & Briars, 1990). Additionally, 6-year old children have special needs as interviewees, when planning questioning techniques and environment. Their cognitive and language skills are less well developed than those of most adults (Barker, 1990; Keats, 1993) and they may have limited experience of responding to questions in a group situation. The difficulty of the situation was acknowledged by Krueger and Casey (2000) with the advice: “Keep your sense of humour, show respect, and be ready to improve” (p. 180).

The interview questions for lessons 2 and 3 were revised to increase the interactions between the researcher and the children, and to give essential information about the children’s thinking processes (Hunting & Doig, 1995). Materials were provided to assist the children to remember what they had been doing and to demonstrate how they had found solutions. The questions for Interviews 2 and 3 commenced with:

What happened when …?
Can you tell me how …?
What would happen if …?

Students responded more positively to these questions, explaining how they had completed the tasks.
Researcher: What happened when you started to cover the area of the shapes?

Cathy: Well you could like. When you were holding it up on the blackboard I didn't know what to do, with that triangle and then put that on the top and put that on the bottom (pointing to the corners of the large triangle).

James: I just figured it out. I can see something in it, it's a circle! (excited, and pointing to a hexagon formed by small triangles within the large equilateral triangle).

Cathy: I know. That's what I worked out. So I put all of the edges like that and then I put this so it's like a diamond shape there and there and there. And I looked at it and I said, this one I might do a different colour.

Researcher: Oh. right.

Cathy: And then with the squares, I said I'll do the same again...

Researcher: Ahh, so that's why you've done the corners first.

Cathy: And then I said oh, that's a Red Cross! And then I looked at that and put these here (Student interview, lesson 2)

A set of questions was specifically planned for Lesson 4:

- What happened when I asked you how many squares would fit into the first stick square?
- Can you tell me an easy way to count the squares?
- What happened when we used six sticks? How could you count these squares?
- What was the best part of the lesson, and why?

3.6.3.3 Field notes

During the lesson trial, a routine was established of completing field notes the evening of each of the four lessons, and transcribing the interview tapes over the next three nights. These observations and thoughts were completed as quickly as possible because, as Minichiello, Aroni, Timewell, and Alexander (1995, p. 216) warn “most experienced field workers note that the quality of their notes diminishes with the passage of time”. This process gave invaluable time to read and reflect on the interviews and to make short notes, before the next lesson and interviews.

The field notes provided an essential component of the data, and an excellent opportunity for the researcher to review the progress of the lesson, to reconsider how and why the children interacted with their friends and the materials, and to identify the learning outcomes of the lesson. This kind of analysis was almost impossible to maintain during a lesson, with the immediate issues of how to manage a class of
children engaged in noisy and active enquiry. The progress of the trial was reviewed after each lesson, to ensure that relevant data was sought and collected.

3.6.3.4 Video recordings

Video recordings were made during lessons 2 and 4. In lesson 2, the camera was operated by a visiting university lecturer who was familiar with recording student work and interviews. This was an ideal situation, as she was also able to record interactions between the researcher and the children. During the student interview, demonstrations and explanations by children were recorded very clearly. In lesson 4, the researcher operated the video camera herself. This was difficult during the lesson, as she was also answering general questions, and reminding children to complete their tasks. During the student interview the camera was left in one position, sitting on the desk. When the researcher attempted to use the video to review the interview it was difficult to see the children’s hand gestures made when the children described how they had made and measured shapes.

3.6.4 Data Analysis

The data collected from the field notes and transcriptions of interviews for each of the four lessons were analysed by identifying references to the achievement of planned student outcomes. These were illustrated through children’s explanations and gestures, the interest that children demonstrated in the tasks and the teacher’s and researcher’s comments. Learning outcomes that the researcher had not predicted were also noted. The data were used to evaluate the effectiveness of the lessons and to comment on the children’s understanding of the attribute of area and the measurement of rectangular area with informal units. Documented discussions between the researcher and teacher about classroom management issues were used in final comments about the suitability of the planned activities for whole-class implementation.
3.7 Part 2: Findings

3.7.1 Analysis of the Lessons

The four lessons were analysed separately, because each lesson had a set of planned learning outcomes and each was modified slightly from the original plan, as the study progressed, to allow for the children’s interests, needs and existing knowledge. The results from each lesson have been presented, followed by summaries referring to the two research questions.

3.7.1.1 Lesson 1

In this lesson, groups of children experimented with making a shape from a 2 m length of braid. Children measured the area enclosed by the braid with square or triangular cardboard tiles with sides of 10 cm. Children then measured the area of the top of a desk. The children returned to a whole class discussion at the conclusion of measuring activities to compare their strategies and results.

Comments from the teacher, children and researcher about achieved learning outcomes were collated to reflect the planned outcomes (Table 3.6).

The comments in Table 3.6 suggest that the planned learning outcomes were achieved. The teacher made positive comments about the suitability of the tasks to a range of abilities and the interest levels stimulated by the tasks.

I think they really got into making the different shapes and working out how they could do - you know - by using their hands and making the points and they were quite fascinated by that (TI1).

However, the teacher and researcher agreed that some aspects of the classroom management could be improved, as this had culminated in:

The classroom teacher had strong words with one group of boys who were disgruntled and arguing with each other (FN1).

and

Researcher: I must have looked as though I was going into shellshock.

Teacher: I didn’t know whether to say anything or not. Richard was going on and on. I thought I was going to explode. I think, when you went to each shape, they couldn’t see. I think it would have been a lot better if somehow we reorganised the classroom and we had a lot more space and we could actually make it like a big circle (TI1).
Table 3.6  

Lesson 1 comments from children, teacher and researcher

<table>
<thead>
<tr>
<th>Planned Outcomes</th>
<th>Comments</th>
</tr>
</thead>
</table>
| **Make a shape with braid and cover the area** | Teacher: First of all, to make the shape and then inside and outside the shape and the surface area inside the shapes (TI1).  
Children: We ... made a shape. Like, we fixed it all at the edges (SI1).  
Researcher: Circles, hexagons and triangles were popular (FN1). |
| **Cover surfaces with tiles** | Teacher: Being able to cover that surface exactly, without overlapping and without having gaps (TI1).  
Children: We covered the shapes up, different shapes (SI1).  
Children: And then we had to cover a desk (SI1).  
Researcher: Covering doesn’t seem to be a problem (TI1). |
| **Discuss covering, overlaps, gaps** | Teacher: Which shapes would cover a surface well, so you wouldn’t have gaps (TI1).  
Children: So it’s not overcovering (SI1).  
Researcher: The class discussed whether it was possible to cover their shapes “exactly” (FN1). |

Comments have been referenced as FN (Field notes), SI (Student interview) or TI (Teacher interview), followed by the lesson number.

During the student group interview, the children demonstrated their ability to count the tiles used to cover area, in rows and columns by counting in multiples. The researcher noted the application of this skill and adjusted her plans for Lessons 2, 3 and 4 accordingly:

- Or you could count by fives because at my desk it had 25 (SI1).
- We went 5, 10, because there was five in each row (SI1).

The researcher observed the children’s ability to make a triangle from the braid using one hand from each of three children. The triangle was made into a hexagon by using each student’s remaining hand.

3.7.1.2 Lesson 2

In lesson 2, children were given templates of a square and an equilateral triangle, each with a side of 15 cm. Children covered the area of the templates by cutting and pasting tiles (sides of 5 cm) from coloured paper and also printing with square and triangular sponges dipped in paint. The children returned to the whole-class group to compare and discuss the patterns made by the tiles, and to count the number of tiles used.
Comments from the children, teacher and researcher’s field notes indicate that the outcomes of covering the shapes and using decorative techniques were achieved by all children. Further comments demonstrate that the outcomes for this lesson were achievable at higher and lower levels:

Out of the whole class, there were very few of them who hadn’t made a pretty good attempt to put the shapes together (TI2).

I think they did very well, I think they’ve all got it (TI2).

Several of the lower-achieving children who were not able to place the squares and triangles precisely, overlayed the shapes until the original shape was well and truly covered (FN2).

Lesson 2 ran smoothly, with no behavioural upsets and only positive comments about children’s responses to management. Children and the teacher commented that the activities had been enjoyable, especially the painting and pasting:

The pasting. I had fun doing this stuff and I looked at it and when I put it up (holding sheet to the light) I can see the triangle (SI2).

The painting was really nice and you got to get messy (SI2).

I thought it all went well. I think the activities were really interesting for them to do (TI2).

Perhaps the most interesting data from Lesson 2 are the video recordings of children’s explanations of how shapes were covered with coloured tiles and also how the total number of square tiles could be counted. When the four children in the interview were asked how to count the total number of square tiles in the given square, each student commenced this in a slightly different way, but three children counted groups of three and one student counted 2, 4, 6, 8, 9.

Researcher: What would be a good way of counting those shapes inside the square?

Mandy: Well, you go by ones, because there's ones around it. 1, 2, 3, then 4, 5, 6, then 7, 8, 9.

Researcher: Right. What do you see when I ask you to count them, James? How would you count them?

James: By threes. 3, 6, 9.

Researcher: I see. Show me which three you see. Show me how the three goes (James points to rows, then columns) (SI2)

The transcript and video recording of the student interview and video of the classroom activity demonstrated that, by the end of the lesson, all children were able
to cover the shapes, from edge to edge, and most were able to calculate the total of square tiles by skip counting.

3.7.1.3 Lesson 3

In this lesson, children worked in groups of four to cut out the squares and triangles that they had decorated in the previous lesson. Each group made a large square and large triangle from their collective shapes. The class returned to sit on the mat at the front of the classroom, and all of the squares were pooled to slowly make one very large (10x10) square. A 2x2 square was made first, and then increased to 3x3 by adding tiles to two sides of the previous square. Each time a larger square was formed, the class calculated the total number of tiles used by counting the rows and columns.

After analysing the teacher’s, children’s and researcher’s comments, it became apparent that this lesson, although very simple in terms of the materials used and classroom management, provided numerous teaching and learning opportunities. Many of the groups of four children made mistakes when assembling the larger squares from their decorated tiles. Children tended to put down more and more tiles, making large rectangles instead of large squares. The researcher and teacher moved around the classroom and questioned these groups of children. This preliminary experience appeared to be important when the children made the whole-class square that was done very quickly and easily.

At the end of the large square activity, groups of children reported on how they had made a large equilateral triangle from smaller equilateral triangles. This activity encouraged experimentation with placing the tiles. Some groups started building from one side of the triangle, and were surprised at how long it took to reach the apex:

Some of them started with a long line and started going upwards. And they didn’t realise how far they had to go up to (TI3).

This lesson was very successful from a whole-class aspect, as all groups were able to make a basic square and triangle, even if they contained only four or nine tiles. Children were very involved in this activity, and they were generally pleased
with their results. Several groups of children enjoyed replacing large triangular tiles in their desk shape with a pattern of nine smaller triangles. When asked if it work when they put additional tiles along one side of their triangle to make it larger, a student answered:

We did, and it made it a bit chunkier (SI3).

Both the teacher and the researcher were excited by the student and teacher learning that was facilitated, and the teacher was very enthusiastic about the activities:

I thought they were all really good activities. They really made them think. You could see the ones who were really catching on quickly (TI3).

Actually, I just loved this one on the floor, because it made sense to me. Sometimes I don’t know how to introduce that area, and I’ve done it with primary classes before, but this just seems so obvious (TI3).

It was good because it made clear the difference between oblong and square. Actually there are probably a few other measurement type activities that I can follow up with (TI3).

3.7.1.4 Lesson 4

In lesson 4, children used 15 cm kebab sticks to make and experiment with the area of shapes that could be measured with the 5 cm square cardboard tiles. The wide black border on the perimeter of the square tiles assisted children to identify the grid pattern formed by the square tiles, and also to draw the outlines of the covering tiles within the shape. Children also made a variety of shapes that had an area of 12 square tiles, and described how the shapes were formed, in terms of rows and columns.

Children’s explanations of how and why they measured area were very prominent, when examining the comments for lesson 4. Student confidence in measuring and counting was the most obvious outcome demonstrated on the video recording of this lesson. This confidence was repeated in the student work samples of the drawing of a 3 X 3 grid, to illustrate covering the square made from four kebab sticks, and was also noted by the class teacher in her comments.

In response to the researcher’s question: Can you show me what you mean by the patterns? One of the interviewed children answered:
Like we can make the rectangles and the squares. We could make like with 12 squares we can make an oblong. And I put, I put 4 here and 4 down there in the middle row and another 4 on the other line (SI4).

The student confidently reconstructed an imaginary rectangle, by drawing with her finger on the desk. She was very sure about the configuration of rows and columns. Each of the four children in the Lesson 4 student interview was able to explain and demonstrate how to quickly count the total number of tiles used to measure an area, by counting in multiples of three or four.

3.7.2 Research Question 1

Can Year 1 children demonstrate an understanding of the measurement of rectangular area?

3.7.2.1 Covering the area of a shape with repeated, tessellating units

Children consistently demonstrated that they could cover surface areas with square and triangular tiles in a methodical manner, and comment on the no gaps, no overlaps rule that their teacher had taught them.

3.7.2.2 Calculating the number of units in multiples (using rows and columns)

Children identified rows or columns of tiles that were used to cover an area and counted in multiples of two, three, four or five, although multiples of two and five were the most familiar to all children. Children demonstrated this skill in the small group interviews and also in whole-class activities.

3.7.2.3 Identifying the pattern of covering units as a grid constructed from continuous lines

This skill was demonstrated in the last lesson, when children recorded the grid pattern of square tiles used to measure the area of rectangular shapes made from 15 cm kebab sticks. Children in the interview group also used continuous lines when gesturing with their fingers to explain how their tiles were arranged within the square and rectangle.
3.7.3 Research Question 2

What are the major elements that need to be included in a sequence of practical and relevant area lessons for Year 1 children, so that children will be assisted to identify and use the pattern of repeated, tessellating units to measure rectangular area?

3.7.3.1 Assisting children to develop the concepts of measuring area with informal units

The first lesson that involved making a shape and identifying the area of the shape, allowed children to revise and demonstrate an understanding of the attribute of area. All children were able to identify the grid pattern of repeated square units used to measure the area of a larger square in lesson 4. However, the teaching program focused strongly on the use of square and equilateral triangle tiles as informal units for measuring area. The children were not given an opportunity to trial and comment on the suitability of other shapes, such as circles, when used as informal units to measure area. The teacher and researcher discussed that this was a weakness in the lessons that should be addressed in the major study.

3.7.3.2 Designing activities for a range of student abilities and interest levels

In lessons 1, 2, and 3 the teacher or researcher made positive comments about the suitability of the activities. Perhaps the most successful lesson in terms of encouraging student explanations and participation on a number of ability levels was lesson 3, when the class made a 10 x10 square from their collective decorated tiles. This was possibly the simplest idea for a lesson, when compared with other tasks that the children completed during the four weeks.

3.7.3.3 Planning activities to assist whole-class management

This issue drew negative comments in lesson 1, positive comments in lesson 2, and no comments in lessons 3 and 4. Lessons 2, 3, and 4 were modified slightly, following suggestions from the class teacher and the researcher’s own reflections on the noisy and frustrating lesson 1. The modifications were successful and were noted for use in the major study.
3.7.3.4 Utilising inexpensive and readily-obtainable materials

The readily available and inexpensive materials were successful in terms of having sufficient for all children to participate at the same time, and being simple to replace through loss or damage. The bags of triangular and square cardboard tiles were easily distributed and collected, and the children appreciated having tiles of different sizes for measuring. Using the braid to make shapes (Lesson 1) was a novel exercise for the children and teacher.

I thought that was a really good idea doing it with the braid. The fact that you could sort of play around with that braid, and make it into such unusual shapes (T1).

3.8 Part 2: Implications for the Major Study

The lesson 2 video recording made with the assistance of a university colleague was very useful in providing a broad perspective on the problem solving strategies used by individual children as well as small group interactions. The researcher identified the order used by most children when they were pasting the small squares, and gauged general working behaviour and small group interactions. Such close observation was almost impossible while teaching was in progress. In lesson 4 the researcher operated the video camera herself and found that she was unable to simultaneously teach or interview children and coordinate the video camera. The resulting videos were of limited value as the recording did not focus on examples of significant student learning. The trial study demonstrated the rich evidence that can be captured by a video recording, but the researcher was not able to continue to use this method of data collection in the major study, for reasons of practicality.

The student group interviews were effective in providing continuing assessment of student learning. However, this assessment information was confined to just four children. The teacher and researcher discussed the possible advantages of being able to assess all children in a practical way, to determine current knowledge before the lessons commence, to identify children who had difficulty with area concepts, and to provide an indication of learning outcomes achieved during the program. Accordingly, the researcher introduced a whole-class assessment task in the major study.
The children in this trial responded positively to the four lessons and at the end of the program demonstrated an advanced level of understanding of how to measure the area of squares and rectangles with informal units. The class teacher discussed each lesson enthusiastically, and was able to expand on two of the lessons by devising further activities. During the trial, several aspects of the data collection were refined, including more descriptive field notes, and the revision of interview questions for the children. The revised interview questions were used in the second phase of the major study, as suggested models for participating teachers who were released from classroom teaching to interview individual children.

3.9 Conclusion

This chapter has described the researcher’s first steps in the field of mathematics education research. The skills and disciplines of defining and investigating an issue were successfully established. Basic techniques of collecting, documenting and analysing data were practised and reviewed. Principals and teachers welcomed the project as being relevant and useful to the teaching of mathematics in their schools. Teachers readily accepted the invitation to discuss their mathematics planning and teaching, and children responded enthusiastically to the trial lessons.

The researcher had sought an indication of Year 1 and Year 2 children’s understanding of the attribute of area and the measurement of rectangular area using informal units. The results of these early trials demonstrated that a sequence of whole-class lessons could encourage children to identify the pattern of repeated tessellating units and to use this pattern in counting the total number of units used to measure rectangular area. The next chapter of this thesis describes how the effectiveness of the researcher’s sequence of area lessons was compared with area lessons planned from *Mathematics K-6*, the current syllabus, through implementation of the lessons and evaluation of student learning outcomes in Year 1 and Year 2 classes in four schools.
CHAPTER 4 – DESIGN OF THE STUDY

The previous chapter described the researcher’s first steps as a mathematics education researcher, firstly in a study of teachers’ current practices in the teaching of area, and secondly in the design, trial and evaluation of area lessons implemented in Year 1 and Year 2 classes. This chapter describes the overall design of the major study as two phases of research. In the following chapters of this thesis, each phase is treated as a separate study, and comments about the outcomes and implications of the two phases are combined in the concluding chapter. The overarching aims of the research were to identify a teaching sequence of activities that would assist children in Year 1 and Year 2 of primary school to develop concepts of the measurement of rectangular area and then to evaluate models of teacher professional learning that would assist teachers to implement similar lessons in their classrooms.

In this chapter, each of the two phases is briefly described, together with the aims, research questions and collection of data that guided the implementation of the research. Phase 1 describes the development, trial and modification of the area lesson sequences, and phase 2 explains how school-based teams of teachers implemented the modified lessons in their own classrooms, supported by varying levels of professional assistance from the researcher.

4.1 Phase 1: Implementation of a Sequence of Area Lessons

The researcher developed a teaching sequence of four area lessons. The lessons were designed to address the use of iterable units, or repeated congruent shapes in measuring area, and the recognition and use of grid patterns and composite units when covering and calculating rectangular area. The activities were based on the findings of Outhred (1996), Owens and Outhred (1996) Reynolds and Wheatley (1996) and Battista (2003). The aim of the lessons was to lead children to understand and use the underpinning structure of repeated and composite units, rather than rote learning basic procedures or formulae (Lehrer et al., 2003). Research has demonstrated that an emphasis on understanding the structure of units of measure is crucial to the concept of measuring length, area and volume (Outhred & McPhail,
2000; Outhred et al., 2003). In this report, the four lessons thus developed are described as the area research lessons.

Teachers in the trial schools were using the mandatory curriculum document *Mathematics K-6* (New South Wales Department of Education, 1989) for planning mathematics programs. The syllabus approach towards teaching area focused on classroom discussions of areas such as the reading area or canteen area, and practical activities emphasising covering surfaces and irregular shapes with a variety of objects such as blocks, books and circular counters. These activities preceded the introduction of formal measuring units (square centimetre and square metre) and the use of the area formula to calculate the area of a rectangle in the middle and upper primary years. The researcher designed a sequence of four lessons using the suggested focus and activities in *Mathematics K-6* (1989) and titled the sequence the area syllabus lessons.

The two sets of lessons were evaluated and compared by teaching the area research lessons in a Year 1 and a Year 2 class in each of two schools (School 1 and School 2). At the same time, the researcher taught the area syllabus lessons in a Year 1 and a Year 2 class in two other schools (School 3 and School 4). At the conclusion of the area syllabus lessons in Schools 3 and 4 the researcher taught the area research lessons, so that a total of eight lessons were implemented in these four classrooms. Student learning outcomes in each of the programs were monitored to assess the children’s development and use of increasingly sophisticated strategies to describe, measure and compare rectangular area. The researcher taught all of the lessons, with classroom teachers providing support during the small-group activities.

<table>
<thead>
<tr>
<th>Table 4.1</th>
<th>Implementation of lesson sequences</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Schools 1 and 2</strong></td>
<td><strong>Schools 3 and 4</strong></td>
</tr>
<tr>
<td>Area research lessons</td>
<td>Area syllabus lessons</td>
</tr>
<tr>
<td></td>
<td>Area research lessons</td>
</tr>
</tbody>
</table>

At the conclusion of the lessons trial and analysis of all data, the researcher developed a third sequence of lessons titled the area refined lessons. This sequence was comprised largely of the area research lessons, with the addition of the initial activities from the area syllabus lessons which focused on establishing the attribute of
area. The area refined lessons were implemented during the teacher professional learning research in phase 2 of this study.

4.1.1 Aim

The aim of phase 1 was to implement the sequence of area research lessons designed to assist Year 1 and Year 2 children to identify, describe, compare and measure rectangular area using repeated, informal units.

4.1.1.1

The research question outlined an evaluation of the learning activities in the area research lessons.

- How effective were the area research lessons compared to the area syllabus lessons in developing:
  - an understanding of the attribute of area; and
  - an understanding of a grid pattern or array of repeated, informal units to measure area?

4.1.2 Outline of Data Collection

Data used to evaluate and compare the student learning outcomes from the area research lessons and the area syllabus lessons were collected from three sources:

- An individual student assessment activity;
- Interviews with individual teachers immediately following each lesson;
- The researcher’s field notes completed after each lesson.

4.1.2.1 Student activity

Student achievement was measured with a pre-test and post-test activity administered in each classroom by the researcher. The children’s understanding was determined by using an instrument and assessment scale developed by Outhred (1993), based on an area tiling activity. The task was administered to whole class groups, took approximately fifteen minutes to complete (including introduction and
explanation) and provided the researcher and teachers with an indication of individual children’s skills.

The children were asked to draw tiles to cover the area of a given rectangle on an A4 photocopied sheet. The purpose of the tiling activity was to gauge children's understanding and recognition of an array by analysing the strategies they used to draw the covering tiles. Children in Schools 1 and 2 were given the area-tiling task at the completion of their four area research lessons. Children in Schools 3 and 4 were given the area tiling task at the completion of the syllabus lessons, and again at the completion of the area research lessons.

4.1.2.2 Interviews with teachers

Individual teachers were interviewed before the implementation commenced, after each lesson, and at the conclusion of the implementation, to discuss the teaching of area and their perceptions of what the children had learnt during the lessons. The comments made in interviews were compared with the researcher’s field notes. In the final interview, teachers were also asked about the effects of their participation in the study and the consequences for their teaching of mathematics in the future.

4.1.2.3 Field notes

The researcher wrote notes as soon as possible after each lesson to record the children’s reactions to tasks, mathematical terminology used, an evaluation of the appropriateness of tasks, possible modifications, children’s learning outcomes and the teacher’s response to the lesson.

4.2 Phase 2: Implementation of Teacher Professional Learning Models

The second phase of this research concerned the implementation and evaluation of three teacher professional learning models. Seventeen teachers from seven schools attended an initial training session and implemented the area refined lessons in their classrooms. The teachers chose to be supported by the researcher in one of three levels of consultancy support.
• The low support teachers had no contact with the researcher during the implementation of six area lessons.
• The medium support teachers had fortnightly team meetings with the researcher.
• The high support teachers had fortnightly team meetings with the researcher and release from teaching following each area lesson to conduct focused interviews with selected children.

The phase 2 research trialled formats of teacher support investigated by previous research projects. Dawe (1998) encouraged teachers to reflect on and diarise their learning. This strategy, when combined with the four principles of classroom reform outlined by Stephens, Lovitt, Clarke, and Romberg (1989), suggested a number of approaches that could be trialled by varying the levels of consultancy support. The implementation models enabled an investigation of outcomes that result when teachers are encouraged to make choices, investigate and trial (I. Robinson, 1989). The planned emphasis on team meetings in two of the models presented an opportunity to identify professional dialogue within the teams (Wenger, 1998) and to compare teachers’ experiences with those in the low support schools.

4.2.1 Aim

The aim of the phase 2 research was to implement, evaluate and compare three models of teacher professional learning that assisted teachers to change their current practice in the teaching of area.

4.2.1.2 Research questions

The research questions focused on an evaluation of the effectiveness of the professional learning models that assisted Year 1 and Year 2 teachers to change their current practices in the teaching of area.

• How do teachers currently plan and implement learning experiences in area?
• Which of the three models implemented was the most successful for the professional learning of the teachers who participated in this study?
• What factors were important to the success of the professional learning models?
4.2.2 **Outline of Data Collection**

The project aimed to investigate the relationship between successful teacher professional learning and the provision of varying levels of professional support, including teacher release time to conduct student interviews. The teachers’ learning and satisfaction with their participation in the research were correlated with the results of children’s pre-test and post-test assessment tasks. Data were collected from transcriptions of individual teacher interviews and group meetings, as well as the pre-test and post-test assessment task administered to all children.

4.2.2.1 Interviews with teachers

- The 17 teachers were interviewed before and after the lesson implementation. Interviews were audiotaped and transcribed.
- Teachers in the high support and medium support schools met with the researcher in their school teams each fortnight during the study to discuss the progress of the lessons. The group discussions were audiotaped and transcribed.
- The teachers were interviewed again, nine months after the study, to gauge the long-term effects of their participation in the study. The interviews were audiotaped and transcribed.

4.2.2.2 Student pre-test and post-test assessment task.

The researcher administered the pre-test and post-test assessment task that was developed and trialled in phase 1 of the research, to all children in the 17 participating classes. Children’s pre-test and post-test responses were scored using the six-point scale. The class results were discussed with each teacher during the final interview with the researcher.

4.3 **Conclusion**

This chapter has briefly described the context and breadth of the two phases of study that constitute this research. This outline contributes an overview to the major research and indicates how the two phases are linked chronologically and are also related by content. In the next chapter, the methodology and implementation of the phase 1 research is explained in greater detail.
CHAPTER 5 - IMPLEMENTATION OF A SEQUENCE OF AREA LESSONS

The previous chapter described the two components of the research study. In phase 1 of the study, the researcher designed, implemented and evaluated two sequences of area lessons in Year 1 and Year 2 classrooms. In phase 2 of the study, 17 teachers volunteered to participate in a professional learning project that involved an initial training meeting and the provision of varying levels of consultancy support to implement the refined area lessons in their classrooms.

In this chapter the design and implementation of phase 1 are described in detail. The following chapter discusses the results of the lesson implementations, modifications made to the sequence of area research lessons, and implications for further teaching.

5.1 Design

5.1.1 Research Context

The study was implemented in four government primary schools in a metropolitan area. The schools were selected to represent a diversity of size, socio-economic status of the school community and current mathematics teaching programs, and were different from the four schools in the initial trial. The school enrolments varied from 200 to 700, and the socioeconomic status ranged from low to middle. The selected schools represented a range of teaching staffs, from young and mobile at School 1, to older, established and very settled at School 2. In Schools 1 and 3, teachers used the syllabus and a variety of teacher texts to plan mathematical activities and were accustomed to collaboration and grade-based programming. In Schools 2 and 4, mathematics lessons were taught from a student textbook purchased by parents, teachers had few discussions about teaching mathematics and a greater emphasis was based on procedures and skills. The diversity of children’s and teachers’ familiarity with mathematics programs was sought to ensure that the area research lessons were evaluated against a wide range of pre-existing experiences and attitudes towards teaching and learning mathematics.
The researcher was familiar with the schools’ profiles and targeted programs and was known to the principals and teachers through her role as the District Mathematics Consultant. The researcher met with the principal at each school to discuss the parameters and intended outcomes of the research, commitment requested of class teachers and the parent permission forms to be completed for all participating children. The principals discussed the implementation with their teachers and asked for one teacher from Year 1 and one teacher from Year 2 to volunteer to participate. Individual teachers met with the researcher and were given full details of the research and syllabus lessons, timelines and interviews, before agreeing to proceed and signing permission forms. In each of the four schools, all children in a Year 1 class and a Year 2 class participated in the project.

### 5.2 Method

#### 5.2.1 Designing the Lesson Sequences

The forty-five minute lessons were designed to a basic structure. The lessons commenced with a whole-class discussion and revision and then continued on to small group or individual activities. Each lesson was concluded with whole-class reporting back and further discussion of strategies used to complete the tasks. Documented lesson plans included lists of outcomes, materials, the organisation of children into groups, the sequence of activities and suggestions for teacher questioning. The activities were designed for children in both Year 1 and Year 2. The tasks were constructed to be appropriate for children aged from five to eight years, with a range of experiences, skills and understandings.

##### 5.2.1.1 Area syllabus lesson sequence

The design of the syllabus lessons focused on addressing the Statement of Principles outlined at the beginning of *Mathematics K-6* (New South Wales Department of Education, 1989). These included recommendations that children be given opportunities to interact with their environment, to investigate, to use appropriate mathematical language and to build on previous knowledge, experiences and achievement.
The sequence of syllabus lessons (Table 5.2) was planned from the Main Ideas in the syllabus Teaching and Learning Units. The Main Ideas for Area Units 2, 3, 4 and 5 described the concepts and understandings that would typically be studied by children in Years 1 and 2.

Area 2: Area is a measure of the amount of surface
Area 3: Areas can be compared
Area 4: Informal units can be used to measure area
Area 5: Three or more areas can be ordered


The lesson sequence commenced with a revision of the concept of area as the amount of surface within a closed line (Area 2 Main Idea). The two tasks selected from the suggested activities in the syllabus involved making closed shapes with a length of braid and identifying shapes within a drawn pattern of crossing lines. In the second lesson, the strategy of superimposing shapes to identify the larger area was introduced (Area 3 Main Idea). Children superimposed pairs of cardboard shapes, and also cut and pasted a triangle and a circle to compare the areas. The following lesson progressed to a discussion of the conservation of area, and children cut one of two identical cards into a jigsaw pattern. Children also experimented with covering a given area with tiles cut in a range of shapes including circles, hearts, triangles and squares (Area 4 Main Idea). The lessons concluded with an investigation of areas made on a geoboard, by counting and recording the number of square units enclosed in each shape made with elastic bands (Area 5 Main Idea).

5.2.1.2 Area research lesson sequence

The program was designed to assist children revise area concepts and investigate the measurement of rectangular area through covering a shape with a tessellating pattern of congruent units and then counting those units (Table 5.1). The lessons focused on establishing the attribute of area, the skills and language to compare areas directly, and the use of informal units to measure and compare areas indirectly. The lessons encouraged children to use the structure of repeated units, array patterns and skip counting to calculate the total number of informal units required to measure an area (Outhred, 1993; Owens & Outhred, 1996; Reynolds & Wheatley, 1996). The
lessons also aimed to assist children to develop a concept of the measurement of area that would transfer to understanding and using the formula for the calculation of rectangular area in later primary classes.

The lessons were based on research in teaching and learning mathematics that suggests that children who are actively engaged in meaningful tasks will learn more effectively and will be able to apply mathematical concepts in a range of contexts (E. Robinson, Robinson, & Maceli, 2000). Selection of the tasks and teaching strategies focused on principles defined by large-scale research projects such as those funded by the National Science Foundation (E. Robinson et al., 2000) and the Mathematics Curriculum and Teaching Project (Lovitt & Clarke, 1988). These principles included identifying the mathematical foci of the activities, determining whether the mathematical content was familiar and ensuring that the tasks were suitable for less-capable and more-capable children, and that children could find several different legitimate solutions to the problems. The tasks were designed with an emphasis on engaging and encouraging children to contribute answers, to explain their thinking and to reflect on the success of chosen strategies (McDonough, 2001; Montgomery & Cheeseman, 2000).

Table 5.1  Comparison of two sequences of lessons

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Area syllabus lessons</th>
<th>Area research lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make shapes</td>
<td>Experiment with making lines and shapes with a length of braid.</td>
<td>Make shapes</td>
</tr>
<tr>
<td></td>
<td>Make a shape to be measured with 10 cm square tiles.</td>
<td>Experiment with making lines and shapes with a length of braid.</td>
</tr>
<tr>
<td></td>
<td>Draw lines that cross, find the shapes made by the crossing lines, and colour these.</td>
<td>Make an area that can be measured with 10 cm square tiles.</td>
</tr>
<tr>
<td>Lesson 1</td>
<td>Superimpose shapes</td>
<td>Cover an area with patterns</td>
</tr>
<tr>
<td></td>
<td>Superimpose pairs of shapes to find which has the larger area.</td>
<td>Cover a 15 cm square with 5cm square tiles and an equilateral triangle with triangular tiles.</td>
</tr>
<tr>
<td></td>
<td>Superimpose, cut and paste two paper shapes to find which has the larger area.</td>
<td>Arrange into colour patterns, then paste.</td>
</tr>
<tr>
<td>Lesson 3</td>
<td>Area syllabus lessons</td>
<td>Area research lessons</td>
</tr>
<tr>
<td>----------</td>
<td>-----------------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td><strong>Make a jigsaw</strong></td>
<td>• Cut a jigsaw from one of two identical pieces of card. Discuss which card now has the larger area.</td>
<td><strong>Make a large square and triangle</strong></td>
</tr>
<tr>
<td><strong>Cover a shape with tiles</strong></td>
<td>• Choose tiles to cover a large shape. Report back on which shape tiles were appropriate, and how many were needed.</td>
<td>• In small groups, cut out squares and triangles from lesson 2. Use these with a bag of tiles to make the biggest square and triangle possible (13 squares and 13 triangles).</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Whole class make a large square shape from the coloured squares. Commence with one tile and stop at 100. Count rows and columns, in multiples.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Make a large triangle in small groups. Combine to make one large class triangle.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lesson 4</th>
<th>Make shapes of a given area</th>
<th>Make and measure area with tiles</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Make shapes of a given area</strong></td>
<td>• Pairs of children make shapes on a geoboard, each with an area of eight squares. Record the shapes on dot-grid paper.</td>
<td><strong>Make and measure area with tiles</strong></td>
</tr>
<tr>
<td></td>
<td>• Draw other shapes on the dot-grid paper that have an area of eight squares. Try to fill the whole grid paper.</td>
<td>• Make a square with 15 cm kebab sticks, cover the area with 5 cm tiles. Record the shape made and arrangement of tiles.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Use more sticks to make other shapes that can be measured with the tiles. Predict and count the number of tiles.</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Investigate and record arrays</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Make and record arrays with twelve square tiles. Count the number of tiles, using multiples if possible.</td>
</tr>
</tbody>
</table>

5.2.3 *Implementation of lessons*

The researcher was given access to the classes at the same time each week for four weeks, to teach the lesson sequences. Where possible, the lessons were implemented immediately before morning tea, lunch or afternoon dismissal time, to provide an opportunity for the teacher and researcher to discuss children’s responses to the tasks immediately after the lesson. The researcher supplied all teaching materials and resources for children’s individual and small group tasks. Teachers were given copies of the lesson notes for further planning and to provide a record for programming purposes.

In schools 3 and 4, the researcher taught the four syllabus lessons and then returned to the classes to implement the sequence of area research lessons. The teaching timetable enabled a comparison of children’s learning as a possible result of the two sequences of lessons, but was also negotiated because the teachers in Schools
3 and 4 were interested in observing and commenting upon an alternative way to teach area concepts (Table 5.2).

<table>
<thead>
<tr>
<th>Week</th>
<th>Lessons</th>
<th>Teacher Interviews</th>
<th>Student assessment</th>
<th>Lessons</th>
<th>Teacher Interviews</th>
<th>Student assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Area research 1</td>
<td>Initial interview</td>
<td></td>
<td></td>
<td>Initial interview</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Area research 1</td>
<td>Lesson 1 interview</td>
<td>Area syllabus 1 interview</td>
<td>Lesson 1</td>
<td>Area syllabus 1 interview</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Area research 2</td>
<td>Lesson 2 interview</td>
<td>Area syllabus 2 interview</td>
<td>Lesson 2</td>
<td>Area syllabus 2 interview</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Area research 3</td>
<td>Lesson 3 interview</td>
<td>Area syllabus 3 interview</td>
<td>Lesson 3</td>
<td>Area syllabus 3 interview</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Area research 4</td>
<td>Lesson 4 interview</td>
<td>Area syllabus 4 interview</td>
<td>Lesson 4</td>
<td>Area syllabus 4 interview</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>Final interview</td>
<td>Concluding assessment</td>
<td></td>
<td>Final interview</td>
<td>Pre-test</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>Area research 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>Area research 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>Area research 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>Area research 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Post-test</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Concluding interview</td>
</tr>
</tbody>
</table>

5.2.4 Collection of Data

The aim of this study was to identify, record and analyse the children’s learning during a specific period of intervention. The methods selected to collect this information needed to capture student learning and a record of the classroom teachers’ observations and perceptions. Several data collection methods were used, including pre-test and post-test assessment tasks for children, interviews with teachers, and the researcher’s field notes made during the study.
5.2.4.1 Interviews with teachers

Individual teachers were interviewed before the implementation commenced, after each lesson in the first two sequences, and at the conclusion of the implementations (Table 5.2). Teachers in Schools 3 and 4 were not interviewed after each of the area research lessons, as these lessons and interviews had not been included in the initial arrangements.

Seven of the eight participating teachers consented to having their interviews audiotaped. The researcher transcribed the recorded interviews and gave a copy to the teacher the next week. Teachers were asked to confirm that the transcriptions were a true record of their responses, and were also invited to add explanatory comments or clarifications if necessary, to confirm that the data sources were accurate (L. Cohen et al., 2000). When speaking with the teacher who did not want to have her interview recorded, the researcher made extensive hand-written notes that were later typed and given to the teacher for confirmation. The checked transcriptions of interviews were used as a data source for analysis.

In the initial interview, teachers were asked to describe the area lessons that they had taught that year, and how they thought that children learnt about the measurement of area.

- Can you tell me about the area lessons that you have taught this year?
- What do you think that children need to learn about area, what sort of skills and understandings are you trying to encourage?
- How do you plan your area lessons?
- What do you think is the best way to teach children about area?
- What will your children need to learn about area in the next three years?

At the conclusion of each lesson, teachers were asked for their perceptions of the effectiveness of the activities in assisting children’s learning. The researcher planned several introductory questions to commence the interview, and further discussion was prompted by the teachers’ comments and observations. The advice that “an interview is a conversation with a purpose” (Maykut & Morehouse, 1994, p. 79), was heeded
by encouraging teachers to suggest changes to the lessons, or questioning the researcher’s intentions when planning specific tasks.

- What did you see or hear the children learning today?
- Was there anything that struck you as being very useful?
- What worked best, and why?
- Is there anything that could have been better?

During the final interviews, conducted approximately one week after the lessons had concluded, teachers were asked about the possible effects of their participation in the study and the consequences for their teaching of mathematics in the future.

- Thinking back to the learning that has happened, are there any specific things that you can remember, when you thought, I really saw something happening there?
- Were there any groups of children who surprised you with what they were doing?
- Do you think that the lessons have advantaged any particular group of children or did you see learning across the whole class?
- How will your participation in this project affect the way you teach area in the future?
- Thinking about your observation of children’s learning, and the way they were learning, will that affect the way you teach mathematics in the future?
- Have you seen any carryover of skills that we’ve covered during these lessons to other parts of mathematics?

In Schools 3 and 4, the researcher implemented both sequences of lessons. When all lessons had been completed, the researcher requested a meeting with the two teachers in each school for a final discussion of the lessons and the results of the children’s assessment tasks. A group interview, in preference to individual interviews, was chosen to stimulate a discussion of the lessons (Watts & Ebbutt, 1987) and to assist with minimising disruption (L. Cohen et al., 2000). The student learning outcomes demonstrated during the two sequences of lessons were compared and teachers gave their opinions of how the area research lessons could be modified and improved. These data formed the basis of changes to the area research lessons,
in preparation for implementation during the professional learning study the following year.

5.2.4.2 Researcher’s field notes

The researcher wrote field notes as soon as possible after each lesson, to record her impressions of the lessons, including the general atmosphere in the classroom, difficulties she experienced with the children or materials and children’s responses to the tasks. The objective was to record a picture of what had taken place, through the researcher’s eyes, without trying to judge the outcomes of each individual lesson. Interpretation of events described in the field notes, together with other data, was left until the final evaluation when all data was analysed (Maykut & Morehouse, 1994).

5.2.4.3 Student assessment task.

The children were asked to draw tiles to cover the area of a given rectangle on an A4 photocopied sheet. A single, square tile was drawn in the top left hand corner of the rectangle, to indicate the size of the tiles. Children were told that tiles of the same size and shape as the sample tile should cover the whole area of the rectangle. They could draw the tiles in any way they chose, but the area inside the sides of the rectangle was to be covered. Children commenced by completing a practice shape on the reverse of the rectangle. This was a small square, that could be covered in a 3 x 3 array of tiles. Children were not allowed to use rulers to measure or draw lines. Some children measured the length of the sample tile against their finger and used the finger length as a guide.

Children in Schools 1 and 2 completed the area-tiling task at the completion of their four research lessons. Children in Schools 3 and 4 were given the area-tiling task at the completion of the syllabus lessons, and again at the completion of the research lessons.
5.2.5 Analysis of Data

5.2.5.1 Analysis of teacher interviews

The teachers’ responses to each of the interview questions were analysed to find appropriate categories and codes that would assist in identifying the outcomes of each lesson. The lessons were evaluated individually and teachers’ suggestions for modifications to the materials or tasks were noted. Each lesson sequence was then evaluated and considered with the results of the student assessment tasks. These data were considered with teachers’ comments about the two lesson sequences, in the final interviews in Schools 3 and 4, to identify the benefits of each of the lesson sequences.

5.2.5.2 Coding of student responses, drawing the tiles

The purpose of the tiling activity was to gauge children’s understanding and recognition of an array, by analysing the strategies they used to draw the covering tiles. A scale (Figure 5.1) for assessing the children’s outcomes on the task was developed, based on the work of Outhred (1993) to describe the grid structure of the children’s drawings:

1. Draws tiles as individual entities
2. Draws tiles in vertical columns as "u" shapes
3. Draws tiles onto previous tiles, as a reverse "L" shape
4. Draws horizontal or vertical lines and then draws a separate line for each tile
5. Draws horizontal and vertical grid lines across or down the rectangle, but does not have the correct number in rows or columns (too few or too many)
6. Draws horizontal and vertical grid lines, and has the correct number of tiles in rows and columns.

The most sophisticated strategy (6) was to draw continuous lines in two directions for the columns and rows. Children needed to be able to see the tiles in an array pattern of composite units, before the tiles could be drawn in this way. It may be argued that children who are unaware of the array pattern may choose a less sophisticated strategy of drawing the tiles, such as individual tiles, rows of individual tiles or columns of individual tiles.
1. Tiles drawn as individual entities  
2. Tiles drawn as “u” shapes in columns  

<table>
<thead>
<tr>
<th>1. Tiles drawn as individual entities</th>
<th>2. Tiles drawn as “u” shapes in columns</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="1" alt="Image" /></td>
<td><img src="2" alt="Image" /></td>
</tr>
</tbody>
</table>

3. Each tile drawn onto previous tile  
4. Horizontal or vertical lines then a separate line for each tile  

<table>
<thead>
<tr>
<th>3. Each tile drawn onto previous tile</th>
<th>4. Horizontal or vertical lines then a separate line for each tile</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="3" alt="Image" /></td>
<td><img src="4" alt="Image" /></td>
</tr>
</tbody>
</table>

5. Horizontal and vertical grid lines, but incorrect number of rows or columns  
6. Continuous horizontal and vertical lines with correct number of tiles  

<table>
<thead>
<tr>
<th>5. Horizontal and vertical grid lines, but incorrect number of rows or columns</th>
<th>6. Continuous horizontal and vertical lines with correct number of tiles</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="5" alt="Image" /></td>
<td><img src="6" alt="Image" /></td>
</tr>
</tbody>
</table>

Figure 5.1  Coding of children’s responses to tiling assessment task

5.3 Conclusion

This chapter has described the methodology of phase 1, the implementation of the two sequences of lessons. Although the lessons were implemented in only eight classes, the sample of schools, teachers and children provided a diversity of
backgrounds, experiences and teaching styles. This diversity, together with the range of data collected from the children, teachers and researcher provided a basis on which to compare and judge the effectiveness of the area research lessons and their suitability to children and teachers in a range of educational settings.

The next chapter discusses the results of the implementation of the two sequences of lessons together with implications for phase 2 of this study and the teaching of area in the mathematics program in primary schools.
CHAPTER 6 – RESULTS OF IMPLEMENTATION OF AREA LESSONS

The previous chapter described how data were collected during phase 1 of the research to assist in conducting an evaluation of the two sequences of area lessons. The methods used to examine the data, including an analysis of teacher interviews and student assessment tasks, were also explained. In this chapter, the findings from each aspect of the data are studied, and comparisons are made between the outcomes of the syllabus lessons and the outcomes of the area research lessons. Guidelines for the modification of the area research lessons are listed. Finally, the amended lessons are documented in preparation for implementation as a component of phase 2 of this research.

6.1 Outline of Data Collection

The data were collected from three principal sources: transcriptions of interviews with teachers; the researcher’s field notes; and the children’s pre-test and post-test assessment tasks. During analysis of the data, the transcriptions of interviews and the researcher’s field notes were studied concurrently to facilitate an examination of each individual lesson from the perspectives of both the teachers and the researcher. The impacts of the sequences of lessons were also evaluated by the researcher at the conclusion of each program. These results were compared with the children’s responses to the post-test task to enable the final evaluation of the efficacy of each lesson sequence.

6.1.1 Data Collected from Interviews with Teachers and Researcher’s Field Notes

Teachers were interviewed before, during and after the implementation of the two sequences of lessons. In schools 3 and 4, after the researcher had implemented the second sequence of lessons, the two teachers met with the researcher to compare and discuss the effectiveness of each of the lesson sequences. The researcher wrote field notes after every lesson to record the classroom atmosphere, her own teaching, and children’s responses to the tasks.
6.1.2 Data Collected from Children

Children in Schools 1 and 2 completed a concluding area assessment task after lesson 4 of the area research lesson sequence. Children in Schools 3 and 4 completed a pre-test at the conclusion of the syllabus lessons and a post-test at the conclusion of the research lessons. In each of the eight participating classes, the task was administered to all of the children who were at school on the day of the assessment.

<table>
<thead>
<tr>
<th>Number of children who were involved</th>
</tr>
</thead>
<tbody>
<tr>
<td>School 1</td>
</tr>
<tr>
<td>Number</td>
</tr>
</tbody>
</table>

6.2 Findings

6.2.1 Results from Interviews with Teachers and Researcher’s Field Notes

Following each lesson, teachers were asked to comment on their children’s reactions to the activities, the effectiveness of the tasks in developing the concept of area measurement and suggestions for possible modifications to the lessons.

6.2.1.1 Labelling of comments

Each comment was labelled to enable retrieval and comparison in the next stage of analysis. The label TS1 referred to a teacher’s comment from area syllabus lesson 1 (Tables 6.4 to 6.7) and TR1 indicated a teacher’s voice from area research lesson 1 (Tables 6.12 to 6.15). In addition, each comment was numbered so that individual comments could be identified, as in comment TS1.1 from syllabus lesson 1 (Table 6.4).

Teachers were also interviewed at the conclusion of the sequence of four lessons. The transcriptions were examined and comments listed in Tables 6.28 and 6.30, using the labelling Final S.1 for area syllabus lessons and Final R.1 for area research lessons. The aspects that featured in the previous tables were used, together with an additional grouping labelled Teachers’ future directions. This category collated responses to the interview question: How will your participation in this project affect the way you teach area in the future?
Comments from the researcher were collected from two sources: remarks made during the interviews with teachers and also the researcher’s field notes. The interview comments were included in the tables of teachers’ comments and labelled with C, as in CS1.3 (Table 6.4). The researcher’s field notes were collated in a similar manner to the teachers’ comments and labelled as FS1 for area syllabus lesson 1 (Table 6.5) and FR1 for area research lesson 1 (Table 6.19), with an additional number for each comment. A summary of the labelling scheme for identifying comments is given in Table 6.2.

Table 6.2  Labelling of teachers’ and researcher’s comments

<table>
<thead>
<tr>
<th>Area syllabus lessons</th>
<th>Comments from Interviews</th>
<th>Researcher’s comments from field notes</th>
<th>Teachers’ comments from final interviews</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Teachers</td>
<td>Researcher</td>
<td></td>
</tr>
<tr>
<td>Area research lessons</td>
<td>TS1.1</td>
<td>CS1.1</td>
<td>FS1.1</td>
</tr>
<tr>
<td></td>
<td>TR1.1</td>
<td>CR1.1</td>
<td>FR1.1</td>
</tr>
</tbody>
</table>

6.2.1.2 Coding scheme

The interview comments from teachers, the researcher and the researcher’s field notes, were collated, analysed and presented in Tables 6.4 to 6.31. Each table was arranged in three major categories (Learning outcomes; The lessons; Children’s responses to tasks) that were selected and trialled to assist with comparison of the effectiveness of the two sequences of lessons. A number of schemes to code comments within the three major categories were designed and tested. Initial coding separated the learning outcomes achieved and the learning outcomes not achieved. This was found to be too repetitive and unwieldy and did not assist in comparing the outcomes of the two lesson sequences. In the final version of coding (Table 6.3), all learning outcomes are listed as one category, and aspects of identifying and measuring area are listed as codes within this category.

Additionally, initial coding prompted by the researcher’s efforts to minimise the total number of individual codes, was too restrictive in describing specific aspects of
the learning outcomes. For example, the code *Alignment* referred to children’s ability to identify, choose and use appropriate informal units in an array with no gaps and overlaps. A trial of the codes indicated that finer detail was necessary to adequately record teachers’ comments about the use of informal units, and to reflect the intent of the tasks. The codes *Choose appropriate units* and *Align units correctly* were introduced.

Category 2 (*The lessons*) was introduced to identify modifications or changes to lessons that may have been suggested by teachers or the researcher. Suggested modifications were to be addressed in the revision of the sequences before implementation the following year. The earliest version of this category included coding for *Unfamiliar medium*, as well as *Resources*, as initial reading of the transcriptions indicated that many children had difficulty with the patterning task in research lesson 2. During trialling the two codes were found to be duplicitous, and were reduced to one code (*Resources*) that described comments on the suitability of the resources to address children’s skills and the intended mathematical outcomes of the lesson.

The inclusion of Category 3 (*Children’s responses to tasks*) was prompted by initial reading of the transcriptions and observation of the large number of comments made by teachers about how their children reacted to the learning experiences. The code *Enjoyed* was included to register the many comments about children enjoying their mathematics. The initial codes included separate codes for *Engagement*, *Group work* and *Success*. After trialling, these were reduced to a single code of *Engagement and perseverance* that described children’s engagement with the task as well as the underlying mathematical concepts, and their readiness to persist with an activity until it was completed. The comments about group work were more relevant to Category 2, and were included in the *Management* code.

Category 4 (*Teachers’ future directions*) was an additional category devised for Tables 6.16 and 6.30 that present comments from the final interviews with teachers. This category captured the principal ideas that teachers described in answer to the question “How will your participation in this project affect your teaching of mathematics in the future?” Initial examination of the transcriptions indicated:
• Teachers thought they should spend more time on ensuring that mathematical concepts are understood by all children before progressing to the next stage;
• Teachers were impressed by their children’s cooperation and discussion in small working groups;
• Teachers had used the program as an opportunity to observe individual children and to make decisions about children’s understanding of mathematical concepts.

Initially, the three codes used in this category were Teaching, Grouping and Responding. In the final version, these codes retained their original groupings, but the descriptions were refined to reflect the thoughtful and insightful comments made by teachers (Table 6.3).

Table 6.3 **Coding scheme**

**Category 1 Learning outcomes**

<table>
<thead>
<tr>
<th>1.1 Identify attribute</th>
<th>Discriminate between a line and a shape, and identify the area of a closed shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2 Conservation</td>
<td>Recognise that area is conserved when rearranged</td>
</tr>
<tr>
<td>1.3 Counting to calculate area</td>
<td>Count the total number of informal units used to measure area</td>
</tr>
<tr>
<td>1.4 Choose and use appropriate units</td>
<td>Identify and use suitable units to measure the area of given shapes</td>
</tr>
<tr>
<td>1.5 Align units</td>
<td>Align tessellating units accurately to measure rectangular area, with no gaps and no overlaps</td>
</tr>
<tr>
<td>1.6 Superimpose</td>
<td>Superimpose two or more shapes to compare and report the areas</td>
</tr>
</tbody>
</table>

**Category 2 The lessons: tasks, resources, suggested extensions**

| 2.1 Tasks | The suitability of the tasks in assisting children to achieve the intended learning outcomes |
| 2.2 Resources | The suitability of the teaching resources to the children’s skills and the intended outcomes of the lesson |
| 2.3 Management | Classroom management strategies and grouping and their success |
| 2.4 Suggested extensions | Suggestions for additional mathematical concepts that could be introduced to extend the basic task |

**Category 3 Children’s responses to tasks**

| 3.2 Enjoyment | Children enjoyed their engagement with the materials and the task |
| 3.3 Engagement and perseverance | Children were engaged with the task and the underlying mathematical concepts and persisted with the activity to find a solution |
| 3.3 Discussion | Children’s participation in the activities facilitated discussion of understanding and observations |
6.2.1.3 Analysis of interviews and field notes comments

The lessons were analysed individually by categorising and coding comments from the interviews with teachers conducted at the conclusion of each lesson and also the researcher’s field notes. The coding and categorising was reviewed and discussed with two academic colleagues.

To enable the outcomes of the area syllabus lessons and the area research lessons to be compared against the intended outcomes and the research questions, the coded comments were reviewed. Agreement between the comments made during interviews and the researcher’s field comments was reported to demonstrate verification of the interpretation of the data. The emerging major themes which described the teachers’ and researcher’s evaluations of the lessons in terms of children’s learning outcomes, tasks, management and students’ responses to the experiences were identified and described under the headings of:

Category 1: Learning outcomes

Category 2: The lessons: tasks, resources, suggested extensions

Category 3: Children’s responses to tasks.

These statements were then used to compose a final summary of each lesson.

6.2.1.4 Presentation of data for each lesson and the groups of lessons

The presentation of tabled data for area syllabus lesson 1 indicates the model used for each lesson, consisting of:

- Table 6.4 Interview comments area syllabus lesson 1
Teachers were interviewed again at the conclusion of the lesson implementation, to comment on the success of the lessons and the student learning which they had observed. These comments were categorised and coded and then summarised (Tables 6.16 and 6.17 for the area syllabus lessons). Finally, an evaluation of the four area syllabus lessons was written, using the summary from each component of the data. This summary appears in 6.2.2.

The comments and responses relating to the area research lessons were analysed in the same way as the area syllabus lessons, and resulting data is presented in Tables 6.18 to 6.31.

6.2.1.5 Area syllabus lesson 1

In this lesson, children experimented with making shapes from a 2 metre length of braid, using square cardboard tiles with a side of 10 cm to measure the area of the shapes. Children also drew criss-crossing lines on a sheet of paper, and identified shapes to colour.

<table>
<thead>
<tr>
<th>Table 6.4 Interview comments area syllabus lesson 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Category 1 Learning outcomes</strong></td>
</tr>
<tr>
<td><strong>1.1 Identify attribute</strong></td>
</tr>
<tr>
<td>I think there was a bit of an ambiguity towards what a line and a shape was. They could actually make it, no problem, but verbalising what the difference between the two was quite interesting to me. I sort of saw that as assumed knowledge but when it came down to the crunch, it was like, yes, this is a shape but I don't know why it is a shape, that type of thing (TS1.8).</td>
</tr>
<tr>
<td>Using the term ‘surface’. (Class teacher had introduced this word, but the children found it difficult to remember, when asked what the word to describe Area was.) (TS1.10).</td>
</tr>
<tr>
<td>By the end of it a lot of them had worked it out, that you needed to close the line to make a shape, but probably there are still some of them, who are probably not quite clear on it (TS1.19).</td>
</tr>
<tr>
<td><strong>1.4 Choose appropriate units</strong></td>
</tr>
<tr>
<td>They sort of noticed that some shapes were not useful to cover the shapes. They were quite good with that (TS1.1).</td>
</tr>
<tr>
<td>They made a big heart out of their line and then tried to use little hearts to cover and said, hang on a minute; it's not working (TS1.3).</td>
</tr>
<tr>
<td>So they were picking up different tiles as they were going and realised quite early in the piece that it was going to work (TS1.4).</td>
</tr>
<tr>
<td>One group started with a rectangle, stretched it into a rhombus, and then</td>
</tr>
</tbody>
</table>
changed their squares to triangles, so they could be used for covering (TS1.5).

One group had some gaps left over. They worked out that they needed three halves to finish, giving a total of thirteen and one half squares. They told me that they would need to buy fourteen squares (TS1.6).

Making shapes with the braid, and filling these in with the cardboard tiles. I was really pleased with what the children did, when they were trying different shapes (TS1.7).

I noticed that some wanted a heart straight away. They put two hearts down and said no, no, no, that's not going to work. And just threw that idea out (TS1.17).

If they picked up a square tile, they made a big square. So they filled it in with little squares. Or they made a big triangle and tried to fill it in with little triangles (TS1.18).

1.5  
**Align units**

They seemed to pick up straight away on not overlapping (TS1.2).

James and his group did well to put the triangles together, by placing the next row of triangles onto the bases of triangles in the previous row (TS1.16).

**Category 2 The lessons: tasks, resources, suggested extensions**

2.3  
**Management**

Now we didn't do inside and outside the shape, I just had too much in that lesson (RS1.1).

Organise the groups a bit better, because I noticed that some groups had kids who weren't working together very well, and that was because I sort of did it on the spot (TS1.12).

I liked the idea of the little groups, having groups of three. Because they got to sort of share and cooperate and toss ideas together, but it was a small, workable group. So that was good (TS1.13).

**Category 3 Children’s responses to tasks**

3.1  
**Enjoyment**

We all enjoyed it. I enjoyed it and the children enjoyed it (TS1.14).

3.2  
**Engagement and perseverance**

Working cooperatively, working out which shapes would and wouldn’t fit. Just trying shapes. The children were willing to take risks. The fact that children persisted with the task until they had a solution. James and his group did well to put the triangles together, by placing the next row of triangles onto the bases of triangles in the previous row (TS1.15).

3.3  
**Discussion**

The little groups, having groups of three - because they got to sort of share and cooperate and toss ideas together, but it was a small, workable group. So that was good (TS1.20).

**Table 6.5 Field notes area syllabus lesson 1**

**Category 1 Learning outcomes**

1.1  
**Identify attribute**

Not many knew how to make a line into a shape, but once they started, they made squares, rectangles and circles, with a heart as well (FS1.1).

Shapes made with the braids were ordinary: just triangles, squares, and rectangles. Lines were also fairly stilted. Straight lines and zigzags were the most common (FS1.3).

Making the lines and shapes with braid went well, and the children
recorded four lines before they started making shapes. The shapes were mundane, circles, squares, and triangles (FS1.5). Kim told me later that the children had difficulty in finding the shapes which were bordered by lines, rather than those which had the edge of the paper as one of the boundaries (FS1.8).

1.3 Counting to calculate area
Most were counting the number of tiles used by ones (FS1.4).
One girl told me she had 16 small squares, because four groups of four make 16. Another boy in her group tried to count by fours, but missed out 12, so he got a total of 20. Other children counted by twos and threes (FS1.6).

1.4 Choose appropriate units
The two groups, who used hearts, were able to work out why the hearts were not suitable for covering (FS1.2).
The next part of the lesson, where the children covered their shapes with tiles, was just terrific. The children worked very enthusiastically. (FS1.17).
It was good giving children a choice of which shapes they wanted to work with. Making the squares, rectangles and triangles was good for this class (FS1.18).

1.5 Align units
Putting the triangles together was a real challenge for several groups (FS1.7).

Category 2 The lessons: tasks, resources, suggested extensions

2.1 Tasks
I showed the children how to make crossed lines drawing. I told them to do one line at a time from each side. I think that the method I used with Lyn’s class, of drawing 6 lines from each side, would have been better (FS1.10).
The children were reluctant to try other arrangements of the tiles once they were in place (FS1.14).
I was hurrying by the time I got to the crossed lines explanation, and could have taken longer to do this. I told the children to draw 6 lines one way and six lines from the other direction. Instructions to add one line at a time may have been better (FS1.13).
One boy asked if he could draw the lines as he worked, and I said that all groups could record if they wished. This was very popular with the children: they took this seriously, and also drew the shapes that they made (FS1.15).

2.2 Resources
We could have used more triangles: we ran out (FS1.12).

2.3 Management
I found that they didn’t listen very well on the mat. I would also try to keep things moving briskly next time (FS1.11).

Category 3 Children’s responses to tasks

3.1 Enjoyment
They enjoyed the opportunities to talk and experiment in their groups, and were very cooperative (FS1.18).
We demonstrated on the mat how many girls, then boys, we could fit inside the shape made by braid. The boys fitted in more, but they then collapsed in a big heap, causing hilarity. This was good for 'inside, outside' (FS1.19).
3.2 Engagement and perseverance

The children settled quickly to their group work, and most wanted to report back when we came back to the mat (FS1.16).

3.3 Discussion

We were given an excellent explanation of why the hearts did not work (FS1.18).

Table 6.6 Analysis of area syllabus lesson 1

<table>
<thead>
<tr>
<th>Category 1: Learning outcomes</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children were able to make both lines and shapes (TS1.8, TS1.19, FS1.1, FS1.3, and FS1.5).</td>
<td></td>
</tr>
<tr>
<td>Children were able to choose appropriate tiles for measuring, some through trial and error (TS1.1, TS1.3, TS1.4, TS1.7, TS1.17, TS1.18, FS1.2, FS1.17, FS1.18).</td>
<td></td>
</tr>
<tr>
<td>Children in two classes were able to align tiles correctly, with a third class finding this a challenge (TS1.2, TS1.16, FS1.7)</td>
<td></td>
</tr>
<tr>
<td>One class had difficulty with finding the shapes inside the crossed lines pattern (FS1.8).</td>
<td></td>
</tr>
<tr>
<td>Children counted the tiles by ones, twos, threes and fours (FS1.4, FS1.6).</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Category 2: The lessons: tasks, resources, suggested extensions</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group work was successful, but children needed to be grouped carefully (TS1.12, TS1.13).</td>
<td></td>
</tr>
<tr>
<td>Not all of the planned material was implemented during the lesson (RS1.1)</td>
<td></td>
</tr>
<tr>
<td>The crossed lines task was difficult to explain, with the researcher trialling two different explanations (FS1.10, FS1.13).</td>
<td></td>
</tr>
<tr>
<td>The tiling task did not encourage some children to trial alternative arrangements of tiles, although other children enjoyed recording their tile patterns through drawing (FS1.14, FS1.15).</td>
<td></td>
</tr>
<tr>
<td>More triangular tiles were needed (FS1.12).</td>
<td></td>
</tr>
<tr>
<td>One lesson was not brisk enough to maintain interest (FS1.11).</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Category 3: Children’s responses to tasks</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children enjoyed the tasks (TS1.14, FS1.17, FS1.19).</td>
<td></td>
</tr>
<tr>
<td>Children worked cooperatively taking risks and persisting with the tasks to find a solution (TS1.15, FS1.18).</td>
<td></td>
</tr>
<tr>
<td>Some discussion was evident at the small group and whole class levels (TS1.20, FS1.16, FS1.18).</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Summary and comparison with intended outcomes</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children identified the difference between a line and a shape, and chose appropriate tiles for covering the shapes.</td>
<td></td>
</tr>
<tr>
<td>Children in three classes aligned the tiles correctly; children counted the tiles by ones. In one class, multiples of two and three were used.</td>
<td></td>
</tr>
<tr>
<td>The crossed line task was too difficult to explain to be consistently successful.</td>
<td></td>
</tr>
<tr>
<td>The children enjoyed the tasks and worked cooperatively.</td>
<td></td>
</tr>
</tbody>
</table>
6.2.1.6 Area syllabus lesson 2

Children superimposed pairs of cardboard shapes to find which had the larger area. Children also superimposed, cut and pasted paper copies of a triangle and a circle to determine which had the larger area.

Table 6.7 Interview comments area syllabus lesson 2: Superimpose shapes

<table>
<thead>
<tr>
<th>Category 1 Learning outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.6 Superimpose</strong></td>
</tr>
<tr>
<td>Comparing the two shapes, the initial exercise was quite good. Most of them could do that, so they were able to see which one had the larger area (TS2.1). The blindfolding was fine. They seemed to understand. A lot of them used the palm of their hand and felt the whole shape, and worked out which one had the larger area by doing that (TS2.2). They all understood the superimposing (TS2.3). When we were playing blindfold, the children explained how to superimpose (TS2.5). I guess they do it naturally but now they are aware of why they are doing it. And they learnt what it is called, as well. (superimposing) (TS2.6). I don’t think a lot of them actually understood what they were actually proving and how, until they went back to their desks and did it. And at the end they all knew that the circle had the larger area (TS2.7). A lot of them didn't understand that concept and as you saw, they cut out the triangle into little bits and pieces and discarded them because they didn't realise that they needed the whole shape (TS2.8). We need to use the vocabulary again, especially superimpose (TS2.9). The more able children understood it, but that was the minority, which was quite surprising (TS2.18).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Category 2 The lessons: tasks, resources, suggested extensions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2.1 Tasks</strong></td>
</tr>
<tr>
<td>A few were confused after they made the first cut, and then they still had to cut again, to fit the pieces of the triangle onto the circle. The two different colours helped, and also having one plain shape and one patterned shape (TS2.4). Some of the slower children were confused with the pasting. They pasted the triangle the wrong way up, and some pasted on the back of the square. They need some different shapes to cut and paste (TS2.11).</td>
</tr>
<tr>
<td><strong>2.2 Resources</strong></td>
</tr>
<tr>
<td>I really liked your bags of shapes. You have inspired me to collect cereal boxes (TS2.10). We did (make shapes to superimpose) at my last school, but not here. It has to be a grade commitment, so we can share (TS2.12). Two colours in the shapes to be cut and superimposed would have been better, so the children could see the shapes more easily (TS2.13). That's half the problem, isn't it? Getting your resources together. And I mean half the time you might do it, but to a lesser scale. Like you might have five bags, which means you need bigger groups, and then it just becomes a total chaos (TS1.19).</td>
</tr>
</tbody>
</table>
Working in pairs, because each one had something to cut and hold and to talk about. You might have noticed that I put the more able children with those who needed assistance (TS2.14).

### Category 3 Children’s responses to tasks

#### 3.2 Engagement

I can certainly see them all thinking about the concept a lot more deeply, just by some of their answers. They’re starting to relate the different things you’ve been talking about, they’re starting to use them to solve the problems that you’re posing to them (TS2.16).

The pairs worked well. They were loud, but they were on task. The discussion part, where they had to explain to each other (TS2.15).

---

### Table 6.8 Field notes area syllabus lesson 2: Superimpose shapes

#### Category 1 Learning outcomes

<table>
<thead>
<tr>
<th>1.1 Identify attribute</th>
<th>I am still concerned at the number of children who refer to Area as “space” (FS2.10).</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6 Superimpose</td>
<td>The comparing of two shapes went well. Most pairs did all three bags of shapes. I moved around to as many children as possible, and couldn’t find any who weren't superimposing (FS2.1).</td>
</tr>
<tr>
<td></td>
<td>I moved around the room and spoke to each child as they held the shapes and compared. They were all able to give an explanation of what they were doing (FS2.3).</td>
</tr>
<tr>
<td></td>
<td>The children were good with their cutting and pasting. The final discussion of why the circle’s area was bigger than the triangle’s area was really good. We had some great explanations from the children (FS2.4).</td>
</tr>
<tr>
<td></td>
<td>All children that I spoke to were correct, and all were able to superimpose, align shapes, and point to the difference in area (FS2.5).</td>
</tr>
<tr>
<td></td>
<td>Lyn was happy with the way the children were able to find the larger area, while wearing a blindfold (FS2.6).</td>
</tr>
<tr>
<td></td>
<td>When they actually sat down and started pasting and cutting, the children seemed to realise what they were actually doing, and this part of the exercise went smoothly. We didn’t have anyone who dropped pieces of the triangle onto the desk (FS2.8).</td>
</tr>
<tr>
<td></td>
<td>As Jan and I moved around, we found that several pairs had &quot;forgotten&quot; small pieces of triangle that were cut off and left on the desk. We still had a number of children who did not seem to understand why all of the little bits were important (FS2.9).</td>
</tr>
</tbody>
</table>

#### Category 2 The lessons: tasks, resources, suggested extensions

<table>
<thead>
<tr>
<th>2.1 Tasks</th>
<th>Some children pasted parts of the triangle onto the back of the hexagon. Two pairs came up with the wrong answer. Certainly they had missed the point of comparing the two areas (FS2.11).</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The cutting and pasting activity was too difficult for some children. I would suggest giving two shapes which are quite obviously different, next time, and also perhaps in two colours (FS2.13).</td>
</tr>
<tr>
<td></td>
<td>Some children thought that the comparison activity was too easy. I asked them to do at least one of the bags of shapes, but most children</td>
</tr>
</tbody>
</table>
completed all three pairs (FS2.16).
Generally, the recording helped to focus the children on what they had been asked to find. It also helped with naming the shapes, because I heard several children who comfortably referred to “trapeziums” at the end of the lesson (FS2.17).

2.2 Resources
When I asked Lyn if she would make the resources for her own lessons, she said no, that she would just make several, and have children demonstrate what to do at the front of the classroom (FS2.14).

2.3 Management
As Chris said later in the interview, it was good that her class is later in the week, so that I have eliminated the possible difficulties by the time I get to her (FS2.12).

Category 3 Children’s responses to tasks

3.1 Enjoyment
Chris did the blindfold activity with the children and they all seemed to enjoy this (FS2.15).

3.2 Engagement and perseverance
Most pairs completed the three bags of shapes, and I asked if I could have the recording sheets for my report (FS2.7).

Table 6.9 Analysis of area syllabus lesson 2

Category 1: Learning outcomes
Children were able to superimpose the bags of similar shapes (TS2.1, TS2.2, TS2.3, and TS 2.6, FS2.1, FS2.5, FS2.6).
Children readily explained what they were doing (TS2.5, FS2.3, FS2.4).
The children had varied success with the cutting and pasting with a triangle and circle (TS2.7, TS2.8 TS2.18).
The mathematical terminology needed to be revised in following lessons (TS2.9, FS2.10).

Category 2: The lessons: tasks, resources, suggested extensions
The original cutting and pasting task was too difficult and a revised version of the task was more successful (TS2.4, TS2.14, FS2.8, FS2.9, FS2.11, FS2.13).
Superimposing the shapes was too easy for one Year 2 class (FS2.16).
Recording the results of the superimposing assisted with learning the names of the shapes (FS2.17).
Teachers commented they could not prepare the superimposing shapes (TS2.10, TS2.12, TS1.19, FS2.14).
Paired working was successful with peer tutoring used in one class.

Category 3: Children’s responses to tasks
Children enjoyed the whole-class blindfolding activity (FS2.15).
Children completed all four of the superimposing tasks (FS2.7).
Paired work was successful and enabled discussion (TS2.14, TS2.15).
Children engaged in whole-class discussions about area (TS2.16, FS2.4).

Summary and comparison with intended outcomes
Children successfully superimposed the shapes.
The cutting and pasting activity was problematic.
Teachers would need to prepare the superimposing resources cooperatively.
Children enjoyed the tasks, worked cooperatively and participated in whole-class discussions about area.
6.2.1.7  Area syllabus lesson 3

In lesson 3, children were given two identical pieces of card and were asked to cut one piece of card into a jigsaw. The class discussed which of the two cards now had the greater area. Children also covered a large shape with differently-shaped tiles and reported on which tiles were best for covering, and how many were needed.

Table 6.10  Interview comments area syllabus lesson 3

<table>
<thead>
<tr>
<th>Category 1 Learning outcomes</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Attribute</td>
<td>The children were very good during the revision questions, at the beginning, and showed that they are remembering the ideas and names each week (TS3.8).</td>
</tr>
<tr>
<td>1.2 Conservation</td>
<td>They all thought that the area would be the same, which is right, but don’t statistics show that all kids think that if a shape is cut up, then it’s the conservation of matter, isn’t it (TS3.11).</td>
</tr>
<tr>
<td></td>
<td>So then I got the pieces together and arranged them in a blob and said: OK, which area is the larger now, the blob or the original rectangle? And Ron said: They’re the same. You just put it in a different way. So I couldn’t fool him (TS3.12).</td>
</tr>
<tr>
<td>1.3 Counting units to calculate area</td>
<td>A lot of them are still doing that one-to-one sort of counting. Some of them did, counted by fours or twos, but they weren't able to verbalise how they got it I suppose (TS3.2).</td>
</tr>
<tr>
<td></td>
<td>I pleased to see that all of them today were able to count quickly and well. I know that some children in the bottom maths group have trouble with one-to-one, but everyone here was really good. They should also be able to count in fives and tens, because they have started to do this in number (TS3.6).</td>
</tr>
<tr>
<td></td>
<td>A lot of them couldn’t count it properly. They, just going back to Jeremy’s group, they got different answers. One got twenty and one got twenty-one and I did it in front of them and I got twenty-four and then they sort of realised that they hadn’t counted. I think they were so excited, and the fact that they weren’t doing it in a constructive sort of way (TS3.16).</td>
</tr>
<tr>
<td>1.4 Choose appropriate units</td>
<td>They found these squares and the rectangles quite easy, because they just grabbed that shape (TS3.1).</td>
</tr>
<tr>
<td></td>
<td>The children gave you some good reasons why you shouldn’t use circles and blobs. I was pleased to see that they both understand and are able to describe why these shapes should not be used (TS3.4).</td>
</tr>
<tr>
<td></td>
<td>It was interesting from where I was sitting to see the looks on their faces when I held up the blobs, because you could see that they all thought that blobs would be silly to use (RS3.5).</td>
</tr>
<tr>
<td></td>
<td>Selecting the shape of the tiles isn’t really a problem for them at all, anymore. They’re doing it without any thinking (TS3.9).</td>
</tr>
<tr>
<td></td>
<td>But hexagons threw them right off. A lot of them used the squares and tried to angle them so they could fit, which was quite surprising, I thought. So yes, I think you're right: they might need a bit more work on that (TS3.13).</td>
</tr>
</tbody>
</table>
And they knew why they didn't want to go near them (circles and blobs for use as tiles). They could express themselves; they could say that they didn't want gaps (TS3.23).

The first activity of measuring showed improvement in covering and fitting. The children were also more confident about choosing tiles for covering (TS3.24).

<table>
<thead>
<tr>
<th>1.6 Superimpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes. I was quite impressed that they remembered the things they did from two weeks ago. The terminology and the superimposing. They were quite good with that, so I was surprised at that (TS3.3).</td>
</tr>
</tbody>
</table>

Category 2 The lessons: tasks, resources, suggested extensions

<table>
<thead>
<tr>
<th>2.1 Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>The conservation activity was just too difficult for many children, because it was too hard to replace the pieces of the jigsaw. They were further confused because they had written their names on different sides of the pieces, so they could see names on some pieces and not on others (TS3.15).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2.3 Management</th>
</tr>
</thead>
<tbody>
<tr>
<td>They really needed more time for this activity, because they were enjoying the basic idea. Some children need a lot more explanation, particularly in drawing the lines (TS3.17). I think the groups worked quite well. They're getting used to each other because they're in the same groups now, so they were quite cooperative. Yes, I was pleased with that (TS3.18).</td>
</tr>
</tbody>
</table>

Category 3 Children’s responses to tasks

<table>
<thead>
<tr>
<th>3.1 Enjoyment</th>
</tr>
</thead>
<tbody>
<tr>
<td>They’re really enjoying the classes. They’re really enjoying the extra attention. I never do a maths lesson that goes for an hour and a half and consists of three different activities (TS3.21).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3.2 Engagement</th>
</tr>
</thead>
<tbody>
<tr>
<td>They were very focused about what they were doing (RS3.22). They loved that activity. They didn’t get bored after doing one, they wanted to do all of the shapes there, and they worked really well together (TS3.20). I thought they’d be bored by that first bit, after one. I thought they’d think: Oh we’ve done this and we did it last time. But they were just really happy to go and do every shape there was (TS3.22).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3.3 Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td>The children were also talking to each other. I noticed that Pip had a totally different answer to her partner’s answer. She was able to justify her answer by describing how some spaces would be filled by extra tiles (TS3.19).</td>
</tr>
</tbody>
</table>
Table 6.11  *Field notes area syllabus lesson 3*

**Category 1  Learning outcomes**

| 1.2 Conservation  | When we spoke about the jigsaw activity, the children already had a good idea that the two areas would be the same, regardless of being cut up or left whole (FS3.5).  
  When I started to explain the cutting jigsaw activity, all of the children except one thought that the areas would still be the same. They also gave good reasons (FS3.7).  
  When we discussed the jigsaw activity, the children immediately decided that both areas would be the same, and gave good reasons for this (FS3.14). |
| 1.3 Counting to calculate area  | The rectangles were easy to estimate. Most children had to count the whole lot to find the number used, but one girl counted the top row and then doubled it (FS3.2).  
  Two pairs had worked out that the hexagon needed 24 tiles, by recognising the halfway mark, and doubling the number from there. The children found estimating the square tiles fairly easy, and told us how they estimated: both by eye and also by using fingers (FS3.6).  
  Counting the tiles was slow, with children counting by ones unless I specifically asked them to count another way. Then they invariably used twos, both horizontally and vertically (FS3.8). |
| 1.4 Choose appropriate units  | Another pair tried to use triangle tiles on the square. When they found that the equilateral triangles wouldn’t fit, they changed the square shape to a hexagon (FS3.1). |
| 1.6 Superimpose  | I asked them what we had done in Area. They remembered the action for superimposing, and some remembered the term (FS3.4).  
  Drawing and naming and cutting went well. Children were able to superimpose to prove that the areas were still the same (FS3.3). |

**Category 2 The lessons: tasks, resources, suggested extensions**

| 2.1 Tasks  | I said to Lyn, as we were completing the covering activity, that really one activity would have been enough for the lesson, without trying to do two (FS3.10). |
| 2.2 Resources  | I could have used more hexagons again today. One pair covered half the hexagon and then doubled the number for the whole area, but another pair couldn’t at first see that they had covered a half, when I spoke to them (FS3.9).  
  Covering the large shapes was very busy. I could have used more hexagons: the children were queuing up to use them (FS3.11). |
| 2.3 Management  | The pairs worked well and the children moved quickly from one shape to the next (FS3.14). |

**Category 3 Children’s responses to tasks**

| 3.1 Enjoyment  | The children were pleased to see me and looked in disbelief when I said we were doing geography (FS3.15). |
| 3.2 Engagement and perseverance  | The activity of covering the shapes was very busy and very excited. The children initially wanted to show Chris and I everything that they had made, and were quite insistent that we should look at what they had (FS3.12). |
Estimating first then counting was a real highlight for a lot of the children. They became quite excited about reporting back to Lyn or myself about what they estimated and what they counted (FS3.13).

Table 6.12  
*Analysis of area syllabus lesson 3*

| Category 1: Learning outcomes | Children used the mathematical terminology and concepts introduced in previous lessons (TS3.8, FS3.4).  
Children used superimposing to demonstrate the areas were the same (TS3.3, FS3.3).  
Children predicted that the area would remain the same (TS3.11, TS3.12, FS3.5, FS3.7, FS3.14).  
Children counted the total number of tiles used (TS3.2, TS3.6, TS3.16, FS3.2, FS3.6, FS3.8).  
Children chose appropriate tiles for covering given shapes (TS3.1, TS3.4, RS3.5, TS3.9, TS3.13, TS3.23, FS3.1). |
| Category 2: The lessons: tasks, resources, suggested extensions | Children needed more time to complete the activity, so one activity would have been sufficient for the lesson. (FS3.17, FS3.10).  
Children in one class had difficulty with too many small pieces of jigsaw (TS3.15).  
More small hexagons were needed to cover the large hexagon (FS3.9, FS3.11).  
Group work was cooperative and successful (TS3.18, FS3.14). |
| Category 3: Children’s responses to tasks | Children enjoyed the activities (TS3.21, TS3.20, FS3.15, FS3.12).  
Children stayed on-task for an extended period of time (TS3.21, RS3.22, TS3.20, TS3.22).  
Children in one class enjoyed the opportunity to estimate, then count, the number of tiles used (FS3.13).  
Children discussed the tasks and were able to justify their answers (TS3.19). |
| Summary and comparison with intended outcomes | The lesson enabled revision and use of terminology and concepts introduced in previous lessons.  
Children predicted that the two areas would remain the same, before the jigsaw cutting commenced.  
Children chose appropriate tiles for covering the shapes, although the large hexagon was difficult in one class.  
Children counted the tiles by ones, with one instance of doubling and grouping.  
One activity for the lesson would have been sufficient.  
Cooperative group work and discussion were successful.  
Children enjoyed the tasks and remained engaged for an extended period of time. |
6.2.1.8 Area syllabus lesson 4

In lesson 4, pairs of children made shapes with elastic bands on a geoboard, each shape having an area of 8 squares, and recorded the shapes on dot-grid paper. Children then drew other shapes with an area of 8 squares on the dot-grid paper, trying to fill the whole paper.

Table 6.13 Interview comments area syllabus lesson 4

<table>
<thead>
<tr>
<th>Category 1 Learning outcomes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.1 Attribute</strong></td>
<td>I was pleased to see that the slower children picked up the concept of area as the measure of surface (TS4.14).</td>
</tr>
<tr>
<td><strong>1.2 Conservation</strong></td>
<td>When you asked them, do they have the same area? They were really confident, and they were going, Yes, they’ve got the same area, they’ve got eight squares. A lot of them really grasped that quite easily and they understood that, even though it was a different shape, they had the same area because of the squares in it, so that was quite good (TS4.1). The children seemed to understand the concept that their shapes still had an area of eight squares, regardless of the shape. It is really good that most of the children have achieved the expected aims, and not just the top few (TS4.3). I asked Jenny last week about whether the areas were the same, regardless of their shape. Today I looked at all her shapes with an 8 square area. I asked her &quot;Which one is the biggest?&quot; And straight away she just said, &quot;They're all the same area.&quot; So that was good (TS4.4). This was the best lesson out of them all. We can see that so many more of them understand (conservation) now. (TS4.2). He said: “That's half a square, so that's only half. So if we want to do a big, long diagonal, it would have to be 16 triangles”. (This was) as opposed to 8 squares. So he knew that the area of 16 triangles was the same area as 8 squares (TS4.12).</td>
</tr>
<tr>
<td><strong>1.5 Align units</strong></td>
<td>They've got covering and the gaps and overlaps and that stuff. Using informal units and counting those, that seems to be quite OK, doesn't it? (RS4.5).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Category 2 The lessons: tasks, resources, suggested extensions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2.2 Resources</strong></td>
<td>The geoboard was a good introduction to it, although some of the shapes were actually a bit hard to manipulate on the geoboard (TS4.6). It (shapes on the geoboards) was something that could be changed (clicks fingers) immediately. So it wasn’t like putting this area in front of them and then putting another one in front of them and they had to switch. They could actually change the area in front of their eyes and get the area in more than one form. They were doing it (TS4.7). I thought that activity was excellent. I liked the way they were all working and they all seemed to know exactly what they were doing (TS 4.18).</td>
</tr>
<tr>
<td><strong>2.3 Management</strong></td>
<td>Perhaps the instruction to number the squares assisted them to keep on track, and also focused them on how many to draw (CS4.17).</td>
</tr>
</tbody>
</table>
The children in my maths group have used the geoboard to make a variety of shapes. Perhaps they could copy these shapes onto the dot paper, the same way as they did today (TS4.16).

**Category 3 Children’s responses to tasks**

<table>
<thead>
<tr>
<th>3.1</th>
<th>Enjoyment</th>
<th>This lesson was really good, and they enjoyed it (TS4.9). And really enjoyable! (TS4.11).</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2</td>
<td>Engagement and perseverance</td>
<td>They were right into it. It was great! (TS4.10)</td>
</tr>
<tr>
<td>3.3</td>
<td>Discussing</td>
<td>The children were discussing what they were doing, amongst themselves (ST4. 13).</td>
</tr>
</tbody>
</table>

Table 6.14  *Field notes area syllabus lesson 4*

**Category 1 Learning outcomes**

<table>
<thead>
<tr>
<th>1.1</th>
<th>Identify attribute</th>
<th>One child suggested the shape could have turned at right angles, which was a good idea, and obviously set the others thinking (FS4.2).</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>Conservation</td>
<td>When we stopped and came back to the mat, the children had lots of suggestions for different shapes to be made with an area of 12 squares (FS4.4).</td>
</tr>
<tr>
<td>1.3</td>
<td>Counting to calculate area</td>
<td>They did not seem to be thinking of any shapes beyond the 8x1, at that stage and I had to push quite hard to get a suggestion of a T shape (FS4.5). The children were quick to catch onto the idea of shapes with an area of 8 squares (FS4.14). The children started work very quickly. And most did the rectangles first, 1x8 and 2x4, and then started to do cross shapes (FS4.13).</td>
</tr>
</tbody>
</table>

**Category 2 The lessons: tasks, resources, suggested extensions**

<table>
<thead>
<tr>
<th>2.1</th>
<th>Tasks</th>
<th>I didn’t see any problems with the drawing from the geoboard (FS4.3). Those who needed assistance seemed to have trouble in counting the squares. I showed these children how to put their finger tip into the middle of each square (FS4.6). Several numbered the dots, instead of the squares. I helped one girl who could count the squares, but couldn’t see how to draw the outline of the shape she was pointing to on her dot paper (FS4.7). This seemed to be a good exercise for looking at a set area in many different orientations and shapes (FS4.8). Explaining what to do with the geoboard was no problem, although it took several children to find one who could tell us that the dot paper and the geoboard were related by the placement of the dots and pegs (FS4.).</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3</td>
<td>Management</td>
<td>Some children slipped a little and put two numbers in a square: I must remember to warn children against that when I do the other two classes (FS4.10).</td>
</tr>
</tbody>
</table>
It was good to have the demonstration on A3 dot paper, so we could talk about making shapes and lines as I drew, and then were able to number the squares (FS4.11).

**Category 3 Children’s responses to tasks**

<table>
<thead>
<tr>
<th>3.1</th>
<th>Enjoyment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kim returned the box of superimposing shapes and the blindfolds to me. She had been using them for lesson breaks and the children also had access to them for spare time activities. Kim said they loved to just play with them (FS4.16).</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3.2</th>
<th>Engagement and perseverance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Everyone had at least one shape when I recalled the class to the mat. One student had filled her dot paper entirely, and was very proud of herself (FS4.1). Once they started working, the whole class was quieter than usual. I think they found it challenging, but most children were able to draw the shapes unassisted (FS4.9). When they came to the mat to show us their shapes, several children had completely filled the sheet (FS4.12).</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3.3</th>
<th>Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td>When the children started working, I was impressed by the way they shared the geoboards, and really talked to each other, about what they were doing (FS4.13). The children started quickly and most made and drew a long, 8x1 rectangle, first. They shared and discussed the geoboards and their shapes – the cooperation and collaboration was good (FS4.15).</td>
<td></td>
</tr>
</tbody>
</table>

---

**Table 6.15 Analysis of area syllabus lesson 4**

<table>
<thead>
<tr>
<th>Category 1: Learning outcomes</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Category 2: The lessons: tasks, resources, suggested extensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Some children needed assistance with counting and drawing the squares (FS4.6, FS4.7, FS4.10). Children copied the geoboard shapes onto the grid paper (FS4.3). The task focused on looking at area in different orientations and shapes (TS4.7, FS4.8). The task required specific instructions and a demonstration (CS4.17, FS4.9, FS4.11). Geoboard was a useful resource (TS4.6, TS4.7, TS4.16).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Category 3: Children’s responses to tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children had used the bags of superimposing shapes as an enjoyable spare-time activity (FS4.16). Children were engaged and completed the task (TS4.18, TS4.10, FS4.10, FS4.9, FS4.12). Paired discussion was very good (FS4.13, FS4.15) Children enjoyed the task (TS4.9, TS4.11).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Summary and comparison with intended outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>This activity was popular with both the children and the teachers. Teachers commented that using the geoboard allowed children to see clearly how the shapes changed but the area remained constant. The task encouraged children to identify shapes in different orientations,</td>
</tr>
</tbody>
</table>
but required careful explanation and demonstration.
Children were engaged and some children completely filled their grid recording paper.
Children discussed the activity with their partners.
Children enjoyed the activity.

6.2.1.9 Area syllabus lessons final teacher interview

Teachers were interviewed after the sequence of four lessons was implemented, inviting comments about the success of the lessons and the student learning which they had observed. The interview transcriptions were analysed using the same categorising and coding systems.

Table 6.16 Teachers’ final interview comments: Area syllabus lessons

<table>
<thead>
<tr>
<th>Category 1 Learning outcomes</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.1 Identify attribute</strong></td>
<td>A lot of them had no idea when they started with like, what is Area? By the end of it, they really understood it, I think, especially by the final lesson that we had this morning. It was like, my God-they seemed to get it (Final S1). I was pleased to see that the slower children picked up the concept of area as the measure of surface (Final S4). The children have learnt how to compare the areas of shapes (Final S8). Overall, most of the children have learnt new concepts. The children in the top group have consolidated the understandings they already had (Final S9). As an entire class, I think they actually would be able to define what area is now, most of them. Before it was probably something that you did in maths, now they can see the uses of it and they would be able to tell you it was a space covering something and they really understand what it is (Final S10). When I ask questions, my children know a lot more. I am finding that in the area questions in Mentals, which they certainly know about area (Final S16).</td>
</tr>
<tr>
<td><strong>1.2 Conservation</strong></td>
<td>When you were explaining the lesson today about the area of the shape being eight squares, I thought there's no way they can do this. But most of them were quite good. And they could understand that they needed eight squares in their shapes and they were coming up with these interesting shapes while maintaining that the Area had to be eight squares. So that was quite interesting (Final S2).</td>
</tr>
<tr>
<td><strong>1.4 Choose appropriate units</strong></td>
<td>In lesson 3, Morris told Carrie that she was better off with larger tiles than the small triangles she was using, because they wouldn't take so long, and she wouldn't have enough of the small ones (Final S3). In general, they have a much greater understanding of area. They are very happy to discuss their work; particularly why they chose different shapes to use as tiles (Final S6).</td>
</tr>
</tbody>
</table>
When we were painting, the children asked would we have enough finger paint to cover all of the surfaces. I heard the comment, when we were pasting raindrops on umbrellas in a craft lesson, that we would need fewer if they were not pasted on top of each other (Final S5).

Category 2 The lessons: tasks, resources, suggested extensions

2.2 Resources

The resources are very simple, cheap, and well organised in the plastic bags. At my last school we bought a lot of resources and they were very expensive (Final S20).

2.3 Management

When I made up the pairs I sort of tried to do it across the board, so a less able student with a more able student. I think that was quite encouraging, and they were really cooperative (Final S13).

It was good to have the better children paired with a slower student, because the better student was able to explain what to do, and what was happening (Final S15).

Category 3 Children’s responses to tasks

3.3 Discussion

The children cooperated very well in the small groups of two or three. I often have them working in groups, but these are usually larger, often six children. The children were discussing what they were doing, amongst themselves (Final S14).

Discussing and sharing ideas with each other. I was watching Ron and Jim talking about their work. The activity was wonderful for Jim, because he had Ron explaining it all to him. It was also good for Ron, to be able to explain to someone else (Final S23).

Category 4 Teachers’ future directions

4.1 Focus on mathematical concepts

There were quite a few follow-up activities that were to do with lesson 1, which I don't do. I tend to just pick out a lesson in isolation, but that was good because things were following up and the learning seemed to be more effective in that sense (Final S18).

I find that when I'm teaching maths that I don't plan sets of lessons well enough. Like that is a fantastic set of lessons that will naturally flow from each other, and I tend more to teach, not single, but maybe a set of two lessons. And I probably would get a lot more out of the children if I taught four. Or if I actually kept extending, and I would keep extending those lower ones, and let those top ones fly, really. That's one thing that it's taught me (Final S24).

4.2 Classroom management and grouping

And just back on that working in pairs, I think that was particularly important because we do group work, but there will be a group of four, and some of them just tend to go into the background and not get that hands-on experience, whereas in this way, they all get to sort of experience the lessons (Final S17).

I liked the way the groups of two or three worked. (Final S19).

4.3 Responding to children’s needs

Maybe going more slowly and making sure that children understand everything before we progress to the next unit. Maybe correlating more maths substrands into lesson planning (Final S22).

I realise that we need to spend more time on basic understanding and not assume they know concepts, such as the properties of 2D shapes. Perhaps I need to spend more time on area. But this is in the context of needing to use the school’s text book. The textbook is quite good, but you do have
to use the materials which they suggest for the activities. We need to educate parents to the concept that we don’t need to use every page in the children’s textbooks (Final S21).

The first thing that I’ll always remember is the string and the shape. It’s not about opening where they're up to and being told Year 2 does this part of the syllabus. It's about working out where they're up to and then going from there (Final S25).

Table 6.17  
Analysis of final interview comments: Area syllabus lessons

<table>
<thead>
<tr>
<th>Category 1: Learning outcomes</th>
<th>Children understood the attribute of area and the concept of measuring area by comparing (Final S1, Final S4, Final S8, Final S9, Final S10, Final S16).</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Children understood that the same area can be presented in different shapes (Final S2, Final S3, Final S6).</td>
</tr>
<tr>
<td></td>
<td>Children understood the importance of aligning units correctly (Final S5).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Category 2: The lessons: tasks, resources, suggested extensions</th>
<th>The resources were well-organised and economical to produce (Final S20).</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The teaching strategy of working in small groups of mixed ability was very successful in providing opportunities for discussion and cooperative working (Final S13, Final S15).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Category 3: Children’s responses to tasks</th>
<th>The children responded positively to working in small groups and sharing ideas and knowledge through discussion (Final S14, Final S23).</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Category 4: Teachers’ future directions</th>
<th>To implement sequence of lessons, rather than just one or two area lessons, to ensure successful consolidation of the concept of area (Final S18, Final S24).</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>To make groups of two or three children instead of larger groups to allow for more interaction with the materials and each other (Final S17, Final S19).</td>
</tr>
<tr>
<td></td>
<td>To ensure that children understand concepts before new work is presented (Final S21, Final S22, Final S25)</td>
</tr>
<tr>
<td></td>
<td>To be more aware of integrating mathematics topics (Final S22).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Summary</th>
<th>The syllabus lessons were successful in assisting children to understand the concepts of the attribute of area and the measurement of area by comparison.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Children understood how to align units correctly to measure area.</td>
</tr>
<tr>
<td></td>
<td>The teaching strategy of small and mixed-ability groups was successful and allowed for more interaction and discussion than the teachers’ use of larger groups.</td>
</tr>
<tr>
<td></td>
<td>Teachers found the sustained focus on area, through the sequence of lessons, to be a successful teaching strategy.</td>
</tr>
<tr>
<td></td>
<td>Teachers articulated the importance of identifying children’s current and continuing understanding when teaching mathematical concepts.</td>
</tr>
</tbody>
</table>
6.2.1.10 Area research lesson 1

During lesson 1, children experimented with making lines and shapes with a length of braid. They measured the area of their shapes with cardboard tiles which had a side length of 10 cm. Children predicted how many tiles would be needed to measure the area of the top of their desks and then checked with tiles.

Table 6.18 Interview comments area research lesson 1

<table>
<thead>
<tr>
<th>Category 1 Learning outcomes</th>
<th>1.1 Identify attribute</th>
<th>1.3 Counting to calculate area</th>
<th>1.4 Choose appropriate units</th>
</tr>
</thead>
<tbody>
<tr>
<td>I saw them saying, Well we want to make a shape, so it can’t be a line. So we need to fix this, you know, I’ve got the wrong bit of equipment kind of thing they were saying (TR1.3). And then they realised that no, they were just lines, they were just open. And then they would realise they had to actually join them to make the shapes (TR1.5). These children seem to talk about the word area more naturally than what I’ve heard other children do (CR1.6). We basically had the discussion first about Area, and when I was listening to them, I got the feeling that Area is somewhere where you’re not allowed to go (CR1.10). I’m surprised they didn’t know the word “Area”. Because I have used the word “Area”. I haven't written it on the board as such, maybe because it was written and they were slightly puzzled by that, I don’t know (TR1.11).</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The counting by fives. I think that was the one that I sort of thought, Well I haven’t done that yet, but maybe they’re ready for it now. Counting by fives and tens definitely (TR1.4). A lot of them are picking up the idea of not counting by ones. They were looking at counting by fives, or, some of them were just counting by twos to start with, and then they were trying to work out a faster way. I could see that the ones who have a very good concept of number, very quickly worked out ten rows of five or whatever. (TR1.7). I was surprised by their counting in tens (TR1.17). Perry said three rows of three, well that’s nine. We have only just touched on that once incidentally, and that’s sort of a little bit further down the track, is doing rows of, but that impressed me straight away, that stood out (TR1.28).</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No, it was interesting, all the different shapes and then seeing what the other groups found and then trying to change what the other group found to make theirs different, so they weren’t exactly the same (TR1.24). We were talking about the shapes and why they were making the shapes and they said, Oh, because these are square and they would fit easier into a square and in a triangle they would be sticking out (TR1.18). And then when they were actually putting the squares to cover the area of the shapes that they'd made, they realised that something that had curved edges was difficult because they had to use the squares (TR1.26).</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1.5 Align units

They thought that it was important that the shapes didn't overlap because that wouldn't give them a true reading of how many shapes, and they were really concerned with whether there were any spaces left and what they could do (TR1.1).

The covering doesn't seem to be a problem. When they had that very structured shape, like the desk, there were no overlaps, so that's not a problem either. The gaps - there were minimal gaps (CR1.2).

Some I noticed weren't very concerned about the fact there were gaps or overlaps. But some of it was also because they had difficulty manipulating the cardboard, getting it to stay in the right spot. But I'm sure you'll be revising that, anyway (TR1.12).

They all started along the edges. I asked them why and they said, Well if you start in the middle, there will be some left over and you will have to do it all over again (CR1.19).

When they were using the big squares on the floor, some of them had just piled them on and you could see them bouncing off other children saying, No, you can't just put all the squares on, they need to be no gaps, like that one's got a gap and no, that one's overlapping (TR1.21).

Some of them on the tables with the small squares were still overlapping (TR1.22).

I asked how were they arranged on the desk, was there anything that you could really see? He said, oh yes, lines, lines. Lines everywhere (pointing at the grid lines formed by the borders of the aligned tiles). Lines this way and lines that way (CR1.23).

1.6 Superimpose

So my impression of the afternoon was that they can all cover, they know about gaps and overlaps, they know about comparing, but only by putting one on top of the other, like superimposing, but there was no idea at all about measuring one with something and comparing that (CR1.8).

Category 2 The lessons: tasks, resources, suggested extensions

2.1 Tasks

For an introduction, I thought that today was fine. It was something that they could succeed in, that wasn't too hard for them. They came to some sort of ideas themselves, so they were successful at it. And it was very open ended too, the first one especially with the shapes (TR1.14).

The problem solving and trying to make sure there weren't gaps with the closed braid was interesting. We've always done it the other way, where we've given them a fixed shape and they've had to cover it, and this time it was changing the shape to suit the covering, the area. So that was interesting (TR1.25).

2.3 Management

I like the way you were coming back and actually asking them to explain exactly what they learnt and telling you in their words. And you were checking to see that you understood what they were saying, or getting them to draw on the board, if you didn't quite understand their description (TR1.17).

2.4 Suggested extensions

Maybe confront them with the problem of how, if it's not like a nice neat square shape, how you are going to measure it accurately, and they could maybe come up with the suggestion of cut the squares in halves, and go by halves, because there's a little bit of..... (TR.1.13).
3.2 Engagement and perseverance

It was great. I loved the braid, using the braid for lines. Interesting to hear them talking about whether they were lines or whether they were shapes. And then making the shapes, and wanting to name the shapes and then actually using them... It was all just because it was really hands on and they were thinking together and working together. There was a lot of busy activity (TR1.16).

They just worked really well. And they were all on task, too. There wasn't anyone who wasn't interested (TR1.20).

I liked it the way they all were, most of them were working quite well together and taking it in turns to have a turn at making a shape (TR1.27).

3.3 Discussion

One group on that lot of tables up the back fitted on 55 squares. And the other group only had 50 and then they were looking at each others’ groups and saying well, why have you got more? And then they started looking, comparing the two tables (TR1.15).

Table 6.19 Field notes area research lesson 1

<table>
<thead>
<tr>
<th>Category 1 Learning outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Identify attribute</td>
</tr>
<tr>
<td>The children easily were able to make appropriate shapes with their braid, to be measured with the triangles and the squares. (FR1.1).</td>
</tr>
<tr>
<td>The children made excellent lines and shapes (FR1.22).</td>
</tr>
<tr>
<td>1.3 Counting to calculate area</td>
</tr>
<tr>
<td>Counting by tens was no problem for the children, but a surprise for Sue (FR1.2).</td>
</tr>
<tr>
<td>One pair of children folded their square tiles to fit them into the remaining space inside the braid shape. Toni... was impressed with the way the children were then able to count up the half shapes to give a total (FR1.3).</td>
</tr>
<tr>
<td>I asked the children to estimate how many would cover. Mark estimated 50, (five groups of ten) and the next closest was 40. One student said 32 and when I asked why, replied four groups of eight (FR1.6).</td>
</tr>
<tr>
<td>We later discussed different ways of counting, although most children had reverted to counting by ones when counting on their desk (FR1.24).</td>
</tr>
<tr>
<td>The activities worked really well, and the children were already demonstrating their ability to skip count and estimate numbers (FR1.28).</td>
</tr>
<tr>
<td>1.4 Choose approp. units</td>
</tr>
<tr>
<td>The children easily were able to make appropriate shapes with their braid, to be measured with the triangles and the squares (FR1.27).</td>
</tr>
<tr>
<td>1.5 Align units</td>
</tr>
<tr>
<td>The children were very impressive when they came back to the mat and reported on the shape they had made, the arrangement of squares, and the total number of squares (FR1.5).</td>
</tr>
<tr>
<td>We also found that the children to able to recall the pattern of the square tiles, quite easily (FR1.18).</td>
</tr>
<tr>
<td>Children worked really hard at making sure there were no gaps and no overlaps (FR1.19).</td>
</tr>
<tr>
<td>The small squares were completed quickly. Some children had spaces on their desks, because there had been slight overlapping (FR1.25).</td>
</tr>
</tbody>
</table>
Category 2 The lessons: tasks, resources, suggested extensions

2.1 Tasks

2.2 Resources The braid was obviously easy to handle and of an interesting length (FR1.11).

The braid was obviously easy to handle and of an interesting length (FR1.11).

The children loved the braids. The lines they made were ordinary (no loops), but they also did some other interesting things with the braids, including measuring each other and objects in the classroom (FR1.26).

2.3 Management The children found working with a small group and sharing materials quite difficult. I spoke with Toni about having the children in pairs, next time, and producing more packets of materials so this would be possible. When I asked several desks to work together and pool their tiles, the children weren’t very happy (FR1.8).

Category 3 Children’s responses to tasks

2.2 Resources The resources are very simple, cheap, and well organised in the plastic bags. At my last school we bought a lot of resources and they were very expensive (Final S.20).

3.1 Enjoyment The children really enjoyed working together, particularly in the first activity in groups of four (FR1.16).

3.2 Engagement and perseverance The children made interesting lines and shapes. The lines had very precise curves or zigzags, which were modelled carefully on the floor. It took some groups some hard thinking to work out how to make a shape (FR1.4).

They were also uncertain of how to commence working, when they had to solve a problem for themselves. Few hands were raised when questions were asked. (FR1.7).

All groups worked together well, except for Mark and two girls. Mark only wanted to do things one way, and the girls eventually ignored him and did it their chosen way, while Mark sulked. I felt quite cross with him and told him so. Fortunately, he had decided to cooperate by the end of the lesson, and was able to give us some excellent answers (FR1.13).

As I walked away, to commence the first interview, Matt came with me and very sincerely said that he felt as though he had been doing maths all day, and it still wasn’t morning tea time (FR1.14).

3.3 Discussion The lesson was very busy, talkative and cooperative. The children really enjoyed working together, particularly in the first activity in groups of four (FR1.10).

The whole-class discussion was keen and the children wanted to talk about area (FR1.12).

The children were enthusiastic, very communicative, very easy to give instructions to, and came up with some excellent ideas. They really wanted to talk about what they were doing and seeing. (FR1.17).

The talking and sharing from the rest of the class was excellent (FR1.21).
**Table 6.20 Analysis of area research lesson 1**

| Category 1: Learning outcomes | Children made lines and shapes (TR1.3, TR1.5, FR1.1, FR1.22).  
|                              | Children discussed the concept of area (CR1.6).  
|                              | Children were unfamiliar with the concept of area (CR1.10, TR1.11).  
|                              | Children counted tiles in multiples of twos, threes, fives, eights and tens (TR1.4, TR1.7, TR1.28, FR1.2, FR1.6, FR1.24, FR1.28).  
|                              | Children chose appropriate tiles to measure the shapes they made (TR1.18, TR1.26, FR1.3, FR1.27).  
|                              | Most children aligned units correctly, explained why “no overlaps” was important (TR1.1, CR1.2, CR1.19, TR1.21, TR1.22, CR1.8, FR1.5, FR1.19).  
|                              | Children described the grid pattern of the repeated square tiles (CR1.23, FR1.18). |

| Category 2: The lessons: tasks, resources, suggested extensions | Tasks weren’t too difficult and were open-ended (TR1.14, TR1.24, FR1.11, FR1.26).  
|                                                               | Task was a reversal of the usual area activity, where children are given shapes to be measured (TR1.25).  
|                                                               | Children were encouraged to explain their actions (TR1.17).  
|                                                               | Cooperative groups needed to be arranged carefully (FR1.8). |

| Category 3: Children’s responses to tasks | Groups of children discussed the shapes they made (FR1.4, FR1.10).  
|                                          | Children were engaged and worked cooperatively (TR1.16, TR1.20, TR1.27).  
|                                          | Children discussed shapes in their groups and as whole class (FR1.13, FR1.12, FR1.21).  
|                                          | Children persevered with the task (TR1.15).  
|                                          | Children enjoyed the activities (FR1.16, FR1.17). |

| Summary and comparison with intended outcomes | Children in some classes were unfamiliar with the concept of area.  
|                                              | Children selected appropriate tiles and aligned them correctly to cover the area of the shapes. The researcher commented that the children were familiar with covering with units and superimposing, but had no concept of using the tiles as units of measure.  
|                                              | Children counted the tiles in multiples of twos, threes, fives, eights and tens.  
|                                              | Children made reasonable estimations of the number of tiles needed to cover shapes.  
|                                              | Children impressed their teachers with counting by multiples and also adding half units.  
|                                              | Teachers and children appreciated the open-ended nature of the braid activity.  
|                                              | Children enjoyed the tasks and worked cooperatively.  
|                                              | The activity was excellent for encouraging discussion in groups and as a whole class. |
6.2.1.11 Area research lesson 2

In lesson 2, children used 5 cm square paper tiles to cover a cardboard square with sides of 15 cm. Children also covered an equilateral triangle with small coloured paper triangles. Pairs of students were encouraged to experiment with and discuss colour variations, before pasting the paper tiles.

Table 6.21 Interview comments area research lesson 2

<table>
<thead>
<tr>
<th>Category 1 Learning outcomes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3 Counting to calculate area</td>
<td>The estimation I thought was very good because we've done estimation just very briefly and you could see them sitting there and trying to calculate the rows of, because we've only done rows of a couple of times incidentally. I thought the estimation were fairly reasonable (TR2.2). They explained that they could see when they had six that there was room for another row or another column, and it would be nine (CR2.24).</td>
</tr>
<tr>
<td>1.4 Choose appropriate units</td>
<td>I saw two children who started putting the triangles in the square and then they thought, No there's a better way. And then they put the triangles back and continued on with the squares (TR2.28).</td>
</tr>
<tr>
<td>1.5 Align units</td>
<td>“What could you have done differently when you placed them, when you glued them down?” And they'd say, “Well, I can see that that's overlapping there, maybe I should have moved it up” (TR2.1). But they did find it hard to make a pattern with the triangles. I think some of them found it hard to make a pattern, they just pasted. Some of them found it hard to have no gaps as well. Bit of a worry (TR2.6). And those that had gaps were mainly because they had difficulty with their fine motor skills or they started right up the top of the triangle, above the black line and when they got to the bottom there was a little space (TR2.8) So it wasn't like she was actually thinking like this is a triangle, and that's a triangle and we could align the both sides and it might fit (TR2.18). I really wanted them to start seeing the repeated unit, and I'm sure they're doing that now (CR 2.21). You had said You have to be as accurate as possible. They've got the idea (of aligning). They understand the concept (TR2.32).</td>
</tr>
</tbody>
</table>

Category 2 The lessons: tasks, resources, suggested extensions

| 2.1 Tasks | If they haven't printed like this before, maybe they don't really know what's going to happen when they do print. It's not something that they're seeing from above; you have to actually maybe look underneath it to see what's going to happen (CR2.4). I don't think they quite knew what to expect when they started stamping (CR2.5). They pasted three down, and then they went to reverse it but they saw |

140
the middle colour being the same when they reversed it, so they thought, oh that's not a pattern (TR2.25).

Following a lesson that I had with a Year 2 last week, the teacher suggested that they do the paper first and then the stamps, and then they'll see the relationship between the stamps and the paper. And all of your kids did. Most of them did the square first, and just put the sponge in exactly the same places as they had done with the paper. Whereas with other classes, particularly Year Ones, they just sort of go out there and it's open slather (CR2.7).

From seeing Wayne go from his pasting to his painting, it might have been good to have them all paste first, and then paint. If we wanted to develop them using a pattern and then seeing just those nine squares, they might have picked up the idea from the pasting and transferred it to painting (TR2.10).

Maybe the painting was more suitable for some of the other kids. I just thought the gluing focussed them more, to what they were doing, or what they had to do. Whereas the other one they were looking more at explaining (TR2.13).

I thought that the pasting was really good and the shapes; the sponge painting was good, but not as good as the piecing together like a jigsaw. I suppose it's the same thing, that they're using stamps (TR2.12).

I thought they approached the pasting a little bit quickly. That maybe they might have thought a little bit more about the colours (CR2.17).

I could've had more containers of the shapes ready, so they were able to spread out a bit more. And maybe if they had been able to spread out a bit more and sort of sit more comfortably, they might have looked more carefully at the patterns (CR2.9).

Have you ever used other shapes, like triangles fitting into hexagons or specifically do you use the triangle and the square? (TR2.11).

Carol, who likes structure. She likes things to be organised and she's very methodical, loved it (TR2.23).

It was fun and it was nice to see them enjoying and learning, which is good to do (TR2.27).

Because they actually worked it out for themselves, rather that someone saying, This is what you do (TR2.3).

I think working on a table with other children and watching what they did. And seeing people making patterns and they doing something and looking and saying, Oh wow! Look what I could have done. So just learning from working with others as well (TR2.15).

They all individually had to put the shapes onto the larger shapes. They were busy the whole time. They all had something to do. They could experience for themselves, it is sometimes good to work on their own rather than working in a group. Most of them had to discover for themselves and they weren't really copying each other, and they were manipulating the triangles (TR2.16).

I think she looked around and thought What was I thinking, I've done it wrong. And she just started again and did it properly (TR2.22).
They were very helpful to each other. Passing bits of paper. They were good. I sort of forget that it's Friday afternoon (CR2.26).
Ann sat straight down (in free time) and decided to cover with the cubes. You think well they've obviously enjoyed what they've done, so they want to keep going, and that's really good (TR2.30).

### Discussion

A few of them were talking about the patterns that they were making. Oh, I've run out of this colour, what can I do? (TR2.33).

### Table 6.22 Field notes area research lesson 2

#### Category 1 Learning outcomes

<table>
<thead>
<tr>
<th>1.3 Counting to calculate area</th>
<th>I asked the children to estimate how many stamps and was surprised to hear thirty, then nine. Mat estimated twelve, in four rows of three (FR2.8)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>We revised what we had done last week - Area, rhombus, the number of squares needed to cover the desk, how they were arranged, counting the squares by ones, fives, tens, twenties (FR2.9).</td>
</tr>
<tr>
<td>1.4 Choose appropriate units</td>
<td>The early finishers had a great time. One child was trying to continue the theme of our lesson, by covering a triangle with counters. She decided that they didn’t work, so she took them off (FR2.13).</td>
</tr>
<tr>
<td>1.5 Align units</td>
<td>Of all the children I observed, only one started pasting the squares along the bottom row. All of the others started in the top left hand corner. Pasting went really well, with only a couple of children needing to pull up shapes and paste again, when they discovered large spaces left (FR2.10). The Year 1 girl pasted hers on quickly and then took it all off again when she saw what the other children were doing, so carefully and exactly (FR2.14).</td>
</tr>
</tbody>
</table>

#### Category 2 The lessons: tasks, resources, suggested extensions

| 2.1 Tasks                     | Not all children made repeated patterns of colour, and I think they may need more practice at pattern making and discussing to do this better (FR2.3). |
| 2.3 Management               | My instructions were really clear and slow. We had one area for squares and one for triangles. The children moved really well between the containers, they were very cooperative and sensible (FR2.2). They really didn’t have time to think much about the patterns they could make with the shapes and patterns. I decided to give more specific instructions about what to do, with the next class (FR2.4). Overall, the lesson was too rushed and a little disorganised, with children moving around the room to find the colours and shapes they particularly wanted (FR2.5). I didn’t have enough containers of shapes, so the children had to move around a bit and find the colours they wanted (FR2.6). I had decided, after talking to Stephanie, that it would be a good idea to paste first and sponge second, and perhaps making the similarity more obvious to the children (FR2.11). The children tried to clean up the sponges, and they took a long time, got themselves wet, and also made a bit of a mess at the sink. (FR2.16). |
### 3.2 Engagement and perseverance

The slowest children seemed quite unconcerned about the time they were taking, and just kept plodding along. They couldn't be hurried to finish (FR2.7).

### Table 6.23 Analysis of area research lesson 2

| Category 1: Learning outcomes | Children estimated the number of tiles by using rows and columns and multiples (TR2.2, CR2.24, FR2.9).  
Children revised the previous lesson by recalling the multiples of tiles used (FR2.9).  
Children chose appropriate tiles for covering (TR2.28, FR2.13).  
Children aligned tiles correctly (TR2.1, TR2.32, FR2.10, FR2.14).  
Children found the triangular tiles difficult to align exactly (TR2.6, TR2.8, TR2.18).  
Children identified the pattern of the repeated unit (CR2.21). |
| Category 2: The lessons: tasks, resources, suggested extensions | The tasks indicated that the children need more practice with pattern making (TR2.25, FR2.3).  
The materials needed to be carefully managed with clear instructions and ready access to the painting and coloured tiles (TR2.10, CR2.7, CR2.17, CR2.9, FR2.2, FR2.4, FR2.5, FR2.6).  
The printing was messy, difficult to manage (CR2.4, CR2.5, FR2.16).  
Children didn’t automatically see the relationship between the paper tiles and the printing (TR2.13, FR2.11).  
The task could be extended by using triangles which cover hexagons (TR2.11). |
| Category 3: Children’s responses to tasks | Children persevered with the task until they were satisfied with their product (FR2.7).  
Children enjoyed the activity (TR2.23, TR2.27).  
Children observed peers for additional ideas (TR2.15, TR2.22).  
Children enjoyed the individual work (TR2.16).  
Children helped each other by passing materials (TR2.26).  
Children chose a similar task in free time (TR2.30).  
Children discussed their patterns in table groups (TR2.33). |
| Summary and comparison with intended outcomes | Children discussed the previous lesson in terms of rows, columns and multiples used to calculate the number of tiles used to cover an area.  
Children aligned units correctly and systematically to cover the area.  
The triangular shape was more difficult than the square shape to cover with tiles.  
The printing task was problematic because it was difficult to organise ready access to the paints and sponges. Children didn’t see the relationship between the paper tiles and the stamped shapes, possibly because the task was unfamiliar and they couldn’t predict the outcome of the stamping.  
Children enjoyed the tasks.  
Children worked individually, but helped each other in table groups and discussed their patterns. |
6.2.1.12 Area research lesson 3

Lesson 3 focused on using the decorated squares and triangles from lesson 2 to make a group square and triangle and then a class square and triangle. The cumulative number of tiles in the large class square was counted as each row and column was added, to a total of 100 tiles. Children were encouraged to skip count or stress count the number of tiles in each row or column.

Table 6.24 Interview comments area research lesson 3

<table>
<thead>
<tr>
<th>Category 1 Learning outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Identify attribute</td>
</tr>
<tr>
<td>I'm reasonably confident that they've got the picture that it was an area that we were covering and we could say how many squares were in that area (CR3.1).</td>
</tr>
<tr>
<td>1.3 Counting to calculate area</td>
</tr>
<tr>
<td>And I was thinking Wow! That's great. The tens they were fine, they're used to counting in tens, but to do the ones that you don't usually at that grade, 8's and 9's, there were a few voices that stood out (TR3.2). The counting. Definitely the counting. That by far was the one really knockout, noticeable thing (TR3.3). Like you said yesterday, it's a really good strategy for practising the skill of multiplication. It's a nice way of doing it (TR3.4). I've basically only briefly just done counting by 10s, and this week we started counting by twos to 20. But we haven't done counting by any other number, so that surprised me too. So I think I'll be working on that and trying to push that little talent that they've got there (TR3.11). I got some cards and we did it on the desk. And when we got up to 5 x 5, I said, OK, You count by yourselves, and they counted by fives quite naturally. It was good (CR3.12). I just like the correlation so much between number and space and measurement, the whole, and how it's important for them to understand their number first, before they can work on something like Area. And counting rows of (TR3.16).</td>
</tr>
</tbody>
</table>

Category 2 The lessons: tasks, resources, suggested extensions

| 2.1 Tasks | I saw children making rectangles, because they thought they had to use all the squares up to make the biggest square. And they were using all the squares because they thought they had achieved what you'd asked (TR3.9). I thought it was excellent the way we made that square together, and kept enlarging it, and then counting. That was fantastic (TR3.15). That was excellent. I want to do that lesson again. I'll start collecting cereal boxes! (TR3.17). |
| 2.2 Resources | So what I'd like to do is to see if I can get some cards or whatever organised and have them in the room so that they can go on with it themselves. So that's something that I'd like to do (TR3.14). |
2.3 Management
I think the twos worked really well. I can see that with threes in here, there are too many dominant kids that they would've taken over (TR3.13).

2.4 Suggested extensions
It might be interesting to start with a rectangle and look at putting squares on each end, and seeing what happens there (CR3.5). Maybe the children probably could have gone on for a lot longer (counting by higher multiples), but it might also have been lost on some of the lower ability kids (TR3.6). Some of these children might even be capable of looking at the square numbers and sorting out the pattern of them (CR3.16).

Category 3 Children’s responses to tasks

3.1 Enjoyment
It was like some challenge that they all wanted to attempt to solve and they love that, with maths and with other areas of the curriculum (CR3.10).

3.2 Engagement and perseverance
I thought it was quite good the way there were several couples who actually had to sit there and work it out. And you could see that they were quite happy just to sit there and wait just a little bit longer, and take a bit longer to do it. I thought that was good (CR3.7).
I was sitting there looking at them thinking, My goodness that's good. That they're so on task and everybody was on task. You know, there wasn't someone fiddling with their shoes, or whatever (TR3.8).

3.3 Discussion
They were talking to each other about what I think it's this much, or I think it's that much. So it was all about what was going on (TR3.18).

Table 6.25 Field notes area research lesson 3

Category 1 Learning outcomes

1.1 Identify attribute
Children had lots of suggestions about how to cut out the squares and triangles carefully, slowly, so as not to cut off any of the Area, and so they would all fit together properly (FR3.18).
I felt I should have emphasised the total area covered, but Sue thought that this was understood by the children, even Sam, who was sitting next to her, and who made a comment about Area (FR3.28).

1.3 Counting to calculate area
There were no problems in seeing which multiple to start counting in each time. We stopped after 64, and I asked them to estimate how much next time. One boy worked out 81 by using his times tables, as he explained. Stephanie looked surprised that he knew his times tables to 9's (FR3.5).
I told them we would make a very large square with all of the squares. When I asked how many squares we might use altogether, we had a big variety of answers, from at least 14 to 150. They thought 150 was hilarious, not realising that we would easily go to 121 (FR3.12).
I didn't count with them when we started each time - just pointed the ruler and let them lead themselves. It was obvious that Lyn knew her times tables: her voice was strong every time (FR3.14).
To count the 11's I told the children to listen to the pattern of the numbers as I said loudly, 22,33,44,55. I then invited them to do it, and
they were great - they obviously thought they were clever too (FR3.15).
Making the squares went well, with children able to count in 2, 3, most
4, 5, but then having difficulty with other multiples (FR3.21).
The children did not seem to have much idea of how many squares we
could possibly use, when I asked them to estimate. Perhaps their
experiences with number have been held to smaller numbers (FR3.22).
I stopped when we had done the 8's one way, and asked what the total
would be the other way. One student thought it would be different. He
and everyone else laughed when we counted the other way and found it
was still 64 (FR3.24).
Interesting that all the children I asked later were able to remember that
it was a 3x3 square (FR3.26).
Children counted easily for 2, 3, 4, 5 and then had some trouble with
the other numbers, until we got up to ten (FR3.27).

| 1.4 Choose appropriate units | It was interesting watching the children who tried to fit their coloured
triangles into their large paired square, and their simple report: They
won't go (FR3.4).
It was interesting watching two boys trying very hard to use their
equilateral triangles to fill a square space (FR3.13). |
|-----------------------------|---------------------------------------------------------------------|
| 1.5 Align units             | Each time we asked a child to lay on the next squares, they did this
without prompting, and apparently easily, along two sides (FR3.2)
There only appeared to be two pairs who needed assistance. The others
all worked out how to make the largest square from their 12 or 13
pieces (by using 9 of the available squares and discarding the
remainder) (FR3.7).
Not all of the children observed that we were only putting on extras on
two sides at a time. Some seemed to think that each time we were going
all the way around. Perhaps the classroom teacher could explore this
later. Several children needed to be guided by the others when they
attempted to put rows onto the wrong sides (FR3.29). |

**Category 2 The lessons: tasks, resources, suggested extensions**

| 2.1 Tasks                  | All pairs were able to make the square, although Vicki and Sally were
helped by Melanie. Some children were quite puzzled by the extra
squares, and asked what they should do with them. Melanie and I
exchanged smiles at their reactions (FR3.11).
Some children didn't want to believe that I would have given them the
extra squares, which were not used (FR3.23). |
|---------------------------|---------------------------------------------------------------------|
| 2.2 Resources             | The children put all of the squares face up, so that we were looking at
paint or paper patterns, or the cereal box patterns. No-body commented
at this, and I didn't realise it myself until I commented that it looked
like a rug (FR3.10). |
| 2.3 Management            | Toni suggested that children work at desks, with their usual desk
partner. This paired work at desks went really well. There only
appeared to be two pairs who needed assistance (FR3.19).
I had cut an additional 50 squares the night before, after Sue and I
decided that maybe three to a group wasn't so good (FR3.20). |

**Category 3 Children’s responses to tasks**
3.2 Engagement and perseverance

The children were really “buzzy” between counts, as we were waiting for the next lot of squares to go down. Sam took ages to put his down (may have been contrived for effect) but otherwise the children completed this really quickly. Other children leaned over and slightly adjusted squares which may have been a little crooked (FR3.8).

I was impressed by the number of children who were happy to keep placing and moving the squares until they had made a square (FR3.9).

I felt really happy when I walked out of the classroom, as the children had been happy, and very focused, and had done some good problem solving (FR3.17).

The others all worked out how to make the largest square from their 12 or 13 pieces. I was impressed by the number of children who were happy to keep placing and moving the squares until they had made a square (FR3.19).

3.3 Discussion

Children seemed happier to see me, and were ready to start talking straight away. Really wanted to share the thoughts they had about Area, and what we had done the previous week (FR3.6).

I asked two groups to join together - this was very well received and stimulated lots of negotiating and talking, so I repeated this with other pairs, to make groups of four. I then asked one group of four to join with the initial four. Again, this worked really well, so eventually we were all clustered around one large triangle. After loud instructions, and crawling over sides, they decided on one last side to increase, and the triangle was made larger (FR3.16).

Table 6.26 Analysis of area research lesson 3

<table>
<thead>
<tr>
<th>Category 1: Learning outcomes</th>
<th>Children discussed the attribute of area (FR3.18, FR3.28).</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Children understood the concept of measuring an area with tiles (CR3.1).</td>
</tr>
<tr>
<td></td>
<td>Children understood the need to align tiles correctly (FR3.2).</td>
</tr>
<tr>
<td></td>
<td>Children counted the rows of tiles in multiples to 10 (TR3.2, TR3.3, TR3.11, CR3.12, FR3.5, FR3.14, FR3.21, FR3.27).</td>
</tr>
<tr>
<td></td>
<td>Teachers commented on the relationship between counting the rows of tiles and multiplication (TR3.4, TR3.16).</td>
</tr>
<tr>
<td></td>
<td>Children experimented with making equilateral triangle tiles into a large square (FR3.4, FR3.13, FR3.16).</td>
</tr>
<tr>
<td></td>
<td>Children estimated the total number of tiles which could be used (FR3.12, FR3.22).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Category 2: The lessons: tasks, resources, suggested extensions</th>
<th>The small group task required problem solving strategies (TR3.9, FR3.11, FR3.23).</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Teachers commented that the lesson was excellent or they would like to organise tiles themselves for further activities (TR3.15, TR3.17, TR3.14).</td>
</tr>
<tr>
<td></td>
<td>Teachers suggested extensions for the whole class activity (CR3.5, TR3.6, CR3.16).</td>
</tr>
<tr>
<td></td>
<td>Small groups functioned better with two members rather than three (TR3.13, FR3.19).</td>
</tr>
</tbody>
</table>
Children enjoyed the challenge (CR3.16, FR3.19).

Children were engaged (CR3.7, TR3.8, FR3.8, FR3.16).

Children discussed the number of tiles in the large square (FR3.8).

Teachers made positive comments about this lesson:

- They would like to organise tiles themselves for further activities;
- Suggestions for extensions, including a whole-class large rectangle, continuing counting in multiples beyond tens, or investigating the square numbers;
- The relationship between calculating rectangular area and multiplication was made very clear during the whole-class making of a large square;
- Children enjoyed the problem solving aspect of the group task. Children enjoyed predicting the number of tiles in the large square, with each new row.

Children participated in counting the total number tiles, in multiples. Year 1 children were familiar with multiples to 5. Year 2 children showed some familiarity with multiples to 10.

Children understood that they were measuring an area by counting the number of tiles used.

The whole-class square provided an excellent opportunity to investigate number patterns, using larger numbers.

Children enjoyed the problem solving nature of the small group task. Children enjoyed the tasks and discussions and were engaged in both the small group and the whole-class activities.

6.2.1.13 Area research lesson 4

Children worked in pairs to make shapes with 15 cm kebab sticks and measure the area of the shapes with tiles of side length 5 cm. Children recorded the shapes made and the arrangement of the tiles. Children were encouraged to experiment with other shapes which could be measured with the 5 cm tiles. As an extension, children made different rectangular shapes with 12 tiles and counted the number of tiles used by skip counting or stress counting.

<table>
<thead>
<tr>
<th>Category 1 Learning outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3 Counting to calculate area</td>
</tr>
<tr>
<td>It was interesting that they were starting to look more at the L shape again when you were asking them how many squares were in there and the rows and going along that way. So they were thinking about the L, you know how many going down and how many going across they tried to work it out that way (TR4.1). Most of the kids I spoke to were talking about, well there's that many</td>
</tr>
</tbody>
</table>
going down there and that many across, you know (TR4.7).
But the three lots of nine really surprised me that she knew it was
twenty-seven and the way, like I was asking her to tell me how she
worked it out. And I was like WOW! You know. Coming from her
(TRR.9).
They seem to have that nine quite firmly embedded in their minds now.
Other children just picked up that nine, and said nine, eighteen
(CR4.10).
At one stage I said, You've got twelve, we've done two rows of six, and
we've done three rows of four, what else can you make? And some of
them didn't want to use twelve. They wanted to make groups of ten
(TR4.17).
The children knew very clearly that each multiple of that square was
going to be nine. A couple of them had some trouble working with
three nines, but they certainly knew that it was a repetition of that basic
unit of nine (CR4.18).

<table>
<thead>
<tr>
<th>1.4</th>
<th>Choose appropriate units</th>
</tr>
</thead>
<tbody>
<tr>
<td>They had made hexagon shapes. And I was just watching and he was saying Hmmm I don't think this is going to work (TR4.6).</td>
<td></td>
</tr>
<tr>
<td>Children were making things like hexagons and pentagons and they sort of knew that the squares weren't going to fit, but they still had to do it anyway (CR4.16).</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1.5</th>
<th>Align units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Before) he was more interested in making sure it was neat and well set out, rather than being no gaps, no overlaps. But towards the end he just knew straight away, that it was meant to be no gaps, no overlaps (TR4.14).</td>
<td></td>
</tr>
</tbody>
</table>

**Category 2 The lessons: tasks, resources, suggested extensions**

<table>
<thead>
<tr>
<th>2.1 Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>I think last week's lesson was really, really helpful for them, to start looking at it in that way, and it was a quicker way to work out (TRR.8).</td>
</tr>
<tr>
<td>I like the idea of using the sticks, when they got to make the shapes and then putting in one square and estimating how many (TR4.18).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2.2 Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>It made me wonder if I should have given them time to play with the materials first, before you get them to actually work with them, so they can get it all out of their system (CR4.5).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2.3 Management</th>
</tr>
</thead>
<tbody>
<tr>
<td>No, it was good that they were working in pairs. If it was done in a group, someone would have dominated (TR4.4).</td>
</tr>
<tr>
<td>I think my other problem with this one was that I should have paired them up better, rather than just getting them to work with the person next to them (TR4.13).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2.4 Suggested extensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maybe even putting other shapes in there like triangles and then make it that they can put the triangles together to make a square. See if they could work that out (TR4.3)</td>
</tr>
</tbody>
</table>

**Category 3 Children’s responses to tasks**

<table>
<thead>
<tr>
<th>3.2 Engagement and perseverance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I think when you asked them to make another shape and to do it by themselves and to figure it out themselves, using more problem solving skills, that worked the best (TR4.2).</td>
</tr>
<tr>
<td>We could make the square smaller so they made it really, really small, you know, and they had all these leftover squares. But then they worked it out eventually (TRR.8).</td>
</tr>
</tbody>
</table>
There were too many children who weren't really concentrating. Maybe it was because they were smaller and more fiddly, whereas the bigger squares were more easily manipulated (TR4.12).

But yours were very, very much on task. They made lots of different things (CR4.15).

d | Field notes area research lesson 4
--- | ---
### Category 1 Learning outcomes

| 1.3 Counting to calculate area | Children who had made multiples of the square, as long rectangles, were able to work out that they had multiples of 9 tiles to cover. Counting by 9's was done mentally and on fingers (FR4.1). Interesting watching children make multiples of squares, and then discover that three small squares only need a fourth to make a bigger square. Counting was in multiples, and there was an easier recognition of counting in multiples of nine for rectangles, than I had seen with the Year 1 class in the morning (FR4.3). The children were able to make rectangles which were multiples of the square, but found it difficult to find strategies for counting the number of tiles needed for covering. Toni said later that the class had not made numbers beyond 20 (FR4.13). I asked them to go back to the square, then to hold one cardboard tile, and estimate how many tiles would fit into the square. I think they all decided 9 (FR4.18). |
| 1.4 Choose appropriate units | They made triangles, houses, pentagons, and I saw one boy make a 3D picture of a cube. When asked if the tiles would fit into the triangles and houses, they invariably looked at the shape for a moment and then said "no" (FR4.2). |
| 1.5 Align units | When I asked children to estimate how many squares would fit into the stick square, we had correct answers and descriptions of how to fit in nine tiles. We talked about rows and columns here (FR4.11). |

### Category 2 The lessons: tasks, resources, suggested extensions

| 2.1 Tasks | This was a simple idea that really worked with this class. Perhaps the small numbers, ownership of 12 tiles, relaxed lesson, and opportunity to see what others were doing, all made it work (FR4.14). |
| 2.2 Resources | The children could have used more tiles - I need to carry the green ones as well, next time (FR4.7). They showed that they really needed more exploration time with the sticks, to see what they could make (FR4.8). |
| 2.3 Management | Movement to desks was very organised, children stood up as their desk partner was handed a bag. Desk work went really well (FR4.15). I asked Sue to make pairs of children, and gave each pair a bag of 12 sticks and 36 tiles. The children went to their desks and were asked to make a square with 4 sticks (FR4.17). |

### Category 3 Children’s responses to tasks

| 3.1 Enjoyment | Making the stick square was fun, and they were really excited when told to make the square into a rhombus. This was a real experience for them (FR4.10). |
3.2 Engagement and perseverance

Even after they saw that the squares would not fit into someone else's hexagon, they still had to try for themselves. When asked if the squares would fit, all children answered 'No'. But they still seemed to have to try (FR4.12).

<table>
<thead>
<tr>
<th>Table 6.29</th>
<th>Analysis of area research lesson 4</th>
</tr>
</thead>
</table>
| **Category 1:** Learning outcomes | Children calculated the area of shapes using multiples (TR4.1, TR4.7, FR4.13).
Children repeated a square of nine tiles to make and measure rectangles, an L shape, and larger squares (TR4.9, CR4.10, TR4.18, FR4.1, FR4.3).
Children estimated how many tiles would cover a rectangular area (TR4.17).
Children identified shapes which could not be easily covered by the square tiles, confirmed by trialing a variety of shapes (TR4.6, CR4.16, FR4.2, FR4.12).
Children calculated how many tiles would be needed to cover the area, without placing all of the tiles within the shape (FR4.18).
Children aligned tiles correctly to cover the area (TR4.14).
Children described the covering tiles in terms of rows and columns (FR4.11).
| **Category 2:** The lessons: tasks, resources, suggested extensions | Paired work with a kit of materials for each pair was successful and organised (TR4.18, FR4.15, FR4.17).
Children needed exploration time with the sticks (CR4.5, FR4.8).
One teacher reported that the squares were too small and fiddly (TR4.12).
Stick shapes enabled children to estimate the area in tiles.
Teacher suggested an extension of using triangular tiles (TR4.3).
| **Category 3:** Children’s responses to tasks | Children cooperated with a partner and enjoyed the tasks (TR4.4, FR4.10).
Children observed other pairs to gain new ideas (FR4.14).
Children persevered with the task to make and measure suitable shapes (CR4.15, FR4.12).
| **Summary and comparison with intended outcomes** | Children used addition and finger strategies to calculate the area of rectangles in multiples of a square of nine tiles. In one Year 1 class, children found this difficult and the teacher commented that the class had not previously counted beyond 20.
Additional time was needed for the children to become familiar with the sticks, by experimenting with different shapes.
Children enjoyed the activity, worked cooperatively with a partner and persevered with the task.

6.2.1.14 Area research lessons final teacher interview

Teachers were interviewed at the conclusion of the lesson implementation to comment on the success of the lessons and their observations of student learning.
### Table 6.30  Teachers’ final interview comments: Area research lessons

<table>
<thead>
<tr>
<th>Category 1 Learning outcomes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.1 Identify attribute</strong></td>
<td>Line and the shape. That was good to see that happening. That was the first bit of learning that I saw (Final R3). But the braid lesson, even as a first lesson, I was impressed with that (Final R6).</td>
</tr>
<tr>
<td><strong>1.3 Counting to calculate area</strong></td>
<td>She was having difficulty in number. And for her to click and be looking at the box and saying There's nine here, because there's three along here and three along here. Three columns and three rows. So there's nine there. And three nines. That was really great. Three nines are 27 (Final R1). The main points that stood out to me were how they looked at it with that L shape and thought about how many squares are in that area by just trying to work out in a quick way (Final R2). The counting impressed me (Final R5). The children have been doing a lot of number patterns and that sort of thing, and we've got some of them who are actually beginning to count by other numbers and they're trying to sort of test each other. So that's good (Final R11).</td>
</tr>
<tr>
<td><strong>1.4 Choose appropriate units</strong></td>
<td>It was good for them to be able to make a shape themselves, rather that giving them specific shapes, and realising that to be able to use a square, to measure the Area, they had to have straight lines (Final R8).</td>
</tr>
<tr>
<td><strong>1.5 Align units</strong></td>
<td>That was fantastic. Seeing them adding on or building up two sides. And also seeing people not let them build on another side, not understanding. Some children could only see the concept of putting on two sides, and some could see that wherever you put it, it was growing and it didn't matter (Final R4). I still think that the one where they printed the shapes and used those shapes to make the big square, I think that one seemed to sink in a little better (Final R6).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Category 2 The lessons: tasks, resources, suggested extensions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2.1 Tasks</strong></td>
<td>I think it's definitely across the class. I think it was catering for all of the levels, and it's let them develop to their next stage of learning. (Final R10).</td>
</tr>
<tr>
<td><strong>2.3 Management</strong></td>
<td>The working in pairs for the first 2, maybe 3 sessions, impressed me (Final R6).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Category 3 Children’s responses to tasks</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3.2 Engagement and perseverance</strong></td>
<td>Little Cindy who worked out the hexagon with the sticks, but then realised that she couldn't do it. So she tried manipulating the sticks. Whereas on the floor she's very submissive and she won't try to manipulate conversations to suit it. But at the table in her own little non-threatening world, she's quite happy to manipulate and change things. (Final R7). They didn't baulk at the idea of, they were quite open to that, they weren't put off, they were challenged by it (Final R9).</td>
</tr>
</tbody>
</table>
### Category 4 Teachers’ future directions

<table>
<thead>
<tr>
<th>4.1</th>
<th>Focus on mathematical concepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>I think I would try to do it very similar to the way you were doing it, because it started with a real, not an abstract level, but it just grew from something that was really informal to something that was formal and it was building. Whereas I probably would have just started at the last lesson and just hoped for the best (Final R13).</td>
<td></td>
</tr>
<tr>
<td>I've done the bit where they cover a set area with the pieces, but I haven't had them construct their own item for measurement. And that, I think, was, has pointed out to me that children need a lot more hands on with that sort of concept (Final R15).</td>
<td></td>
</tr>
<tr>
<td>If a lesson looks like it's going well, then letting them go for that little bit longer, and sort of being a little bit more relaxed and making sure that - yes, sure, you know that you're supposed to do (Final R18).</td>
<td></td>
</tr>
<tr>
<td>I think I can see a lot more of the importance of the progressive steps, and to keep repeating what you’re doing, but then moving onto the step further (Final R19).</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4.2</th>
<th>Classroom management and grouping</th>
</tr>
</thead>
<tbody>
<tr>
<td>I suppose that it was a reminder of how important it is for the children to explain. I like them to be discovering for themselves, but for them to explain exactly what they've discovered and to give them time, if they haven't quite worked out the idea then, to give them that time to go back and think about it. So that was a good reminder (Final R23).</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4.3</th>
<th>Responding to children’s needs</th>
</tr>
</thead>
<tbody>
<tr>
<td>I think I would do a lot more problem solving and give the onus on them, more so. I’d start at something really simple (Final R12).</td>
<td></td>
</tr>
<tr>
<td>They were more involved in construction of the things that they were using to measure. And I found that that has pointed out to me that these children prefer to do a bit of construction, as well as manipulation of the equipment. They like to construct it as well (Final R16).</td>
<td></td>
</tr>
<tr>
<td>I will definitely be doing more hands on, because obviously I've seen how some children who were borderline at grasping concepts have just picked them up very quickly, and kept going (Final R17).</td>
<td></td>
</tr>
<tr>
<td>Up until now, I suppose that area had been something that we've had to teach, and I did the activities and everything that were in the book and in the syllabus, but I didn't go that little bit further. Whereas having you in, made me see someone else do it that little bit more complicated, doing an Area lesson over a couple of sessions, rather than get down, do it and go onto the next session. (Final R20).</td>
<td></td>
</tr>
<tr>
<td>Just listening to their thought processes, and then watching them problem solving, you know. It was really interesting (Final R21).</td>
<td></td>
</tr>
<tr>
<td>We were talking about mathematics outcomes in our grade meeting yesterday and I commented that this is the first time I have been able to observe children demonstrating some of the outcomes, because I have had the opportunity to observe some of my children (Final R22).</td>
<td></td>
</tr>
</tbody>
</table>
| Category 1: Learning outcomes | Children identified lines and shapes (Final R3, Final R6).  
| | Children counted the covering tiles in rows and columns (Final R1, Final R5).  
| | Children realized there was a quick way of counting the tiles used (Final R2, Final R11).  
| | Children were able to choose appropriate tiles to cover shapes (Final R8).  
| | Children understood how to align tiles in rows and columns to cover a rectangular surface (Final R4, Final R6). |
| Category 2: The lessons | Tasks were suitable for a range of ability levels (Final R10). |
| Category 3: Children’s responses | Children found the tasks challenging but engaging (Final R7, Final R9). |
| Category 4: Teachers’ future directions | Teachers saw the need to identify children’s current knowledge and to build lessons from there (Final R12, Final R13).  
| | Teachers commented on the importance of measuring areas which the children had constructed (Final R15, Final R16, Final R17).  
| | Teachers thought they should proceed more slowly, encourage children to explain, and observe children’s responses to ensure that children understood the concepts before planning further work (Final R18, Final R19, Final R20, Final R21, Final R22, Final R23). |
| Summary | Children chose appropriate tiles and aligned these correctly to measure area.  
| | Tasks were challenging but engaged children at their working level.  
| | Children used the composite units of tiles in columns and rows to assist in counting the total number of tiles used to measure rectangular area.  
| | Children used skip counting and some multiplication to calculate the number of tiles used to measure the area of rectangles and squares.  
| | Teachers saw the importance of identifying children’s current knowledge and understanding before introducing new concepts, proceeding slowly if necessary and monitoring understanding carefully. |

6.2.1.15 Concluding discussions in schools 3 and 4

In schools 3 and 4, the researcher implemented the syllabus lessons and then the area research lessons, as requested by the class teachers. At the conclusion of the teaching program, the two teachers from each school met with the researcher to discuss and compare the two sequences of lessons. At each of these meetings, the teachers stated that they would recommend a combination of both sequences of lessons, because each sequence was successful in achieving the planned outcomes.
Teachers commented that children found the area syllabus lessons easier than the area research lessons, but the research lessons were still necessary for establishing the basic understandings and the direct comparison of area:

When we started, some of the children commented that this was too easy, but really it was easy only for some of them (School 4).

The first lessons covered area very thoroughly and this is what the children needed (School 4).

However, the teachers stated that the area research lessons provided a natural progression to the use of skip counting and eventually multiplication to calculate the number of tessellating units used to measure rectangular area.

You sort of forget that the formula has to be proven and taught. I mean that’s a perfect way to do it, in terms of a regular polygon (School 3).

I mean the kids in my class that didn’t know their tables, and there aren’t many of them, they just got so much more out of the whole thing, because they saw the patterns (School 3).

When asked if a combination of the syllabus lessons and area lessons, to make a total of six or seven lessons, would be too time-consuming to plan in the crowded mathematics curriculum, teachers stated that they would prefer the longer program:

If you have to do them, you just have to (School 4).

The consensus from the four teachers was that the two lesson sequences could be combined to provide an effective program incorporating the basic concepts of the attribute of area, comparison and ordering of areas by superimposing, and the use of informal, tessellating, congruent units to measure and compare rectangular areas.

I think the ones from the syllabus, I mean I thought you taught all of them really well, and the kids’ concept of area has just gone through the roof, but I thought the ones from the syllabus actually taught the concept of what area actually is, and I thought your trial area lessons taught how to calculate area with regular polygons and I mean there’s benefit in both of them, obviously. The first lot I thought was pretty much necessary so they knew what they were doing for the second lot (School 3).

6.2.2 Evaluating and Comparing the Lesson Sequences

The summaries of comments from all interviews with teachers, the researcher’s field notes, and the final discussions with teachers from schools 3 and 4 were used to evaluate the effectiveness of the lesson sequences and individual lessons. The main outcomes of the lessons and the success or failure of specific tasks were identified.
This information, combined with the students’ results, was used to modify the area research lesson sequence, in preparation for phase 2 of the major study.

6.2.2.1 Evaluating the tasks: Area syllabus lessons

Categories and comments in Table 6.17 (Analysis of final interview comments: Area syllabus lessons) were reviewed to summarise the outcomes, tasks, and teachers’ future directions.

Outcomes:
- Children were able to discuss the concept of area as a measure of the amount of surface.
- Children superimposed shapes from the bags of shapes and discussed which areas were larger.
- Children demonstrated a ready ability to select and use appropriate tiles for covering the shapes they made.
- Children predicted that the areas of the jigsaw pieces would remain the same, before the pieces were cut.
- Children were able to count the number of tiles used to cover the area.
- Responses to some tasks indicate that children are capable of achieving a higher level of understanding than that expected by the syllabus.

Tasks:
- Children enjoyed the lessons and engaged with the activities. The management strategy of asking the children to work in small groups of two or three, facilitated group discussions and interactions with the materials.
- Two tasks taken from the syllabus were problematic:
  - The crossed lines task (Lesson 1) did not assist children to work with the concept of area as a measure of the amount of surface. The lesson was introduced in several different ways, but not all children understood the focus of the activity;
  - The cutting and pasting of two shapes (Lesson 2) was difficult to explain to some children. These children were following the researcher’s or teacher’s instructions to complete the activity, rather than engaging with the underlying concepts.
• Making shapes on the geoboard was an engrossing task that attracted a large number of positive comments from teachers.

**Teachers’ future directions**
• Teachers commented that the sustained focus on area concepts, through four sequenced lessons, was successful in assisting all children to understand the attribute and measurement of area.
• Teachers articulated the need to identify children’s current knowledge and understanding and to address these when planning lessons:

6.2.2.2 Evaluating the tasks: Area research lessons

Categories and comments in Table 6.31 were reviewed to summarise the outcomes, tasks, and teachers’ future directions.

**Outcomes:**
• Children selected appropriate tiles and aligned them correctly and systematically to cover the area of the shapes.
• Children counted the tiles in multiples of twos, threes, fives, eights and tens and also added half units.
• Children made reasonable estimations of the number of tiles needed to cover shapes.
• In the last lesson, children calculated the area of large squares, rectangles and L shapes by working with repeated squares that had an area of nine tiles, without putting all of the tiles onto the stick shape. Children used addition, counting and finger strategies to calculate the multiples of nine.
• Children demonstrated that they were able to achieve the planned outcomes, even though several tasks incorporated concepts that were not included in the syllabus.

**Tasks:**
• The children enjoyed the tasks and most activities were suitable for children working at different levels.
• The materials were economical to produce and efficiently packaged, ready for classroom use.
• The printing task was problematic because it was difficult to manage so that all children had ready access to the paints, and the sponges were very messy to
clean. Teachers and the researcher commented that children didn’t automatically see the relationship between the paper tiles and the stamped shapes, possibly because the task was unfamiliar and they couldn’t predict the outcome of the stamping.

- The whole-class square provided an excellent opportunity to investigate number patterns, using larger numbers.

**Teachers’ future directions**

- Teachers explained that the experience of observing children’s working, listening to explanations, and reflecting on the progress of the lessons had shown them that they needed to make some changes to the way in which they planned and taught mathematical concepts. These included an initial identification of current knowledge and then a more relaxed pace in introducing further concepts.
- Teachers commented that they would give children more time to explain what they were doing and thinking, both to the teacher and also to other children.
- Teachers saw the need for careful observation during mathematics lessons, to gauge student knowledge and to identify achievement of the syllabus learning outcomes.

### 6.2.3 Results from Children

Children completed the area tiling task at the completion of the implementation of the lesson sequences. The children’s responses were assessed using the coding scheme described in Figure 5.1. The strategies which children used to draw tiles to cover the area of a given rectangle ranged from individual entities (scored as 1), to each tile drawn onto a previous tile (scored as 3), to continuous horizontal or vertical lines (scored as 4) and continuous horizontal and vertical lines with the correct number of tiles (scored as 6). The scores were collated and data representing all students from schools 1 and 2 were compared with data from all students in schools 3 and 4.

A direct comparison between the results from the two groups cannot be made as the schools and children were not matched. In spite of this limitation, it is interesting to see differences between the 95 children from schools 1 and 2 who had
experienced the area research lessons, and 94 children from schools 3 and 4 who had worked with the syllabus lessons (Figure 6.1). The results indicate that more children from schools 1 and 2 gained the higher scores of 5 and 6 in the performance scale.

Figure 6.1 Results at completion of research lessons and syllabus lessons

Children in schools 3 and 4 completed the area tiling activity again, at the completion of the area research lessons. Their responses to the task indicate a significant increase in learning, in terms of using a grid pattern to measure area, during the area research lessons teaching program.

Figure 6.2 Post-syllabus and post-research results in schools 3 and 4
The learning outcomes demonstrated by individual children during the area research lessons were exciting for both the classroom teachers and the researcher. The two work samples, representing strategies used by a Year 1 child from School 4, following the syllabus lessons and then the research lessons demonstrate the child’s growth in skills and understanding (Figure 6.3).

![Post- syllabus lessons, Level 2](image1)
![Post- research lessons, Level 6](image2)

Figure 6.3 Post- syllabus lessons and post- research lessons, Year 1 child

### 6.3 Research Question

6.3.1 *How effective were the area research lessons compared to the area syllabus lessons in developing an understanding of the attribute of area?*

Both sequences of lessons focused on the identification, discussion and measurement of area. However, the area syllabus lessons appeared to have a more leisurely pace than the area research lessons, and a greater emphasis on the attribute of area. In the final interview, teachers spoke very positively about the children’s understanding of the attribute of area:

As an entire class, I think they actually would be able to define what area is now, most of them. Before it was probably something that you did in maths, now they can see the uses of it and they would be able to tell you it was a space covering something and they really understand what it is (Final S10).

A lot of them had no idea when they started with like, what is Area? By the end of it, they really understood it, I think, especially by the final lesson that we had this morning. It was like, my God-they seemed to get it (Final S1).
By comparison, the nature and focus of the area research lessons resulted in the majority of teacher comments in the final interviews relating to counting, choosing and organising informal units. Teachers acknowledged the importance of the line and shape lesson in establishing an area bounded by a line, but were keener to describe the children’s counting patterns:

- Children identified lines and shapes (Final R3, Final R6).
- Children counted the covering tiles in rows and columns (Final R1, Final R5).
- Children realized there was a quick way of counting the tiles used (Final R2, Final R11).
- Children understood how to align tiles in rows and columns to cover a rectangular surface (Final R4, Final R6).

In conclusion, the area syllabus lessons placed a greater emphasis on identifying and discussing the attribute of area. The children’s successful responses to this focus were identified and commented upon by the four teachers who experienced both sequences of lessons (6.2.1.15 Concluding discussions in schools 3 and 4). The teachers suggested that a combination of both sequences of lessons would take advantage of the demonstrated strengths of each.

6.3.2 How effective were the area research lessons compared to the area syllabus lessons in developing an understanding of a grid pattern or array or repeated units to measure area?

The area tiling activity completed by all Year 1 and Year 2 children at the conclusion of the area research lessons demonstrated that a basic understanding of the structure of repeated units to measure area was achievable by almost all children. In comparing results between the two sequences of lessons, more children from the area research lesson classes gained the higher scores of 5 and 6, indicating a more sophisticated strategy for partitioning an area and drawing the array pattern of repeated units (Figure 6.1). The results in Figure 6.2 indicate the excellent improvement in drawing skills in Schools 3 and 4, when children had been exposed to firstly the area syllabus lessons and secondly the area research lessons.

Both the researcher and the participating classroom teachers were impressed by the success of the research lessons. The developmental sequence of concepts,
progressed from basic revision of identifying area (Reeves et al., 1981; Zacharos, 2006), to investigations involving the skills of selecting and measuring with informal units (Grant & Kline, 2003; Hirstein, 1981; Owens & Outhred, 2006; Reynolds & Wheatley, 1997). Children also experienced several opportunities to make, discuss, count and record the array patterns of repeated units (Lehrer, 2003; Lehrer et al., 2003). The provision of rectangular, congruent units ensured that the children were using materials appropriate to the task (Zacharos, 2006). A comparison of the children’s results in the assessment at the completion research lessons and the syllabus lessons indicated an enhanced ability to partition an area into repeated, identical units. The sequence of learning experiences echoed the foundational concepts in learning to measure area as described by Stephan and Clements (2003). Teachers also commented that the research lessons encouraged them to integrate Measurement with Number and Space, and to look at wider and more encompassing mathematical understanding.

The results of this study have implications for the teaching of length, area and volume in the primary mathematics curriculum, and the development of concepts that will enable children to understand and use the multiplicative model of repeated, tessellating units as expounded by Anderson and Cuneo (1978). This understanding is crucial to learning and using the formulae to calculate area and volume.

### 6.4 Modification of the Area Research Lessons

To conclude phase 1 of the study, the area research lessons were modified to reflect comments and suggestions made by the teachers and researcher and were given the title area refined lessons in this report. The research literature assisted in defining the desired learning outcomes. However, a specific sequence of whole-class tasks and lessons which would assist young children to firstly understand and identify the attribute of area and secondly to develop an understanding of using and recording iterated, congruent, rectangular units to measure area, was not located within the mathematics education research literature. Various aspects of the area refined lessons were based on the literature, but the complete sequence of activities represented an innovative plan to achieve the identified learning outcomes within a classroom program. The data collected from teachers, children and the researcher during the
trials of the area syllabus lessons and area research lessons indicated that the activities were successful in assisting the majority of children to achieve the desired skills and understandings.

The teachers who had observed both the syllabus lessons and the research lessons suggested that a combination of the two sequences would provide a very comprehensive study of area. Teachers were impressed by the slower pace of the syllabus lessons that allowed them to feel confident that all children understood the attribute of area, before further concepts were introduced. Teachers also commented that they would be willing to extend the series of lessons to a total of six or seven lessons, instead of the planned four.

The area refined lessons:

- involved the identification and comparison of area, and then the measurement of rectangular area with tessellating, congruent units;
- commenced with activities that reinforced or revised earlier classroom tasks to identify the surfaces of objects and define area of the shape enclosed by a closed line;
- provided opportunities for children to measure area by arranging tiles of regular shapes;
- included activities that emphasised the identification and documentation of the grid patterns of tiles used to measure area;
- highlighted opportunities for counting tiles in multiples;
- encouraged teachers to emphasise the links between mathematics strands;
- incorporated tasks that were simple to explain to children and also allowed children to experiment and investigate at their own level;
- focused on creating opportunities for small group interactions and whole class discussions.
Table 6.32  *Area refined lessons*

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson 1</strong></td>
<td>Make lines with a piece of braid, and record. Make shapes with a piece of braid. Measure the area of the shapes with cardboard tiles. Count and record.</td>
</tr>
<tr>
<td><strong>Lesson 2</strong></td>
<td>Compare the areas of pairs of shapes by superimposing. Cut and paste two shapes to find which has the larger area.</td>
</tr>
<tr>
<td><strong>Lesson 3</strong></td>
<td>Paste coloured squares and triangles to cover the area of outlines, in a pattern.</td>
</tr>
<tr>
<td><strong>Lesson 4</strong></td>
<td>Cut out the pasted squares and triangles and use to make a larger square in a small group, then the whole class.</td>
</tr>
<tr>
<td><strong>Lesson 5</strong></td>
<td>Use four sticks to make a shape and measure the area of the shape with square tiles. Make other areas that can be measured with the square tiles. Predict each area, then measure. Record the shapes.</td>
</tr>
<tr>
<td><strong>Lesson 6</strong></td>
<td>Find the area of shapes that have been drawn onto a grid. Order the shapes from largest to smallest. Draw own shapes onto the grid: one shape larger than others, one shape smaller than the others.</td>
</tr>
</tbody>
</table>

### 6.5 Conclusion

This chapter has described the process of trialling and modifying the research area lessons. The researcher and her teacher co-researchers felt confident that the resultant sequence of six lessons were a successful amalgamation of the syllabus outcomes of establishing the concept of the attribute of area, and the research-based goal of using the pattern of repeated, congruent, tessellating units to measure and compare rectangular area. Children were assisted to use the structure of repeated units and skip counting to calculate the number of informal units needed to measure an area. This approach to teaching the measurement of rectangular area has been described in recent research (Outhred, 1993; Reynolds & Wheatley, 1996; Owens & Outhred, 1996; Outhred & McPhail, 2000). The lesson sequence was successful in encouraging children to understand how area is measured, and to use early multiplication and division skills, before the introduction of the formula for the calculation of rectangular area (McPhail, 2004a).

The results of the research trial indicated success not only in terms of student learning, but also in terms of the teachers’ professional learning, reflections of observations and potential for positive changes to their teaching practices:
Oh, it was great. It was really good in the sense of seeing another..... I'm scared of teaching Maths and teaching it the wrong way. It's not something that I'm really comfortable with teaching, mainly because I don't understand a lot of it myself, or I know it, but to actually say it, to explain the process to someone (School 1).

Maybe I don't think I would have looked at area in that way. I mean, have a look at mine. I was sitting there looking at mine, and it's saying. It took 26 hands to cover the surface of this window. And look at the spaces. I'm thinking this is in total contrast. There are lots of overlapping and lots of spaces. So I'm going to rip that down now (both laugh) (School 1).

The researcher was also involved in a continuing learning process. The procedures of designing and implementing the study, collecting student data, analysing interviews with teachers, discussing results with academic colleagues and reflecting on the results provided were encountered, engaged with, and achieved. The researcher felt she was privileged to share impressions, ideas, and thoughtful discussions with eight very committed and professional teachers and was both optimistic and excited about the phase 2 of the major study.

I really enjoyed my interview time with Sue at the end of the lesson. Sue is really focussed on the role of the co-researcher, and she is quite refreshing to talk to. It is fascinating to work with someone who is so engaged with the children’s learning (FR2.17).

The next chapter outlines the methods used in phase 2 of the study, the implementation and evaluation of teacher professional learning strategies designed to assist teachers of Year 1 and Year 2 to teach area within the mathematics program.
CHAPTER 7 - IMPLEMENTATION OF TEACHER PROFESSIONAL LEARNING MODELS

The previous chapter described the results of phase 1 of this study - the development and implementation of a teaching sequence of six whole-class lessons. This chapter outlines the process of implementation of phase 2 of the study – a teacher professional learning project implemented with 17 teachers in seven schools. The teachers’ project employed three models of professional development, based on the teachers’ selection and participation in one of three levels of consultancy support for implementing the area lesson sequence. Two of the three professional learning models were designed to encourage teachers to reflect on their students’ learning (Dawe, 1998), to make choices, investigate and trial (I. Robinson, 1989) and to conduct professional discussions with colleagues (Wenger, 1998). The third model did not explicitly encourage teacher collegiality and discussion. The outcomes of the three professional learning models were measured and compared by assessing student learning and teacher learning in each of the schools, using a case study approach.

7.1 Design

7.1.1 Aim

The aim of the phase 2 research was to implement, evaluate and compare three models of professional learning designed to assist teachers to change their current practices in the teaching of area. The research questions focused on an evaluation of the effectiveness of the professional learning models.

7.1.1.1 Research questions

The research questions for this study were:

- How do teachers currently plan and implement learning experiences in area?
- Which of the three models implemented was the most successful for the professional learning of the teachers who participated in this research?
- What factors were important to the success of the professional learning models?
7.1.2 Research Context

The research was implemented in seven government primary schools in a metropolitan area. The researcher’s name was known to the participating teachers as the District Mathematics Consultant, but she had not previously made personal contact with some of the teachers. For the remainder of this report, the schools and teachers are given fictitious names to provide anonymity.

7.1.2.1 Teachers

The 17 teachers who participated in this study were all volunteers. The researcher advertised the project in the district and fifteen teachers responded to the invitation for school teams to attend an after-school training and information session. The researcher’s study and the choices of consultancy support models available to participants were outlined and discussed. The final number of teachers in the project increased to seventeen, after the teams returned to their schools and discussed the program with colleagues and school executive (Table 7.1).

<table>
<thead>
<tr>
<th>School</th>
<th>Teacher’s name</th>
<th>Years of teaching</th>
<th>Class taught</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Kinder</td>
</tr>
<tr>
<td>Avenue</td>
<td>Vicki</td>
<td>35</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>Sally</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Deb</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Mall</td>
<td>Beth</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Jan</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Vista</td>
<td>Kara</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cathy</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Hills</td>
<td>Gemma</td>
<td>15</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>Mark</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Plains</td>
<td>Scottie</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Celia</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Matilda</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Forest</td>
<td>Nadia</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Karen</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mary</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Station</td>
<td>Milly</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Kelly</td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.1 Participating teachers’ experience and class taught

167
Each school-based team consisted of two or three teachers who were teaching Year 1 or Year 2 classes, although a Kindergarten teacher and a Kindergarten/Year 1 composite teacher made special requests to join the project. The teachers had a broad range of teaching experience, from a beginning teacher to an assistant principal with 35 years of experience.

7.1.2.2 Schools

The schools were representative of the range of metropolitan schools in terms of size, socioeconomic factors, multicultural diversity, and the experience of teaching staff.

Table 7.2  Schools represented by participating teachers

<table>
<thead>
<tr>
<th>School</th>
<th>Students</th>
<th>Teaching focus</th>
<th>Influences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avenue</td>
<td>496</td>
<td>Emphasis on literacy in professional learning programs. No ability grouping for mathematics. Student textbooks used to teach mathematics.</td>
<td>Medium to low socioeconomic area, experienced teachers, conservative programs. Parents interested, but leave schoolwork to the teachers.</td>
</tr>
<tr>
<td>Mall</td>
<td>334</td>
<td>History of trialling innovative literacy and numeracy programs. No ability grouping. No student textbooks for mathematics in K-3 classes.</td>
<td>Middle class suburb with supportive parents. Mixture of young and more experienced teaching staff, culture of professional sharing and using own initiative.</td>
</tr>
<tr>
<td>Vista</td>
<td>118</td>
<td>Concentration on literacy, numeracy and providing cultural experiences for children. Significant classroom behaviour issues. No ability grouping. No student textbooks for mathematics.</td>
<td>Low socio-economic area, school receives additional funding from the Australian Government. High mobility of children. Young teaching staff and a teaching principal in her first role as principal.</td>
</tr>
<tr>
<td>Hills</td>
<td>297</td>
<td>Concentration on basic literacy and numeracy. Significant classroom behaviour issues. No ability grouping. No student textbooks for mathematics.</td>
<td>Low socio-economic area, school is adjacent to a public housing estate and receives additional funding from the Australian Government. Experienced, stable teaching staff. Some mobility of children.</td>
</tr>
<tr>
<td>Plains</td>
<td>589</td>
<td>Focus on achieving measurable results in literacy and numeracy. Ability grouping in mathematics. Student textbooks used to teach mathematics.</td>
<td>Middle class suburb with supportive parents. Experienced, stable teaching staff. Teachers use school-based programs for teaching.</td>
</tr>
</tbody>
</table>
7.1.2.3 Children

The focus of this study was the professional learning experienced by the seventeen participating teachers. However, one of the factors useful for the validation of teachers’ perceptions of the effects of positive changes to their teaching was the collection of data from children in those seventeen classes. Accordingly, data was collected from the 381 children who participated in the area lessons and completed both the pre-test and the post-test activity.

Table 7.3  

<table>
<thead>
<tr>
<th>School</th>
<th>Kindergarten</th>
<th>Year 1 and Year 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avenue</td>
<td>-</td>
<td>73</td>
</tr>
<tr>
<td>Mall</td>
<td>-</td>
<td>49</td>
</tr>
<tr>
<td>Vista</td>
<td>9</td>
<td>23</td>
</tr>
<tr>
<td>Hills</td>
<td>-</td>
<td>49</td>
</tr>
<tr>
<td>Plains</td>
<td>24</td>
<td>51</td>
</tr>
<tr>
<td>Forest</td>
<td>-</td>
<td>55</td>
</tr>
<tr>
<td>Station</td>
<td>-</td>
<td>48</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>33</strong></td>
<td><strong>348</strong></td>
</tr>
</tbody>
</table>

7.2 Method

7.2.1 Professional Learning Models

During the training and information meeting, the teams of teachers were asked to select one of three models of professional learning consultancy support that were described and discussed. All teams were successful in gaining their first choice of professional learning model, as outlined in Table 7.4. The opportunity to select was
made to address preferred methods of learning, needs and available time (Irwin & Britt, 1999).

Table 7.4  Three models of teacher professional learning support

<table>
<thead>
<tr>
<th>High support model (Avenue, Mall, Vista)</th>
<th>Medium support model (Hills, Plains)</th>
<th>Low support model (Forest, Station)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial meeting and introduction to the refined area lessons and materials</strong></td>
<td><strong>Initial meeting and introduction to the refined area lessons and materials</strong></td>
<td><strong>Initial meeting and introduction to the refined area lessons and materials</strong></td>
</tr>
<tr>
<td>Initial training session.</td>
<td>Initial training session.</td>
<td>Initial training session.</td>
</tr>
<tr>
<td>Overview of lesson plans shared and explained.</td>
<td>Overview of lesson plans shared and explained.</td>
<td>Overview of lesson plans shared and explained.</td>
</tr>
<tr>
<td><strong>Implementation of refined area lessons</strong></td>
<td><strong>Implementation of refined area lessons</strong></td>
<td><strong>Implementation of refined area lessons</strong></td>
</tr>
<tr>
<td>Class teachers implemented sequence of six lessons.</td>
<td>Class teachers implemented sequence of six lessons.</td>
<td>Class teachers implemented sequence of six lessons.</td>
</tr>
<tr>
<td>Teachers released for 30 minutes following each lesson to interview four children and write notes.</td>
<td>Teachers released for 30 minutes following each lesson to interview four children and write notes.</td>
<td>Teachers released for 30 minutes following each lesson to interview four children and write notes.</td>
</tr>
<tr>
<td>School teams meet with researcher each fortnight for approximately 20 minutes to discuss progress of lessons and children’s responses.</td>
<td>School teams meet with researcher each fortnight for approximately 20 minutes to discuss progress of lessons and children’s responses.</td>
<td>Access to researcher to meet or answer queries, if requested.</td>
</tr>
</tbody>
</table>

All of the support models commenced with a training session delivered by an “expert” facilitator (Putt & McLean, 2002). All teachers were provided with lesson notes and hands-on materials to implement the lessons. The lesson notes included suggestions for open-ended questioning to encourage student explanation, discussion and reasoning (D. S. Mewborn & Huberty, 1999; Vale, 2003).

The high support and medium support models were designed to encourage teacher collaborative dialogue and reflection (Garet et al., 2001; Hiebert, 1999; Tytler et al., 1999) through the scheduling of fortnightly team review meetings with the researcher. The high support schools had an additional strategy, in the form of teacher release at the conclusion of every lesson to interview a small group or individual students. Suggested question sheets for the interviews following each lesson were included in the teaching package. This strategy was designed to supplement the lesson plan questions and provide further practice in asking questions.
to assess student understanding and to encourage higher order thinking (Sullivan et al., 2000).

7.2.2 Initial Meeting and Training

The introductory meeting and training were conducted after school (3:30 pm to 5:00 pm) at the District Office, following an advertising flyer sent to all primary schools in the District, inviting teachers to participate in an area project. The program was launched during a period of prolonged industrial unrest in government schools. An indication of the level of interest and motivation was the attendance by 15 teachers despite a directive from the teachers’ union not to attend events held out of school hours.

Teachers were introduced to the focus of the project, the history and rationale of the lesson sequence, the three levels of consultancy support and the teachers’ roles as participants in the project. The teachers discussed their participation within their school teams and chose the model which was appropriate to teaching timetables and current commitments. Teachers were given explanatory notes and were shown a sample kit of materials for the practical activities. The two participating teachers who did not attend the initial meeting were visited by the researcher prior to the initial teacher interviews.

7.2.2.1 Rationale of the research area lesson sequence

The development of concepts underpinning the lesson sequence was discussed. The teachers were given explanatory notes which detailed the sequence of students’ acquisition of early understandings of how to identify and measure rectangular area using informal units:

- Demonstrating an understanding of the attribute of area and identifying an area within a defined boundary;
- Comparing areas by superimposing;
- Measuring a rectangular area by covering methodically with informal, tessellating, congruent units in an array. Using the structure of the array of units to count the total with a stress count (1, 2, 3, 4, 5, 6, 7, 8…) or skip count (4, 8, 12…).
• Recording, describing and recalling the array structure.

7.2.2 Lesson notes and guidelines

Specific implementation and management issues were also discussed. The lessons were described as whole-class lessons, which integrated the syllabus strands of Space, Measurement and Number, with the relevant Syllabus outcomes referenced within the lesson notes (Appendices 2 to 7). Particular note was made of integrating the counting of informal units with the children’s early skills in multiplication and division, by using appropriate stress or skip counting to calculate the total by rows or columns.

Teachers were advised to implement the lessons by forming mixed ability groups of two, three or four children to engage with the tasks. The importance of encouraging children to discuss their actions, suggestions and findings, in small groups and also in whole class discussions, was emphasised. Specific mathematical language to be encouraged by explicit teacher modelling was also summarised.

Teachers were reminded that the principal differences between the these area lessons and a typical measurement program planned from the current syllabus were the use of tessellating, congruent units and the recognition and use of an array structure, when placing and counting units to measure rectangular area.

7.2.3 Teaching Materials Provided for Each School Team

The researcher visited the seven schools to gain written approval from the Principals, and to collect a signed participation letter from each teacher. The teachers were given a folder containing lesson notes for all lessons (Appendices 2 to 7) and master activity sheets for lessons 2, 3 and 6. The folders for teachers in the high support schools (Avenue, Mall, Vista) also included suggested questions for individual student interviews following each lesson (Appendices 8 to 13).

Each school team received a kit of materials sufficient for a class of 27 students working in groups of three to implement the area refined lessons. The kits consisted of clearly-marked bags packed into a crate to allow for easy carrying between the two or three classes involved in each school. The kits contained: five 2 metre braids,
packs of 20 cardboard 10 cm square tiles and 20 cardboard equilateral triangles with 10 cm sides for lesson 1; 12 pairs of shapes for superimposing, including rhombuses, rectangles, squares, trapeziums and irregular hexagons, and a master sheet for the cutting and pasting for lesson 2; master sheet of squares and equilateral triangles for lessons 3 and 4; 10 sets of seven kebab sticks and packs of twenty-five 5 cm square tiles for lesson 5 and a master sheet of grid shapes for lesson 6.

7.2.4 Collection of Data

Data to assist in assessing both the short-term and the longer-term effects of the teachers’ participation in the professional learning study were collected from both teachers and students. The researcher interviewed each teacher before the lessons commenced. Teachers were interviewed again at the conclusion of the six lessons, and again nine months later. The audiotapes of the interviews were transcribed by the researcher and returned to each teacher for checking. The interview material included information about each teacher, including motivations for volunteering and perceptions of individual and team learning during the program. The interviews also questioned the teachers’ current methods of planning and teaching area and the effectiveness of the lessons.

The researcher audiotaped and then transcribed the team meetings held with teachers in the high support and medium support schools. Again, the researcher gave copies to teachers for checking. Students in each of the seventeen classes completed a pre-test and a post-test.

7.2.4.1 Initial interviews with teachers

An initial interview was conducted with each teacher after the introductory meeting and before the teacher commenced implementing the area lessons. The questions were:

- Can you tell me about the area lessons that you have taught this year?
- What do you think children need to learn about area, thinking about skills as well as understandings?
- What do you think is the best way to teach children about area?
- When you plan your lessons, do you work from the syllabus or the text book or a combination of those?
- What do your children need to learn about area in the next three years?
- How often do you teach area, and would you do that in a single lessons or a series of lessons?
- How many years have you been teaching?
- So that I can get back to you at the end of the project, to see whether it was successful in your terms, why did you volunteer for this?

7.2.4.2 Final interviews with teachers

A final interview was conducted with each teacher within two weeks of the completion of the six area lessons. The questions were:
- Has your participation in the project made any difference to the way that you plan or teach mathematics?
- I’d like you to think about the aspect of the project that was richest for you. Was there any professional development strategy that was more successful for you than the others?
- Do you think the project will make any difference to the way that you teach mathematics in the future?
- What are the sorts of things that you have learnt in the last six weeks?
- At the beginning, I asked you what you hoped to get out of participating in the project, and this is what you said … (quote from each teacher’s initial interview inserted here). Did you achieve what you wanted to or expected to?

7.2.4.3 School-based team meetings

In the high support and medium support schools, fortnightly school team meetings were conducted at a time chosen by the teachers, usually before school, or at morning tea or lunch and typically lasted for 15-20 minutes. The teachers talked about the area lessons that they had implemented in the previous fortnight, and discussed the success of the lessons, student responses and learning, suggested modifications to the lessons, and implications for the next lessons. Teachers also used the team meetings to give advice to colleagues who had not implemented the lessons, to make generalizations about student understanding of area, or to reflect on their own learning.
The researcher played a minimal role in the meetings. She started the discussion with an open question such as *how did the lessons go?* She confined her comments and queries to specific questions on children’s reactions to the lesson content, such as:

- How did your counting go? Did they see the need to use multiples to count? (Avenue School, Group discussion 3)
- You did the geoboards as well? (Hills School, Group discussion 2)
- And the triangles - they didn’t have any trouble turning the triangles around to make them fit? (Vista School, Group discussion 2)

The discussions from the team meetings were audiotaped and transcribed by the researcher. Printed copies were given to teachers with a request to check that comments had been interpreted correctly, and to make additional comments if necessary.

### 7.2.4.4 Children’s assessment task

The tile-drawing task which was used in phase 1 of this study was applied again as a pre- and post- test to assess children’s learning within each class. The researcher visited each class to administer the task, before the lessons commenced and again after the six lessons had been completed.

### 7.2.4.5 Follow-up interviews with teachers

The follow up interviews were held nine months after the refined area lesson implementation with fourteen of the original seventeen teachers, as three teachers were absent from their schools. The interviews were conducted with individual teachers in their classrooms before or after school. The interview questions were designed to provide information about the longer-term effects of the project on teaching strategies and activities, including whether the lessons had been taught again, and whether the suggested lesson plans had been modified by teachers. Modifications may have suggested that teachers felt confident enough to be able to adapt the lessons to specific classroom needs or interests. The questions were:

- I would like you to think about the area project lessons that you implemented last year in your classroom. At the time, we discussed the student learning that
you had observed, and also your own learning. Now, nine months later, have you taught any of the lessons again or used the materials again?

- Do you know if your colleagues have used the lessons or materials this year?
- Has your participation in the project had any effect on your teaching?
- What were the most successful professional development strategies?
- In retrospect, what was the best part of the project for you?
- Is there any one single idea or memory that you have retained from the project?
- Did you have any assistance from your principal or the executive?

7.2.5 Analysis of Data

The aim of this study was to identify the most effective professional learning model by examining children’s learning and teachers’ learning. The sample of seven schools and 17 teachers came from one metropolitan district, but the schools represented many differences in terms of leadership, size, teaching experience and socio-economic status of the communities. To enable a thorough analysis of data and identification of factors which assisted teacher professional learning, a case study approach was adopted.

Case studies involve a detailed investigation of a social unit and include an analysis of the multiple variables which are relevant to the participants being studied (Polit & Hungler, 1983). The case study approach focuses on an individual case, and an understanding of the conditions within a chosen site, rather than a comparison of effects and forces between sites. This approach aims to identify everyday or natural factors which will allow a true description of the social unit within its own setting (Stake, 1995). As Merriam (1988) commented, this form of research received recognition during the 1960s and 1970s when in-depth investigations of teaching practices and interventions were conducted by the United States government, but the approach has many applications including health, technology, and social science:

Case studies using qualitative methods are most valuable when the question being posed requires an investigation of a real life intervention in detail, where the focus is on how and why the intervention succeeds or fails, where the general context will influence the outcome and where researchers asking the questions will have no control over events (Keen & Packwood, 1995, p. 444).
Merriam (1988) differentiated case studies in terms of their end products and listed the different forms as primarily descriptive, interpretive or evaluative. Merriam’s (1988) warning was that a limitation of case studies, especially evaluative case studies, was the possible bias of the researcher in selecting data to illustrate a chosen viewpoint. To ensure validity, Merriam (1988) advised researchers to present the view which informants have of themselves and their experiences, as well as applying the processes of triangulation, member checks, long-term observations and stating the researcher’s bias at the outset of the study.

A case study approach would allow conditions and results at each school site to be considered, resulting in a rigorous examination to enable an informed judgement of both the success of the professional learning model and the reasons for that success.

7.2.5.1 Initial interviews with teachers

The data from these interviews were collated and analysed to assist in responding to the research questions. The teachers’ self-reported current planning and teaching practices in measurement and area were listed, to enable statements to be made referring to commonalities and differences in the range of teachers’ beliefs and traditional area lessons.

7.2.5.2 Final interviews with teachers

The transcriptions of the teachers’ final interviews were analysed and responses to the questions were collated and compared. Teacher’s responses were grouped to examine the immediate changes teachers had made to their mathematics programs, their intentions to make changes in the future, and statements of what they felt they had learnt. The teachers’ statements of whether they had achieved their planned professional learning goals were also analysed and grouped in the three levels of support.

7.2.5.3 School-based team meetings

The transcriptions of the team meetings were initially examined to identify the key experiences which teachers described. The transcriptions were then studied to
identify the possible changes in teachers’ thinking as the program of lesson implementation progressed. The approach used to analyse the teachers’ discussions was based on the grounded theory method as described by Glaser and Strauss (1967) and Corbin (1986). Silverman (2000) described a simplified model of grounded theory as:

- an initial attempt to develop categories that illuminate the data
- an attempt to ‘saturate’ these categories with as many appropriate cases in order to demonstrate their relevance
- developing these categories into more general analytic frameworks with relevance outside the setting (p. 144).

The researcher had extensive experience in providing consultancy support to achieve teacher change and curriculum implementation, and had also assisted schools to plan and implement major professional learning programs. As Silverman (2000) warns, a shortcoming of grounded theory is its failure to acknowledge the researcher’s theories that may impact on the early phase of analysis. To guard against such a bias, a three-stage methodology was used. A projected model of professional learning, based on previous experiences and research, was recorded before commencing the analysis. This model consisted of three major categories or stages (Teach, Discuss and Evaluate). The model was tested against the data and revised.

7.2.5.4 Children’s pre and post assessment task

The children’s rates of learning during the refined area lessons implementation were compared by noting the difference in individual achievement between pre-test and post-test tasks. Each school was treated as a case study and the results of the children’s collated performance scores were combined with data from the teachers’ interviews to identify the sites where the greatest measurable learning had occurred.

7.2.5.5 Follow-up interviews with teachers

Teachers’ responses to interview questions were initially recorded in a large table, and then quantitative and qualitative data were collated. These included the number of teachers who had implemented the lessons again and the number of...
schools in which other teachers had accessed the lessons. The data also provided a
gauge of the continuing effect of the teachers’ participation on their current teaching
strategies, teachers’ impressions of the project as successful professional learning,
and a single memory which teachers had retained as a result of the implementation
of the refined area lessons.

7.3 Conclusion

This chapter has explained the methodology of phase 2, the implementation of
three teacher professional learning models. The next chapter describes how the
analysis of the data collected from teachers and students during the study was
effective in enabling individual school case studies to be developed. This material
was then refined and compared, leading to the identification of the school sites
where the most significant teacher learning and children’s learning was achieved. In
addition, the factors influencing the effective learning were identified from the
teachers’ post- and final interviews.
CHAPTER 8 - RESULTS OF IMPLEMENTATION OF TEACHER PROFESSIONAL LEARNING MODELS

The previous chapter described the methodology of phase 2 of this study – the evaluation of professional learning strategies which assisted Year 1 and Year 2 teachers in seven schools to implement the area refined lessons. The resultant teacher professional learning was described and compared by considering each of the seven schools as a case study.

This chapter examines each component of the research data and relates these to the appropriate research questions. Teachers’ current planning and teaching in area were studied. The effectiveness of the teacher professional learning strategies at each school site was evaluated, allowing statements to be made about the impact of specific factors on individual school teams.

8.1 Outline of Data Collection

Data were collected from three main sources: initial, final, and follow-up interviews with teachers; transcriptions of school-based team meetings in the high support and medium support schools; and the children’s pre- and post- assessment tasks.

8.1.1 Teacher Initial Interviews

After the list of participating teachers had been finalised and school principals had given written permission for the project to proceed, the researcher visited each teacher to conduct an initial interview, to collect the teacher’s participation consent letter, and to deliver the teaching materials. The initial interviews were held before or after school and were usually located in the teacher’s own classroom. The audiotapes of the interviews were transcribed by the researcher, and a copy returned to individual teachers to be checked.

8.1.2 Teacher Final Interviews

During the three weeks following the conclusion of the implementation of the six refined area lessons, the researcher visited each teacher to conduct a final
interview. The interviews were held in teachers’ classrooms and audiotaped. Transcriptions were returned to teachers for checking. Teachers were reminded that the researcher wished to interview them again the following year, to evaluate any lasting effects of their participation in the area project.

8.1.3 School-based Team Meetings

In the high support and medium support schools, team meetings were held each fortnight during the area project. The researcher asked each team to organise a classroom or small office space and a mutually-suitable time, which was usually before school or at lunch time. The meetings were audiotaped and later transcribed by the researcher. Copies of the transcriptions were returned to teachers for checking.

8.1.4 Children’s Pre- and Post-Assessment Task

The researcher visited each class before and after the lesson implementation to conduct the children’s pre- and post-assessment tasks. Each child’s pre-test and post-test responses were compared to identify individual children’s achievement. Class lists of pre-test and post-test results were collated and given to teachers for discussion during the final interview.

8.1.5 Teacher Follow-up Interviews

The teacher follow-up interviews were conducted with individual teachers, 9 months after the conclusion of the lesson project. The researcher visited each teacher, including one teacher who had transferred to a school in another metropolitan region. Of the original 17 teachers, three were not teaching when the interviews were conducted. Interviews were audiotaped, transcribed by the researcher, and copies of the transcriptions were sent to teachers for checking.

8.2 Findings

8.2.1 Results of Teacher Initial Interviews

The transcriptions of the teacher initial interviews were analysed in order to answer the research question: How do teachers currently plan and implement
learning experiences in area? Teachers’ responses to the interview questions were examined in three aspects. The first aspect represented the teachers’ practical guidelines of planning and teaching area lessons, including the materials used and the source of lesson ideas. These comments were in response to the initial interview questions:

- Can you tell me about the area lessons that you have taught this year?
- When you plan your lessons, do you work from the syllabus or the textbook or a combination of those?

The second aspect listed the teachers’ stated frequency of lessons within their mathematics programs and was collated from the initial interview question:

- How often do you teach area, and would you do that in a single lesson or a series of lessons?

The third aspect examined the teachers’ own contexts within which area lessons were planned and taught, including their rationales for specific teaching approaches as well as an overview of how current learning would assist in developing concepts of the measurement of area in later years. These responses were taken from the initial interview questions:

- What do you think children need to learn about area, thinking about skills as well as understandings?
- What do you think is the best way to teach children about area?
- What do your children need to learn about area in the next three years?

8.2.1.1 Teachers’ practices in the planning and teaching of area measurement

“Covering” shapes with informal units was the most common activity described by teachers, with 15 responses involving areas of book covers, desks, shapes made from skipping ropes and body shapes. Most teachers gave children square-based blocks or rectangular tiles to use as informal units, with only two classes given non-tessellating shapes as well as blocks. Two teachers stated that their area lessons included superimposing, and one teacher had discussed playground area and classroom areas as an introduction to the topic.
When describing the source of area activities and programs, 13 of the 17 teachers stated that they used the mathematics syllabus together with a textbook. Three of the teachers stated that they used the syllabus only. One of the early career teachers had never taught area.

8.2.1.2 Frequency of area lessons

When asked how often they taught area, and whether the lessons were in a sequence or single lessons, 14 of the teachers stated that they planned a sequence of three or four lessons, to be taught once each term for three or four terms each year. Two teachers had planned three or four single lessons to be taught at intervals throughout each term.

8.2.1.3 Teachers’ rationale in planning and teaching the measurement of area

In response to the question asking what they thought that children needed to learn about area in terms of skills and understandings, 13 of the teachers focused on learning to cover a given area with informal units, leaving no gaps or spaces. Two teachers explained the need to know how to compare areas, one teacher said that understanding the purpose of measurement was important, and one teacher referred to the need to learn how to use an array for quickly calculating the number of informal units used to measure a rectangular area. Only one teacher mentioned an understanding of the attribute of area, and her definition was a little confused:

The surface that something takes up, the amount of space that something covers (Matilda, initial teacher interview).

When commenting on the best way to teach area, all 17 teachers stated that practical activities contributed to the most appropriate teaching method. Four teachers used the specific description “concrete materials” and nine teachers used the term “hands on”.

Using lots of concrete materials (Jan, initial teacher interview).

Practical, concrete materials (Cathy, initial teacher interview).

Hands on, trial and error (Gemma, initial teacher interview).

Hands on and practical (Mary, initial teacher interview).
One teacher stated that children needed to work with tessellating units to cover an area, and two thought that children needed opportunities to discover or investigate area concepts. One teacher commented that children should cover surfaces and make comparisons.

Teachers made very mixed responses to the question asking what children needed to learn about area in the next three years. One teacher replied that she had not looked in the syllabus and didn’t know. Five teachers stated that calculating by using the formula for rectangular area would be learnt. Four teachers replied that formal units of area measurement would be introduced. Other comments included tessellation of informal units (two responses), an understanding of the attribute of area, comparing areas and visualising tiling, practical applications, and finding the areas of irregular shapes.

8.2.2 Results of Teacher Final Interviews

The transcriptions of the teachers’ final interviews were analysed and responses to the questions were collated and compared. This information was later added to data from the follow-up interviews to answer the research question: Which of the three models implemented was the most successful for the professional learning of the teachers who participated in this research? Teacher’s responses were grouped to examine the immediate changes teachers had made to their mathematics programs, their intentions to make changes in the future, and statements of what they felt they had learnt. The teachers’ reflections on whether they had achieved their initial goals of participation were also examined. The questions from the final interview that provided these data were:

- Has your participation in the project made any difference to the way that you plan or teach mathematics?
- Do you think the project will make any difference to the way that you teach mathematics in the future?
- What are the sorts of things that you have learnt in the last six weeks?
- At the beginning, I asked you what you hoped to get out of participating in the project, and this is what you said … (quote from each teacher’s initial interview inserted here). Did you achieve what you wanted to or expected to?
The remaining research question for this study was: *What factors were important to the success of the professional learning models?* During the final interview, teachers were asked:

- I’d like you to think about the aspect of the project that was richest for you. Was there any professional development strategy that was more successful for you than the others?

Responses to this question were listed and analysed to identify themes within the three groups of schools.

### 8.2.2.1 Changes that teachers had already made to teaching mathematics

When the teachers were asked whether their participation in the project had made any differences to the way they planned or taught mathematics, responses fell into three main themes: working cooperatively and sharing with colleagues, the use of questioning strategies during teaching, and planning lessons to ensure that children understood the concepts (Table 8.1).

<table>
<thead>
<tr>
<th>Theme</th>
<th>High support schools (7 teachers)</th>
<th>Medium support schools (5 teachers)</th>
<th>Low support schools (5 teachers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working cooperatively and sharing with colleagues</td>
<td>3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Teacher’s questioning techniques</td>
<td>5</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Planning lessons to ensure that children understand the concepts</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Teachers from the high support and medium support schools commented on teamwork and professional collaboration, whereas this strategy was not reported from the low support schools. Vicki, Deb and Sally from the Avenue school commented in their final interviews that they were sharing materials, exchanging advice and planning ahead and that this level of collaboration was novel to the teachers. Teachers from the Avenue and Hills schools also explained that the sharing and discussion had extended beyond their immediate team, so that other classes in the school were introduced to the lessons and rationale of the project.

We also have another new teacher who is trying everything we do even though she's not working with you. So that's been a really good vehicle for me to say, oh you know, Diane wants me to do
I can sort of force the issue without making them feel threatened that I'm questioning them sort of thing. So that's been really excellent (Gemma, Hills School, Teacher final interview).

The teachers in the older years, and we've spoken to them and told them what we're doing, and they think it's fantastic because they're having problems at the moment with their older kids. And they're sort of saying, well they're trying to teach that to their kids at the moment, in a different way, and they said they're having a lot of difficulty (Deb, Avenue School, Teacher final interview).

Eight of the 17 teachers said that participation in the project had resulted in a focus on their questioning techniques, prompted by the suggested questions in the lesson notes. Teachers commented that they were trying to present more open-ended questions, and were listening more carefully to students’ responses.

The questioning - when I was doing these area lessons, I would always bring the class back and consolidate what we were talking about and say go back again and just try to extend it a bit further and then bring them back again. I was doing that more than I usually do in maths (Karen, Forest School, Final teacher interview).

At Vista School (high support model), Kara explained that the experience of interviewing students at the conclusion of the lesson had led her to review children’s learning during other lessons, and then she and Cathy implemented a new teaching strategy:

But having a chance to interview the children after a lesson made me slow down and realise that what I say doesn't always go in, the way you thought it did. And that really made me start to think that you really need to have something after a maths activity if it’s going to be a hands on thing that sort of gives you an indication of how they went. Whether it is a talk to them or whatever. So one of the things that Cathy suggested because the interviewing was - you couldn't always interview after every maths lesson, it just wouldn't be practical - is a maths journal that we've started … The idea is not for them to have a turn at writing, but just to get their ideas out. And I found that very useful because I go around and just quickly say, what did you do and why did you do it? And it's been very, very helpful (Kara, Vista School, Final teacher interview).

Seven of the teachers stated that a major change in their teaching was a greater emphasis on planning to ensure that all children understood the mathematical concepts being presented. Teachers thought that they needed to take smaller steps, to revisit concepts if children were unsure, and to present the same concept in a number of different ways. These responses were prompted by the program of area lessons, including the sequence of ideas in the area lessons, the sustained focus on one aspect of measurement, the revision at the beginning of every lesson, and the concluding whole-class discussions.

Doing the series of lessons was a good idea. Really helped to consolidate. Really gave the children time to work with a concept instead of doing it in one-off type lessons (Milly, Station School, Final teacher interview).
It’s clarified a lot of how to do things. And I feel more comfortable with it. I also think in terms of how to present a similar concept in several different ways has also been very beneficial for my professional development (Nadia, Forest School, Teacher final interview).

Analysis of teachers’ responses to the question of changes already made to teaching mathematics indicates questioning and planning strategies were common to teachers across the three levels of consultancy support, and cooperative planning was stated in the high support and medium support schools only.

8.2.2.2 Intended future changes to be made to teaching mathematics

In the final interview, teachers were asked if they envisaged a change in the way they taught mathematics in the future (Table 8.2).

Table 8.2 Frequency of responses to intended future changes

<table>
<thead>
<tr>
<th>Theme</th>
<th>High support schools (7 teachers)</th>
<th>Medium support schools (5 teachers)</th>
<th>Low support schools (5 teachers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>More hands-on tasks</td>
<td>1</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Cooperative planning for other units of work</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Continue open-ended questioning strategies</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Use the lessons again</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Integrate mathematics strands where possible</td>
<td>1</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Encourage children to discuss and reflect on concepts</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Use the correct mathematical terminology</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Think more carefully about achieving the outcomes when planning</td>
<td></td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

The wide range of responses to this question appeared to relate to the rationale of the area lessons, rather than the different models of implementation. Teachers commented on their intention to plan more hands-on tasks, to use the lessons again, to integrate the mathematics strands, to use more mathematical terminology, to focus on open-ended questioning strategies, and to encourage children to reflect on and discuss mathematical concepts (Table 8.2). The only response which was common across the three levels of consultancy support was the intention to teach the lessons again. The teachers’ statements of what they had learnt during the project were varied both in the range of topics and the number of items listed. Several teachers from the
high support group listed two items in their response, so the frequency of comments
could not be compared across the three groups.

8.2.2.3 What teachers learnt during the area project

Responses were categorised into four themes. The most popular theme of
teaching and programming strategies included integrating area and multiplication,
using tessellating informal units, commencing each lesson with a revision of previous
activities and guiding children to look for a pattern in an array.

The very important part of the lesson for my class was the little bit of blackboard work first to
psych them up. A bit of feedback from the kids even before you begin. To see where you have to
start, to see how much they know or how little they know (Mary, Forest School, Final teacher
interview).

I’ve learnt that children often need to be told to look for a pattern. With some children it doesn’t
come automatically to them. Once they are given that kind of net they can see it after a while
(Nadia, Forest School, Final teacher interview).

The second theme identified was questioning techniques. Teachers used the
suggested questions in the lesson notes and six teachers commented the experience
had shown them the value of focused questioning:

I always look at the language but I must admit I don't think I concentrate on that part of it very
well, so with the questioning it made me realise that that was one of my down points and I needed
to work on that (Celia, Plains School, Final teacher interview).

I think I learned from that they still need a lot more guidance and a lot less of just being left to
their own devices … I think the thing that you have to work out is that the questions you ask are
very important, because if you don’t ask the right questions, you're not sending them along the
right path that you want them to go along (Scottie, Plains School, Final teacher interview).

I didn’t have those (questioning techniques) at all, it certainly helped them. I'm still finding that
hard. I think that's probably the hardest thing personally, my questioning techniques and just the
way to structure the question. But yes, it definitely helped (Jan, Mall School, Final teacher
interview).

And for me personally it's been a big eye opener. I mean I've done lots of other things since with
more open ended questions and they've really surprised me with the results (Gemma, Hills
School, Final teacher interview).

The third theme related to teacher’s increased awareness of children’s
understanding of the concept of area measurement:

Having the interviews was a real eye-opener because, as I said before, the ones that you were
expecting are learning it are the ones who aren't getting the full picture. So it was good to have
that opportunity (Sally, Avenue School, Final teacher interview).
To actually see what the students got out of it. I was amazed by what they did, and sometimes by what they didn't get out of something (Deb, Avenue School, Final teacher interview).

In the final theme, several teachers commented on their new understanding of area and volume or increased confidence in teaching the topic of area:

I’ve actually understood myself the difference between covering the surface and the space that you have to fill up with volume (Matilda, Plains School, Final teacher interview).

The frequency of responses within the four themes were organised against the levels of consultancy support to determine any differences between results from the three groups of schools (Table 8.3). Gemma from the Hills School (Medium support model) gave a response that was different from all others, when she stated that she had learnt to discuss mathematics, rather than the content of the syllabus units.

The teachers from the high support and medium support schools placed greater emphasis on the questioning techniques and the need for children to acquire and use mathematical language. It may be argued that these teachers had a greater opportunity to reflect on and communicate their own learning and impressions through the team discussions. Teachers indicated that talking about mathematics lessons with colleagues was a new experience for them.

Table 8.3  **Frequency of responses to teacher learning**

<table>
<thead>
<tr>
<th>Theme</th>
<th>High Support Schools (7 teachers)</th>
<th>Medium support schools (5 teachers)</th>
<th>Low support schools (5 teachers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teaching and programming strategies</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Teacher’s questioning techniques and emphasis on mathematical language</td>
<td>4</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>The need for children to understand the concept of area measurement</td>
<td>2</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Teacher content learning and confidence</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

8.2.2.4 Teachers’ achievement of planned professional learning goals

During the initial interview teachers were asked why they had volunteered for the area project. Eleven of the 17 teachers referred to an opportunity to gain new ideas or new ways for teaching area (Tables 8.4, 8.5, 8.6). Deb (Avenue School) commented that she had been impressed by the *Count Me In Too* project (New South Wales Department of Education and Training, 2003a) and she assumed that the area
project would offer a similarly-effective professional learning experience. Two teachers (Kara and Celia) explained that they were hoping to boost their confidence in teaching area. Milly (Station School) saw her participation as a personal challenge, and Kelly (Station School) wanted to confirm that she was teaching area effectively. Gemma from Hills School was very emphatic and focused in her reply:

Better skills. It's my goal this year to improve my maths teaching. T&D to support ME and my teaching (Gemma, Initial teacher interview).

Both Gemma and Mark (Hills School) had made a decision, before the area project was advertised in the district, to participate in mathematics professional learning during the year. This approach was different from the other 15 teachers, many of whom expressed a weak knowledge of how to teach area. These teachers did not explain that their participation in the area project was a component of a conscious decision to seek ongoing mathematics professional learning experiences.

<table>
<thead>
<tr>
<th>Teacher &amp; school</th>
<th>Teacher's reasons for volunteering (stated during initial interview)</th>
<th>Achievement of teacher’s goals (response to initial interview statement, during final interview)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vicki (Avenue School)</td>
<td>I feel that's one area that I really need some help with and I'm guilty of not doing as much as I should. And I thought it might give me some ideas, a little bit more enthusiasm to keep going.”</td>
<td>Well, it has. It has. I've done much more hands on things and less text book type things, and the kids have had so much more experience since we've started on the area and followed through with the measurement.</td>
</tr>
<tr>
<td>Deb (Avenue School)</td>
<td>Impressed by what kids are doing in CMIT, they see what they are doing, enthusiastic, love it.</td>
<td>Definitely. It has. I mean I can ask the children now, you only have to mention the word area, and they automatically look behind, because we displayed some of the tiles. The word tile comes to their minds. Oh yes. You see I've always done my maths in groups where I've rotated them through and having, when I first started Count Me In Too I thought Oh, I'm going to have to change the way I teach my maths. But it’s been good.</td>
</tr>
<tr>
<td>Sally (Avenue School)</td>
<td>Good for me to learn and get new ideas. Help kids to learn.</td>
<td>Yes, definitely it gave me some sorts of feedback on the ways I teach it to make it more successful. Yes, gave me ideas and things to look for and when they're not actually understanding it and how to take it from there.</td>
</tr>
<tr>
<td>Jan (Mall School)</td>
<td>Ideas and things to look for when the children don't understand.</td>
<td>Yes, it was. It was a good experience, even the other teachers, because we always talked about how good the lessons were. She's doing them next door, and as soon as she did the first one, she said it was really good.</td>
</tr>
<tr>
<td>Beth (Mall School)</td>
<td>Learn new ideas and different ways of teaching. Interviewing the children to find which lessons were most successful.</td>
<td></td>
</tr>
</tbody>
</table>

Table 8.4 Teachers’ achievement of planned goals in high support schools
<table>
<thead>
<tr>
<th>Teacher &amp; school</th>
<th>Teacher's reasons for volunteering (stated during initial interview)</th>
<th>Achievement of teacher’s goals (response to initial interview statement, during final interview)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Kara (Vista School)</strong></td>
<td>Understand how to teach area. Not confident with measurement</td>
<td>Yes, I think it has been. Area, measurement is a strand that I don't feel confident with because I never really grasped the concepts myself. So I think just even doing the activities myself I found helpful. The actual content, and that's pretty embarrassing to say.</td>
</tr>
<tr>
<td><strong>Cathy (Vista School)</strong></td>
<td>Different ways of teaching area, especially to younger children.</td>
<td>Being able to talk to them about their learning I thought was really good. And seeing them progress through activities that had already been prepared, already been sequenced, for a purpose.</td>
</tr>
</tbody>
</table>

### Table 8.5 Teachers’ achievement of planned goals in medium support schools

<table>
<thead>
<tr>
<th>Teacher &amp; school</th>
<th>Teacher's reasons for volunteering (stated during initial interview)</th>
<th>Achievement of teacher’s goals (response to initial interview statement, during final interview)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gemma (Hills School)</strong></td>
<td>Better skills. It's my goal this year to improve my maths teaching. T&amp;D to support ME and my teaching.</td>
<td>It’s done that, exactly that. Obviously I've still got a long way to go, I don't think we're ever perfect…. But unless you change the way you teach, nothing is going to change. And I really think that I am changing the way I am teaching.</td>
</tr>
<tr>
<td><strong>Mark (Hills School)</strong></td>
<td>I don't know if I'm doing it right. Get some good ideas, lessons, how to structure lessons.</td>
<td>Yes it was. I often still feel like there's an awful lot to cover. It’s just a gradual getting things together. But it has given me a lot of help in that respect. So it’s been very useful.</td>
</tr>
<tr>
<td><strong>Scottie (Plains School)</strong></td>
<td>To get help rather than just the paperwork. Learn better and new ways, add to knowledge.</td>
<td>Yes. Definitely change the way I'll teach area. Children understood covering.</td>
</tr>
<tr>
<td><strong>Celia (Plains School)</strong></td>
<td>To feel more comfortable teaching area. More confident. Make it enjoyable for these kids.</td>
<td>Changed my whole attitude to teaching area. I felt comfortable, and would love to do volume too.</td>
</tr>
<tr>
<td><strong>Matilda (Plains School)</strong></td>
<td>New ideas. Kids can do more than we are expecting. Help me to help other teachers.</td>
<td>Yes. I learnt that terminologies and language are very important. I didn’t think I would get that much out of it. Children have surprised me.</td>
</tr>
</tbody>
</table>

### Table 8.6 Teachers’ achievement of planned goals in low support schools

<table>
<thead>
<tr>
<th>Teacher &amp; school</th>
<th>Teacher's reasons for volunteering (stated during initial interview)</th>
<th>Achievement of teacher’s goals (response to initial interview statement, during final interview)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nadia (Forest School)</strong></td>
<td>Thought it would be interesting, learn some new things and it will transfer down to the kids</td>
<td>I was looking for integration, impact on my teaching and professional development and I think that has taken place. I think the children have learnt from it and I’ve got some new ideas.</td>
</tr>
<tr>
<td><strong>Karen (Forest School)</strong></td>
<td>Looking for new ideas and how to teach measurement.</td>
<td>Yes. Learnt different ways to approach area and measurement, built up my ability to teach area.</td>
</tr>
</tbody>
</table>
Mary
(Forest School)

Some new ways and new ideas.

I feel I learnt a lot. The children thoroughly enjoyed it.. And particularly my lower group I think they’ve probably improved more than the others, because they didn’t have the concepts and now perhaps they’ve got the concepts.

Milly
(Station School)

I’m always ready to take on a bit of a challenge and that’s what I saw it as, a challenge.

Oh definitely! I think I’m sort of looking at those strands of maths now, with a bit more motivation, a bit more creativity, in planning what to do.

Kelly
(Station School)

I think it sort of forced me to look at what I’m doing, to see whether I’m covering; giving the children the skills they need to know

Yes. Fresh ideas and children have all made some progress in their understanding

The teachers’ responses to the question in the final interview asking if they had achieved their goal were examined to identify any impact of the three levels of consultancy support. All of the teachers stated that their goals had been realised. Seven of the 17 teachers also commented that their children had benefited, Kara from Vista School admitted that she had learnt how to measure area herself and Celia (Plains School) reported that she now felt “comfortable” about teaching area. No difference in teachers’ stated achievement of goals between the three models was identified, but a powerful contrast between the 7 schools was found. Most of the teachers gave a definite response that their participation in the project was successful and would impact positively on their teaching of area in the future (Jan, Beth, Vicki, Cathy, Scottie, Matilda, Milly, Nadia, Karen). By comparison, Gemma and Mark from Hills School explained that the project marked the beginning of positive changes to their teaching, but there was still room for improvement before they would be completely satisfied with their teaching strategies and programs.

Obviously I’ve still got a long way to go; I don’t think we’re ever perfect. … You can keep on getting better and better at doing something the way you teach. But unless you change the way you teach, nothing is going to change. And I really think that I am changing the way I am teaching (Gemma, Final teacher interview).

I often still feel like there’s an awful lot to cover. … It’s just a gradual getting things together. But it has given me a lot of help in that respect (Mark, Final teacher interview).

8.2.2.5 Teachers’ perceptions of successful professional learning strategies

Teachers were asked which professional learning strategy had been the most helpful in reviewing classroom strategies.
Table 8.7  
*Frequency of responses to successful professional learning strategies*

<table>
<thead>
<tr>
<th>Theme</th>
<th>High Support Schools (7 teachers)</th>
<th>Medium Support schools (5 teachers)</th>
<th>Low Support schools (5 teachers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson notes and materials</td>
<td>5</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Student interviews</td>
<td>6</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Meetings with consultant</td>
<td>2</td>
<td>2</td>
<td>N/A</td>
</tr>
<tr>
<td>Initial meeting</td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Overwhelmingly, teachers stated that the provision of lesson notes and materials were successful strategies to support their professional learning. The lesson notes were straightforward: “Your lesson notes are very simple. You can’t really muck them up, can you?” (Nadia, Forest School, Final teacher interview). Teachers used the suggested questions in the lesson notes and stated that this material showed them how to pose open-ended questions in other areas of mathematics:

I actually did the questions too because going back to getting the language out of the kids and focusing on the language, I actually found the questions were helpful to get me on the right track to ask the right things of the children (Matilda, Plains School, Final teacher interview).

I’ve been asking them, Well how did you get the answer? This just hasn’t been with area but its been with a lot of things, getting them to explain how they got that answer. Some of them have trouble, even with just simple addition questions (Sally, Avenue School, Final teacher interview).

Having the researcher to visit was important to teachers from Avenue school who explained that it gave them additional confidence to trial the new program. The researcher was careful to play a minimal role in group meetings. Vicki, the most experienced teacher in this team, retired from teaching the year following the trial. She used the researcher as a sounding board after every visit and later agreed that the researcher listened to her but didn’t actively respond. Vicki explained that she needed the debriefing sessions to give her courage to trial the area teaching material which she considered very new and even risky:

But even if you were very non-committal about what I said, even if you said nothing, I knew I was talking to someone who could understand and maybe just file it away (Vicki, Avenue School, Final teacher interview).

At Hills school, knowing that the researcher would visit provided additional motivation to implement the lessons:

I think things very quickly fade into, I'll just do this or I'll just do that, but because you're there, not that you're an authority figure, but because you come to talk to us … (Gemma, Hills School, Final teacher interview).
Six of the seven teachers in the high support group of schools stated that the release from teaching to interview four children at the conclusion of each lesson had been a major factor in professional learning. Teachers felt that the interviews gave them: a “fascinating” insight into what the children understood; excellent feedback; and models of different kinds of questions. The teachers from Mall School were so impressed with the strategy that they made a submission to the school planning committee for a budgetary allowance for release time to interview children during the following year:

But see the interviews were good for me because when you're observing and asking questions around the room, I think, She understands this. But sometimes you ask them a question and they say something totally different and I think Ooh. She didn't really understand that. And see I didn’t notice that from what she was doing. I thought the interviews were very valuable (Beth, Mall School, Final interview).

The only formal strategies available to the teachers in the low support group were the initial meeting and the provision of lessons plans and materials. The five teachers selected the lesson materials as effective strategies, and one teacher noted that the initial training session had been successful.

8.2.3 Results of School-based Team Meetings

The school team meetings in the high support and medium support schools provided a forum for discussions of observations of children’s responses to the learning experiences and teachers’ suggested variations to the activities. The meetings also assisted teachers to clarify their own thinking about the development of children’s skills and knowledge and how to address children’s specific needs in the classroom.

The transcriptions of the meetings were analysed in two ways. Firstly, the major themes in the teachers’ thinking and sharing were identified using a grounded theory approach as described by Glaser and Strauss (1967) and Corbin (1986). The themes were listed with specific comments from the transcriptions, and included the teachers’ descriptions of student learning and knowledge, teachers’ surprise at some student knowledge, collaborative sharing of ideas and suggestions, the teachers’ own learning about mathematics and the possibility of integrating area lessons with other aspects of the curriculum. Some of the teachers’ responses were applicable to more
than one category, so the researcher selected the most dominant application for each comment or suggestion.

Secondly, the researcher sought to design and test a model that would reflect the teachers’ emerging cycle of professional dialogue and learning.

8.2.3.1 Teachers’ descriptions of children’s learning

Teachers accurately described children’s understanding of mathematical concepts based on careful observations of students’ responses to the tasks. Teachers described the specific strategies children had used to make the square and triangle patterns, and focused on children’s use of array patterns and counting patterns when measuring area with tiles. Teachers were also able to describe the boundaries of students’ knowledge.

Some of them did a really good job with working out the area of the irregular shapes. Like they’d section it off and say, that bit there is 4, 8, 12, 16, and you add on 4. (Mall School, Discussion 2)

It was easy because it was visual, to see the squares with the dots for instance, they can see that. But once you extended it to more than just the four dots they found it very difficult. (Hills School, Discussion 2)

They were quite good at seeing the pattern that some are up and some are down. Like they had different ways like the pointy end first and the flat end, and they had different ways of describing. But they all had the right idea like one had to be up and one had to be down. (Vista School, Discussion 2)

The top boy in my class, here I am thinking "No gaps, no overlapping" and his was the only group that had big gaps everywhere…No, he couldn’t see it, that there were gaps. I’ll have to work on you, I think. (laughing) And he's like, if you give him any number work, like base 10, he picks it up. (Plains School, Discussion 2)

8.2.3.2 Teachers’ surprise at student knowledge

Teachers were surprised at student knowledge – either a lack of knowledge or understanding, or very good, pre-existing knowledge. Cathy at Vista School assumed that all of her Year 2 students had a working knowledge of how to make a square, but was surprised to find some confusion:

We had a problem identifying what a square was. I just assumed, because I had talked about it a hundred times before that they would know what a square was. So we spent a lot of time, why is it a square and we talked about that for a really long time (Vista School, Discussion 2).
Beth at Mall School found that her Year 2 students were able to use the structure of rows and columns to calculate the area of a large rectangle in tiles, without covering the entire rectangle:

I was quite surprised by this that a lot of mine made a really big rectangle, and they said they were going to run out of tiles. And I said, what could you do? And they said, they put the first one down, I think it was five. And then they used all their tiles up to 35, say, and they started using the tiles from before, taking those ones off and using them again. And they said, we don’t really need these tiles, we know they go in rows of five. They were skip counting by fives (Mall School, Discussion 2).

8.2.3.3 Teachers gave and accepted advice from colleagues

During the meetings, teachers supported and encouraged colleagues by listening sympathetically when teachers reported problematic lessons, and making helpful suggestions of how to approach specific tasks. The teams from Avenue, Mall, and Hills adopted a routine of one teacher trialling each new lesson and then passing advice to the other teachers of how to manage the tasks and materials, at an informal meeting the next day:

It probably helped Sally to talk to me before she did her lesson, to find out all my mistakes (Avenue School, Discussion 3).

I said to David, he asked them to estimate, but I would have made a bit more of that estimation than David did, and we talked about it at the time. Because when he asked them to estimate how many they thought would fit, that’s where we want them to start thinking, that’s where they can really think, can I picture this, can I think one, two, three, four, five, six (Hills School, Discussion 2).

And it’s good for Vicki when she does it on a Monday, because we can then ask her how it went or how long to allow things like that (Avenue School, Discussion 1).

The team from Plains school did not meet informally and did not discuss the lessons outside of the meetings with the researcher: “The three of us never really got together” (Celia, Plains School, Final teacher interview).

8.2.3.4 Teachers described their own learning

Teachers described their learning about teaching mathematical concepts, noting that teachers need to have explicit plans of what and how children learn. Teachers also talked about their own school experiences of learning mathematical formula, with limited understanding, comparing this approach with the understandings that their students were acquiring.
I think I’ve learnt a lot. I mean I know I keep saying this, but it’s really made me think about what I’m asking them to do, to be more explicit. Because when you’re not explicit, you get the rubbish that you … I mean you really have to be clear in your mind (Hills School, Discussion 2).

And looking at the squares, how easy it is for them to get the idea when it comes to the multiples, length by breadth or whatever we used to call it. It’s just so – we learnt it as a rule. You did it but you didn’t understand why (Avenue School, Discussion 2).

Just the learning. I’m learning more and more about how kids are thinking and about how I should be teaching them (Hills School, Discussion 1).

8.2.3.5 Teachers discussed integrating area with other mathematics

Teachers commented how area concepts integrated into other aspects of mathematics teaching, such as space and number. Teachers demonstrated their understanding of how the teaching of area concepts can be integrated with number and space and their readiness to recognise and use opportunities when presented.

Mine were very quick to point out that it was the same number of triangles. We then went on to make bigger triangles and made triangular numbers and squared numbers, which was interesting (Hills School, Discussion 1).

We've only just started counting by fives, by tens and by twos. And then I remember one little boy went, That's easy. We'll go, and you could see his head going, we'll go two, then we'll go six, ten, twelve. He was pointing to them and how he was going to do it. That's counting by twos. But no, he wanted it his way. And then I had to show them the pattern, like let's practice our twos counting (Plains School, Discussion 3).

8.2.3.6 Teachers sustained a focus on area concepts throughout the lessons

None of the participating teachers had previously implemented such a long sequence of area lessons (see 8.2.1.1). The six lessons and the continuing emphasis on understanding the attribute of area represented a sustained focus on measuring and discussing area and the use of appropriate terminology. Teachers reported that children reacted very favourably to their teaching emphases and strategies.

They definitely understand that area is surface because we've drummed that every time. I say, What is area? And they still, the amount of space that is covered, they still use that. Which is OK, but the idea is surface (Plains School, Discussion 3).

I’ve got an excellent comment. We’ve done four lessons. I ask them every time, ”Who can tell me what area is”, because I always start the lesson with revision. And Nathan said, ”You draw a shape and then when you fill inside of it, then that's the area.” That was from a Kindergarten child. It was just a brilliant description, because he's actually verbalising what an area is. It is a difficult thing for even an adult to do (Plains School, Discussion 2).

My slowest, slowest child in the class. He can now tell me what area is and what it means in really simple terms (Avenue School, Discussion 2).
8.2.4 Modelling the Professional Learning

The researcher recorded a projected model of professional learning consisting of three major stages (Teach, Discuss, Evaluate), before commencing analysis of the team meeting transcriptions, as illustrated in Figure 8.1.

**Contributing concepts**

<table>
<thead>
<tr>
<th>Stage 1: Teach</th>
<th>Stage 2: Discuss</th>
<th>Stage 3: Evaluate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teaching the lessons; Watching children; Listening to children.</td>
<td>Listening to teachers; Discussing with teachers; Achieving group consensus.</td>
<td>Accepting or rejecting lesson content; Making changes to lessons.</td>
</tr>
</tbody>
</table>

Figure 8.1 Projected model of teacher professional learning

To commence open coding (Strauss & Corbin, 1990), one transcript from each of the five schools was examined. All of the concepts from the projected model emerged but the list also included:

- Expressing surprise at student achievement;
- Integrating area concepts in other mathematics lessons;
- Reporting student learning in terms of outcomes;
- Reflecting on their own learning;
- Committing to teaching the lessons again;
- Variations made to the documented instructions or methods when teaching the area refined lessons.

All of these concepts were included in the revised categories and a tentative diagram was designed (Figure 8.2). The titles of the three Stages were progressively modified as the properties of each Stage, focus of the teacher discussions, and the relationships between the Stages, became more explicit. In the tentative model, the Stages were titled Focus on student learning, Collaboration and Teacher learning.
Stage 1 of the tentative model emphasised student learning from the teachers’ perspectives. Teachers clearly described the learning outcomes, the strategies used by children, and results that were different from the teachers’ expectations. The transcriptions indicated that teachers made a conscious effort to prepare for the team meetings by thinking carefully about the lesson outcomes and student responses.

Stage 2 of the tentative model reflected the teachers’ interactions and the interplay of describing, advising, and consensus-building that took place during the meetings. Teachers were more proactive and creative about giving and taking advice than the simple discussing as listed in the projected model.

Stage 3 of the tentative model indicated the frank and personal way that teachers discussed their own learning. Teachers demonstrated confidence in their understanding of how children were developing area concepts, and discussed how to implement the next lessons in the sequence. Teachers also commented on their plans to revise and teach the lessons again the next year.

As the transcriptions were coded and reviewed, it became apparent that teachers were using the meetings to voice reflections about all of the lessons to date. They used this material to make decisions about how to modify the next lesson if necessary, how to cater for children experiencing difficulties, and also how the lessons would be modified for an implementation in the following year. The three
stages of the tentative model portrayed the interactions of teachers with the lessons, children and each other.

The tentative model was then modified (Figure 8.3) to better reflect an action research cycle of teaching, observing, reflecting, collaborating with colleagues and then using new knowledge and strategies to commence the cycle again. The categories and concepts from the tentative model were retained, but the linear relationships between categories were amended.

![Action research model of teacher professional learning](image)

Figure 8.3 Action research model of teacher professional learning

The final model of professional learning (Figure 8.3) represented the dynamic process that revolved around the teachers in their teaching and observations of student learning, discussing with colleagues and relating new knowledge to old before continuing to implement the lesson sequence and modifying lessons to address the children’s needs.

Focus on student learning: Teachers described children’s responses to the tasks in detail. They compared the demonstrated learning outcomes with the planned outcomes and tested children’s knowledge against previous expectations. Teachers’ reflections on classroom observations and interviews with individual children provided the basis for explicit and confident dialogue during the team meetings.
Collaboration: The focused and enthusiastic nature of the team meetings indicated that the teachers had taken ownership of the program. Teachers compared responses from class groups, explained the modifications to lessons they had introduced, supported colleagues who had difficulties, and described how they had integrated the area concepts into other mathematics lessons. Teachers also used colleagues’ advice and issues discussed in the meetings to assist in more focused observations during subsequent lessons.

Teacher learning: The team meetings provided a conduit for teacher learning, as demonstrated by an improved understanding of how to teach area concepts, a move away from teacher-centred descriptions and the improved ability to integrate area concepts across other mathematical areas. Teachers were personal and forthright in statements describing their own learning. The confidence that teachers demonstrated in their understanding of how to plan and teach area lessons progressed with the lessons, and differed markedly from comments about teaching area in the initial teacher interview. The teachers also consistently spoke of changes they would make before implementing the lessons the next year. Teacher’s new knowledge was applied to the next phase of the cycle, student learning.

The model of teacher professional learning reflected a dynamic, interactive, and ongoing picture of inquiry, collegial support and interaction. The richness and enthusiasm of the teachers’ interactions were a positive indication of the success of this aspect of the study.

8.2.5 Results from Children’s Pre- and Post-Assessment Tasks

Children completed a pre- and post-assessment task before and after the implementation of the refined area lesson sequence. The researcher analysed the children’s responses in terms of the children’s ability to visualise and draw tiles covering the area of a rectangle. Children who drew an array of tiles using the most sophisticated strategy of continuous horizontal and vertical lines, in the correct proportions, received a score of 6. Children who drew individual tiles, using the least sophisticated strategy, were scored at 1. The performance scale was used in Phase 1 of this study and is described and illustrated in Chapter 5. The researcher compared
and scored each child’s two drawings, to determine the growth in understanding of how to structure an array to cover the area of a rectangle.

An examination of the results from each school indicates children’s growth within their cohort. At the three high support schools of Avenue, Mall and Vista (Figures 8.4, 8.5 and 8.6) the majority of children scored 3 or below on the pre-test and improved to 4, 5, or 6 on the post-test.

Of the two medium support schools, Hills (Figure 8.7) and Plains (8.8), Hills indicated a higher percentage of children had demonstrated growth from a score of 3 to a score of 5 or 6. The growth at Plains school is not as strong as the growth in the previous four schools.

Results from the two low support schools, Forest (Figure 8.9) and Station (Figure 8.10) show a higher number of children scoring 3 or 4, rather than 5 or 6 in the post-test, as demonstrated in the other schools and particularly at Hills school. The results from all pre- and post-test tasks indicate that the children from a medium support school, Hills, demonstrated the strongest growth in learning.

![Figure 8.4  Avenue school results of pre-test and post-test](image)
Figure 8.5  Mall school results of pre-test and post-test

Figure 8.6  Vista school results of pre-test and post-test

Figure 8.7  Hills school results of pre-test and post-test
Within each class, results were collated to indicate how many children had improved by 0, 1, 2, 3 and 4 points (Table 8.8). These results indicate that children
from Plains, Forest and Station schools demonstrated the least growth and the children from Hills school exhibited the highest growth with the maximum difference of 4 points between the pre- and post-tasks. This table does not indicate the children’s pre-test scores, so does not specify the number of students who received a higher score on the pre-test and therefore had limited opportunity to demonstrate improvement.

Table 8.8  
Students’ improvement between pre-test and post-test

<table>
<thead>
<tr>
<th>Group</th>
<th>School</th>
<th>0 points on scale</th>
<th>1 point on scale</th>
<th>2 points on scale</th>
<th>3 points on scale</th>
<th>4 points on scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>High support school</td>
<td>Avenue</td>
<td>30.1%</td>
<td>24.7%</td>
<td>23.3%</td>
<td>20.5%</td>
<td>1.4%</td>
</tr>
<tr>
<td></td>
<td>Mall</td>
<td>28.6%</td>
<td>16.3%</td>
<td>40.8%</td>
<td>14.3%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>Vista</td>
<td>21.7%</td>
<td>4.3%</td>
<td>56.5%</td>
<td>17.4%</td>
<td>0%</td>
</tr>
<tr>
<td>Medium support school</td>
<td>Hills</td>
<td>20.4%</td>
<td>16.3%</td>
<td>30.6%</td>
<td>28.6%</td>
<td>4.1%</td>
</tr>
<tr>
<td></td>
<td>Plains</td>
<td>56.9%</td>
<td>23.5%</td>
<td>17.6%</td>
<td>0%</td>
<td>2%</td>
</tr>
<tr>
<td>Low support school</td>
<td>Forest</td>
<td>56.4%</td>
<td>20%</td>
<td>14.5%</td>
<td>9.1%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>Station</td>
<td>62.5%</td>
<td>12.5%</td>
<td>20.8%</td>
<td>2.1%</td>
<td>2.1%</td>
</tr>
</tbody>
</table>

8.2.6  Results of Teacher Follow-up Interviews

After teachers were interviewed again, nine months after the implementation of the refined area lessons, the interview transcriptions were analysed to determine the continuing effects of participation in the project. These data were combined with previous data to respond to the research questions about the most effective professional learning model and the factors important to the success of the teacher professional learning. Fourteen teachers of the original 17 were available for a follow-up interview. One teacher had retired, one had taken twelve months Leave without Pay, and one was on Maternity Leave and had not returned to full-time teaching.

Teachers’ responses to each question were collated and grouped in high support, medium support and low support tables (Tables 8.9, 8.10 and 8.11). Common themes from responses were identified within headings of whether the teachers had taught the lessons or concepts again, the effects on their teaching, and their perceptions of the professional development strategies. Teachers were also asked if their school principal or executive had given assistance with the implementation of the program and these affirmative and negative responses were tallied.
### Table 8.9  Results of high support teacher follow-up interviews

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Lessons taught again?</th>
<th>Concepts taught again?</th>
<th>Effect on teaching strategies</th>
<th>Successful professional learning strategy/ies</th>
<th>Single memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>Yes</td>
<td>Yes</td>
<td>Planning, Integrating</td>
<td>Lessons, Materials</td>
<td>Asking questions</td>
</tr>
<tr>
<td>Beth</td>
<td>Yes</td>
<td>Yes</td>
<td>More hands on</td>
<td>Dialogue, Lessons</td>
<td>Student learning Tiles</td>
</tr>
<tr>
<td>Vicki</td>
<td>Yes</td>
<td>Yes</td>
<td>More hands on</td>
<td>Dialogue, Commitment</td>
<td>Experiments</td>
</tr>
<tr>
<td>Sally</td>
<td>Yes</td>
<td>Yes</td>
<td>Planning, Progress</td>
<td>Dialogue, Reflection</td>
<td>Planning, Integrating Hands-on</td>
</tr>
<tr>
<td>Kara</td>
<td>No*</td>
<td>Yes</td>
<td>Questions, Practice</td>
<td>Dialogue, Team work</td>
<td></td>
</tr>
<tr>
<td>Cathy</td>
<td>Yes</td>
<td>Yes</td>
<td>Listening, Students sharing</td>
<td>Dialogue, Team work</td>
<td></td>
</tr>
</tbody>
</table>

* Another teacher was taking Kara’s class for Measurement

### Table 8.10  Results of medium support teacher follow-up interviews

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Lessons taught again?</th>
<th>Concepts taught again?</th>
<th>Effect on teaching strategies</th>
<th>Successful professional learning strategy/ies</th>
<th>Single memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scottie</td>
<td>No*</td>
<td>Yes</td>
<td>Planning</td>
<td>Lessons, Materials</td>
<td>Student success</td>
</tr>
<tr>
<td>Matilda</td>
<td>No*</td>
<td>Yes</td>
<td>Structure of area, Arrays</td>
<td>Lessons, Materials</td>
<td>Student counts by 7’s</td>
</tr>
<tr>
<td>Celia</td>
<td>Yes</td>
<td>Yes</td>
<td>Students’ capabilities</td>
<td>Facilitator, Lessons</td>
<td>Student success</td>
</tr>
<tr>
<td>Gemma</td>
<td>Yes</td>
<td>Yes</td>
<td>Arrays</td>
<td>Reflection, Dialogue</td>
<td>Stress counting, arrays</td>
</tr>
<tr>
<td>Mark</td>
<td>Yes</td>
<td>Yes</td>
<td>Thinking about teaching</td>
<td>Commitment, Lessons</td>
<td>Student success</td>
</tr>
</tbody>
</table>

* Scottie and Matilda were teaching Kindergarten classes
Table 8.11  Results of low support teacher follow-up interviews

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Lessons taught again?</th>
<th>Concepts taught again?</th>
<th>Effect on teaching strategies</th>
<th>Successful professional learning strategy/ies</th>
<th>Single memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milly</td>
<td>No*</td>
<td>Yes</td>
<td>Tessellating shapes for area</td>
<td>Lessons, Materials</td>
<td>Integration</td>
</tr>
<tr>
<td>Nadia</td>
<td>Yes</td>
<td>Yes</td>
<td>Raised expectations</td>
<td>Lessons, Materials</td>
<td>Student working with array</td>
</tr>
<tr>
<td>Karen</td>
<td>No*</td>
<td>Yes</td>
<td>Changed area teaching</td>
<td>Materials</td>
<td>Large squares, triangles</td>
</tr>
</tbody>
</table>

* Milly and Karen were teaching Kindergarten classes

8.2.6.1 The number of teachers who had taught the lessons again

Of the fourteen teachers, ten had taught the sequence of lessons again. Of the remaining four teachers, three were teaching Kindergarten classes and had not attempted to implement the lessons again as they were not considered appropriate to Kindergarten in the first half of the year. One teacher was not teaching Measurement, as the Relief from Face-to-Face teacher in her classroom had responsibility for these lessons. However, all fourteen teachers commented that the concepts around which the area lessons had been designed, including the emphasis on array patterns and skip or stress counting had been repeated in their current teaching.

The two teachers from Hills school were the only participants who had made significant modifications to the original lessons. One of the Hills teachers, Mark, had also implemented the pre-test and post-test. This was the only difference in responses to these two questions, between the three levels of consultancy support.

8.2.6.2 Effects of participation on teaching

When asked what effect participation in the project had made on their current teaching, there were no major differences between responses from the three levels of consultancy support. Within each group, teachers mentioned teaching strategies such as using hands-on tasks, arrays and tessellating shapes. Teachers also described planning and programming issues and raised expectations of student achievement.
Two teachers from the high support schools were the only participants to mention questioning and listening to students.

8.2.6.3 Teachers’ perceptions of the professional learning strategies

When asked which professional development strategies had been the most successful during the project, most teachers listed two items, such as professional dialogue and the provision of lessons. The most common responses from the high support teachers were dialogue and teamwork. By contrast, the medium support and low support teachers focused on the lessons and materials. One teacher from each group listed the researcher’s presence as a significant strategy in providing confidence and support to complete the project.

One teacher from each of the high support and medium support groups stated that they had made a commitment to complete the lessons when they chose to participate in the project. Again, one teacher from each of same two groups commented that their participation in the project encouraged them to reflect on their teaching program and strategies, and this had provided significant professional learning.

8.2.6.4 Evidence of teamwork at each school site

The teachers’ interviews were examined for specific references to teamwork or working with a colleague as the lessons progressed (Table 8.12). Teachers described their teamwork in two different forms. At Avenue, Mall and Hills schools, a more experienced teacher provided support and guidance for the less experienced teachers. In each case, the teacher assumed leadership and some responsibility to ensure that team members had access to materials and kept to the lesson implementation schedule. They also gave advice of how to present the tasks.

Speaking to her before I actually did the lesson. When Vicki would say, make sure you do this because this didn't quite work for me. That was probably the best thing, especially with a subject like the area where none of us really had a lot of experience in teaching it. I found that I could very easily, I used to steer away from those sorts of topics. And now, I would have no problem going in (Deb, Avenue School, Final teacher interview).

At Vista school neither of the two teachers assumed leadership, but they regularly checked with each other to ensure that materials were accessible and the
lesson implementation was progressing. At Plains school the teachers met with the researcher for team meetings, but had no other interactions about the lessons. At Station and Forest schools (low support schools), the teachers did not meet or discuss the lessons during the implementation period. The project had originally been advertised for school teams but no suggestions for team meetings or support had been suggested to the participants, and these teachers chose not to function as teams.

Table 8.12  *Schools with an active team or teacher leader*

<table>
<thead>
<tr>
<th>School</th>
<th>Team support</th>
<th>Teacher leader</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avenue</td>
<td>Yes</td>
<td>Vicki</td>
</tr>
<tr>
<td>Mall</td>
<td>Yes</td>
<td>Jan</td>
</tr>
<tr>
<td>Vista</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Hills</td>
<td>Yes</td>
<td>Gemma</td>
</tr>
<tr>
<td>Plains</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Forest</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Station</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

8.2.6.5 Assistance given by the principal

Teachers were asked whether their principals had given assistance or made specific enquiries about the lessons during the implementation. Kara and Cathy from Vista school were the only teachers who regularly spoke to their principals about the progress of the lessons. Vista school was a small school with a teaching principal who was very involved with the academic progress of all students. None of the other teachers had been given assistance with the project or had discussed specific details of students and lessons with their principals.

8.3 Research Questions

8.3.1 *How do teachers currently plan and implement learning experiences in area?*

Teachers stated that the best way to teach area was through the provision of practical activities, and the lessons which teachers described provided examples of the application of this belief. Teachers placed a great emphasis on the notion that measuring area is synonymous with covering. The skill of covering was given high
importance, with only one teacher out of 17 commenting that children need to understand the attribute of area. The majority of the covering was done with square-based blocks or books, with two teachers commenting that non-tessellating shapes were also used.

The teachers had attended the introductory session of the Area Project, before the initial interviews. The introductory session focused on the rationale behind the refined area lessons, with an explanation of the importance of identifying the structure of repeated, tessellating units and the formation of composite units. Only one teacher referred to this concept in her response to the question asking what children needed to learn about area in the next three years. The remaining 16 teachers did not appear to link their ideas of the emphasis on covering in the early years with their understanding of what would be taught in years 3 and 4.

All teachers used the syllabus for planning lessons, with the majority of teachers also using a textbook for further ideas and tasks. Most teachers preferred to plan a series of three or four lessons, and to implement such a series three times during the year.

Teachers were unanimous in their stated belief that area lessons should involve practical, hands on materials using some form of concrete materials. However, the teachers’ emphasis on covering an area was more important than establishing the concept of the attribute of area and an understanding of how to choose, organise and calculate the number of informal units used to measure a rectangular area.

8.3.2 Which of the three models implemented was the most successful for the professional learning of the teachers who participated in this study?

This study trialled three models of teacher professional learning, based on three levels of consultancy support. The most successful school, in terms of teacher learning and student achievement, was Hills, which received the medium level of support. Given that all of the participating teachers indicated that their initial goals were realised, there were several differences between the Hills teachers (Gemma and Mark), and other participating teachers:
• Gemma and Mark had previously identified a need to make changes to their mathematics programs:
• Gemma and Mark were the only teachers who reported they made modifications to the area refined lessons in the second year of implementation;
• During the final interview, Gemma and Mark indicated that their changes had not been finalised, and they expected to continue making changes.

At the Hills School, the single most important fact or determining success appeared to be teacher disposition. Gemma and Mark were ready to apply and critically appraise new ideas, and to persist with the innovation until they were satisfied with the results. Their professional interactions were productive and their expectations of student achievement were very high. Gemma integrated the concept of an array structure to many facets of her teaching, including discussing arrays when leading the whole-school assemblies.

In the pre- and post- assessment tasks, the children from the Hills school demonstrated the strongest growth in understanding of the structure of an array to measure rectangular area.

In this study, the medium support model was the most successful strategy. The success was partially attributable to the single-mindedness and high level of professional collaboration exhibited by the two teachers, but was supported by the structure of the model, including the provision of teaching plans and materials and a program of professional dialogue.

8.3.3 What factors were important to the success of the professional learning models?

The factors which ensured the success of the study included the high level of participation from every teacher in the project, the lesson notes and materials, teamwork and dialogue, release time to interview students, the presence of a supportive facilitator and the emergence of teacher leaders.

The volunteer teachers who committed to the project contributed to its success through ensuring that they implemented all of the lessons and materials, met with the researcher as planned to share and discuss their personal experiences and made their
classes available for pre- and post-testing. A valid comparison of the three professional learning models would not have been possible without the full participation of all of the teachers, and an assessment of the professional learning models should acknowledge the commitment, perseverance and motivation which the 17 teachers contributed to the study.

The professional learning strategy which was common to all schools was the provision of refined area lesson notes and teaching materials. Teachers consistently nominated the notes and materials as being effective aids to professional learning. The lesson program was praised because it showed teachers how to:

- Plan a sequence of lessons on a sustained theme;
- Begin each lesson with a revision of previous work;
- Use open-ended questions to encourage children to think, to create their own knowledge and to contribute to classroom discussions;
- Use practical, group-centred and engaging tasks;
- Concentrate on establishing children’s understandings of concepts.

Teachers in the high support schools and medium support schools met with the researcher each fortnight for a short team meeting. This strategy was successful in four of the five schools, in establishing a collaborative and supportive professional group which met informally during the week and ensured that team members were on track and confident of being able to present the lessons. These teachers nominated the teamwork and dialogue as successful professional learning strategies during their interviews with the researcher. Teachers from Plains, Station and Forest schools, where no team interactions were reported, commented that informal chats and suggestions from team members would have enhanced the implementation.

Teachers in the high support schools were given release to conduct interviews with a small group or individual students following each lesson. The teachers nominated this strategy as the most important in their final interviews and several noted it changed the way they conducted whole-class lessons.

Several teachers commented that they appreciated the presence of the researcher as a trusted colleague to provide support when they were feeling uncomfortable and
unsure of how to present new material. In these cases, the researcher listened sympathetically, but did not try to give advice.

The researcher noted that in each of the more successful schools, a teacher leader was self-appointed. The leader took responsibility for trialling the lessons first and giving hints of how to manage the materials and present the content, to other team members. Leaders also provided advice to assist children who did not appear to understand the area concepts presented in the lessons.

8.4 Conclusion

The area project was successful in providing the participating teachers with access to professional learning opportunities. Teachers articulated their own learning and their children’s learning and applied their new knowledge to teaching area and other aspects of mathematics. Analysis of teacher interviews and student achievement on the pre- and post-tests indicates that teacher disposition and in-school collaboration and support between teachers were the most powerful agents in teacher learning and student outcomes.

The project presented an excellent opportunity for teachers to engage in a shared enterprise: they worked from identical lesson plans, used the same teaching materials, and were working towards agreed and clearly articulated student learning outcomes. The discussions of the lessons were focused and precise, and presented opportunities for teachers to comment not only on student learning, but also on their own learning. For several teachers, the opportunity for extended dialogue about teaching mathematics was a unique and inspiring experience. However, an assessment of the success of the professional learning models must also acknowledge that the participating teachers were unstinting in their commitment to the project and contributed their classroom time and professional skills and judgments.
CHAPTER 9 - CONCLUSIONS AND RECOMMENDATIONS

This study was prompted by the researcher’s concern with the teaching of area within the mathematics program in the early years of school. The researcher’s experience, confirmed by many studies (M. A. Clements & Ellerton, 1995; Ferrer et al., 2001; Hart, 1981a; Outhred & McPhail, 2000), was that children confused the measurement of area and perimeter and also the use of linear and two-dimensional units of measure. The first phase of the study addressed the design and trial of a program of lessons to introduce Year 1 and Year 2 children to the concept of measuring rectangular area with repeated, tessellating informal units. Researchers (D. Clements & Stephan, 2004; Outhred & Mitchelmore, 1992) have suggested that establishing an understanding of the spatial pattern of an array and the use of the array structure to calculate the number of units used form an introduction to understanding the rectangular area formula in later years.

The researcher was an experienced teacher who was a newly-appointed District Mathematics Consultant as the study commenced. She was later promoted to a statewide role in coordinating numeracy programs. In each of these positions, the imperative for delivering practical, accessible, and economical teacher professional learning programs was apparent. The second phase of this study described the researcher’s investigation, design and trial of teacher professional learning strategies which would assist teachers to adopt successful methods of teaching young children to measure area.

The two phases of the study have been addressed separately in this chapter. The research questions and findings of each phase are described, together with the implications of these outcomes for firstly, the design of area teaching programs and secondly, the design and implementation of teacher professional learning strategies.

9.1 The Outcomes of Phase 1 and Implications for the Design of Area Teaching Programs

The research question addressed in phase 1 of this study related to the trial and evaluation of two sequences of four area lessons. The area syllabus lessons were
based on lesson suggestions in the then current syllabus, *Mathematics K-6* (New South Wales Department of Education, 1989), which teachers used to plan mathematics lessons and programs. The lessons commenced with the identification of the attribute of area by drawing lines and shapes and superimposing and comparing areas. Further activities involved the conservation of area, covering shapes with tiles and making shapes on a geoboard.

The *area research lessons* were designed by the researcher to address the findings from researchers that an understanding of the formula $L \times B$ to calculate area is dependent upon a knowledge and use of the array structure of repeated, tessellating, congruent units (Battista et al., 1998; Nunes et al., 1994; Outhred, 1993; Outhred & Mitchelmore, 1992, 2000). The lessons commenced with using lines and shapes to identify area and then progressed to arranging and counting square and triangular tiles to measure or decorate the areas of given shapes. Children were asked to draw the array structure of their square tiles and to experiment with different ways of counting the number of tiles used. The effectiveness of the two sequences of lessons was compared by the administration of an individual task with participating children and the analysis of teacher interviews and the researcher’s field notes completed after every lesson.

### 9.1.1 Phase 1 Research Question and Findings

The first phase of this study aimed to design and trial a sequence of lessons to assist Year 1 and Year 2 students to identify, describe, compare and measure rectangular area using repeated, informal units.

How effective were the area research lessons compared to the area syllabus lessons in developing:

- an understanding of the attribute of area; and
- an understanding of a grid pattern or array of repeated, informal units to measure area.

The analysis of teachers’ and researcher’s comments indicated that both sequences were successful in developing an understanding of the attribute of area. Effective features of the lessons included the sustained focus on area to ensure that all children understood the concepts and the small group, interactive tasks.
The students’ responses to the tiling assessment task and teachers’ observations of classroom activities indicated that the area research lessons were more effective in developing an understanding of a grid pattern to measure rectangular area. However, teachers who had observed both sequences of lessons suggested that a combination of both the area syllabus and the area research lessons would provide a thorough study of area. The revised sequence of lessons, the area refined lessons, were implemented in phase 2 of the study.

9.1.2  Implications for the Design of Area Teaching Programs

The results of the area research lesson trial indicated that whole-class lessons can assist young children to develop an awareness of the attribute of area and an understanding of how to structure and use an array of tessellating informal units to measure rectangular area. The specific teaching strategies which featured in the lessons included the early and sustained attention to the attribute of area and a focus on identifying, discussing, counting and drawing the rows and columns in the spatial structure of repeated tiles. Key materials were the 2 metre length of braid used to make shapes, the 5 cm and 10 cm square tiles with a solid black border (Appendix 14), and the 15 cm square tiles made by the children.

9.1.2.1 Establishing the attribute of area

The introduction of the 2 metre length of braid in lesson 1 was successful in helping children to define the difference between a line and a shape and to explain the attribute of area. This understanding was crucial to further lessons (Nitabach & Lehrer, 1996; Nunes et al., 1993; Primary Mathematics Project Team Curriculum and Development, 1989). The children were immediately engaged with discussing strategies, trialling and explaining in a task which had no one answer (McDonough, 2001; Montgomery & Cheeseman, 2000). As one teacher commented, the children were making, modifying and measuring their own areas, instead of measuring a given area.

The emphasis on understanding the attribute of area was sustained for the lesson series: every lesson commenced with an explanation and discussion of area, to revise and reinforce previous concepts. The teacher’s questions listed in each lesson
provided opportunities for children to use their own words to describe, discuss and compare areas (Grant & Kline, 2003).

9.1.2.2 Focusing on the array structure of repeated, tessellating units

The focus on constructing an array from square or triangular tiles in every lesson assisted children to identity and describe the rows and columns in the arrays (Reynolds & Wheatley, 1996). The activities were designed to present a range of contexts from the exploratory shapes in lesson 1, to the structured patterns in lesson 2, the large class square made from 15 cm squares in lesson 3 and the shapes made from kebab sticks in lesson 4. The children used 5 cm and 10 cm cardboard squares with 0.5 cm black borders and were asked to record their kebab stick shapes and tiles in lesson 4. The borders on the tiles assisted in counting the tiles and identifying the grid lines of the array. In the refined area lessons, the geoboard lesson from the area syllabus sequence was added to give an additional emphasis on identifying and recording the structure of the units (F. Kidman, 1997; G. Kidman, 1999).

9.1.2.3 Integrating area measurement concepts with counting strategies

The lessons emphasised the organisation of units into composite units, by identifying rows and columns in every lesson (Battista, 2003). Children were encouraged to point to and describe composite units and to count by multiples or use a stress count (Outhred & Mitchelmore, 2000, 2004). Teachers observed both the integration of area with their multiplication lessons and also the children’s growing skills in skip counting and stress counting. Teachers also commented that in the last lesson, some children were able to calculate the area of their kebab stick shapes by placing one column of tiles and skip counting by the number in the top row.

9.1.3 Further Development of Lessons

The refined area lessons focused on measuring small areas within the classroom such as book covers and desks. Further lessons should include an exploration of the children’s meso space (Reynolds & Wheatley, 1997) to encourage the development and application of concepts of constructing and co-ordinating units on a larger scale. Activities which require children to measure and compare several large areas such as
a vestibule or handball court will include the need to choose larger and physically-appropriate units (Zacharos, 2006). Children would also need to develop cooperative small group strategies for moving, marking and counting the units.

The area research lessons did not include an application of the inverse relationship between the size of the unit and the number of units used to measure area (D. Clements & Stephan, 2004; Outhred et al., 2003). An understanding of this relationship is important to converting between formal units of measure in the primary years. Outhred et al. found that young children were able to demonstrate this concept with specific tasks in the Count Me Into Measurement project.

9.2 The Outcomes of Phase 2 and Implications for the design of Teacher Professional Learning Programs

The research questions addressed in phase 2 of this study investigated the teachers’ current practices in the planning and teaching of area as well as the effectiveness of three professional learning models. The vehicle for these investigations was the implementation of a sequence of six lessons, the refined area lessons. Seventeen teachers from seven schools in the researcher’s metropolitan district volunteered to participate in one of the three professional learning models offered: high support; medium support; and low support. All teachers attended an introductory meeting held after school, and all were given lesson notes and hands-on materials to implement the lessons. Teachers in the high-support model of consultancy met with the researcher for 15-20 minutes for a school-based meeting each fortnight and were also given 30 minutes release at the conclusion of each lesson to interview a small group of students. Teachers in the medium support model met with the researcher each fortnight, and teachers in the low support group did not have any contact with the researcher during the implementation of the lessons. The models were evaluated by analysing the transcriptions of interviews conducted with individual teachers and the transcriptions of group meetings, and through an analysis of students’ responses to a pre- and post- area assessment task.
9.2.1 Phase 2 Research Questions and Findings

The aim of the phase 2 research was to implement, evaluate and compare three professional learning models designed to assist teachers to change their current practices in the teaching of area. The research questions for this phase were:

- How do teachers currently plan and implement learning experiences in area?
- Which of the three models implemented was the most successful for the professional learning of the teachers who participated in this research?
- What factors were important to the success of the professional learning models?

Teachers reported that “covering” shapes with informal units was the most common activity in area lessons. All of the teachers used the syllabus for planning and 13 stated that they used a text book as well. Fourteen teachers planned and taught a sequence of three or four lessons once each term, for three or four terms each year. When asked what skills and understandings the children needed to learn, 13 teachers focused on covering area with no gaps or overlaps. All 17 teachers advocated practical activities. The teachers gave mixed responses to the question asking what children would learn about area in the next three years.

The most successful professional learning model, measured in terms of teacher learning and student learning, was the medium support model implemented at Hills school. The success of the implementation was heavily reliant on the disposition of the two Hills teachers, their attitudes towards learning to teach mathematics well, their professional collaboration and the leadership role assumed by one of the teachers.

The effective professional learning strategies and factors identified by the teachers and researcher included the lesson notes and materials, teamwork and dialogue, post-lesson interviews with students, the presence of a supportive facilitator, and the emergence of teacher leaders.
9.2.2 Implications for the design of Professional Learning Programs

The extensive literature on successful school change reports that a number of potential issues must be addressed when planning professional learning programs. These include a reminder that real improvement comes from within a school community that is focused on teaching and learning (Stoll, 1998), that a positive school culture is essential for supporting and maintaining change (Fullan, 1991), effective change takes time (Stoll & Myers, 1998) and that the turbulence of change processes needs to be understood and addressed (Hargreaves, 2003).

This study has demonstrated that the implications for initiating and maintaining change in mathematics teaching practices also include the provision of suitable teaching materials, the promotion of collegial planning and discussion and the acknowledgment and nurture of teacher leaders.

9.2.2.1 Providing Teaching Resources to Implement a New Program

This study demonstrated the positive impact of providing teachers with a conceptual overview of teaching content, and guidance in implementing sequences of lessons based on a sustained theme. In the measurement strand of the newer 2002 mathematics syllabus (Board of Studies New South Wales, 2002), emphasis was placed on developing an understanding of the pattern and structure of iterated units used to measure rectangular area. The syllabus document did not include a clear description of the rationale that underpins the learning outcomes including the need to encourage children to identify the links between the calculation of rectangular area and their early use of multiplication facts. Research has indicated that an understanding of how to form and use repeated composite units provides the link to using multiplication to calculate rectangular area (Battista et al., 1998; McPhail, 2003; Outhred & Mitchelmore, 2004; Reynolds & Wheatley, 1996). Teachers who were not aware of the intent of the new learning sequences may still have taught the calculation of area and volume as procedural use of the formulas, in much the same way as they were taught at school (Garet et al., 2001; Loucks-Horsley et al., 1998).

The Teaching measurement books (New South Wales Department of Education and Training, 2003c, 2004) were based on the lesson trials and lesson designs of this
study. The books have provided a popular and practical teaching resource for New South Wales primary teachers. The guidelines in the books strongly encourage teachers to plan blocks of lessons, to observe and question children’s skills and understandings and to focus on establishing linear measurement concepts before area and volume measurements are commenced (Bragg & Outhred, 2000b; Stephan & Clements, 2003). The two *Teaching measurement* books (New South Wales Department of Education and Training, 2003c, 2004) included a conceptual framework which described the early teaching and learning stages for the spatial organisation of length, area and volume with informal units before the introduction of formal measurement units and the investigation of formulae. During the statewide trials of the framework and lesson ideas in the *Count Me Into Measurement* project, teachers commented that the framework provided a sequence for their planning, the lists of skills and knowledge at each stage encouraged them to observe, question and record student understanding, and the expanded lesson notes motivated them to implement the lessons with confidence (Outhred, 2001; Outhred & McPhail, 2000; Outhred et al., 2003).

9.2.2.2 The Role Played by Teacher Leaders

This study highlighted the crucial role played by teacher leaders in leading and supporting the lesson implementation. These teachers were self-appointed and had a great influence on the implementation of the area lessons and the professional development of the teachers. They assisted team members with practical and emotional support by sharing classroom management advice for each lesson, initiating discussion and ensuring that teachers had access to materials. The teacher leaders at Avenue, Mall, Vista and Hills schools also spoke to other teachers in the school and were successful in achieving the implementation of the refined area lessons in a number of classes apart from those formally participating in the project. This outcome was not planned nor suggested by the researcher, but was a strategy chosen by the teacher leaders. However, there was no teacher leader in three schools (Plains, Station and Forest). The participating teachers at these schools did not report collaborative dialogue and group assistance and two of the teachers commented during the final interview that discussions with colleagues may have assisted with the implementation.
The recognition of the role of teacher leader would appear to be a practical strategy in schools that are embarking on curriculum or pedagogical interventions. Research suggests that teacher leaders deserve more acknowledgment for the role they play in contributing to high quality student learning and school-based teacher professional learning (Crowther et al., 2002; Moller & Katzenmeyer, 1996; Urbanski & Nickolaou, 1997). The challenge may be for schools to identify teacher leaders, recognise their potential to mentor and support, and provide access to relevant resources. These resources include additional time allocations to mentor, team teach and plan with colleagues as well as access to professional development which enhances their leadership, interpersonal and pedagogical skills.

9.2.2.3 Promoting collegial sharing and discussion

Analysis of the results of this study indicates that one of the factors influencing teacher learning and student learning was the presence or absence of collegial support and discussion. In the high support and medium support schools, the consultant met with school teams each fortnight. The consultant facilitated, rather than led the discussion. In four of these schools, teachers continued the dialogue at informal meetings throughout the week. At Plains school, teachers did not continue the conversations informally and one commented in the final interview that collegial support would have helped. In four schools, the strategy of setting a time and place to meet and discuss the lessons helped to establish or reinforce the collegial interactions. The consultant’s role at these meetings could be undertaken by an executive staff member or school facilitator. The meetings held with school teams discussed specific lessons within a sequenced program and provided teachers with opportunities to engage with new ideas and materials, encouraged group ownership of the program, and encouraged teachers to question, reflect on and openly discuss their practice (Garet et al., 2001; Hiebert, 1999; Tytler et al., 1999).

Primary teachers who plan, program and teach six Key Learning Areas explain that allocating time for discussions and planning in just one small component of a crowded curriculum is very difficult (Bobis, 2004). Such a program would require negotiation with teams of teachers to ensure teacher commitment to the program, allocation of time, and coordination of teaching programs across grades or stages.
This study has indicated a need for a professionally supportive culture within the school to ensure that teachers have access to support not only from peers, but also from school executive and the principal (Wideen et al., 1996). Several teachers in this study explained to the researcher that they valued the opportunities to talk to her about their lessons. During these conversations, the teachers were not asking for advice but acknowledgement that they felt uncomfortable and professionally challenged with the unfamiliar lessons.

This research has focused on the implementation of a teaching program and the resultant professional learning in a small team in each of the seven project schools. To achieve ongoing change and learning across the school, the initial impetus must be continued within the team and also expanded across year levels. This form of change may require considerable leadership intervention and planning, and needs to become a priority for school executive. Educational researchers have noted that strong leadership is crucial to school improvement, therefore the school principal has a leading role in guiding and supporting teachers in their professional community (Fullan, 1991; Stoll, 1998). In this study, only one of the seven school principals took an interest in the area lessons and the children’s responses to the tasks. It may be argued that greater emphasis on curriculum leadership and knowledge of classroom programs would assist principals to provide knowledgeable support and effective leadership within their school communities.

9.3 Conclusion

This study combined research into students’ learning within a context in which teachers were also learning, by being actively involved in teaching that was based on observing, analysing and reporting on student responses. As Clements et al. (1998) noted, such an approach to curriculum development, classroom teaching and mathematics education research has the potential to make a rich contribution to all three areas.

The study was successful on a personal level for the participating teachers and the researcher. All of the seventeen teachers said they had achieved their initial goals at the end of the implementation of the refined area lessons. The children demonstrated
a growth in understanding of how to use an array structure to measure area with informal units. The researcher took her first steps as an educational researcher and she formed close bonds with many of the teachers.

This study investigated the knowledge and skills which underpin an understanding of the L x B formula for calculating rectangular area. Those factors were then used to plan a teaching program for Year 1 and Year 2 children, focusing on establishing the attribute of area, making, describing and drawing the spatial structure of arrays of repeated informal units to measure areas, and methods of counting to determine the total number of units. The combination of teaching materials and sequence of activities was successful in assisting children to visualise and draw the array of repeated units used to measure rectangular area.

The study also investigated professional learning strategies and was successful in encouraging all of the participating teachers to develop their content knowledge and to modify teacher-centred teaching practices. The key strategies and factors which contributed to this success included ongoing school-based professional dialogue and support, the provision of a teaching program which emphasised students’ conceptual development within a sequence of activities, the role played by teacher leaders within each team, opportunities to develop questioning techniques and the motivation and disposition of the participating teachers.

This study demonstrates the rewarding outcomes of working closely with teachers and students to identify practical solutions to problematic areas. An implication for mathematics education researchers is that research which seeks to solve the difficulties of everyday mathematics teaching and learning will add rich knowledge to the discipline and empower our teachers and children.
LIST OF APPENDICES
Appendix 1: Teacher interview schedule for First Steps Part 1

Teachers’ Knowledge and Practices in Teaching Area

1 Tell me about the Area lessons you have taught this year.

2 What do you think the children need to learn about Area, including skills and understandings?

3 How do you plan your lessons - do you work from the syllabus, or do you have textbooks or activity books, or old programs?

4 What do you think is the best way to teach children about Area?

5 Do you cover skills or knowledge from other sub-strands or Key Learning Areas in your area lessons?

6 How often do you teach Area? Are these single lessons, or a series of lessons?

7 How many years have you been teaching?
   How many years have you taught this grade?
Appendix 2: Refined area lesson 1

Activities
Make lines with a piece of braid, and record.
Make shapes with a piece of braid. Measure the area of the shapes with cardboard tiles, count and record.

Outcomes
M 1.3 Estimates, compares and orders the areas of shapes using informal units
S 1.3 Recognises, explains and uses objects and symbols to produce patterns.
WM 1.5 Recognises what worked and what did not work while answering mathematical questions.

Syllabus References: Area 2, Space 2D 2, Space 2D 4

Objectives: Children will:
• Make lines and shapes with a long piece of braid;
• Identify the area of a shape;
• Cover the area of the shape with cardboard tiles (squares, triangles, “hearts”) and count the tiles;
• Recognise and name circles, squares, triangles, rectangles;
• Additional - Estimate how many children’s bodies will be needed to cover the area of a shape made by joining several braids.

Materials: Length of braid for each group of three students; bags of cardboard tiles (squares, triangles, and one bag of "hearts"); pencil and recording paper.

<table>
<thead>
<tr>
<th>Discussion and activities</th>
<th>Suggested questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>Can you tell us where the area of this shape is?</td>
</tr>
<tr>
<td>Discuss “What is area?”- ask for explanations, give definition as the amount of surface of a given shape.</td>
<td></td>
</tr>
<tr>
<td>Step 1 Groups of two or three</td>
<td>What kinds of lines could you make with this braid?</td>
</tr>
</tbody>
</table>
| Children take a braid and experiment with lines on the floor/desks. Students record the different lines made. Report back for a whole class discussion. | How would you draw your line? What name will you write on the line?  
Tell us about the lines which you have made. |
<table>
<thead>
<tr>
<th>Discussion and activities</th>
<th>Suggested questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 2  Groups of three</strong>&lt;br&gt;Children make a shape with their braid and cover the shape with tiles (squares, triangles or hearts). The tiles should cover the area exactly.&lt;br&gt;&lt;em&gt;Children draw the shape they made and write the number of tiles used. Some students may like to draw the tiles.&lt;/em&gt; Make a different shape if time allows. Report back to the class.</td>
<td><em>How can you make shapes with your braid?</em> (Discuss how to make a shape by tying the ends of the braid.)&lt;br&gt;<strong>How could you measure the area of the shape you have made?</strong> (Remind students about &quot;no gaps, no overlaps&quot;).&lt;br&gt;What shape did you make, what did you use to cover, and did the tiles cover exactly?&lt;br&gt;How did you make the tiles cover exactly?&lt;br&gt;How were the tiles arranged? (Describe the array)&lt;br&gt;Were any of the tiles folded?&lt;br&gt;What kind of shape have we got?&lt;br&gt;What could we cover it with?&lt;br&gt;Can you estimate how many children we would need to cover this area if they were lying down?&lt;br&gt;Can they fit better if we put them another way?&lt;br&gt;How many children did we need to cover the area?</td>
</tr>
<tr>
<td><strong>Step 3  Whole class</strong>&lt;br&gt;&lt;em&gt;Whole class sit in a circle, several students make a shape from five or six joined braids on the floor in the middle of the circle.&lt;/em&gt; Children estimate how many children would be needed to cover the area of the shape by lying on the floor.&lt;br&gt;Children direct volunteers to lie in the shape (experimentation with orientation may be necessary)&lt;br&gt;Count the bodies.</td>
<td></td>
</tr>
</tbody>
</table>
Appendix 3: Refined area lesson 2

Activities
Compare the areas of pairs of shapes by superimposing.
Cut and paste two shapes to find which has the larger area.

Outcomes
M 1.3 Estimates compares and orders the areas of shapes using informal units.
S 1.2 Recognises names and makes simple two-dimensional shapes and describes their properties using everyday language by observing similarities and differences.
WM 1.5 Recognises what worked and what did not work while answering mathematical questions.

Syllabus References: Area 3, Space 2D 2, Space 2D 4.

Objectives: Children will:
• Compare two shapes by: handling, superimposing, cutting and pasting
• Recognise and name simple 2D shapes
• Order two-dimensional shapes according to size.

Materials: Pairs of shapes in plastic bags for students working in pairs; stencil of two shapes to be cut out, superimposed, cut and pasted by each student; scissors and paste

<table>
<thead>
<tr>
<th>Discussion and activities</th>
<th>Suggested questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Introduction</strong></td>
<td></td>
</tr>
<tr>
<td>Whole class discussion about area and explanations from last week.</td>
<td>What is area?</td>
</tr>
<tr>
<td>Discuss how the area of two shapes can be compared simply by superimposing.</td>
<td>How could you tell which area was larger?</td>
</tr>
<tr>
<td>How is this form of measurement called?</td>
<td>When would this method be unsuitable?</td>
</tr>
<tr>
<td>What would you do if you couldn’t easily fit one shape on top of the other?</td>
<td></td>
</tr>
<tr>
<td><strong>Step 1 Pairs</strong></td>
<td></td>
</tr>
<tr>
<td>Each pair of students take a plastic bag with four shapes inside.</td>
<td>How can we work out which shape has the larger area?</td>
</tr>
<tr>
<td>Compare the pairs of shapes, which have the same name.</td>
<td>How could you record your results?</td>
</tr>
<tr>
<td>Record results by using headings &quot;Larger area&quot; and &quot;Smaller area&quot;, or similar. Students may have time to compare all pairs of shapes. Report back for a whole class discussion.</td>
<td>Tell us how you found out which shape had the larger area.</td>
</tr>
<tr>
<td><strong>Discussion and activities</strong></td>
<td><strong>Suggested questions</strong></td>
</tr>
<tr>
<td>--------------------------------</td>
<td>------------------------</td>
</tr>
</tbody>
</table>
| **Step 2 Individuals**  
Cut out the triangle and the circle on the stencil and compare areas by superimposing, cutting and pasting (The triangle will not fit neatly over the circle, so at least two corners will need to be cut off and pasted onto the circle). | *What could you do if the shapes don't fit exactly, one on top of the other?* |
| **Step 3 Conclusion**  
Individual students volunteer to stand with eyes closed, in front of the class, and handle two shapes to find which one has the larger area. | *What can you feel?*  
*How will you work this out?* |
Appendix 4: Refined area lesson 3

Activities
Paste coloured squares and triangles to cover the area of outlines, in a pattern.

Outcomes
M 1.3 Estimates compares and orders the areas of shapes using informal units.
S 1.3 Recognises, explains and uses objects and symbols to produce patterns.
WM 1.5 Recognises what worked and what did not work while answering mathematical questions.
N 1.4(b) Models numbers and number relationships in a variety of ways while answering mathematical questions.

Syllabus References: Area 2, Space 2D 2, Addition 3 and 5, Multiplication 2.

Objectives: Children will:
• Cover shapes by using a given paper tiles, ensuring the entire surface is covered.
• Count the number of units used when covering a shape.
• Make arrays with 12 square tiles and report on the array patterns.

Materials: Duplicated outline of a 15cm square and an equilateral triangle for each student; square and triangular paper tiles, in bright colours to be pasted onto the outlines; paste or glue sticks; bag of 15-20 square tiles for each student; 15cm squares, with a black line around the perimeter.

<table>
<thead>
<tr>
<th>Discussion and activities</th>
<th>Suggested questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Introduction</strong></td>
<td>What can you tell me about Area?</td>
</tr>
<tr>
<td>Whole class discussion about definition and explanations from previous lessons.</td>
<td>What shapes could we use to cover the area of this square or triangle? (Duplicated shapes)</td>
</tr>
<tr>
<td><strong>Step 1 Individual working</strong></td>
<td>Can you make your covering go right to the edges?</td>
</tr>
<tr>
<td>Students take the duplicated square and triangle and cover with own choice of coloured tiles. Encourage students to place tiles within outlines and experiment with patterns before pasting down.</td>
<td>What will happen if you don’t?</td>
</tr>
<tr>
<td><strong>Step 3 Whole class</strong></td>
<td>What do you remember about overlapping?</td>
</tr>
<tr>
<td>Compare the patterns. Discuss how the square and triangle were covered, encouraging children to describe the pattern of the tiles. Model the use of terminology: rows, columns, grid. Store the triangles and squares in preparation for the next lesson.</td>
<td>What kinds of patterns did you make with the small squares?</td>
</tr>
<tr>
<td>Where was a good place to start? Why?</td>
<td>What kinds of patterns did you find in your triangles?</td>
</tr>
<tr>
<td>How many tiles would I need to cover the next size square and triangle?</td>
<td></td>
</tr>
</tbody>
</table>
Appendix 5: Refined area lesson 4

Activities
Cut out the pasted squares and triangles and use to make a larger square in a small
group, then the whole class.

Outcomes
M 1.3 Estimates compares and orders the areas of shapes using informal units.
S 1.3 Recognises, explains and uses objects and symbols to produce patterns.
WM 1.5 Recognises what worked and what did not work while answering
mathematical questions.
N 1.4(b) Models numbers and number relationships in a variety of ways while
answering mathematical questions.

Syllabus References: Area 4, Space 2D 2, 4 & 7, Numeration 10, Multiplication 2.

Objectives: Children will:
• Cover a shape with informal units and count the number used.
• State the properties of squares and triangles.
• Model and describe equal groups or rows and find their totals.
• Make groups of 20 to 99 objects.

Materials: Decorated squares and triangles from previous lesson; scissors for each
student; pack of 15 square tiles (each 15cm square) for each pair of students (10
packs in the school kit); pack of 30 triangle tiles for groups of 4-6 students. (5 or 6
packs in the school kit).

<table>
<thead>
<tr>
<th>Discussion and activities</th>
<th>Suggested questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>What did we do last time?</td>
</tr>
<tr>
<td>Revise instructions and activity from last week.</td>
<td>What have we been measuring with tiles?</td>
</tr>
<tr>
<td>Step 2 Groups of three</td>
<td>Section 1.01 How do you know it is a square?</td>
</tr>
<tr>
<td>Children work in small groups to cut out their own decorated squares and triangles.</td>
<td>Where can you put more tiles to make it bigger?</td>
</tr>
<tr>
<td>Groups of children make the biggest square possible, using the 15 tiles in the pack and their own squares. (Some students may be confused, when they find they have too many tiles for an exact square)</td>
<td>How many more will you need to make it bigger?</td>
</tr>
<tr>
<td>Step 3 Whole class- Make a class square</td>
<td>How many squares have you used, and how are they arranged?</td>
</tr>
<tr>
<td>Children return to the mat, bringing their squares and sit in a large circle.</td>
<td>What’s the smallest square we can start with?</td>
</tr>
<tr>
<td></td>
<td>What will be next?</td>
</tr>
<tr>
<td>Discussion and activities</td>
<td>Suggested questions</td>
</tr>
<tr>
<td>---------------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>Class makes the biggest square possible from the collection of squares (decorated squares and also from the packs). Start with a 2 x 2 square. Count the rows of tiles in both directions. Increase the size of the square by adding tiles along two sides, increasing to 16, 25, etc. Each time, count the total number of tiles by using a skip or stress count, and pointing to each row as it is counted. Stop when children become tired of counting.</td>
<td>Can you estimate how many squares we will use to cover the area of a square in the middle of our circle?</td>
</tr>
<tr>
<td><strong>Optional Step 4 Whole class</strong> Make the largest triangle possible from the decorated triangles and packs of triangles.</td>
<td></td>
</tr>
<tr>
<td><strong>Optional Step 5 or follow-up activity</strong> Groups of 4-6 children make their triangles into the largest triangle possible.</td>
<td>How do you know it is a triangle? Where can you put more tiles to make it bigger? How many more will you need on the next row to make it bigger?</td>
</tr>
</tbody>
</table>
Appendix 6: Refined area lesson 5

Activities
Use four sticks to make a shape and measure the area of the shape with square tiles. Make other areas that can be measured with the square tiles. Predict each area, then measure. Record the shapes.

Outcomes
M 1.3 Estimates compares and orders the areas of shapes using informal units.
WM 1.5 Recognises what worked and what did not work while answering mathematical questions.
N 1.1 Approximates, counts, compares, orders and represents whole numbers and groups of objects up to 100.
N 1.4(b) Models numbers and number relationships in a variety of ways while answering mathematical questions.

Syllabus References: Area 3 & 4, Space 2D 2 & 4, Addition 3 and 5, Multiplication 2, Numeration 10.

Objectives: Children will:
• Make a square using four sticks, and cover the area with cardboard tiles.
• Make shapes of own choice and cover the area with cardboard tiles.
• Report to the class on shapes made and the pattern of covering tiles.
• Optional Make arrays with 12 squares and report on the array pattern.

Materials: Kit containing 12 sticks (length 15 cm) and approximately 36 squares (side 5 cm) for each pair of children (15 stick kits in the school kit); pencils and paper.

<table>
<thead>
<tr>
<th>Discussion and activities</th>
<th>Suggested questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Introduction</strong></td>
<td><strong>What would this shape look like?</strong>&lt;br&gt;<strong>Can you draw it for us?</strong>&lt;br&gt;(Children should describe a square and a rhombus)</td>
</tr>
<tr>
<td>Whole class discussion: What shapes could you make with four sticks of equal length? (Teacher holds up 4 sticks from a stick kit)</td>
<td><strong>Can you and your partner estimate how many tiles you will need to cover your shape?</strong>&lt;br&gt;<strong>How did you estimate?</strong>&lt;br&gt;<strong>How close was your estimate?</strong>&lt;br&gt;<strong>Did you have any trouble filling the corners? Why not?</strong></td>
</tr>
<tr>
<td><strong>Step 1, Groups of 2</strong></td>
<td></td>
</tr>
<tr>
<td>Pairs of children are given a stick kit, containing 12 kebab sticks and 36 small square tiles, with black lines on the perimeter. Children make a shape with square corners, using 4 sticks. Children estimate how many tiles would be needed to cover the area of the shape. Children cover with tiles, and record the array.</td>
<td></td>
</tr>
</tbody>
</table>
### Discussion and activities

#### Step 2 Pairs of children
Children work with partner to make their own shapes, using as many sticks as they wish.

Each shape must be covered by the tiles, and the total counted. Record each shape.

Children will find that triangles, pentagons and hexagons cannot be measured with square tiles.

Some pairs may find that their shapes are too large for the number of tiles they have. Encourage children to think about how to count the missing tiles.

#### Step 3 Whole class
Report back to the class:
- The name of the shape made
- The number of tiles needed for covering
- How the tiles were counted.

#### Optional Step 4 Pairs
Children use 12 tiles from their kits to make an array of 12 on the desk.
Take another 12 tiles to make a different array.

### Suggested questions

- How many tiles do you think you will need? Why?
- How will you count the tiles? Is there an easier way?
- How many rows do you have, and how many in each row?
- Can you use a stress count or a skip count to check the total?
- Can you think of a different array?

- Tell us about your shape. How would you describe your shape?
- Did anyone find some quick ways of counting the rows or columns?
- Did anyone work out what to do when you ran out of tiles?
Appendix 7: Refined area lesson 6

Activities
Find the area of shapes that have been drawn onto a grid. Order the shapes from largest to smallest.
Draw own shapes onto the grid: one shape larger than others, one shape smaller than the others.

Outcomes
M 1.3 Estimates compares and orders the areas of shapes using informal units.
S 1.2 Recognises names and makes simple two-dimensional shapes and describes their properties using everyday language by observing similarities and differences.
WM 1.3 Explains simple mathematical situations using everyday language, actions, materials and drawing.
WM 1.4 Supports answers to mathematical questions by explaining or demonstrating how the answer was obtained.

Syllabus References: Area 4, Space 2D 2, Numeration 8.

Objectives: Children will:
• Recognise and name simple 2D shapes (square, triangle, rectangle,)
• Cover a shape with informal units and count the number used.

Materials: Worksheet for each student (1 cm grid with grid missing within marked shapes). Children find the area of each shape, and record in the margin, from smallest to largest.

<table>
<thead>
<tr>
<th>Discussion and activities</th>
<th>Suggested questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Introduction</strong></td>
<td></td>
</tr>
<tr>
<td>Show the worksheet to the children and discuss how to find the area of each shape. Explain the second part of the activity: children draw one shape which has a smaller area than all of those shown, and one which has a larger area.</td>
<td>What do you think you have to find? What will you use to help you? How will you use the clues? Could you make it easier if you get stuck? (Perhaps draw the grid lines) What kind of counting could you use?</td>
</tr>
<tr>
<td><strong>Step 1 Individual</strong></td>
<td></td>
</tr>
<tr>
<td>Children complete the activity.</td>
<td></td>
</tr>
<tr>
<td><strong>Step 2 Whole class</strong></td>
<td></td>
</tr>
<tr>
<td>Report back to the class. Display and discuss some of the shapes which have an interesting outline. Children may have used half-squares, or irregular shapes. Ask children to explain how they calculated the area of each shape.</td>
<td>Can you tell us how you found the area of this shape? Is there another way to find the area of this shape?</td>
</tr>
</tbody>
</table>
Appendix 8: Suggested student interview, lesson 1

Suggested interview questions for high support teachers to implement with individual children or a small group of children during the release time following refined area lesson 1

(Have braid and tiles on hand for discussion)

Can you tell me what we did in the mathematics lesson today?
(This question is to remind children what they did. Ask for quick outline of the activities)

How would I know whether I had made a line or a shape?

How did you work out what shape to make with the braid?

Do you remember how you covered the area with tiles, and how many there were?

Was there an easy way of counting the tiles you used?

How could I cover the area of this table top with tiles?)
Appendix 9: Suggested student interview, lesson 2
Suggested interview questions for high support teachers to implement with individual children or a small group of children during the release time following refined area lesson 2
(Have superimposing kits and a sample cut-and-paste task ready for discussion)

Let's start by talking about what we did in the mathematics lesson today. (Quick, revision question)

How can I work out which of these two shapes has the larger area?
(Ask children to explain in words, without touching the shapes)

These two shapes look about the same size. Can you show me if one has a larger area? How can you tell the area is larger?
(Children may talk about bits "left over")

Can you feel if one of the shapes is bigger, with your eyes closed?
(Ask student to close eyes, then put two shapes into their hands)

Why have we been putting the shapes one on top of the other?

Is there another way to work out which shape has the larger area?
Appendix 10: Suggested student interview, lesson 3

Suggested interview questions for high support teachers to implement with individual children or a small group of children during the release time following refined area lesson 3
(Have a blank stencil, some coloured tiles, and the children’s completed patterns ready for discussion)

First of all, let's talk about the things we did in mathematics today. (Quick recall)

Why was it easy to guess that we would need 9 tiles to cover the area of the square?

Is there a clever way to count the tiles?

Did you find some patterns inside your square and triangle? Tell us about them.

How did you make sure that you covered the area of the shapes?

If there was one more row of tiles on the square, how many would there be altogether? If there were two more rows, how many altogether? (Extend to three or four rows for Year 2 children)
Appendix 11: Suggested student interview, lesson 4

Suggested interview questions for high support teachers to implement with individual children or a small group of children during the release time following refined area lesson 4
(Have some square and triangular tiles ready to discuss)

First of all, let's talk about the things we did in mathematics today. (quick recall)

When your group made a square, why couldn't you use all of the squares which you had? Did you make any other shapes, while you were trying to make the square?

How many tiles were in your square and what did the array (pattern) look like?

Did your group find a way to make the triangle? How did you work this out?

Why did we count by 2's, then 3's, then 4's, then 5's?

When I asked you to make a large triangle with your group, how were you able to make it larger and larger?
Appendix 12: Suggested student interview, lesson 5

Suggested interview questions for high support teachers to implement with individual children or a small group of children during the release time following refined area lesson 5
(Have a stick kit ready for discussion)

We did lots of things today in mathematics. Can you remember what they were?

If we made a shape using 6 of these sticks, how can we work out how many squares will cover that shape?

If we start with 8 sticks, what will happen to the number of small squares that we use?

Did you make any shapes that were impossible to measure the area with these square tiles? Why was it impossible?

Tell us about the different kinds of counting that you heard today.

What will happen if you start with an odd number of sticks? (3 or 5 or 9 sticks)
Appendix 13: Suggested student interview, lesson 6

Suggested interview questions for high support teachers to implement with individual children or a small group of children during the release time following refined area lesson 5
(Have a worksheet ready, and children bring own worksheets)

What did we do in mathematics today?

Finding the area of these shapes must have been fairly tricky. Can you explain to me how you did it?

Which shape was the easiest to count? Why? Which was the hardest? Why?

When you drew your large area, how did you work out how many squares to have in each row?

What kind of shape could you have drawn, and did you have to use whole squares?

Why do you think we did this activity?
Appendix 14: Cutting template for 5 cm tiles
REFERENCES


Murphy, C. (2005). The role of subject knowledge in primary trainee teachers' approaches to teaching in the topic of area. In D. Hewitt & A. Noyes (Eds.), *Proceedings of the Sixth British Congress of Mathematics Education held at the University of Warwick* (pp. 113-119).


