A THEORETICAL AND EMPIRICAL ANALYSIS OF
THE WAGNER HYPOTHESIS OF PUBLIC
EXPENDITURE GROWTH

BY

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DEDICATION

This thesis is dedicated to the memory of my father. I also dedicate this thesis to my mother, my sisters, and my brothers. They mean everything to me. Finally, I dedicate this thesis with love to Cathleen Finn, Medeni Finn, Michaela Finn, Mark Gray, R.J. Gray, and George Gray, who all make me feel that Australia is my second home.
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STATEMENT OF AUTHENTICATION

The work presented in this thesis is, to the best of my knowledge and belief, original except as acknowledged in the text. I hereby declare that I have not submitted this material, either in full or in part, for a degree at this or any other institution.

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ABSTRACT

The debate over the role and the size of state activity in public economics remains controversial despite more than a century of active research. As a result, several explanations have been suggested to explain the expansion of the size of the public economy and a large amount of literature has been produced. In the late 19th century, the German economist Adolph Wagner was perhaps the first to propose a direct economic explanation of the expansion of the size of the public economy. Wagner predicted that economic development would be accompanied by a relative growth in the public sector. Empirical testing of the Wagner Hypothesis continues to produce mixed results and no consensus has been reached about the explanatory power of the Wagner Hypothesis.

This thesis argues that the way that the Wagner Hypothesis has been interpreted in the existing literature has been incomplete both on theoretical and empirical grounds. This thesis provides a comprehensive framework of the Wagner Hypothesis. The framework suggests that with the development process represented mainly by per capita income, the fiscal state, represented by the share of government expenditure in national income, will increase at a higher rate than that of the increase in per capita income until it reaches a certain limit. This limit is, however, not pre-determined, but is a function of the balance of forces driving government involvement in a market economy. To capture this process, the thesis employs two sigmoid curves, the logistic and the Gompertz equations. The study uses a cross-section data set of 88 countries during the period from 1990 to 1997.
The current results tend to confirm a general convergence of state expenditure share in GDP relative to economic development. That is, the economic development process explains some of the observed variation in state expenditure growth. It is suggested that the sigmoid functions perform quiet well in describing the observed path of government growth in the economy across countries. As the model performs well, and the countries in the sample seem to follow a non-linear process, then the results can be interpreted as consistent with the suggested framework of the WH. This, in turn, supports the use of a non-linear approach to model state government growth over relatively short periods.

Finally, the data shows that the independent variable is an important determinant of the growth of government share in income. This result is generally consistent with the WH and lends support to the logistic and the Gompertz processes of government growth. Following the discussion above, the principal conclusion is that the rise of the share of government expenditure in income in the cross section sample of 88 countries can be partly explained by the changes in the levels of real GDP per capita for those countries. The thesis concludes with a discussion of some of the implications of these results and suggestions for further research.
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CHAPTER ONE

1 Chapter 1: Introduction and Overview

1.1 Background

One of the most enduring topics in public economics literature concerns the debate over the role and the size of state activity. It is notable that the size of state activity relative to the economy has expanded in most industrial economies over the twentieth century. This development has attracted the attention of both economists and political scientists. As a result, several explanations have been suggested to explain the expansion of the size of the public economy and a large amount of literature has been produced. Most of the analyses have dealt with the formulation and empirical testing of the expansion over time and across countries. The explanations have focused on different aspects of state activity including economic, fiscal, political, institutional, and international.

Within the economic and fiscal aspects of public expenditure expansion, economic research has taken two main lines in analysing the expansion of the public economy. One line of research has attempted to determine the level of government expenditure at which the rate of economic growth is optimized, a level that is then referred to as the “optimal” size of government. The second line of research investigates the forces that drive the expansion of the size of the public economy. The current thesis is interested in the second line of research. In the late 19th century, the German economist Adolph Wagner was perhaps the first to propose a direct economic explanation of the expansion of the size of the public economy. An important theme here is that, while most of the extant nineteenth century literature was concerned
with the appropriate role of government, Wagner directed his attention to the size of government by proposing an hypothesis which predicted that economic development would be accompanied by a relative growth in the public sector. In particular, the ‘Wagner Hypothesis’ (hereafter the WH), which has also been referred to as the ‘Wagner Law’ (hereafter the WL), suggested that during industrialization the share of government activities in the economy would increase at a rate greater than that of national income.

As with many hypotheses that are proposed in general terms, several interpretations of the WH have been proposed. The current thesis addressed 7 of these interpretations, Peacock and Wiseman (1961), Gupta (1967), Goffman (1968) and Goffman and Mahar (1971), Pryor (1968), Musgrave (1969), Man (1980), and Florio and Caulatti (2003).

The testing of the different interpretations of the WH has been quite extensive and, while many studies have found support for the general tendency of state expenditure to grow with income, the hypothesis has not been without criticism. Peacock and Wiseman suggested that state activities may increase, but not for the reason or in the way that Wagner hypothesized. Then they developed their own hypothesis, the ‘Displacment Effect’. There have since been several other theories offered for observed increases in the state expansion in the economy such as: Fiscal Drag Aaron (1976), Degree of Openness Hypothesis Cameron (1978), and Public Choice Theory Buchanan and Tullock (1962). While these rival explanations have emerged as a challenge to the universality of Wagnerian type explanations, the Wagner Law
remains a viable and, perhaps still the most widely tested explanation of the relative growth in state activity.

Interest in the subject declined in the late 1970s and early 1980s after an initial translation into English by Cooke (1958). However, the developments in time-series econometric techniques and changing patterns of public expenditure growth in the late-twentieth century, combined with a new translation into English by Biehl (1998), have led to a revival of research interest in this field and the development of more techniques for testing. Particularly, Henrekson (1993) was critical of the prevailing empirical studies in modern time-series econometrics. He noted that many tests of the WH using time-series data are spurious since the time-series involved are likely to be non-stationary. Thus, he suggested that the results of these tests are biased toward finding high “t” ratios and do not confirm any economic relationship between the economic variables.

The WH has attracted a great deal of interest in the public economics literature and has been empirically tested for different economies. The empirical testing of the WH has commonly used two broad types of analysis: time-series and pooled cross-section time-series. In the first type, scholars have tested the WH for a single country over a long time span while in the second type, scholars have tested the WH for groups of countries over relatively short time periods. It appears that most of the existing studies have used time-series analysis. Within time-series analysis these studies have tested the WH on developed or industrial countries and, to a lesser extent, on developing countries. Two candidate reasons for such a choice might be the absence
of accurate and comprehensive data on developing countries, or the general belief that the WH does not apply to the case of developing countries.

On the other hand, pooled cross-section studies of the WH, where considered, in general appear to have reported relatively more conflicting results than time-series studies. Some of these studies concluded that there exists a significant positive relationship between public sector growth and economic development, as Wagner hypothesized, and others have not found support for the relationship.

This thesis argues that the way that the WH has been interpreted in the existing literature has been incomplete both on theoretical and empirical grounds. The thesis provides a comprehensive framework for the WH. This framework suggests that with the development process represented mainly by per capita income, the fiscal state, represented by the share of government expenditure in national income, will increase at a higher rate than that of the increase in per capita income until it reaches a certain limit. This limit is, however, not pre-determined, but is a function of the balance of forces driving government involvement in a market economy. To capture this process, and without misspecifying Wagner’s relationship, this thesis employed two sigmoid curves, the logistic and the Gompertz equations. It is to be expected that the sigmoid curves will be consistent with observation and that they will give a better fit to the observed data. Looking at public expenditure through this perspective gives a new insight of the patterns of public expenditure growth in the twentieth century.

It is notable that the WH has generated much debate between scholars, which makes it an interesting case for study, and further investigation. This research makes several
contributions to the literature of the WH. It provides a consolidation of the WH based on a comprehensive look at Wagner’s work. The research investigates the key elements of differences and similarities in the previous interpretations of the WH and discusses how these interpretations have contributed to the WH. Moreover, this thesis will re-establish the regulatory aspect of state activities in the WH even though it will not be able to measure it for empirical research. Furthermore, the thesis aims also to establish the concept of an upper limit to the proportion of government expenditure in a market economy. This is based on the logic that government share cannot grow forever in a market economy and there must be some limit for government growth that will be set inside the model resulting in the modern mixed market capitalist economy. Finally, this thesis contributes to the literature of the WH by establishing several summary tables that describe the existing empirical studies of the WH, which can be useful in future research.

1.2 Research Questions and Justification for the Research

Unlike the existing studies of the WH, a comprehensive review of the literature on the WH is established in this thesis and a number of research questions have been raised:

1. What are the appropriate measures and variables to be used in the context of the WH?
2. What are the key elements of differences and similarities in the existing interpretations of the WH?
3. How do the existing studies contribute to the literature of the WH?
4. What are the major reasons that contributed to the asymmetric results on the WH in the existing empirical literature?
5. Do the existing linear models, either in variables or parameters, explain the stages of development in government expenditure in the context of Wagner’s relationship?

6. How does one determine the upper limit of the share of government expenditure in aggregate income?

7. Are non-linear forms more appropriate to model the WH and the best way to work within sensible boundaries that Wagner envisaged in a market economy more than century ago?

8. Is it possible to establish a recasting of the WH in a way that avoids the shortcomings, if any, in the existing studies?

These research questions show the gaps in the existing literature on the WH, and this thesis attempts to fill these gaps.

One might ask why is this topic so important? The fact is that the growth of government, and of its involvement in the lives of people, has become more than just the concern of political economists. It is now the concern of ordinary citizens as well as the focus of an increasing number of political movements. Policy makers are concerned with government growth, but they have little ability to control its expansion. Therefore, measurement and analysis of the growth of government in a single country over long time periods, and across-countries over short periods of time, seems to be important. This study presents some figures that show the upward historical growth of government expenditure through the period from 1950 to 1997 in individual and groups of countries (Chapter 2, section 2.2).
Further justification for conducting this research is that, despite the considerable amount of literature on the WH, most of the existing studies have not recognized the limits of government expansion relative to the economy in the WH. Also, there has been no attempt at producing a comprehensive taxonomy of the existing research methodologies. The methods used in existing studies of the WH have tended to miss-simplify Wagner’s relationship by employing linear models either in variables, or at least in parameters, which could result in unrealistic parameter estimates and range over the measure of government activity. All of these research problems form a justification for conducting this research.

1.3 Thesis Outline

This thesis is presented in 7 chapters. This chapter lays the foundations for the thesis. It introduces the research problem and research questions. Chapter 2 presents the theoretical framework and relevant previous work on the WH. It begins by presenting a general introduction on the topic. The chapter then traces the WH using translations of Wagner’s original writings. The translations shed light on the forces of state expansion, limits to state expansion in market economies, and the measurement issue of state activity in the WH. The chapter also presents the development of the different interpretations of the WH through the existing literature. Seven different interpretations will be discussed individually and their contribution to the WH will be acknowledged. Finally, this chapter will lay down a recasting of the WH that will be adopted and tested in this thesis.

Chapter 3 provides an overview of the variety of the existing methodologies followed in testing the WH. It begins with a brief introduction on the existing
empirical tests and testing procedures. Then the chapter distinguishes between time-series and cross-section analyses of the WH. The existing analysis of the WH has differed in a number of key elements while testing for the WH. These elements are mainly: type of economy; time span; measures of government size; measurement of the explanatory variables in Wagner’s relationship; current and constant price data; type of models of the WH; and test and estimation methods. All of these issues will be analysed in terms of the existing empirical literature and supported with tables and indices. The chapter then traces the non-linear attempts to model the WH in the existing literature and establishes the rationale for using pooled cross-section time-series analysis for testing the WH.

Chapter 4 presents the empirical model that will be employed in the current thesis to test the WH. It begins by presenting a general introduction to the empirical modeling of the Wagner relationship and reviews the test procedures that have been followed in existing studies. The chapter then presents the linear models of the WH and provides detailed discussion of the variables used to test the WH drawing on chapters 2 and 3. This section also discusses the identification problem accompanying linear models of the WH. Based on the theoretical framework of the WH established in chapter 2, this chapter employs an empirical model captured by two different sigmoid curves, one symmetric and the other asymmetric, to examine the effect of growing levels of real GDP per capita on the share of government expenditure in GDP. Both the logistic and the Gompertz equations will be discussed in detail and their historical application will be presented. Finally, the chapter presents the stochastic model and the estimation method to be used in the thesis.
Chapter 5 is devoted to the source, transformation and description of the sample data used in this study. It begins with an introduction on the importance of data selection in economic research generally, and in cross country comparison studies specifically. Then the chapter discusses the source of the base data and data definitions. The data employed in this study is derived from two sources: the International Financial Statistics\(^1\); and the Penn world tables Heston, et al. (2002)\(^2\). A pooled data set is constructed over an eight year period from 1990 to 1997 for 88 countries. This data comprises different types of countries which reflect a great diversity in aggregate income levels. This chapter describes carefully how the raw data were transformed into suitable variables for the model established in chapter 4. The chapter also gives the usual descriptive statistics on the selected variables so that readers may acquaint themselves with the scale and variation in the data used.

Chapter 6 presents the analysis of the results obtained and discusses the significance of these results. The analysis covers the results obtained from estimating the four and three parameter logistic and Gompertz functions. The chapter also gives some robustness checks to confirm the findings. The chapter begins by introducing the major results achieved in this study. Then the chapter presents the empirical findings of testing the four-parameter logistic and Gompertz functions. These results will be analysed and discussed in depth and a critique of these results will be presented. The following section analyses and discusses the findings of testing the three-parameter logistics and Gompertz functions. Specifically, these parameter estimates will be presented along with the estimated marginal effects of the explanatory variable on the dependent variable. Further, the stability of the estimated parameters will be

\(^1\) Available online [http://ifs.apdl.net/imf/ifsbrowser.aspx/](http://ifs.apdl.net/imf/ifsbrowser.aspx/)
\(^2\) Available online [http://pwt.econ.upenn.edu/](http://pwt.econ.upenn.edu/)
examined. The final section of the results chapter provides diagnostic tests for heteroskedasticity.

The thesis ends with a concluding chapter. This chapter gives an overview of the main results. It also outlines potential avenues for research in this area, stemming from the outcomes of the current study.

1.4 Expected Outcomes

It is expected that the WH will hold over the data set. There is a general belief that the WH has been mainly deployed to attempt to explain public expenditure growth in the developed countries. Therefore, less attention has been given to the cases of developing countries in the existing literature. In part, this may be due to the problems of data availability and accuracy, but is also partly due to circumstances specific to such countries. With limited resources, the distributional objectives of public spending in developing countries are varied, and there is often a role for government to provide certain types of physical and human infrastructure that would be otherwise underprovided. This would include: the provision of basic levels of education, preventive health care services, and basic infrastructure services such as clean water and sanitation. Much of the role of government in such countries can be viewed as establishing infrastructure in its broadest sense: educational, technological, financial, physical, environmental, and social. To this can be added a well-maintained and efficient infrastructure of airports, roads and ports, electricity, telecommunications, water, waste disposal, and other similar facilities.
One of the main factors that can place pressure on government spending is demographic change, especially the growth of population, since a rapidly growing population will increase the pressure on hospitals, schools, roads, public housing, sewers and water supply, and so forth. Moreover, rapid increases in population are frequently associated with movements from rural to urban areas, as standards of public services tend to be higher in urban areas. Other demographic features, like very young or elderly populations, may affect the fiscal development of a country by increasing government spending, decreasing the tax revenues, and creating additional burdens on the social security budget (Hondroyiannis and Papapetrou, 2000). The emergence of strong world opinions and strong local opinions can also put pressure on government to spend, since it may produce a worldwide demand for improvements in living standards and social services of all kinds.

“Expenditures may differ in their outcome in the long run since some expenditures are expected to yield income in the longer run, like education and public works, and others will not yield any additional income in the long run like defence”(Prest, 1985, pp.7-8).

A rationale for studying public expenditure growth is that empirical test outcomes are less likely to contradict the WH for industrialised countries compared to the developing countries. This seems to indicate that economic development, measured by indicators such as GDP per capita, might easily be linked with public expenditures in the industrialised countries. However, for developing countries, the relationship might be more complex, reflecting the complexity of development itself.
However, this distinction may only be important in terms of the methods and models currently used to test the WH. If Wagner truly has a “law”, then that law must apply across all countries. The suggested non-linear models combined with pooled short time-series in this thesis should be able to explain the behavior of public expenditure growth across countries, and the expected outcome is a valid test of Wagner, irrespective of empirical results in terms of rejection and non-rejection of the WH.
2 Chapter 2: Theoretical Literature on the Wagner Hypothesis

2.1 Introduction

As noted in the opening of chapter 1, one of the most enduring topics in public economics concerns the debate over the role and the size of government activity in the economy. Over the twentieth century, both the role and the size of government activity in the economy have expanded in most industrial countries. Economic research has taken two main approaches to this development. One has been to take the size of state activity as exogenous to the economic development process, and to ask what effect state activity has on economic growth. This approach has often been termed the ‘Keynesian approach’. The line of research following this approach has studied the level of government activity at which the rate of economic growth is optimized, and this level is referred to as the “optimal size of government”. This approach has been adopted and tested extensively in public economics literature including, for example, Yavas (1998) who observed that the size and type of expansion of government activity in an economy differs according to the stage of development. He observed that in underdeveloped countries a significant portion of government activity is directed to building economic infrastructure and this type of government activity will have a stimulating effect on private sector production and, consequently, will stimulate the growth of the economy. In contrast to underdeveloped countries, Yavas’s study suggested that developed countries already have most of their infrastructure established and a major part of their government spending is on welfare programs and various social services. Dar and AmirKhalkhali
(2002) predicted that there is an optimal size of government activity in the economy and if this optimal size exceeds a maximum then a negative effect on economic growth should be expected in those countries.

Other scholars have adopted a similar approach by investigating the relationship between government activity and economic growth as a non-linear process, a proposition that was first empirically tested in endogenous growth models. Heitger (2001), for instance, hypothesized that government activity on public goods has a positive impact on economic growth, but this positive impact tends to decline, or even reverse, if government further increases its activity over some optimal size. In this sense, Heitger was hypothesizing that there is an optimal size of government activity in the economy. In addition, some scholars have advocated the use of an allocative efficiency rule to establish the optimal size of government. For instance Gupta, Sanjeev, et al. (2003) suggested that the size of government spending is optimal when the social marginal cost of public resources is equal to their social marginal benefit. It is not the intention here to cover the literature on these approaches to the relationship between government activity and economic growth. Instead, the main concern of this thesis is with the approach which considers the expansion of government activity as endogenous and being driven by economic development. In some ways, the focus of the approach of this thesis potentially reverses the causality envisaged in the Keynesian approach.

Since at least the late nineteenth century, works in public economics literature have tried to establish criteria by which the revenue and activity policies of government should be evaluated. While other scholars had written on the topic before this time,
they did so generally as part of a wider analysis of the determinants of economic
growth, such as Smith (1776) in his famous book “the Wealth of Nations” and Mill
(1848) the author of “Principles of Political Economy”. Smith and Mill intended to
explain the principles by which revenue and expenditure policies could be
determined as part of their wider investigation of the relationship between the state
and economy. The German economist Adolph Wagner was perhaps the first to
propose a direct hypothesis that the expansion of government activity responds
positively to changes in economic development, so that as a country’s income rises,
the size of that country’s public sector, relative to the whole economy, rises too.
Wagner observed a growing role of the state as a provider of social services in areas
such as transportation, education and infrastructure. He also noted that technology,
such as steam technology, was making it easier for the state to organize its own
production plants more efficiently than the private sector and that the demand for
public goods was growing faster than the demand for private goods.

An important theme here is that, while most nineteenth century literature was
concerned with the appropriate role of government, Wagner directed the attention to
the size of government by proposing an hypothesis which predicted that economic
development would be accompanied by a relative growth of the public sector in the
economy. In particular, Wagner suggested that during industrialization the size of
government activity relative to the economy would grow at a rate greater than the
rate of growth of income. That is, the WH, which has also been referred to as the
Wagner’s law, proposes a monotonic increase in public activity relative to national
income in a way that incorporates the ratio approaching some maximum. The WH
has attracted a great deal of interest in the public economics literature and has been
tested for different economies both over time and across countries. And, as with many hypotheses that are proposed in general terms, several interpretations of the WH have been proposed and tested in the existing literature. Some of these studies have supported the WH as an explanation of the expansion of the size of government activity in the economy, while others have found evidence that does not support it, or contradicts it. The different interpretations and, also, the existing testing procedures and results, will be reviewed in the course of establishing a more comprehensive interpretation and testing procedure employed in this thesis.

This chapter reviews the literature dealing with the evidence of the influence of economic growth on public activity in the context of the WH. The hypothesis put forward by Wagner more than one hundred years ago is still subject to testing and many studies have been carried out in an attempt to support the causation that runs from economic development to public activity. The results so far have generally been mixed. An important preliminary point is that there is no definitive outcome to the existing empirical studies of the WH. This chapter develops the argument that the way that the WH has been interpreted previously has been incomplete both on theoretical and empirical grounds. After reviewing the existing interpretations of the WH, the thesis attempts a recasting of the WH that is designed to provide a comprehensive interpretation of the WH. This should provide a better representation of the WH and thus permit a better basis for testing.

The remainder of the chapter is structured as follows. Section 2.2 revisits Wagner’s original writings according to two main translations, Cooke (1958) and Biehl (1998). This section opens up the discussion about Wagner’s original statement in a way that
will allow for a more comprehensive understanding of the WH, and a basis for evaluating subsequent interpretations. Section 2.3 examines the development of the different interpretations of the WH. This section shows how each interpretation has contributed to the WH. Section 2.4 lays the basis for a recasting of the WH that will be analyzed and tested in this thesis. Section 2.5 summarises the chapter and introduces the next chapter which is a taxonomy of the empirical models of the WH.

2.2 The Wagner Hypothesis

According to Getzler, it is important to recognize that “Wagner was writing at a specific time and place; when many scholars in Bismarckian Germany became filled with nationalism and the desire for a strong state to heal the political and economic disorders affecting German society” (Getzler, 2000, P.13). During this period in Germany there emerged a ‘Social Policy Association’ consisting of a group of eminent scholars, which became known by its liberal critics as the ‘Kathedersozialisten’; the ‘Socialists of the Chair’ (Streeck and Yamamura, 2001, p.55). The ‘Kathedersozialisten’ referred to a number of the younger German professors of political economy who believed in largely state-centered methods to solve social and economic problems in contrast to the methods followed by the Manchester school at that time. For the Manchester school, ‘laissez faire’ and private legal action was the principal way of dealing with economic activity in an economically efficient and socially harmonious way. “The ‘Socialists of the Chair’, by contrast, supported state intervention in industry, especially as a way of promoting the rapid industrialization of the German economy. One common feature of the ‘Socialists of the Chair’ was their attraction to the autocratic and centralizing rule of the state” (Ashley, 1998, p.412).
At the time when the ‘Socialists of the Chair’ views prevailed in Germany, Wagner focused on the relative share of the state activity to national income, on the psychological motivations of the individual, and on population development. In public economics, the role that Wagner assigned to the state, which he called ‘state socialism’, was of primary importance. Wagner based his ‘state socialism’ on an organic concept of the state and on a rejection of laissez faire liberalism. This issue will be revisited in some depth in section 2.2.1 of this chapter, but as Reich puts it: “Wagner advocated the nationalization of sectors that showed a high degree of monopolization. Wagner also formulated an historical hypothesis of ‘growing public and state activity’ as a general consequence of cultural development” (Reich, 1998, p.846).

Wagner proposed that there is a tendency for an expansion of the size of public sector activity relative to the economy along with economic and cultural progress. The theoretical analysis of the WH in this thesis is based mainly on two direct translations of Wagner’s original writings. One translation was produced by Cooke (1958) who translated Wagner’s 1883 text. There, Wagner discusses three extracts on public finance: the nature of the fiscal economy, the basic principles of taxation, and justice in tax distribution. The Cooke translation formed the basis for at least three rival interpretations of the WH of the expansion of state activity. A later, and more comprehensive translation, was produced by Biehl (1998) who translated Wagner’s 1911 version of his hypothesis. It is hoped that, by analyzing these two different translations in some depth, a comprehensive account of the intent of Wagner in his hypothesis of the expansion of state activity will be developed, before moving on to the extant interpretations.
The following sections proceed as follows: section 2.2.1 presents Wagner’s original statement and his organic view of the state. Section 2.2.2 outlines the forces of state expansion in the economy in the context of the WH. Section 2.2.3 discusses the existence of limits to government expansion in the context of the WH. Section 2.2.4 deals with measurement issues of state activity according to the WH. This section firstly recognizes the fiscal measure of state activity in the economy, and then moves on to re-establish the regulatory aspects of the state in the WH.

### 2.2.1 Wagner’s Original Statement and the Organic View of the State

Writing in the midst of a period of rapid urbanization and industrialization in the late nineteenth century, Wagner observed that economic development in countries undergoing industrialization was being accompanied by a growing public activity relative to the economy. He formulated a study of the economic history of the industrialization of European countries such as Britain and Germany, as well as the United States and Japan in the nineteenth century. On the basis of this study he proposed an hypothesis to explain the observed phenomenon of the growth of public activity relative to the economy in industrializing economies. Wagner (1911) stated his hypothesis as follows:

> “Historically there exists a clear tendency for an expansion of public activity together with the progress of the economy...” (Biehl, 1998, p.107)

Wagner based his hypothesis on several underlying assumptions, an organic view of the state being one of them. In the light of the nationalistic purposes of the ‘socialists
of the chair’, of which he was a prominent member, Wagner based this organic view on the principle of state intervention in many phases of national life. Hutter (1982) translated Wagner’s second edition of *Grundlegung*, ‘the foundation of political economy’. In it, Wagner criticized the classical theorists on the grounds that their view of the private system could not deal adequately with the entire task of economic organization and that the private system is often unable to adequately satisfy public needs. Therefore, Wagner proposed that the state is needed to enforce and develop the legal basis of the private system which represents the modern system of free competition. Wagner’s organic view (emphasising the collective economy visible in the public sector) can therefore be contrasted with the individualistic economy visible in the private system (Clark, 1940, p.395). However, Wagner argued that, in a capitalist society, both spheres of activity, public and private, satisfied a wide range of services in the economy. Wagner (1879) noted that:

“In order to maximize the sum of the goods and optimise distribution, a combination of the two systems must be sought. The optimal combination will change as the context of economic activity changes”

(Hutter, 1982, p.134)

It is clear at this point that Wagner views the state, represented by government activity, as an organic part of the social and economic system, and that he expected that the state would tend to grow proportionally with the growth of the economy as a whole. It appears that Wagner sees a relationship between the size of government in the economy and the total level of economic activity in that economy. This relationship is causal with causation running from the level of total activity in the
economy to the level of government activity in the economy. This quite clearly contrasts with the Keynesian view explained briefly at the beginning of this chapter. There, the level of total activity was caused by autonomous changes in the government sector activity.

Wagner’s organic view of the state, where state activity grows in response to growth in total activity of the economy, has created quite a deal of controversy, and formed the basis of opposition to his hypothesis. For instance, a notable critique is Peacock, A.T. and Wiseman (1961). They investigated the WH and rejected both the inherent (organic) theory of the state implicit in Wagner, and the mechanism by which any expansion would take place¹. Such criticism has, however, formed the basis of rival, rather than differing, interpretations of the WH. Since it is the purpose of this section to develop Wagner’s view of the public economy, the next section returns to the WH, concentrating on a discussion of the forces that Wagner thought drove the expansion of the state activity in the economy.

## 2.2.2 Forces of State Expansion

In the previous section, reference was made to Wagner’s view that it was, in part at least, the deficiencies and disharmonies of the private system that lead to a growth in government activity in the economy. This section outlines these forces in more detail. Wagner identified three main forces that lead to more government involvement in an industrializing economy. First, the demand for the enforcement of law and order internally and externally increases as economies grow. Second, the demand for culture and welfare services such as health, education and other services,

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¹ A full discussion of Peacock and Wiseman’s work is presented in section 2.3.1
increases government involvement as a collective producer where government in general is more efficient than private producers. This would lead to an expansion in state activity. The final force that Wagner identified as a driver of the development of government activity is said to be the participation of public ownership in material production.

The WH emphasizes external sources of demand for growth in government expenditure. Wagner identified these sources accompanying industrialization as the key reasons for expansion of the size of the public sector. The WH developed in the late nineteenth century has been criticized as unhelpful in understanding the causes of government growth in a more modern era in the late twentieth century. In addition, it might be true that Wagner’s analysis concerns countries experiencing industrialization more than countries in the industrialized mode of their development (Abizadeh and Gray, 1985). To check whether the WH is still applicable to the industrialized countries, the logic of these forces needs to be re-examined.

Wagner stated that in increasingly complex societies, the demand for public protective and legislative activity grows. Importantly here, the need for the legislature is to establish laws of contract and property and for the judiciary to administrate such laws and ensure the smooth working of the private sector. As the scale and scope of the private sector grows, with increasing complexity, the need for complex legislation and the administration of those laws in contract and property also grows. In addition, Wagner thought that ‘Law and Order’ also includes the maintenance of the nation as a political unit against other nations, and is restricted to
governments\(^2\). The relevance of this aspect of the WH is likely to continue to be a factor in underpinning government activity in industrialized countries.

Wagner expected that governments will respond to increases in income by spending more on goods and services. In this way Wagner proposes that the demand for government spending will increase as a response to increases in personal income. That is, it is expected to increase at a rate faster than income. An illustration here is that the demand for services such as health and education has grown as social income has grown. In other words, a positive elasticity of demand for these services, with high public sector content with respect to aggregate income, tends to produce a positive elasticity of demand for many public sector activities. It may well be, however, that this aggregate income elasticity of demand is greater in the early and middle stages of the industrialization process. Therefore, the countries which fall into such a category will experience a growing involvement of government in the aggregate economy as the demand for such goods and services increases proportionately more than aggregate income. However, this aspect of the WH may also be relevant in explaining the growth of government purchases of goods and services in industrialized countries. That is to say, as there is a high level of societal income in the economy, governments in industrialized countries can provide many of the ‘luxury’ facilities in life to their citizens. This factor may thus contribute to the scope of government spending being positively related to societal personal income even in industrialized countries. This also has important implications for the exact nature of the share in the economy of the fiscal state relative to the size of the economy and this will be revisited in more depth in section 2.2.3 of this chapter.

\(^2\) For more details on this topic see, Clark, E. A. (1940), "Adolph Wagner: From National Economist to National Socialist." *Political Science Quarterly* **55**: 378-411
Wagner himself considered ‘Culture and Welfare’ to be a relatively less important force than ‘Law and Order’ for government involvement in an industrializing economy. Wagner thought that the private system might also serve as a provider of goods and services in areas such as health, education and other services during the industrialization process. Wagner (1911) noted that:

“This development (of an increasing size of state activity) emerges from both domains of state activity, but is more pronounced in the domain of ‘law and power’” (Biehl, 1998, p.108)

In addition to these two factors, Wagner also noted that the participation of public ownership in material production was also contributing to an increase in state activity. Here two factors may be emphasized: new technical processes which often resulted in natural monopolies; and the urbanization of the industrializing economy. In order to prevent monopolistic practices in highly concentrated industries, and because of the seeming impossibility of any individual firm raising sufficient capital to undertake large-scale projects such as water and drainage and public transportation, Wagner observed that the state often had to take over such activity directly. However, in modern states (and in the United States from much earlier), capital markets are more developed and capable of funding large capital-intensive projects and governments are more prepared to engage in regulatory control over private providers of such activities³. In this area Wagner (1911) noted that:

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³ This issue will be revisited in more depth in section 2.2.4.2 of this chapter
“Material goods production and its legal and economic foundations, all represent to a considerable extent the development of production technology, the needs for large enterprises, the evils of speculative private capital and its forms of organisation... All these factors contribute to the fact that public institutions, other facilities, services of the state and of associations as well as of local government take the place of private institutions, of individual and of private enterprises”.

(Biehl, 1998, pp. 109-110)

Wagner also suggests that industrialization enhances the need for production of public goods as a result of the urbanization in that nation. In contrast to the income elasticity effect it seems that this force fits newly industrializing economies more than industrialized economies. That is, the rapid concentration of population in urban areas associated with industrialization was concentrated in the first half of the twentieth century in most of the developed industrial countries. However, when Wagner laid down this aspect of his hypothesis, the demographic changes accompanying industrialization may have then still been critical. However, if the industrialized countries no longer experience this factor in the same way that drove increased public spending on public goods and services, these countries may face demographic changes, other than urbanization movements, that may impact on the demand for government production of goods and services in their economies. The most important demographic shifts in the post-war era are changes in income distribution, household composition, and the age of the population. All, or some, of these factors might influence the demand for government spending in industrialized countries.
However, despite all of the forces discussed above, Wagner explicitly recognized that the economy is a market economy in which private institutions are still active in the economy and provide goods and services beside, or instead of, the state. Therefore, the smooth organic relative development of the state in the economy is not expected to continue forever since Wagner recognized, at least, that certain fields of economic life such as agriculture, trade and commerce would remain outside the reach of public activity expansion.

Having established this point, this thesis suggests that the size of the public activity in the economy, measured by the share of public expenditure in income, cannot grow forever or more than the economy as a whole. The next section re-examines the limits to the growth of state activity in the economy in the context of the WH.

### 2.2.3 Limits to State Expansion in the Economy

Wagner expected the size of state activity in the economy to expand with the progress of the economy. However, Wagner was also interested in the limits to this relative expansion. In his original writings, he recognized that there must be a limit to the growth of the relative size of government in the economy. Wagner (1883) noted that:

> “There is thus a proportion between public expenditure and national income which may not be permanently overstepped. This only confirms the rule that there must be some sort of balance in the individual’s outlays for the satisfaction of his various needs” (Cooke, 1958, p.8)
In other words, Wagner, according to this translation, recognized that there would be an upper limit to the share of public spending in the aggregate economy. That is, he envisaged relative measures of government fiscal activity, such as the ratio of government expenditure to national income, to be asymptotic with respect to national income. However, Wagner was unwilling to set such a limit in advance. For instance, Wagner (1883) noted that:

“All earlier attempts to lay down absolute figures of expenditure or to define an upper limit of its proportion to national income, have always miscarried” (Cooke, 1958, p.8)

One reason Wagner realized that such limits exist is that the increasing proportion of expenditure to national income will impose a burden on taxpayers and at some point they will start to reject any further calls for state revenue. Wagner (1883) addressed the expected limits to future expansion of state activity through the following question:

“Should public expenditure be allowed to become so high that the requisite taxation becomes an oppressive burden on the people?”
(Cooke, 1958, p.107)

The limit of government growth in the economy has been a relatively neglected area in the existing literature on the WH. However, some scholars have recently noted the existence of such limit in Wagner’s writings. For instance, based on the Biehl (1998) translation, Peacock and Scott (2000) recently noted that:
“Wagner recognized there must be an upper limit to the proportion of government growth. The limit would be determined by the resistance that comes from taxpayers if their ‘normal’ personal consumption was to be diminished by tax burdens, but Wagner was scornful of attempts to define such a limit” (Peacock and Scott, 2000, p.3)

Recall from section 2.2.1 that the WH refers to a market economy where state activity expands organically with the national economy. Therefore, employing functional forms that allow the share of government to grow without a limit will incorrectly specify the WH. This could be due to two causes. First, the limitation of earlier translations of his work could lead to misconceptions by those wishing to test the WH about the nature of the relationship between government and the economy envisaged by Wagner. Secondly, the adoption of linear forms, either in variables or at least in parameters, (that is the log-linear specification), has been commonly thought to be a convenient way to estimate the WH. However, such formulations might not allow for a limit to the relative expansion of the government, or, if they do, may lead to an estimation of inappropriate empirical relative limits which has no economic sense (Verbeek, 2000).

Based on Wagner’s writings and the rationality that a share of government expenditure to national income must be estimated between zero and one, this thesis suggests that any estimate must incorporate a limit to government activity growth relative to the economy. Section 2.4 will offer a recasting of the WH which is underpinned by Wagner’s intentions more than 100 years ago but only recently revealed in modern translations of his work. One important aspect of this recasting is
the nature of the limits to relative government growth in the economy and the factors which naturally lead to such limits in the relative organic development of government. However, before this can be adequately undertaken, the issue of the measurement of government activity must be examined in section 2.2.4. Section 2.3 will then examine the various interpretations of the WH. After this task is completed, the thesis will be able to return to the important issue of the limits to government growth in the economy.

2.2.4 Measuring State Activity in the WH

2.2.4.1 Fiscal Activity of the State

Tests of the WH have almost always used a measure of state activity in terms of the state as a fiscal entity. Wagner, indeed, hypothesized that state finances (government expenditure in modern terms) could serve as one measure of the size of public activity in the economy. Wagner (1911) noted, for instance, that:

“The development and expansion of public services as well as the increase of public, in particular State finances reflect the expansion of public activity” (Biehl, 1998, p.108)

Wagner recognized that the state activity needs a certain amount of economic goods. The fiscal economy has to provide for that part of state requirements that consists of goods or money to be used directly in public services. Wagner (1883) noted that:
“This part of total state requirements may specifically be called fiscal requirements” (Cooke, 1958, p.2)

In a monetary economy, fiscal requirements consist mostly of monetary means and appear on the budget as state expenditure. To cover state expenditure, the fiscal economy has to function as an income or revenue economy, where its income appears as state revenue.

Wagner argued that the fiscal aspects of the state represented by state expenditure serve as a measure of the size of the state activity. However, this thesis suggests that measuring state activity as a fiscal entity approximated by government expenditure is only one aspect of state activity in the WH. Wagner also recognized ‘regulatory activity’ as another aspect of state activity within the economy. However, Wagner did not include the non-fiscal effects of state regulatory activity in the measurement of the state activity in his proposition and kept the measurement of the state activity purely as a fiscal entity. The next section examines the regulatory role of government and attempts to rehabilitate this often-neglected dimension of state activity in Wagner’s proposition.
2.2.4.2 Regulatory Activity of the State

Wagner explicitly recognized that fiscal activity represents an important aspect of state activity. Wagner (1883) noted that:

“If the state itself is considered as an economic unit, the fiscal economy may be considered as a department thereof” (Cooke, 1958, p.1)

However, Wagner also recognized that state activity is not restricted to the state as a fiscal entity. Indeed, he recognized that many of the important avenues by which the state affects the economy such as regulatory activity have little or no budgetary consequence. This recognition of the importance of state regulatory activity appears quite clearly in a recent translation of Wagner’s writing. Wagner (1911) noted that:

“The state will act with legislative and administrative measures and with financial means” (Biehl, 1998, p.110)

Here it is quite clear that Wagner recognized two aspects of state activity: the ‘fiscal’ and what might collectively be termed the ‘regulatory’. Yet almost all debate on, and testing of, the WH has focused exclusively on the state as a fiscal entity. The main reason why Wagner excluded regulatory activity from the measurement of the state activity when elaborating on his hypothesis might have been the difficulty of measuring regulatory activity.
This thesis argues that testing the fiscal side of the state only cannot be a definitive test of the WH, since the regulatory side is not included in this testing. Therefore, previous tests of the WH have not actually tested the law fully if they have not incorporated regulatory activity but, instead have tested only part of it. Furthermore, the failure to include the regulatory aspect in the measurement of the state activity may have impacted adversely on the findings of tests of the WH’s fiscal side. That is, in some cases, state regulatory activity might be complementary to fiscal activity. But, in other cases, they may be competing forms. The state may, for instance, provide public roads and regulate traffic. In this case the fiscal and the regulatory activity are complementary activities of the state. On the other hand, the state may also choose between fiscal and regulatory activity. Some countries have regulated railroads run by private corporations, while others have nationalized them. In this case, the fiscal and the regulatory activities are competing forms of state activity. There are obvious fiscal differences between the two forms.

Figure 2-1 is an illustration of the complementary activity of the state, where both fiscal and regulatory activity are growing in a complementary way, so that total state activity has a growth trend which simply combines both aspects of the state. Figure 2-2 shows another case where the fiscal and the regulatory activity of the state are in a competitive mode; that is, one activity is growing at the expense of the other activity which declines. However, the summation of both trends might result in positive total growth in the state activity as shown in Figure 2-2. It is important to note that, in this case, the total growth trend is a net of both trends and could conceivably decline in total. The two figures also show that total state activity in both
are consistent with the WH while, in Figure 2-2 the fiscal measure would appear to contradict Wagner.

**Figure 2-1:** Stylised Accumulative Line Graph of Complementary State Activity

**Figure 2-2:** Stylised Accumulative Line Graph of Competitive State Activity
To support the restoration of Wagner’s regulatory dimension of the state, which is the objective of this section, it is necessary to present some definitions of the regulatory state where it is seen as an alternative form of state activity to production. This will be followed by an exposition of those reasons that might lead to the neglect of the regulatory aspects of the state in the existing literature on the WH. Finally, this section will present some studies that have investigated the rise of the regulatory activity in some of the modern states in the late twentieth century. It should be noted that some of these recent studies have even attempted to measure the size of the regulatory activity through the development of quantitative estimates.

2.2.4.3 The Growth of Regulatory Activity

The existing literature on the state as a regulatory entity has defined regulation in several ways. In its broadest sense, regulation is often equated with governing and, in this sense, it is broadly a combination of the ways in which public purposes are authoritatively decided on and implemented. Regulation is also defined as a particular and separate policy instrument distinguished from other instruments such as spending, taxation, or the direct delivery of services. This definition refers only to that subset characterized by the use of state law by state agencies in instrumental fashion. Such a definition appears in studies such as Hood, et al. (2000) and Scott (2003). In this way regulation is defined to include only given legislation, or rules that are made pursuant to powers granted in a parent law. In other notions, regulation is defined in a sense in which it is seen as an alternative form of state activity to production. However, this thesis does not intend to enter that debate.

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Wagner had recognized the effect of the regulatory activity of the state on the expansion of state activity. But there are several reasons why regulatory aspects of the state have been neglected in most of the existing literature on the WH. Pildes and Sunstein put the case that “Regulatory activity of the state is complex and difficult to measure” (Pildes and Sunstein, 1995, p.127). Another reason for the relative neglect of the analysis of the regulatory activity in the existing studies might be the relatively low visibility of regulatory activity. Majone (1994), for instance, noticed that in the United States the meaning of regulation is fairly clear in terms of independent agencies combining legislative, administrative, and judicial functions. In Europe, the tendency has been to treat regulatory issues either as purely administrative - economic and social regulation - or as judicial. Furthermore, different countries reflect different mixes of regulation and state production. For example, some countries have regulated railroads, while others have nationalized them. Some studies that based their analysis on developing countries have suggested that the increasing role of the regulatory activity of the state has led government size to expand in the economy faster than the increase in national income in that economy. For instance, Thorn (1972) observed that developing economies need more extensive mechanisms of regulatory control and co-ordination which would tend to increase the size of government expenditure faster than the increase in national product. Diamond (1977a) has also suggested that the requirement for greater regulation is one of the many possible explanations for the growing share of the public sector in national income in developing economies.

In the last few decades the analysis of regulatory activity of the state has attracted the attention of scholars in the public economics literature, although not within the
analysis of the WH. This attraction resulted from the international debate about privatization and deregulation. For instance Amann and Baer (2004) analyzed the effects of privatization on the Brazilian economy. They suggested that the privatization process has been accompanied by greater state control and regulation and that the state has retained a key role as a regulator in Brazil. Other studies have investigated the causes behind the rise of the regulatory state in individual countries, Gleaser and Shleifer (2002), for instance, investigated the rise of the regulatory state in the United States during the period from 1887 to 1917. They developed a model of law enforcement in which private legal action, government regulation, a combination of the two, and doing nothing, are considered as alternative institutional arrangements to secure property rights. They observed that, during the first half of the nineteenth century, agriculture represented a large proportion of the economy, and manufacturing was concentrated in small firms. In this period, any social harms of economic activity were left for the judicial system to deal with. However, with the rise of manufacturing and heavy industry, firms grew sharply in size and the legal system that was adequate during the agrarian period, failed when faced with individuals or bodies that had large incentives to subvert it. Therefore, increasing regulatory activity of the state was an efficient response. Gleaser, et al. (2003) suggested that the operation of legal, political and regulatory institutions could be subverted by the wealthy and the politically powerful for their own benefit. They argued that economic inequality, which often comes with industrialization, can undermine the functioning of the institutions of law enforcement through subversion.

Despite all the different definitions of regulation, and the analytical problems that arose in dealing with regulation, some studies have attempted to provide a
quantitative estimation of any increased regulation over time in scale and scope. Using government employment numbers, Walters (1993), for instance produced results that suggested that the growth in federal regulation in the U.S., that began during the 19th century, increased rapidly during the 1930s. Similarly, using government employment numbers Majone (1994) suggested that the growth of administrative regulation in Europe greatly accelerated during the last two decades of the twentieth century. For example, in Britain, the 1970s represented a period of significant modernization, especially in the area of social regulation. Regulatory agencies were set up in the 1980s and early 1990s, partly because it was realized that in many cases privatization would only mean the replacement of public by private monopolies unless the newly privatized companies were subjected to public regulation. Results presented by Moran (2001) suggested that in Britain the regulatory state activity has grown in scale and scope and are expected to continue to grow over time.

Other scholars have even attempted to measure the size of the regulatory activity in quantitative units. Hood, et al. (2000), for instance, examined the number of government employees engaged in regulatory activity in Britain in two different years, 1976 and 1995, and in different parts of the public sector. Their results showed that the number increased from about 135 to over 200 regulatory organizations and the staff size estimate of these regulatory organizations increased from almost 14,000 to almost 20,000. The direct annual running costs of these organizations increased from about £750 million to about £1 billion in real terms. Furthermore, their results suggested that regulatory activity of government in the UK apparently continued to grow and even accelerated after 1995. Walters (1993) also attempted to assess
whether the size of government regulatory activity has grown in the US during the period from 1970 to 1991. He measured the regulatory activity of the US through its federal regulatory spending and staffing. His results showed that federal regulatory spending and staffing levels grew in real terms from about 3 billion dollars in 1970 to about 7 billion dollars in 1980\(^5\), then started to fall to some extent during the 1980s, and by 1991 regulatory activity in the U.S. was at its highest level.

The discussion has intended to restore the regulatory dimension of the state in the late nineteenth century, which appeared to have been neglected by many researchers engaged in testing the WH. It has established that Wagner recognized the effect of the regulatory activity of the state in his original writings. The thesis has also shown, through reference to the works of others, that these regulatory activities of the state might have grown in size and scope in industrialized countries in the last few decades. While it is not the intention of the current thesis to provide tests of this proposition, it does suggest that testing the fiscal activities of the state is only a partial test of the WH. It will be helpful to return to the possible effects of state regulatory activity when interpreting the empirical results of this thesis in chapter 6.

Having established the broad parameters of the intent of the Wagner hypothesis, the next section introduces and reviews the different interpretations of the WH in the existing literature. In so doing, the thesis intends to assess the contribution of the different interpretations of the WH, and provide the basis for a comprehensive version of the WH, one which is suitable for empirical testing.

\(^5\) The base year used is (1982=100)
2.3 Interpretations of the Wagner Hypothesis

Wagner’s general hypothesis has provided scope for a range of different interpretations in the existing literature. It is possible to identify at least six of these interpretations, Peacock, A.T. and Wiseman (1961), Gupta, S. P. (1967), Goffman (1968) & Goffman and Mahar (1971), Pryor (1968), Musgrave (1969), and Man (1980). In addition, a recent attempt has been made by Florio and Colautti (2003) to interpret the WH by synthesizing it with a theory of tax efficiency. There are key differences that appear in these interpretations of the WH, concerned mainly with issues discussed above in the key elements of the WH. These include issues such as: the measurement of the economic variables in the hypothesis, the functional form of the relationship between the key variables in the WH, and the nature of limits to government growth.

The discussion in the following subsections will critically review the interpretations of the WH. A common theme of all of the existing interpretations of the WH involving empirical testing have followed Peacock, A.T. and Wiseman (1961) and measured the state as a fiscal entity using some aggregate measure of government expenditure. None have included the regulatory aspects of the state in the testing of the WH or even considered it in their analysis. Furthermore, all of the different interpretations of the WH, with the two exceptions of Florio and Colautti (2003) and Gupta, S. P. (1968), have specified the WH as a linear relationship between the two economic variables either measured in levels or in logarithms. Unlike the other interpretations, Florio and Colautti measured the fiscal state in a relative way by analyzing the share of government expenditure in GDP to the level of GDP. An important theme here is that all of the interpretations have not recognized explicitly
the limits of government growth in the WH, except for Florio and Colautti (2003). However, the Florio and Colautti analysis of the WH imposed a restriction on government growth in the economy from outside the demand model, which has already been established as unnecessary. Gupta introduced a non-linear model of the WH and suggested that modeling the WH this way might give a better understanding of the behavior of public expenditure in relation to national income over time and across countries. However, Gupta’s results did not contradict the WH and he did not develop his non-linear model further to a sensible form that places boundaries on government share.

 Whilst it appears that there is disagreement on the appropriate interpretation of the WH, the existing empirical studies of the WH have found general evidence in support of the law. An analysis of the empirical findings of the different tests of the WH is provided in chapter 3.

2.3.1 Peacock and Wiseman (1961)

Peacock and Wiseman (hereafter P&W) interpret the WH as:

"The proportion of public expenditures to gross national product must be expected to rise over the foreseeable future"  (Peacock and Wiseman, 1961, p.10)

The P&W interpretation of the WH envisages that public expenditure should increase smoothly and consistently at a rate higher than the rate of increase in national demand.

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6 The Florio and Colautti study will be discussed in some depth in section 2.3.7
income, and assumes that the growth of public expenditure is associated with changes in the demand for public services. Those changes in demand are mainly due to the growth in income per capita and population. However, Wagner considered other factors, such as a steadily developing division of labour, technology and scientific progress, as well as the increasing complexity of transport and communications, would lead to a higher level of government activity. The P&W interpretation states that the level of government expenditure is a function of national income and can be represented in the general relationship shown in equation (2.1)

\[ GE = f(GNP) \]  

(2.1)

where GE represents total government expenditure and GNP represents Gross National Product. A few comments can be made on the P&W interpretation. First, it can be considered to be the first modern analysis to revive the WH, and began the modern measurement of the state as a fiscal activity such as GE. Second, it rejected both the inherent (organic) theory of the state explicit in Wagner, and also rejected the mechanism by which any expansion would take place. The problem they suggest is that state activities may increase, but not for the reason or in the way that Wagner hypothesised. However, they adopted Wagner’s historical approach to study the behavior of British public expenditure by looking at the relevant time-series and historical facts. In rejecting the theoretical foundations of the WH (its organic view of the state and its alleged demand-side focus) and the validity of the WH in explaining the pattern of state activity growth, P&W were in some way proposing a rival model to the WH, based on a supply-side time-pattern approach to public expenditure growth in Britain. They found that state activity displayed a step-wise,
rather than gradual, pattern of government growth. Third, Wagner did not suggest a precise functional form of the relationship between the size of public expenditure and economic development. However, the P&W interpretation of the WH still proposes a linear relationship between the two economic variables as depicted in equation (2.2)

\[ GE = a + b(GNP) \quad (2.2) \]

In modern analysis, a linear functional form of the WH will either have an elasticity of GE with respect to GNP which is less than, equal to, or greater than one. If it is less than 1 then as GNP increases, the elasticity scores will increase towards 1 but will limit to 1. If the elasticity coefficient is greater than 1, then as GNP increases the elasticity scores will fall towards 1. It has been established in the previous section that adopting a linear functional form might not be convenient for the analysis of the WH. In an economic sense neither GE nor GNP can be negative. Yet fitting a simple linear relationship allows the possibility of negative values, which is unrealistic, especially the notion that GE can have a negative intercept. However, GE must have a negative intercept if the elasticity of GE with respect to GNP is greater than 1 and if GE/GNP is to have an upper asymptote. Again, a problem here is that even though such a linear relationship must result in an asymptote for GE/GNP as GNP increases without limit it could result in GE/GNP becoming greater than 1, which again is not realistic and does not make economic sense.

P&W were concerned primarily with the time-pattern of public expenditure growth, and in so doing propose their own stepwise process of public expenditure growth.

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7 For more details on elasticities see appendix B
where they stress the importance of supply side crises such as wars and depressions. They argued that the greater role of government during these times leads to increases in the tolerable burden of taxation, rather than the smooth ‘organic’ demand-led growth that they argue was proposed by Wagner. According to P&W, the crisis level of taxation tends to remain high after the crisis has passed because the expanded bureaucracy will act to ensure its continued new levels of funding, albeit for a different suite of post-crisis expenditures. They further argue that crises, especially war, can concentrate power at the national level. They call their hypothesis the “displacement” effect and it is typified by public expenditure which is rapidly flexible upwards during crises, but is inflexible downwards after the crises. However, the P&W displacement hypothesis was unable to explain the sustained large rise in the role of the public sector after World War II in the United Kingdom or in other countries (Bird, 1972).

Several scholars in the existing literature have tested the P&W interpretation of the WH. Thorn (1972)\textsuperscript{8} tested the WH for 52 countries from 1952 to 1962 adopting the P&W interpretation and found evidence which did not contradict the WH in those countries. Thornton (1999) tested a P&W version of the WH for 6 developed countries during the period from 1850 to 1913\textsuperscript{9}. Thornton’s results again did not contradict the WH during that period for the selected countries. Biswal, et al. (1999) tested the WH for Canada during the period from 1950 to 1995 and found the evidence contradicted the WH. Other studies have followed the P&W version and tested for the WH such as Bairam (1992), Courakis, et al. (1993), Kolluri, et al. (2000), Diamond (1977), Bohl (1999), Bonin, et al. (1969), Nagarajan (1979),

\textsuperscript{8} Originally published in 1967
\textsuperscript{9} Denmark, Germany, Italy, Norway, Sweden and the UK
Nomura (1995), and O'Hagan (1980). A deeper analysis of the results of these studies will be presented in chapter 3.

There are some clear problems with the linear relationship used by P&W, and those that have tested the WH using the P&W version had similar problems. One particular problem is that, to ensure that the ratio GE/GNP is asymptotic from below, the intercept of the GE axis must be negative. This ensures an elasticity of GE with respect to GNP greater than 1, so that, in some range of GNP, the share of government in GNP is increasing. Further, it also ensures that as GNP increases, the elasticity falls to 1 and that GE grows in equal proportion to GNP and the share GE/GNP reaches a maximum. However, there is still an important issue, the asymptote value could be greater than 1. Further, and very importantly, negative values of GE do not make economic sense. This would indicate that perhaps a better direction would be to directly model the relationship of the share of GE/GNP with some measure of the level of the economy such as GNP or GNP per capita. P&W’s functional form could suffer from an endogeneity problem. This problem may occur in P&W’s model if both the Keynesian’s and Wagner are right. That is, GE causes GNP and GNP causes GE. If this is the case then there is a simultaneous determination of both variables in the model so that GDP would be an endogenous regressor in equation (2.2). However, it should be pointed out that all models which test the WH and have GE or any measure in levels of government expenditure on the left hand side of the equation could suffer from an endogenous regressor problem (Verbeek, 2000, p.122). In time-series, a clear way to overcome this problem would be to introduce the lagged variable income, or the lagged variable income per capita,
as an instrumental variable in the two stage least square model (Greene, 2003, p.74).

This issue will be revisited in chapter 4 which reviews the empirical literature.

2.3.2 Gupta, S. P. (1967)

Gupta interpreted the WH by considering the relationship between state activity and national income where:

“Government expenditure must increase at a rate faster than that of the national income” (Gupta, 1967, p.426)

Gupta measured the size of government by GE per capita, and economic development by GNP per capita, as shown in the general relationship depicted in equation(2.3):

\[
\frac{GE}{P} = f\left(\frac{GNP}{P}\right)
\]

(2.3)

where P represents population and other variables are as defined previously.

Gupta examined the time-pattern of public expenditure growth for a group of countries. These countries were: the UK, Germany, the U.S.A., Canada, and Sweden. Gupta suggested that the P&W version of the WH refers only to the shift in the level of government expenditure with relation to national output. He suggested that P&W were looking for the association between social upheaval and the level of government expenditure growth with relation to economic growth. Gupta argued that the concept of a tolerable burden of taxation adopted by P&W can explain shifts in
the level of public expenditure during wars and crises but cannot explain the shift in
the level of public expenditure during a depression since taxes are reduced in this
period. Gupta explained further that including other methods of financing in addition
to taxes, such as deficit financing, in the P&W concept of a tolerable burden, might
provide a better explanation. Gupta may be the first to devise rigorous statistical tests
for a displacement effect, separately testing for a shift in the government expenditure
level and whether social upheaval is associated with the change in the income
elasticity of government expenditure with relation to economic growth.

To test for the WH and the ‘Displacement’ effect, Gupta adopted a double
logarithmic functional form, which is depicted in equation (2.4)

\[
\ln\left(\frac{GE}{P}\right) = a + b \ln\left(\frac{GNP}{P}\right)
\]  

(2.4)

Gupta’s logarithmic form gives a constant elasticity score on the left hand side
variable of the equation \((GE/P)\) with respect to the right hand side variable \((GNP/P)\).
Gupta’s model of the WH is different from that of P&W in a way that the left and
right hand side variables are now represented as ratios to population. Gupta also
tested for the WH using a log-linear function form whilst P&W’s functional work
implies a simple linear form. Because \(GE\) and \(GNP\) exhibit strong simultaneity,
Gupta’s log-linear functional form suffers from an identification problem as did
P&W’s functional form.

The test results suggested that a shift in the level of \(GE\) per capita was associated
with the great depression in the United States and Canada, which could not be
explained by P&W’s displacement effect hypothesis. Gupta justified this on the grounds that the shift associated with the great depression occurred because much new expenditure, such as welfare services and subsidies and assistance generated by the great depression, were mostly deficit financed. The results also suggested that a significant change in the income elasticity of the level of GE per capita with respect to per capita GNP is associated with each major social upheaval but with no generalization of its direction. In the case of Sweden, where there were no social upheavals, the income elasticity of the level of GE per capita also changed positively after the Second World War, which presented some support for the WH in Sweden during that period.

The results of the tests in Gupta’s study suggested a limited acceptance of the WH in most of the countries included in the tests. However, and importantly for this thesis, Gupta suggested that a non-linear analysis of government expenditure with relation to income could be followed to test for the WH, both for time-series and cross-section. Gupta, S. P. (1968) introduced a non-linear model of the WH and suggested that modeling the WH this way might give a better understanding of the behavior of public expenditure in relation to national income over time and across countries. His results did not contradict the WH but he did not develop his non-linear model further to a sensible form that places boundaries on government share. However, Gupta’s non-linear interpretation marked a significant step in the development of the interpretations of the WH, since Gupta was the first to recognize that the growth of government relative to national income would follow a non-linear process.
Gupta’s (1967) linear interpretation has been adopted, and tested for different economies by many scholars in the existing literature. Michas (1974) tested Gupta’s version of the WH for Canada during the period from 1950 to 1961 and found support for the WH during that period. Nomura (1995) also tested Gupta’s version of the WH for Japan during the period from 1960 to 1991 and found support for it, whereas Singh and Sahni (1984) tested Gupta’s version for India during the period 1950-81 and found no support. Other studies have followed and tested Gupta’s version of the WH such as Singh and Sahni (1984) for India, Chletos and Kollias (1997) for Greece, and Ansari, et al. (1997) for three African countries.

2.3.3 Pryor (1968)

Pryor analyzed the growth of public expenditure in market and centrally planned economies. The market economies included were the USA, West Germany, Austria, Ireland, Italy, Greece and Yugoslavia. The centralized economies included were Czechoslovakia, East Germany, the USSR, Hungary, Poland, Romania and Bulgaria. Unlike the two previous works, Pryor (1968) interpreted the WH so that in growing economies, public consumption expenditure becomes an increasingly larger component of the national income. Pryor’s interpretation is different from both the Gupta and P&W interpretations in that Pryor narrowed the definition of government expenditure to include only government consumption expenditure. Pryor’s general relationship of the WH is depicted in equation (2.5)

\[
\frac{GC}{GNP} = f \left( \frac{GNP}{P} \right)
\] (2.5)

---

10 The inclusion of Yugoslavia as a market economy is based upon Pryor’s classification of economies
where GC denotes government consumption expenditure, and other variables as presented previously in the thesis. Pryor tested the WH using a log-linear functional form as depicted in equation (2.6)

\[
\ln\left(\frac{GC}{GNP}\right) = a + b \ln\left(\frac{GNP}{P}\right)
\]  

(2.6)

Pryor modeled a share form of the WH with the dependent variable being the ratio of GC to GNP. The adoption of the share variable may well purge the model of any endogeneity problem. According to Slemrod (1995), it is easy to envisage a causality from GNP/P to GC/GNP. Reversing this causality is not very plausible, so that simultaneity is not likely. Thus the Pryor model does not suffer from the problem of endogenous regressor.

Pryor’s study aimed mainly at comparing market and centrally planned economies, focusing the study on ‘Comparative Economic Systems’. You will recall from section 2.2 that Wagner applied his hypothesis to market economies where free competition prevailed in the market, and democracy is an important feature of the governing of the state’s expenditure process. However, Pryor’s analysis is differentiated from other previous analyses in that it attempted to examine the effects of different types of conditions and variables on the forms of the WH. For instance, he examined the effect of economic development on GC for different economic systems instead of a group of countries with the same economic system. Pryor’s results suggested that two types of countries do not seem to fit his interpretation of the WH: the highly underdeveloped and the highly developed economies.
Pryor’s conclusions may be explained with the use of Figure 2-3. By definition the limits to GC/GNP are 0 and 1 as illustrated in this figure. The line L depicts the type of relationship estimated by Pryor. The S shaped structure labeled NL depicts the growth of GC/GNP which falls within the bounds of 0 and 1. Popular candidates for such structures are the logistics and Gompertz functions Ratkowsky (1990). These forms will be discussed in chapter 4 on the empirical model. Returning to Figure 2-3, it could be noted that the points labeled (U) and (P) on the growth curve NL represent underdeveloped and post-industrial countries respectively. Neither of these countries tracks the line L very well and would therefore seem not to fit with a WH restricted to the linear form of L. Alternatively, stages of economic development between the two extremes (U) and (P) on NL do track the line L and would seem to fit with a WH restricted to the linear form of GC/GNP. That is, any point on the curve NL between the upper and lower asymptotes might well fit a linear Wagner proposition.
Pryor employed both cross-section and time-series data to test for the WH and found that Wagner’s generalization seems applicable on both bases for countries that are in the process of transforming their economies from rural agricultural to urban industrial. He thought that this stage might be described as the beginning of an industrial economy. Pryor also disaggregated GC to observe the behavior of the different components of GC over time along with the development of the economy. Pryor found mixed results when he disaggregated GC into different components and tested with cross-sectional data. On the one hand, he found that empirical tests using the internal security, foreign aid, and research and development categories did not contradict the WH. On the other hand, he found that economic development seemed to have little explanatory power for the military, welfare, education and health expenditures categories. However, in almost all time-series samples, per capita income significantly affected GC.

Pryor’s interpretation of the WH has been adopted and tested by a number of scholars in the existing literature. Abizadeh and Yousefi (1988) tested Pryor’s version of the WH for the USA during the period from 1950 to 1984. Their results support the WH for the USA during that period. Hondroyiannis and Papapetrou (1995) tested Pryor’s version of the WH for Greece during the period from 1951 to 1992. Their results suggested no support for the hypothesis. Iyare and Lorde (2004) tested Pryor’s version for 9 Caribbean countries. They found mixed results for the WH\textsuperscript{11}.

2.3.4 Goffman (1968) and Goffman and Mahar (1971)

Goffman (1968) and Goffman and Mahar (1971) interpreted the WH in the following way:

“The public sector’s share of the community’s output increases with economic development” (Goffman, 1968, p.59)

“As a nation experiences economic development and growth, an increase must occur in the activity of the public sector and the ratio of increase, when converted into expenditure terms, would exceed the rate of increase in output per capita” (Goffman, 1968, p.359)

The Goffman (1968) and Goffman and Mahar (1971) interpretations imply a relationship of the WH as in the general functional form:

\[ GE = f \left( \frac{GDP}{P} \right) \]  \hspace{1cm} (2.7)

where variables are as defined previously. In this general form the dependent variable is the level of government expenditure and the measure of development is the level of GDP per capita. Goffman, and Goffman and Mahar, did not use standard econometrics methods, such as the linear stochastic model; rather they used simple ratios between the dependent and independent variables. Based on these ratios they calculated the elasticity of government expenditure with respect to GDP per capita over points in GE/GDP space. One can only presume that they must have envisaged a linear relationship:
Following Peacock, A. T. and Wiseman (1961, P.10), Goffman measured government growth in absolute levels and suggested that Wagner provided little reason for measuring the rise of public expenditures as proportional to income. Gupta suggested that Wagner’s proportional rise relies on Wagner’s typically Germanic view of the state. In other words, Goffman suggested that Wagner thought that it was desirable for the state to grow at a rate that would increase the share of state functions in output.

Goffman criticized previous studies of the WH in that they presented their results in terms of the rising or falling of the ratios of public expenditures relative to income instead of in terms of the values of the elasticities. Goffman’s view of the elasticity of demand in the WH proposes that the percentage change in income leads to a greater percentage change in expenditures. Goffman did not actually test for the WH; instead, he relied on simple ratios of percentage changes in government spending and GDP, and interpreted the resulting ratios as elasticities.

Whilst Goffman, and Goffman and Mahar, are critical of some previous studies, there are two issues with their work. First, they ignore the potential for an endogenous regressor brought about by potential simultaneity between GE and GDP/P. Second, even though they argue for analysis couched in terms of the elasticity of GE with respect to GDP per capita in favor of the ratio GE/GDP, they appear to ignore some elasticity issues. For the elasticity of GE with respect to GDP/P to be greater than unity, the linear form of their interpretation requires a
negative intercept for GE, implying negative GE scores for low levels of GDP/P. Furthermore, the linear form must mean that the limit to the measure of elasticity described here must approach one. Thus, as GDP/P grows larger, GE growth approaches equi proportions with GDP/P growth so that GE/GDP reaches same maximum level. However, there is no guarantee that this maximum is less than one.

Some studies have adopted and tested the Goffman and the Goffman and Maher version of the WH. Wagner and Weber (1977) tested the version for 34 countries during the period 1950-1972 and found no support for the WH. Courakis, et al. (1993) tested the version for Greece and Portugal during the period 1958-1985 and also found no support for the WH. Bohl (1996) tested the version for the G7\textsuperscript{12} countries during different time periods and found mixed results for the hypothesis.

2.3.5 Musgrave (1969)

Musgrave (1969) interpreted the WH as follows:

\[ \text{“The proposition of expanding scale, obviously, must be interpreted as postulating a rising share of the public sector in the economy. An absolute increase in the size of the budget can hardly fail to result as the economy expands” (Musgrave, 1969, p.74)} \]

Musgrave’s interpretation of the WH assumes a functional relationship between the ratio of total public expenditure to GNP and per capita income as depicted in equation (2.9):

\[ \frac{GE}{GNP} = f \left( \frac{GNP}{P} \right) \]  \hfill (2.9)

This functional relationship proposes that, with the development process represented by per capita income \((GDP/P)\), the share of government expenditures in national income \((GE/GDP)\) will increase at a higher rate than that of per capita income. Musgrave tested for the WH using the linear functional form as depicted in equation (2.10)

\[ \left( \frac{GE}{GNP} \right) = a + b \left( \frac{GNP}{P} \right) + e \]  \hfill (2.10)

This linear form requires that GE/GNP is a positive function of GNP/P if economic development is to lead to a relative increase in government expenditure as posited by Wagner. Clearly, there must be an upper limit to this expansion and the linear form will not control for this limit. Unfortunately, the specific functional form adopted by Musgrave in equation (2.10) is the simple linear form. There are clear problems with this form because it does not place lower and upper bounds of zero and one respectively on the share variable GE/GNP. That is, the linear specification in principle can lead to forecasts of GE/GNP which are negative or greater than one, values which are in reality impossible and have no economic sense. This issue is very well recognized in modern micro econometrics with the use of logistics, logit and probit regressions (Greene, 2003). It could well be that the estimation methods contemporary with Musgrave’s work produced such linear models. Nevertheless, it may be that his results can be explained by using a more appropriate functional form.
This will be revisited later in this section and in some depth in chapter 4 on the empirical model.

Musgrave examined economic factors that might support the hypothesis of a rising share of public expenditure in GNP by studying the development of a country from low to high per capita income in the course of economic growth. Musgrave’s version is differentiated from other versions of the WH in several ways. First, his interpretation considers shares instead of absolute levels and so is less likely to suffer the endogeneity problem. Second, following Wagner, Musgrave considered the cause of particular types of public expenditures. He accepted the distinction between defense and civilian functions but his choice did not conform with Wagner’s choice of expenditure categories: protection, general administration, economic administration, and education. Instead, Musgrave asserted that civilian expenditures might be better examined in economic categories such as public capital formation, public consumption, and transfers.

Musgrave expected that the rise of the public share in total capital formation will be relatively high in the early stages of development, but with less predictable change thereafter, and that the ratio of transfers will tend to decline with rising income. His rationale was that the facilities for private capital formulation are limited in the early stages of development, and public production of certain capital goods might therefore be necessary. However, at a later stage of development, the institutions for private capital formation become more developed and the provision of such capital goods may be left to the private sector. Musgrave suggests that the WH covers only the earlier to middle stages of economic development and does not apply to the post-
industrialized states. However, Musgrave suggests that changing private consumption patterns might call for complementary private investment, so that the net effect on the public share depends on each particular case. While this might have motivated Musgrave to a non-linear version of the WH, he retains the linear form of earlier interpretations.

Following Wagner, Musgrave suggested that there is a rising public to private consumption ratio over the early stages of economic growth as a result of the three main factors discussed in section 2.2.2. Regarding the increasing complexity of economic organization which comes with economic development, Musgrave suggested that this force may generate demand for a new set of basic public services which are of a corrective sort. For example, the emergence of corporations and large enterprises may necessitate the services of regulatory agencies. Musgrave also considered certain conditioning factors of change which have an important effect on the efficient expenditure share (overall public expenditures to GNP ratio) that will result from the mix of the three forces. These factors accounted for demographic change, technological change, social culture and political factors.

Musgrave’s version has been adopted and tested in many studies in the existing literature where most of these studies have generally obtained results supporting the WH. Murthy (1994) tested the Musgrave version for Mexico during the period 1950-1980 and found support for the hypothesis. Lin (1995) tested Musgrave’s version for Mexico during two different periods 1950-80 and 1950-90 and found support for the WH. Islam (2001) tested the Musgrave version for the USA during the period 1929-96 and obtain results that supports the WH. Alleyne (1999) tested Musgrave’s
version for 4 Caribbean countries and obtained results that did not support the WH in those countries.

2.3.6 Man (1980)

Man (1980) tested all earlier interpretations of the WH for Mexico over the period from 1925 to 1976. His results suggested that P&W, Goffman and Mahar, and Gupta’s versions support the WH in Mexico since the elasticity coefficients exceed unity. Opposite results are obtained with the share versions of the WH when compared to Musgrave and Pryor.

Man modified the P&W interpretation into a share version and called it a structural version of the WH. Man interpreted the WH by considering the share of public expenditure in income should increase at a rate higher than the rate of increase in national income. Man’s formulation of the WH translates into the functional relationship:

\[
\frac{GE}{GDP} = f(GDP) \tag{2.11}
\]

where GDP represents national income and the other variables are as defined previously. Man used a log-linear functional form as depicted in equation (2.12) to test his general relationship for Mexican data:

\[
\ln\left(\frac{G}{GDP}\right) = a + b\ln\left(GDP\right) \tag{2.12}
\]

This form is different in that it measures fiscal government relatively in the form of a share as did Pryor and Musgrave but, unlike those authors, Man relates this share to the level of GDP rather than GDP per capita. Man’s results suggested that the WH is supported only between the proportional levels of spending in the overall public sector and the changing industrial and demographic structure in terms of urbanization of Mexico.

2.3.7 Florio and Colautti (2003)

A more recent analysis of the WH has been offered by Florio and Colautti (2003). They tested the secular growth of public expenditures for the US, the UK, France, Germany, and Italy during the period from 1870 to 1990. They interpret the WH by considering that, as there is economic development in the economy, the social demand for public goods will increase at a rate higher than the rate of increase in income. Florio and Colautti rejected the WH arguing that it ignores the social cost of financing government expenditure by taxation and disregards the role of ever increasing distortionary taxation introduced by Pigou (1947). Pigou’s tax effect theorem proposes that the excess burden of taxation constrains the growth of public expenditure. Florio and Colautti combined both the WH and the ‘Pigou’s effect’ and suggested that there is a social demand for public expenditure which increases with per capita income to a certain limit where the marginal social cost of public provision equals its marginal social benefit.

To analyze the Florio and Colautti framework the section now moves on to present the ‘Pigou’s effect’ theorem. Pigou (1947) noted that:
“The raising of [an] additional £ of revenue necessitates increasing the rates at which taxation is imposed ... with some sort of taxes this inflicts indirect damage on the taxpayers as a body over and above the loss they suffer in actual money payment. Where there is indirect damage, it ought to be added to the direct loss of satisfaction involved in the withdrawal of the marginal unit of resources by taxation, before this is balanced against the satisfaction yielded by the marginal expenditure. It follows that, in general, expenditure ought not to be carried so far as to make the real yield of the last unit of resources expended by the government equal to the real yield of the last unit left in the hands of the representative citizen (Pigou, 1947, pp.33-34).”

Pigou argued that the total welfare costs induced by the provision of a public good are higher than its production costs, as government expenditure has to be financed by distortionary taxation. Following Pigou, Florio and Colautti assumed that there is only distortionary taxation available to government to finance public expenditures and that the excess burden of taxation acts as a brake to the supply of publicly provided goods. In other words, Florio and Colautti argued that the optimal supply of a public good is up to the point where the marginal social cost of provision equals its marginal social benefit.

As a starting point, there is no consensus in the public economic literature regarding the effect of the Pigouvian taxation theorem on the optimal supply of public goods. Since Pigou presented his distortionary taxation principle, many studies have been carried out to test his principle on different aspects of the public economy. This
thesis has reviewed some of the major studies that investigated Pigou’s effect on the optimal supply of public goods. However, no attempt will be made here to discuss in detail the contribution of each of these studies, and even less to examine them. A considerable number of these studies support the case where government might supply more public goods even with distortionary taxation. It is argued that even when there is distortionary taxation, it is often offset by the distributional effect. A recent study by Lindert (2004) investigated Pigou’s effect on the supply of public goods and suggested that it is not supported by historical evidence. Lindert suggested that a higher tax to finance social spending does not correlate negatively with the level or even the growth rate of GDP per capita. Lindert supported his suggestion with statistical tests that show near zero estimates of the net damage from social programs on economic growth. Lindert analysed the case of the Swedish welfare state since 1950. He found that Sweden’s tax policies have not harmed growth relative to the policies followed in other OECD countries (Lindert, 2004, p.295)

This thesis argues that it was unnecessary for Florio and Colautti to import a restriction to the growth of government from outside the demand model by introducing the excess burden theorem of taxation. Florio and Colautti suggested that Wagner did not expect an upper limit for the share of government growth in the economy and they proposed to offer a determination of a virtual upper limit of the growth of government in the economy based on the welfare burden of taxation. Logically, there is a clear upper bound to the government share in the economy because, by definition, a share, measured as a ratio, cannot exceed one. Wagner, in his original writings, did recognize that there must be an upper limit to the growth of

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14 To view other major studies on the Pigouvian theorem, see Appendix A
government as a share in the economy and that it comes from the resistance of taxpayers to a further provision of public goods so that government cannot grow forever as was established in section 2.2.3. He recognized that there would always be a demand for goods and services provided by the private sector. That is in the Wagner economy there is always a mixture of the private and public sectors, so that the public share of the economy is limited to some proportion less than one. So whilst Wagner did not necessarily provide a restriction on the relative share of government in the economy through some taxation constraint, he saw much more fundamental demand and supply side forces determining such a restriction. However, Wagner suggested that there is no need to specify this limit of the share of government in the economy exogenously and any attempt to ever do so will fail. Following Wagner’s writings, this thesis suggests that the limits to the share of government in the economy could be determined from factors on the demand side of Wagner’s model.

However, compared with other models, Florio and Colautti have developed a more robust empirical model of government share. Florio and Colautti represented their interpretation in a non-linear functional form captured by the logistic function\textsuperscript{15}. They suggested that the logistic process of the growth of government gives a better fit for the observed data when government is measured in share form. The logistic process of government growth is able to specify stages to the growth of government in the economy and locate a limit for that growth from the data employed. There are some issues with the way that Florio and Colautti apply this model empirically,

\textsuperscript{15} Verhulst was the first to introduce the Logistic function in 1845 to model population growth; see the methodology chapter.
particularly substituting time for GDP per capita as the conditioning variable. This will be returned to in chapter 3.

2.4 Concluding Remarks

The WH proposed that there is a tendency for an expansion of public sector activity relative to the economy along with the economic and cultural progress of the economy. Wagner expected the increase in public sector activity relative to the economy to be greater than the increase in the progress of the economy. Even though Wagner’s statement seems simple, scholars for over a century have debated the appropriate interpretation of the WH. This thesis argues that the way the WH has been interpreted in the existing literature has been incomplete, both on theoretical and empirical grounds, as a result of either the limitations in the translation of his writings or the use of inappropriate tools to test for the WH. This section intends to provide a recasting of the WH that will be adopted and tested in this study.

The translations of Wagner’s original writings indicated the following key elements of the WH:

1. Wagner observed in the late nineteenth century that economic development in industrializing countries has been accompanied by growth in the role and the size of public activity. Those countries from which Wagner drew his observations were in the process of transforming their economies from rural agricultural to urban industrial. This stage of development is described as the beginning of an industrial economy.

2. Wagner tends to measure this growth in fiscal aspects of the state represented by government expenditure. This thesis argues that measuring public activity
in fiscal terms of the state approximated by government expenditure represents only one aspect of state activity. Wagner also recognized ‘regulatory activity’ as another aspect of state activity. However, Wagner did not include the regulatory activity in the measurement of the state in his proposition and kept the measurement of the state as a fiscal entity. Thus, section 2.2.4.2 of this chapter has contributed to the literature of the WH by re-establishing the regulatory aspect of state activity in the WH.

3. Even though the effect of the regulatory activity of the state on the growth of aggregate state activity is recognized in this thesis, there are many problems in moving to a robust test of state regulatory activity. The tests of the WH in this study will proceed to measure state activity in its fiscal aspect and no attempt will be made to include the regulatory aspects in the measurement of the state for the data employed in the study mainly because of the difficulty of measuring regulatory activity for the countries included. Thus the results will be interpreted in fiscal terms of the state. On the right hand side of Wagner’s relationship, Wagner tends to measure economic development by the growth in income. Using per capita national income figures captures demographic changes.

4. Furthermore, the translations of Wagner’s writings indicated that there is considerable support for the views that Wagner intended to measure the rise of public expenditure activity as proportional to income rather than the rise in absolute levels of public expenditure. Wagner’s proportional increase relies mainly on his organic view of the state. Wagner views the public economy, represented by public activity, as an organic part of the social
system which will tend to grow, proportionally, with the growth of the whole economy.

5. This thesis suggests that the size of public expenditure relative to the economy cannot grow forever or more than the economy as a whole. It will be recalled from section 2.2.1, that Wagner proposed a smooth organic growth of public expenditure in the economy. Wagner refers to a market economy where state expenditure expands organically with the national economy. A market economy consists of both public and private sectors. Thus, Wagner recognized that government cannot grow forever and must have limits. Having established this point, this thesis will model the WH in a way that allows for a limit to the share of government expenditure in the economy and this limit will be determined endogenously, from inside the demand model of the WH, by allowing existing data to determine this limit.

The WH proposes a monotonic increase in the ratio of public expenditures to national income. However, this growth is not infinite and should have an upper limit in the long run. A more comprehensive interpretation of the WH should be based on proportionate measures of public expenditure and the development of the economy, taking population growth into consideration. On this basis, the Musgrave (1969) version seems to fit this criterion. Musgrave interprets the WH in a way that proposes that the ratio of government expenditure in income will grow at a rate higher than that of per capita income. The share version of the WH is less likely to suffer from the endogeneity problem noted earlier in this chapter. An endogeneity problem occurs if the Keynesian and Wagner interpretations of the relationship between government and aggregate income are simultaneously correct. That is if government
expenditure causes income and income causes government expenditure then there is a simultaneous determination of both variables in the relationship. This will not occur with the proportionate measures of fiscal government in the economy. The statistical properties of the share version of the WH will be discussed in some depth in chapter 4.

The recasting of the WH in this thesis proposes that with the development process in the economy represented mainly by the growth in per capita income, the fiscal state represented by the share of government expenditures in national income will increase at a rate higher than that of per capita income. However, this rate of increase will fall until it is at the same rate of expansion as per capita income, so that the share of government expenditure reaches a maximum. Furthermore, this saturation point is determined endogenously from demand side forces. To capture this process of rising government share in the economy, the thesis introduces a non-linear specification which allows the data to determine the asymptote of government expenditure to GDP.

The importance of this model is twofold. First, at low levels of GDP per capita it allows for economies where government expenditure increases over time at a rate greater than GDP per capita so that, not only does government share increase, it also increases at a rate greater than GDP. At higher levels of GDP the ratio of government expenditure to GDP may continue to increase, but now, even though this must mean that government expenditure is still growing at a rate greater than GDP per capita, that excess is lower. Moreover, the excess of growth of government expenditure over GDP per capita growth eventually slows until both growth rates are equalized and the
long run share of government expenditure of GDP remains unchanged. That is, in terms of the marginal effect of GDP per capita on the share of government expenditure in GDP, there may be three stages\textsuperscript{16}:

- At low levels of GDP/P, the marginal effect may be positive but growing; that is:

\[
\frac{d (GE/GDP)}{d (GDP/P)} > 0 ; \quad \frac{d^2 (GE/GDP)}{d (GDP/P)^2} > 0 \quad (2.13)
\]

- At higher levels of GDP/P, the marginal effect may be positive but declining; that is:

\[
\frac{d (GE/GDP)}{d (GDP/P)} > 0 ; \quad \frac{d^2 (GE/GDP)}{d (GDP/P)^2} < 0 \quad (2.14)
\]

- At some higher levels of GDP/P, the marginal effect is zero; that is:

\[
\frac{d (GE/GDP)}{d (GDP/P)} = 0 \quad (2.15)
\]

It turns out that empirically, relations (2.14) and (2.15) exist, but the relationship in (2.13) appears not to. Secondly the standard logistic model of, say, Florio and Colautti is naturally nested in the general model that this thesis proposes and it can, therefore, be used to test for the specification adopted by Florio and Colautti.

\textsuperscript{16} For more details see Appendix C.
2.5 Summary

This chapter has contributed to the existing literature on the WH in two ways. Firstly, it has provided a consolidation of the WH based on a comprehensive review of Wagner’s work. Such a review has been lacking in extant literature. Secondly, it has reviewed the existing interpretations of the WH and pointed out the key elements of differences in those interpretations.

Section 2.2 provided a comprehensive review of the WH using original translations of Wagner’s writings and, in so doing, established the critical key elements of the WH consistent as possible with his writings. Wagner’s original statement suggests that the economic development in industrializing countries such as Germany, Britain and the United States in the late nineteenth century was accompanied by a growing role of the state in society. Wagner based his hypothesis on several underlying assumptions, an organic view of the state being one of them. Wagner’s organic view considers the public economy represented by public activity as an organic part of the social system which will tend to grow, proportionally, with the growth of the whole economy. For Wagner, the growing role of the state in the late nineteenth century was consistent with his organic view of the state, a view he shared with a number of other German intellectuals of the time.

Wagner identified three main forces that lead to government involvement in an industrializing economy. First, the demand for the enforcement of law and order internally and externally increases as economies grow. Second, the demand for culture and welfare services such as health, education and other services increases government involvement as collective producers. Third, the participation of public
ownership in material production increases because of new technical processes which often resulted in natural monopolies.

Even though Wagner finds three forces that will drive the expansion of the state in the economy, he explicitly recognized that the economy is a market economy in which private institutions are still active and provide goods and services. Wagner applied his hypothesis to mixed market economies where some activities are provided in the private sector and other activities are provided in the public sector. Both sectors, according to Wagner, function in a complementary relationship. Wagner, in his original writings, recognized that there is a limit to government growth. He recognized that the size of state activity in the economy would not grow without limit or more than the whole economy. Therefore, Wagner recognized that there is a maximum to government growth that, at the least, will result from the taxpayer’s resistance. Wagner did not attempt to specify this limit \textit{a priori} and suggested that any attempt to set a limit to state expansion exogenously will fail.

Wagner tends to measure the state as a fiscal activity represented by state expenditure. However, he recognized the effect of regulation as another type of state activity but did not include it in the measurement of the state activity mainly because of the difficulty of measuring regulatory activity. He did not specify the functional form of the relationship between the two economic variables. However, a sensible functional form representing the WH would need to be able to express a limit for the growth of the share of government expenditure in a market economy in the form of a maximum share of government in the economy.
Section 2.3 reviewed the different interpretations of the WH that have been produced in the existing literature. All of these interpretations have related the growth in public expenditure to economic development which was seen to determine that expenditure. All existing interpretations of the WH have measured the state as a fiscal entity and they have not considered the regulatory aspects of the state in their analysis. The existing interpretations of the WH have not recognized the limits of government growth in the WH with the very notable exception of Florio and Colautti (2003). However, it could be argued here that Florio and Colautti have imposed a restriction to government growth from outside the demand model of the WH in the form of taxpayer resistance, and that a careful examination of the WH leads one to conclude that Wagner envisaged an ‘organic’ limit to government offered by several forces in the economy. The recasting of the WH in this thesis proposes that, with the development process in the economy represented mainly by the growth in per capita income, the fiscal state represented by the share of government expenditures in national income will increase at a rate higher than that of per capita income. However, this rate will fall until it is at the same rate of expansion as per capita income, so that the share of government expenditure reaches a maximum. Furthermore, this saturation point is determined endogenously from demand side forces.

Finally, all the works that have investigated Wagner and have been reviewed in this chapter undertook empirical tests of the WH. Some reference in this review has been made to the nature of these empirical tests and the implications they have for the nature and forms of the empirical analysis undertaken in this thesis. However, no deep analysis of the results of those tests has been undertaken. Such an analysis
would reveal important information useful in formulating the empirical analysis of this thesis. This task will be taken up in the next chapter, chapter 3. There, additional empirical works will be analyzed along with those already reviewed. These works will be classified in a range of ways to reveal important insights about the relevant testing procedures for the WH. The categories include issues such as the nature of the data, the functional form used, the different variables and their definitions used, and the relationship/s between their outcomes and other dimensions. It is hoped that this will add further insights to the empirical model which will be used to test the WH in subsequent chapters of this thesis.
CHAPTER THREE

3 Chapter 3: Empirical Literature on the Wagner Hypothesis

3.1 Introduction

Despite the large volume of literature that has investigated the WH and several literature surveys, a comprehensive review of that literature does not exist. This absence has motivated the thesis in two dimensions. Firstly, an object of the thesis is to trace Wagner’s original writings through translations and the development of the different interpretations of the WH stemming from those translations that are now in the existing English language literature. The second dimension is the examination of the developments in the econometric analysis which has had an impact on the different interpretations of the WH.

The first dimension was dealt with in the previous chapter where the development of the WH was traced using translations of Wagner’s original writings. It was found that recent discussion about Wagner’s original statement, including a new translation, helped to establish a more comprehensive understanding of the WH. The previous chapter examined the development of the different interpretations of the WH and showed how each interpretation has contributed to the WH. The chapter suggested a possible recasting of the WH that will be tested in this thesis. This recasting of the WH suggests that, with the development process represented mainly by per capita income, the fiscal state, represented by the share of government expenditures in national income, will increase at a higher rate than that of the increase in per capita income until it reaches a certain limit. This limit is however not
pre-determined, but is a function of the balance of forces driving government involvement in a market economy. Consequently, in the model of the WH tested here, that limit will be determined from the data inside the model. Empirical tests of such a model should therefore seek to establish two things. First, it should test for a significant positive relationship between the fiscal share of GDP and the development of the economy in terms of per capita GDP. The model should also test for a significant sensible limit to the fiscal share in the economy.

The second dimension, an examination of the different econometric analyses of the WH, is the objective of this chapter. Previous research on the empirical testing of the WH is extensive. This thesis recognizes the contribution of these empirical studies and will build upon them in the development of an empirical model with which to test the WH. These empirical studies have contributed to the WH mainly by applying more modern techniques to test for Wagner’s relationship. However, several issues were raised in the previous chapter which should be borne in mind when undertaking a review of the empirical literature. In the previous chapter the thesis has established Wagner’s recognition of the regulatory aspects of the state as well as the fiscal aspects of the state. However, all the existing econometric analyses of the WH included in this chapter have been directed at examining the fiscal aspects of the state. While there are daunting problems in testing any thesis on the regulatory aspects of the state, it should nevertheless, be recognized that, because of this absence, existing analyses of the WH should be considered as partial. To reiterate, without a simple and robust numerical measure which encapsulates the depth of the regulatory state, all empirical analyses are naturally confined to the fiscal state.
Unfortunately the empirical analysis of this thesis is also constrained by this limitation.

The previous chapter established that Wagner recognized that government cannot grow without a limit. That is, the share of government expenditure cannot grow forever. However, most of the existing empirical studies have not included this limit in their empirical models. Those that have attempted to test for it tend to impose an exogenous limit from outside the demand model of the WH. Finally, Wagner did not specify the functional form of the relationship between the two economic variables. However, the existing empirical studies have mostly specified a linear or log-linear form of Wagner’s relationship. There is solid ground to conclude that a linear relationship of government expenditure against GDP is not an appropriate form for testing the WH, since it allows the government share of GDP to grow to some limiting share greater than one or decline towards zero. Similarly the log-linear form of government expenditure against GDP is not appropriate because it permits either government share to decrease to zero or grow without limit. These relationships are examined in some depth in Appendix B. Put simply then, sensible limits to government share in the economy require careful selection of appropriate empirical functional forms before moving on to testing.

This chapter has found at least 53 empirical studies of the WH that have been carried out since 1960 in the English language. These existing empirical studies have been notable in employing different models of the WH. This chapter aims at identifying some of the key elements that might affect the formulation of the employed models in the existing empirical studies. These elements can be classified into the type of
economies, time-span, measurement of government size, measurement of explanatory variables, models of the WH, and testing procedures. Several tables, categorizing the different studies of the WH, have been developed in this chapter to help in the review of the existing empirical studies within these elements.

The remainder of the chapter is structured as follows: Section 3.2 distinguishes between the different types of econometric analyses followed in the existing studies of the WH. These types of analyses have varied between time-series and cross-section. Section 3.3 reviews the type of economy included in the existing studies and discusses whether it may have affected the modeling of the WH. Section 3.4 reviews the effect of the data time span on the testing of the WH in the existing studies. Section 3.5 reviews the measures of government size in the existing studies of the WH and discusses to what extent it may have affected the results of testing the WH. Section 3.6 reviews the measures of explanatory variables in the existing studies of the WH and discusses whether they have affected the modeling of the WH. Section 3.7 discusses the effect of prices on international comparisons of income in cross-country studies of the WH. Section 3.8 reviews functional forms that have been employed in testing the WH in the existing studies and discusses the limitations of those functional forms. Section 3.9 reviews the different econometric procedures followed in time-series analyses of the WH to date. Further, it distinguishes between studies that used ordinary least squares regression and studies that used techniques in modern time-series econometrics. Section 3.10 introduces some important non-linear models of the WH in the existing studies. Section 3.11 discusses the rationale for preferring time-series to cross-section analysis in tests of the WH. Further, this section intends to establish the rationale for using pooled cross-section, short time-
series data in testing the WH. Finally Section 3.12 summarises the chapter and introduces the methodology that will be followed in testing of the WH in the next chapter.

3.2 Type of Analysis of the Wagner Hypothesis

The existing studies of the WH have followed two main types of analyses, time-series and cross-section pooled over several years. To recapitulate, the WH states that the expansion of the public fiscal economy will respond positively to changes in economic development. Therefore, time-series analysis examines the effect of the growth in income on the growth in government expenditure over time for a particular economy. The alternative to time-series, cross-sectional analysis, which looks at the relationship across different economies at the same point in time, might not be appropriate. Whilst testing a temporal growth pattern, using cross-sectional studies which do not involve convergence to same rate, will not necessarily lead to “Galton’s Fallacy” Quah (1993), Bliss (1999, 2000), and Cannon and Duck (2000), it is still questionable. Even though the cross-section sample may include low-income economies, and the estimated relationship between government expenditure and GDP is positive, it does not necessarily mean that those economies will grow in such a way over time. However, “pooling” the cross-sectional data over several years, particularly when those involve disimilar countries, introduces a time-series element. This makes it possible to represent a country, not at a single point in time, but through several points in time, in the development of government expenditure with GDP. This permits the empirical analysis to better reveal the dynamic process involved. In this chapter and subsequent chapters, the term cross-section will be treated synonymously with pooled cross-section over short time-series.
Testing for the WH in existing empirical studies has generally produced mixed results and no clear pattern can be found. However, for the purposes of this analysis, this section classifies these studies into three groups. The first group, the so called supporting studies, have results that suggest a tendency for government expenditure to increase along with economic development, which is consistent with the WH. The second group of studies, those that do not support the WH, have results that mainly suggest a tendency for government expenditure to either decline with economic development, or to find no relationship between government expenditure and economic development both of which contradict the WH. Finally, the third group, those that find mixed results, have estimated more than one empirical model and have found the model outcomes asymmetric. That is, the outcome of one estimate does not contradict the WH, whilst another does. These studies usually involve time-series models of different economies, but can also involve models using different measures and definitions of variables. Mixed results are, therefore, positive relationships between government expenditure and GDP for some economies and negative relationships for others, or positive relationships between government expenditure and GDP for some classifications of expenditures and negative relationships for other classifications. The three groups of studies are listed in Table 3-1. The table is constructed based on the following criteria: column one (author); column two (data type); and column three (country).

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1 These tables and the other tables included in this chapter are intended to illustrate the diversity of the studies which have been carried out in the context of testing for the WH.
Table 3-1: Existing Studies of the Wagner Hypothesis

<table>
<thead>
<tr>
<th>No.</th>
<th>Author</th>
<th>Type of analysis</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Found support</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Thorn (1972)</td>
<td>Cross-section</td>
<td>52 countries 1952-1962&lt;sup&gt;3&lt;/sup&gt;</td>
</tr>
<tr>
<td>3</td>
<td>Vatter and Walker (1986)</td>
<td>Time-series</td>
<td>USA 1929-1979</td>
</tr>
<tr>
<td>7</td>
<td>Gyles (1991)</td>
<td>Time-series</td>
<td>UK 1946-1985</td>
</tr>
<tr>
<td>11</td>
<td>Oxley (1994)</td>
<td>Time-series</td>
<td>Britain 1870-1913</td>
</tr>
<tr>
<td>17</td>
<td>Thornton (1999)</td>
<td>Time-series</td>
<td>6 developed countries&lt;sup&gt;5&lt;/sup&gt; (1850-1913)</td>
</tr>
<tr>
<td><strong>Do not support</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Lall (1969)</td>
<td>Cross-section</td>
<td>46 developing countries 1962-1964</td>
</tr>
<tr>
<td>23</td>
<td>Goffman and Mahar (1971)</td>
<td>Time-series</td>
<td>6 nations&lt;sup&gt;7&lt;/sup&gt; 1940-1965</td>
</tr>
<tr>
<td>24</td>
<td>Wagner and Weber (1977)</td>
<td>Time-series</td>
<td>34 countries 1950-1972</td>
</tr>
</tbody>
</table>

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<sup>2</sup> Originally published in 1967  
<sup>3</sup> The countries are grouped according to their per capita income.  
<sup>4</sup> Canada, France, Japan, UK, USA and West Germany  
<sup>5</sup> Denmark, Germany, Italy, Norway, Sweden and the UK  
<sup>6</sup> GCC refers to Gulf cooperation council countries. These are Saudi Arabia, the United Arab Emirates (UAE), Kuwait, Oman, Bahrain, and Qatar.  
<sup>7</sup> Six Caribbean countries, Haiti, the Dominican Republic, Costa Rica, Panama, Honduras and Guyana
Table 3-1: Continued: Existing Studies of the Wagner Hypothesis

<table>
<thead>
<tr>
<th>No.</th>
<th>Author</th>
<th>Type of analysis</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>Ferris and West (1996)</td>
<td>Time-series</td>
<td>USA 1959-1989</td>
</tr>
<tr>
<td>33</td>
<td>Alleyne (1999)</td>
<td>Time-series</td>
<td>4 Caribbean countries</td>
</tr>
</tbody>
</table>

**Found mixed results**

<table>
<thead>
<tr>
<th>No.</th>
<th>Author</th>
<th>Type of analysis</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>Man (1980)</td>
<td>Time-series</td>
<td>Mexico 1913-1958</td>
</tr>
<tr>
<td>43</td>
<td>Bohl (1996)</td>
<td>Time-series</td>
<td>G7 countries</td>
</tr>
<tr>
<td>45</td>
<td>Ansari, et al. (1997)</td>
<td>Time-series</td>
<td>3 African countries</td>
</tr>
<tr>
<td>50</td>
<td>Chang (2002)</td>
<td>Time-series</td>
<td>6 countries</td>
</tr>
<tr>
<td>51</td>
<td>Chang, et al. (2004)</td>
<td>Time-series</td>
<td>10 countries</td>
</tr>
<tr>
<td>53</td>
<td>Iyare and Lorde (2004)</td>
<td>Time-series</td>
<td>9 countries</td>
</tr>
</tbody>
</table>

This chapter identified 45 empirical studies in Table 3-1 that have used time-series data to test for the WH. Time-series studies take up approximately 85% of the

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9 Those countries grouped into poor, developing and developed countries.
10 Those countries classified into less developed (LDCs), developed, central/south American LDCs, African LDCs, and Asian LDCs.
11 Australia, Colombia, Germany, Malaysia, Pakistan, Sri Lanka, Thailand, Canada, India, Peru, Sweden, Switzerland, the UK, the USA, Venezuela, Chile, Finland, Greece, Honduras, Italy, and Japan.
14 Three of them are emerging industrialized countries of Asia: South Korea (1954-1996), Taiwan (1951-1996), and Thailand (1951-1995), and three industrialized countries: Japan (1952-1995), USA (1951-1996) and the UK (1951-1996).
empirical studies of the WH found thus far. These studies have in turn followed two broad methods for testing the WH. A large number of these studies have tested the WH in a single country whilst other time-series studies have tested the WH for a group of countries. However, testing the WH using the second method has been done on a time-series basis for each country individually. That is, the data has comprised long time-series with a relatively narrow number of countries and no attempt has been made to pool the data. It is notable from Table 3-1 that time-series studies have been applied to all types of economies across income scales, but mostly testing has been done for developed or industrial countries.

The remaining 15% of the empirical studies of the WH are taken up with pooled, cross-sectional studies. This comprises 8 papers in Table 3-1. In comparison to time-series of multiple countries, the number of cross-sectional units (countries) in the data is large, relative to the length of the time-series in these cross-sectional studies. This constitutes a short wide pooled data set. There is much greater benefit from pooling the data when the many countries included in the sample reflect a wide range of GDP or GDP per capita. Thus, each small time-series for a country forms a small ‘slice’ of the wide distribution of the time-series of GDP generally. Modern time-series methods are restricted to linear models. The pooling of short time-series over a relatively large number of cross-sectional units retains some dynamic characteristics but opens up the analysis to non-linear models normally associated with pure cross-sectional data. Despite these clear advantages, pooled cross-sectional studies of the WH are rare in comparison to time-series, perhaps because the availability of internationally price comparable data is rare. It is the intention of this thesis to use
pooled data to obtain the advantages of cross-sectional modeling using non-linear functional forms.

Whilst outcomes of the empirical studies in aggregate are asymmetric, some significant relationships between the Wagnerian variables have been found in the statistical models of the WH applied to time-series. Thirty nine percent of the empirical studies did not contradict the WH, with 28% producing contradictory results and the remainder producing results which were mixed. However, it must be borne in mind that the sample of cross-sectional studies is small. Such results can easily occur in time-series where more than one country is analyzed and individual models are estimated for different countries. However, mixed results can occur for individual countries where different models are estimated for either different definitions or different categories of government expenditure. For the cross-sectional studies, 38% of the results did not contradict the WH, 12% produced contradictions to the WH, with the remainder being mixed.

The existing empirical studies have been notable in employing different models of the WH. Those models have been affected mainly by the different interpretations of the WH. This chapter will proceed to present the main methodological issues that have affected the formulation of the models used for testing the WH in the existing studies. These issues can be classified into: type of economy and time span, measures of government size, measures of economic development, and actual or real values of the variables. These issues will be discussed in the following sections of this chapter. This will be followed by two further sections on the functional form of the empirical models and the nature of the estimation method used. The implications of these
issues for the choice of the empirical model in the thesis will be discussed in the penultimate section.

3.3 Type of Economy

Existing studies of the WH have differed in terms of which countries were included to test for the WH, namely developed, developing or a mix of both countries. Table 3-2 classifies time-series studies according to their author, country and time-span, and major result found. The table shows two main types of the existing time-series studies of the WH: studies that tested the WH in a single country, and studies that tested the WH in a group of countries. However, both types apply time-series analysis on each country individually. Furthermore, the table lists the outcome of both types in terms of being consistent with Wagner as ‘support’, being inconsistent with Wagner as ‘no support’ and producing asymmetric results as ‘mixed results’.

Table 3-2 shows that the existing time-series studies have varied between developed and developing countries, with developed countries predominating. No attempt has been made to distinguish outcomes between differences in countries regionally, culturally, or politically as such differentiation can be value laden. Over 70% of time-series studies have tested the WH in developed or industrial countries. The results here are mixed. For instance, while Ahsan, et al. (1996) tested the WH in Canada during the period from 1950 to 1988 and obtained support for the WH, Legrenzi, G. and Milas (2002) tested the WH in Italy during the period from 1959 to 1996 and found results that do not support the WH. Biswal, et al. (1999) tested the WH in Canada during the period from 1950 to 1995 and obtained mixed results for the WH using different definitions.
Fewer time-series studies have tested the WH in individual developing countries and, those that have, have obtained either no support or mixed results for the WH. For instance Abdel-Rahman and Barry (1997) tested the WH for the kingdom of Saudi Arabia during the period from 1970 to 1991 and obtained mixed results. Asseery, et al. (1999) examined the WH in Iraq for the period from 1950 to 1980. The empirical results contradicted the WH in Iraq. Burney (2002) tested the WH in Kuwait during the period from 1969 to 1994 and the results showed that there is little support for the WH over this period.

The limited time-series data for less developed countries over long time spans is one possible reason for testing the WH for developed countries more predominatly than less developed countries in the existing time-series studies. This could, in turn, be due to two (possibly related) reasons. Firstly, a lack of resources has often resulted in a low priority being given to data collection. And, secondly, these countries might have not been rich enough in the past to sustain a robust long series. This has motivated some scholars to test the WH using cross-sectional data. Pooling the data over short time-series for different countries has increased the number of observations in cross-sectional analysis. Table 3-3 classifies the existing pooled cross-sectional studies of the WH according to their author, country and time-span, and major result found.
### Table 3-2: Time-series Studies of the Wagner Hypothesis

<table>
<thead>
<tr>
<th>No.</th>
<th>Author</th>
<th>Country</th>
<th>Major result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Vatter and Walker (1986)</td>
<td>USA 1929-1979</td>
<td>Support</td>
</tr>
<tr>
<td>5</td>
<td>Oxley (1994)</td>
<td>Britain 1870-1913</td>
<td>Support</td>
</tr>
<tr>
<td>13</td>
<td>Henrekson (1993)</td>
<td>Sweden 1861-1990</td>
<td>No support</td>
</tr>
<tr>
<td>16</td>
<td>Ferris and West (1996)</td>
<td>USA 1959-1989</td>
<td>No support</td>
</tr>
<tr>
<td>22</td>
<td>Man (1980)</td>
<td>Mexico 1913-1958</td>
<td>Mixed results</td>
</tr>
<tr>
<td>26</td>
<td>Hayo (1994)</td>
<td>Mexico 1950-1980</td>
<td>No support</td>
</tr>
<tr>
<td>30</td>
<td>Thornton (1999)</td>
<td>6 developed countries (1850-1913)</td>
<td>Support</td>
</tr>
<tr>
<td>34</td>
<td>Bairam (1992)</td>
<td>OECD countries 1950-1985</td>
<td>Mixed results</td>
</tr>
<tr>
<td>43</td>
<td>Goffman and Mahar (1971)</td>
<td>6 nations 1940-1965</td>
<td>No support</td>
</tr>
<tr>
<td>44</td>
<td>Wagner and Weber (1977)</td>
<td>34 countries 1950-1972</td>
<td>No support</td>
</tr>
<tr>
<td>46</td>
<td>Alleyne (1999)</td>
<td>4 Caribbean countries 1950-1991</td>
<td>No support</td>
</tr>
</tbody>
</table>

---

Table 3-3: Pooled Cross-sectional Studies of the Wagner Hypothesis

<table>
<thead>
<tr>
<th>No.</th>
<th>Author</th>
<th>Countries included</th>
<th>Major result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Lall (1969)</td>
<td>46 developing countries 1962-1964</td>
<td>No support</td>
</tr>
<tr>
<td>2</td>
<td>Thorn (1972)</td>
<td>52 countries 1952-1962</td>
<td>Support</td>
</tr>
<tr>
<td>5</td>
<td>Ram (1987)</td>
<td>115 countries 1950-1980</td>
<td>Mixed results</td>
</tr>
</tbody>
</table>

Table (3-3) clearly shows that these studies cover a much wider range of countries and include a wide mixture of countries at very different stages of development. An exception to this is Michas (1974) who tested the WH using data from the provincial and municipal level in Canada during the period from 1950 to 1961. Even though the time-series are of short span, there is a significant difference between the relative incomes of the Canadian provinces and municipalities to ensure an adequate test of the hypothesis. Michas’ results did not contradict the hypothesis of the positive relationship between government expenditure and economic development. Wahab (2004), for instance, tested the WH using data for the OECD countries during the period from 1950 to 2000 and obtained mixed results. However, one can note that this is a long time-series pooled across few countries. That is, it is a ‘long, narrow’ pooled sample.

It should be noted that other studies in Table 3-3 have shorter time-series (especially when compared to Table 3-2) and involve a larger number of countries. That is, they are a ‘short wide’ pooled data set and are better suited to pooled regression methods. Other studies have tested the WH for a group of developing countries. Lall (1969), for instance, tested for 46 developing countries during the period from 1962 to 1964 and found no support for the WH. Again, one might argue that there was not a
sufficiently large enough variation in national income and this could have determined these results. Dao (1995) on the other hand tested the WH for 55 middle-income countries during the period from 1980 to 1991 and obtained results that support the WH. Alternatively, some pooled cross-sectional studies tested the WH for a mix of developed and developing economies. Thorn (1972), for instance, tested the WH for 52 developed and developing countries during the period from 1952 to 1962 and obtained results that support the WH. Abizadeh, S. and Gray (1985) tested data for 53 countries grouped into poor, developing and developed countries during the period from 1963 to 1979 and obtained mixed results for the WH. Ram (1987) tested the WH for 115 countries during the period from 1950 to 1980 and obtained mixed results for the WH. Koop and Poirier (1995) tested the WH for 86 developed and developing countries during the period from 1960 to 1981 and obtained mixed results. These results suggest that one cannot say that the WH applies only to certain types of economies, such as the industrialising economies, and not to others, such as developing economies. However, it does call into question the wisdom of limiting cross-sectional studies to economies of certain incomes which can only reduce the variation in sampled incomes. The following section discusses whether the time span selected in the existing studies affected the results of testing the WH.

3.4 Time Span

It is notable from Table 3-2 and Table 3-3 that the time span in both time-series and pooled cross-sectional studies of the WH has varied between the late nineteenth century and the present time. However, over 90% of the existing empirical studies have tested the WH for data after World War II. Further, the results of these studies are asymmetric with some studies supporting the WH, but others not finding a
significant relationship between government expenditure and growth. Some scholars have suggested that the WH might be more applicable to countries in the early stages of their development, as was originally observed by Wagner when he applied his hypothesis to countries transforming their economies from rural agricultural to urban industrial in the middle-to-late nineteenth century. The rationale behind this suggestion is that testing the WH for industrial economies at the early stages of their development (from the late nineteenth to the early twentieth century) might give results that support the WH, whilst testing those same economies at a later stage of their development (late twentieth century) might give results with less support for the WH. Bird (1971), for instance, argued that the WH does not cover most of the countries included in cross-sectional analyses because most of the developed countries included are well past the stage of development which is thought to be applicable to the WH. That is, he argued, if the cross-section does not include a sufficient number of developing countries, the analysis will fail due to the lack of variation in the data. Thus, industrial economies at Wagner’s time could be described as post-industrial countries in the late twentieth century. The important idea here is that the WH is expected to be more applicable in newly industrializing societies, which experience rapidly increasing rates of economic growth and have similar conditions to those countries observed by Wagner (urbanization and modernization).

Some of the existing studies listed in Table 3-1 have tested the WH with data that fitted the time span postulated by Wagner in the late nineteenth century and early twentieth century. Oxley (1994), for instance, tested the WH in Britain during the period from 1870-1913 and obtained results which did not contradict the WH. Thornton (1999) tested the WH in six developed countries (Denmark, Germany,
Italy, Norway, Sweden and the UK) during the period from 1850 to 1913. Thornton’s results indicated that there is considerable support for the WH for European countries in the nineteenth century. On the other hand, some studies, also with long time spans, have obtained results contradicting the WH; see, for example, Henrekson (1993) for Sweden and Bohl (1996) for Germany and Britain. Again, one might argue that all of the studies that tested the WH for long time spans were restricted to developed countries. One main reason is related to the limited availability and accuracy of data that would cover earlier time periods before World War II.

Some studies suggest that the relationship between the two economic variables, public expenditure growth and economic development, may weaken with higher levels of industrialization and development. However, this thesis expects that the WH in post-industrial countries would still apply, albeit in a different way to linear models. The rationale behind this suggestion is that, even though countries experience a transition to higher levels of GDP, the demand for government services will not decline. However, the demand for government expenditure will be such that the share of GDP in government will not now be growing. That is, government expenditure will increase at the same rate as the economy and, therefore, will not expand at the relative expense of the private sector. Some of the forces that Wagner expected would drive public expenditure in the late nineteenth century might change or weaken when those economies moved into their post industrial stage. For example, public expenditure on infrastructure may appear more clearly in newly industrializing countries and the infrastructure in post industrialized countries may be well established. Further, urbanization that leads to the increase in government expenditure in industrial countries had been largely completed in post industrial
countries at the end of World War II. However, urbanization movements might appear more clearly now in the newly developing countries. There are two clear issues here. Firstly, even though government might not be expanding at such a rapid rate in post industrialized countries, this does not negate Wagner. Government expenditure in absolute terms may still be expanding, but may have reached a limit in relative terms. Also, any empirical study must have a sample which allows reasonable variation in the data. A pooled cross-section with short time-series and with economies of similar incomes might well not have enough variation in the data. A similar argument applies to shorter time-series generally.

This section has not found a consistency in the results of testing for the WH employing different data time spans in these tests. Indeed, Table 3-2 and Table 3-3 suggest that the WH may or may not be supported in empirical models with post industrial countries as well as industrializing countries. This chapter will now proceed to introduce other methodological issues that might contribute to the mixed results of testing the WH in the existing studies. The next section will discuss the effect of the different measures of government size on testing the WH in the existing studies.
3.5 Measurement of Government Size

It could well be that the WH may apply to certain categories of government expenditure. One important issue is the inclusion of transfer payments in measures of government expenditure. Table 3-4 classifies the existing studies of the WH according to their author, measures of government size, and major result found. The table shows two main types of measures of government size: those studies that have measured government on an aggregate level, where all government expenditure data are included; and those studies that followed a different methodology by disaggregating government into different components and tested the WH for each component individually.

In addition, some studies have followed another form of disaggregation by measuring government at its different levels; that is, central or federal, state, and local. Others have attempted to measure government size by other than budgetary means, such as public employment. However, all of the existing studies have measured government in terms of a fiscal measure of the state. That is, no studies have attempted a qualitative measure of the regulatory activities of the state. Table 3-4 lists the outcome of each type of measure in terms of being consistent with Wagner, as ‘support’, being inconsistent with Wagner as ‘no support’, and where multiple analysis is involved, producing asymmetric results as ‘mixed results’. Table 3-5 is constructed to interpret the codes used for the different measures of government size in Table 3-4.
### Table 3-4 Measures of Government Size in Existing Studies of the WH

<table>
<thead>
<tr>
<th>No.</th>
<th>Author</th>
<th>Government measure(s)</th>
<th>Major result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Thorn (1972)</td>
<td>GE/GNP</td>
<td>Support</td>
</tr>
<tr>
<td>2</td>
<td>Michas (1974)</td>
<td>GE/P</td>
<td>Support</td>
</tr>
<tr>
<td>3</td>
<td>Nagarajan and Spears (1990)</td>
<td>GE/GDP</td>
<td>Support</td>
</tr>
<tr>
<td>4</td>
<td>Gyles (1991)</td>
<td>GE/P</td>
<td>Support</td>
</tr>
<tr>
<td>5</td>
<td>Ram (1992)</td>
<td>GE/GDP, GE/GNP</td>
<td>Support</td>
</tr>
<tr>
<td>6</td>
<td>Murthy (1993)</td>
<td>GE/GDP</td>
<td>Support</td>
</tr>
<tr>
<td>7</td>
<td>Murthy (1994)</td>
<td>GE/GDP</td>
<td>Support</td>
</tr>
<tr>
<td>8</td>
<td>Oxley (1994)</td>
<td>GE/GDP, RGE</td>
<td>Support</td>
</tr>
<tr>
<td>9</td>
<td>Lin (1995)</td>
<td>GE/GDP</td>
<td>Support</td>
</tr>
<tr>
<td>10</td>
<td>Ahsan, et al. (1996)</td>
<td>GE/GDP, RGE</td>
<td>Support</td>
</tr>
<tr>
<td>12</td>
<td>Thornton (1999)</td>
<td>RGE</td>
<td>Support</td>
</tr>
<tr>
<td>15</td>
<td>Lal (1969)</td>
<td>GE/GDP</td>
<td>No support</td>
</tr>
<tr>
<td>16</td>
<td>Goffman and Mahar (1971)</td>
<td>GE, RGE</td>
<td>No support</td>
</tr>
<tr>
<td>17</td>
<td>Legrenzi, G. and Milas (2002)</td>
<td>GE, GE/P, GE/GDP, GC</td>
<td>No support</td>
</tr>
<tr>
<td>18</td>
<td>Henriksson (1993)</td>
<td>GVE/GDP</td>
<td>No support</td>
</tr>
<tr>
<td>19</td>
<td>Ashworth (1994)</td>
<td>GE/GDP</td>
<td>No support</td>
</tr>
<tr>
<td>20</td>
<td>Hayo (1994)</td>
<td>GE/GDP</td>
<td>No support</td>
</tr>
<tr>
<td>21</td>
<td>Ferris and West (1996)</td>
<td>RGE/GDP</td>
<td>No support</td>
</tr>
<tr>
<td>22</td>
<td>Alleyne (1999)</td>
<td>GE/GDP</td>
<td>No support</td>
</tr>
<tr>
<td>23</td>
<td>Burney (2002)</td>
<td>GE/GNP</td>
<td>No support</td>
</tr>
<tr>
<td>24</td>
<td>Halicioglu (2003)</td>
<td>GE/GDP</td>
<td>No support</td>
</tr>
<tr>
<td>25</td>
<td>Man (1980)</td>
<td>GE/GDP</td>
<td>Mixed results</td>
</tr>
<tr>
<td>26</td>
<td>Abizadeh, S. and Gray (1985)</td>
<td>GE/GDP</td>
<td>Mixed results</td>
</tr>
<tr>
<td>27</td>
<td>Ram (1987)</td>
<td>RGE, RGE/RGDP</td>
<td>Mixed results</td>
</tr>
<tr>
<td>28</td>
<td>Bairam (1992)</td>
<td>RGE</td>
<td>Mixed results</td>
</tr>
<tr>
<td>29</td>
<td>Koop and Poitier (1995)</td>
<td>RGE/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>30</td>
<td>Payne and Ewing (1996)</td>
<td>RGE/RGDP</td>
<td>Mixed results</td>
</tr>
<tr>
<td>31</td>
<td>Ansari, et al. (1997)</td>
<td>RGE/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>32</td>
<td>Karagianni, et al. (1998)</td>
<td>GE, GC, GE/P, GE/GDP</td>
<td>Mixed results</td>
</tr>
<tr>
<td>33</td>
<td>Chang (2002)</td>
<td>RGE, RGE/P, GE/RGDP</td>
<td>Mixed results</td>
</tr>
<tr>
<td>34</td>
<td>Chang, et al. (2004)</td>
<td>RGE, RGE/P, GE/RGDP</td>
<td>Mixed results</td>
</tr>
<tr>
<td>35</td>
<td>Wahab (2004)</td>
<td>RGE, RGE/RGDP</td>
<td>Mixed results</td>
</tr>
<tr>
<td>38</td>
<td>Dao (1995)</td>
<td>GC/GDP</td>
<td>Support</td>
</tr>
<tr>
<td>39</td>
<td>Nomura (1995)</td>
<td>RGE/P, RGCE/P, RGTE/P</td>
<td>Support</td>
</tr>
<tr>
<td>41</td>
<td>Al-Fariss (2002)</td>
<td>GE/GDP, GCE/P, GCAE/P</td>
<td>Support</td>
</tr>
<tr>
<td>42</td>
<td>Wagner and Weber (1977)</td>
<td>GE, GCE</td>
<td>No support</td>
</tr>
<tr>
<td>43</td>
<td>Singh and Sahni (1984)</td>
<td>RGE/P, RGAE/P, RGSDE/P, RGDE/P, RGDSE/P</td>
<td>No support</td>
</tr>
<tr>
<td>44</td>
<td>Courakis, et al. (1993)</td>
<td>RGE, RGCE, RGIE, RGTE</td>
<td>No support</td>
</tr>
<tr>
<td>45</td>
<td>Hondroyiannis and Papapetrou (1995)</td>
<td>RGE, RGCE, RGE/GDP, RGIE/P</td>
<td>No support</td>
</tr>
<tr>
<td>46</td>
<td>Bohl (1996)</td>
<td>GPE/GDP, GIPE/GDP, GSE/GDP, GTE/GDP, GCAE/GDP</td>
<td>Mixed results</td>
</tr>
<tr>
<td>47</td>
<td>Chletsos and Kollias (1997)</td>
<td>RGE/P, RGCE/P, RGIE/P, RGTE/P, RGVE/P, RGME/P, RGMEE/P, RGMSE/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>48</td>
<td>Abdel-Rahman and Barry (1997)</td>
<td>RGE/RGDP, RGCE/RGDP, RGIE/RGDP</td>
<td>Mixed results</td>
</tr>
<tr>
<td>49</td>
<td>Biswal, et al. (1999)</td>
<td>RGUE, RGE, RGE, RGPDE, RGIE</td>
<td>Mixed results</td>
</tr>
<tr>
<td>50</td>
<td>Asseery, et al. (1999)</td>
<td>RGCE, RGTSE, RGPE, RGCAE, RGGAE, RGDE, RGESE, RGSE</td>
<td>Mixed results</td>
</tr>
<tr>
<td>51</td>
<td>Iyare and Lorde (2004)</td>
<td>RGE, RGCE, RGE/RGDP, RGIE/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>52</td>
<td>Bairam (1995)</td>
<td>RGE, RSGE, RFGGE, RGNDE</td>
<td>Mixed results</td>
</tr>
<tr>
<td>53</td>
<td>Vatter and Walker (1986)</td>
<td>TE</td>
<td>Support</td>
</tr>
</tbody>
</table>

18 For the interpretation of these abbreviations see Table 3-5.
Table 3-5: Interpretations of the Abbreviations of Table 3-4

<table>
<thead>
<tr>
<th>Code</th>
<th>Measure of government</th>
<th>Code</th>
<th>Measure of government</th>
</tr>
</thead>
<tbody>
<tr>
<td>GE</td>
<td>Total government expenditure</td>
<td>RGSCE</td>
<td>Real government social expenditure</td>
</tr>
<tr>
<td>GCE</td>
<td>Government consumption expenditure</td>
<td>GE/GDP</td>
<td>Total government expenditure share in GDP</td>
</tr>
<tr>
<td>GTE</td>
<td>Government transfer expenditure</td>
<td>GE/RGDP</td>
<td>Total government expenditure share in real GDP</td>
</tr>
<tr>
<td>GTSE</td>
<td>Government transfer and subsidies expenditure</td>
<td>RGE/RGDP</td>
<td>Real total government expenditure share in real GDP</td>
</tr>
<tr>
<td>GIE</td>
<td>Government investment expenditure</td>
<td>RGCE/RGDP</td>
<td>Real government consumption expenditure share in real GDP</td>
</tr>
<tr>
<td>GPE</td>
<td>Government purchases expenditure</td>
<td>GTE/GDP</td>
<td>Share of transfers expenditures in GDP</td>
</tr>
<tr>
<td>GCAE</td>
<td>Government capital account expenditure</td>
<td>GSE/GDP</td>
<td>Share of subsidies expenditures in GDP</td>
</tr>
<tr>
<td>FGE</td>
<td>Federal government expenditure</td>
<td>RGIE/RGDP</td>
<td>Real government investment expenditure share in real GDP</td>
</tr>
<tr>
<td>SGE</td>
<td>State government expenditure</td>
<td>GPE/GDP</td>
<td>Share of government purchases of goods and services in GDP</td>
</tr>
<tr>
<td>GDE</td>
<td>Government defence expenditure</td>
<td>GCA/GDP</td>
<td>Share of capital account expenditures in GDP</td>
</tr>
<tr>
<td>GNDE</td>
<td>Government non-defence expenditure</td>
<td>GEIP/GDP</td>
<td>Share of interest payments expenditures in GDP</td>
</tr>
<tr>
<td>GGAE</td>
<td>Government general affairs expenditure</td>
<td>RGE/P</td>
<td>Real total government expenditure per capita</td>
</tr>
<tr>
<td>GESE</td>
<td>Government economic services expenditure</td>
<td>RGCE/P</td>
<td>Real government consumption expenditure per capita</td>
</tr>
<tr>
<td>GSCE</td>
<td>Government social services expenditure</td>
<td>RGTE/P</td>
<td>Real government transfers expenditure per capita</td>
</tr>
<tr>
<td>TE</td>
<td>Total employment</td>
<td>RGIE/P</td>
<td>Real government investment expenditure per capita</td>
</tr>
<tr>
<td>RGE</td>
<td>Real total government expenditure</td>
<td>GCAE/P</td>
<td>Government capital account expenditure per capita</td>
</tr>
<tr>
<td>RGCE</td>
<td>Real government consumption expenditure</td>
<td>RGV/P</td>
<td>Real government civilian expenditure per capita</td>
</tr>
<tr>
<td>RGTE</td>
<td>Real government transfer expenditure</td>
<td>RGME/P</td>
<td>Real government military expenditure per capita</td>
</tr>
<tr>
<td>RGTSE</td>
<td>Real government transfer and subsidies expenditure</td>
<td>RGMEE/P</td>
<td>Real government military expenditure on equipment per capita</td>
</tr>
<tr>
<td>RGIE</td>
<td>Real government investment expenditure</td>
<td>RGMSE/P</td>
<td>Real government military expenditure on salaries per capita</td>
</tr>
<tr>
<td>RGPE</td>
<td>Real government purchases expenditure</td>
<td>GDE/P</td>
<td>Government defence expenditures per capita</td>
</tr>
<tr>
<td>RGCAE</td>
<td>Real government capital account expenditure</td>
<td>RGD/P</td>
<td>Government real defence expenditures per capita</td>
</tr>
<tr>
<td>RGDE</td>
<td>Real government defence expenditure</td>
<td>GUE/P</td>
<td>Government current expenditure per capita</td>
</tr>
<tr>
<td>RGUE</td>
<td>Real government current expenditures</td>
<td>GDSE/P</td>
<td>Government dept servicing expenditure per capita</td>
</tr>
<tr>
<td>RGPDE</td>
<td>Real government public dept expenditure</td>
<td>GAEP</td>
<td>Government administration expenditure per capita</td>
</tr>
<tr>
<td>RGGAE</td>
<td>Real government general affairs expenditure</td>
<td>RGAE/P</td>
<td>Government real administration expenditure per capita</td>
</tr>
<tr>
<td>RGESE</td>
<td>Real government economic services expenditure</td>
<td>RGDSE/P</td>
<td>Government real dept servicing expenditure per capita</td>
</tr>
</tbody>
</table>
It appears from Table 3-4 that 65% of the existing studies of the WH have measured
government on the aggregate level of government expenditure GE, where all types of
government expenditures are included. These “aggregate” studies have supported the
inclusion of transfers and all government expenditure types in the measure of
government size. “Both goods and services and transfer payments are usually
financed by taxes and determined by the political decisions about the allocation as
well as distribution functions” (Gupta, 1968, p.29). Berry and Lowery (1987) also
include government transfers in testing for the WH. Berry and Lowery suggested that
the WH might be relevant for explaining the growth of transfers since “income
equity” is itself a collective good that might be purchased through transfer spending.

“There is no obvious reason for the exclusion of transfer payments since Wagner
perceived that workers required protection from the effect of economic change on
employment” (Peacock and Scott, 2000, p.5). That is, individual welfare functions
are interdependent and that, rather than leave transfers through private charities,
which will fail because of “free riders”, redistribution transfers are provided as a
collective good. This study suggests that the inclusion of transfer expenditure, where
the government collects funds from taxpayers and redistributes them directly to
individuals in the form of cash, in the measure of government expenditure might
better represent the definition of the fiscal state in the WH. That is, redistribution is
part of the WH of expanding state activity.

Outcomes of the empirical studies that used aggregate measures of government size
are asymmetric. Some significant relationships in the statistical models have been
found and 40% of these studies gave results which did not contradict the WH, while
28% of the studies produced contradictory results for the WH, and the remainder
gave results which were mixed. On the other hand, some scholars have narrowed their measurement of government size to final government consumption expenditure by excluding transfer government expenditures from aggregate government expenditure. Henrekson (1993), for instance, excluded transfers from the measure of government. He restricted his analysis to government expenditure on goods and services for consumption and investment purposes of the Swedish government and obtained no support for the WH in Sweden using this measure. Ram (1986) excluded transfer payments from the measure of government and found that the results of testing the WH this way did not differ much with the inclusion of transfer payments. Bird (1970) tested the WH and his results suggested that the inclusion of transfer payments ‘overstates’ the size of government expenditure. Table 3-4 shows also that some cross-sectional studies have excluded transfers from total government. Dao (1995), for instance, restricted the measure of government to the share of government consumption expenditure in national income. Dao’s results supported the WH using this measure of government.

Table 3-4 shows that almost 30% of the existing studies have disaggregated government expenditure into different components and test the WH for those components separately. Whilst these indicated that disaggregated measures of government might offer a better explanation of the role of the economic development on each component of government expenditure, it must be noted that the way that government expenditure is disaggregated has varied in these studies. For instance, scholars such as Gandhi (1971), Henrekson (1988), Courakis, et al. (1993), and Biswal, et al. (1999) disaggregated public expenditure data by category: consumption, investment and transfer payments. Other studies such as Chletsos and
Kollias (1997) disaggregated government consumption expenditure into civilian and military. Studies such as Asseery, et al. (1999) classified government expenditure by economic function such as expenditure on general affairs, defence expenditure, economic services, and social services. However, disaggregation produced less optimistic results for the WH. Table 3-4 shows that 37% of the studies that disaggregated government expenditure data have generated mixed results, 25% did not contradict the WH, while 38% did contradict the WH.

Relatively fewer studies which disaggregate expenditure find support for the WH in comparison with those that aggregate government expenditure. Some scholars suggested that disaggregating to the major components of government expenditure is more preferable than excessive disaggregation and that this produces more consistent results for testing the WH. Berry and Lowery (1987), for instance, analysed the changing size of government spending in the United States. They disaggregated government expenditure into transfer payments and purchases of domestic goods and services. Berry and Lowery suggested that the concept of government growth is different from summarising the growth of different components of government.

All the previous measures of government in the studies discussed so far have been based on central government. However, most countries have different levels of government, with centralized constitutions having central and local governments and decentralized or federal constitutions have federal (central), state, and local governments. Table 3-4 shows that some studies have tested the WH on the different levels of government. Bairam (1995), for instance, tested the WH for the USA during the period from 1972 to 1991. Bairam disaggregated government expenditure into
Federal government expenditure and State government expenditure, producing mixed results for the WH in the USA. Unlike previous measures of government, Vatter and Walker (1990) have attempted to measure government size by means other than budgetary accounts. They used employment as the measure to assess the growth of government in the United States. Their results supported the WH for the U.S. Weiher and Lorenz (1991) also used state public employment as a measure of government for 48 states of the USA in the year 1986 and obtained no support for the WH. Once again, the review shows that while testing the WH at a central aggregate level is most favoured, there is little consensus in the literature as to the motivations for different approaches.

Unfortunately, there seems to be no definitive pattern in the outcomes of the empirical tests relative to the measure of government used. Given the data source, and because it is drawn from many countries, the measure of government expenditure used in this thesis will be highly aggregated. An aggregate measure of government expenditure including transfers is used. Further, the data will be restricted to central government and will not include state, regional or local government expenditures. The next section moves on to the other important Wagner variable, the various measures of aggregate economic development.

3.6 Measurement of the Explanatory Variables

The current empirical studies of the WH have also differed in how they measure economic development and the choice of additional explanatory variables in the model. Table 3-6 classifies the existing studies according to their author, explanatory variables, and major result found. The table distinguishes between those studies
which use a single explanatory variable reflecting the development of the economy, and those that use variables additional to this. The additional measures accounted mainly for urbanization and industrialization. Furthermore, the table lists the outcome of both types in terms of being consistent with Wagner as ‘support’, being inconsistent with Wagner as ‘no support’ and producing asymmetric results as ‘mixed results’. Table 3-7 is constructed to interpret the codes used for the explanatory variable(s) in the Wagner relationship.

Table 3-6 shows that 92% of the existing studies have considered the growth in income as the independent variable in Wagner’s relationship. Moreover, 71% of these studies have approximated the growth in income by GDP or GDP per capita, while 29% have used either GNP or GNP per capita as an explanatory variable. Measuring the growth in income by GDP appears in studies such as Kolluri, B. R., et al. (1989), Nagarajan and Spears (1990), Murthy (1993), Nomura (1995), Al-Faris (2002), Legrenzi, G. and Milas (2002), Hondroyiannis and Papapetrou (1995), Chang (2002), and Wahab (2004). The outcomes of the empirical studies that used the growth in income as the only explanatory variable in aggregate are asymmetric. No discernable pattern in the statistical models has been found in the forms of the WH applied in these studies. Thirty six percent of the empirical studies did not contradict the WH, with 28% producing contradictory results and the remainder producing results which were mixed. Further, there appears to be no relative difference in outcome between those studies using domestic product (the majority) to those using national product, as the measure of aggregate income.
<table>
<thead>
<tr>
<th>No.</th>
<th>Author</th>
<th>Explanatory variables</th>
<th>Major result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Abizadeh, S. and Yousefi (1988)</td>
<td>GDP/P, SGP, RGDP/P, RSGP</td>
<td>Support</td>
</tr>
<tr>
<td>3</td>
<td>Nagarajan and Spears (1990)</td>
<td>RGDP/P, RGDP</td>
<td>Support</td>
</tr>
<tr>
<td>4</td>
<td>Murthy (1993)</td>
<td>RGDP/P</td>
<td>Support</td>
</tr>
<tr>
<td>5</td>
<td>Murthy (1994)</td>
<td>RGDP/P</td>
<td>Support</td>
</tr>
<tr>
<td>6</td>
<td>Oxley (1994)</td>
<td>RGDP/P, GDP/P</td>
<td>Support</td>
</tr>
<tr>
<td>7</td>
<td>Lin (1995)</td>
<td>RGDP/P</td>
<td>Support</td>
</tr>
<tr>
<td>8</td>
<td>Dao (1995)</td>
<td>RGDP/P</td>
<td>Support</td>
</tr>
<tr>
<td>9</td>
<td>Nomura (1995)</td>
<td>RGDP/P</td>
<td>Support</td>
</tr>
<tr>
<td>10</td>
<td>Ahsan, et al. (1996)</td>
<td>RGDP, RGDP/P</td>
<td>Support</td>
</tr>
<tr>
<td>12</td>
<td>Al-Faris (2002)</td>
<td>RGDP/P</td>
<td>Support</td>
</tr>
<tr>
<td>13</td>
<td>Legrenzi, G. and Milas (2002)</td>
<td>GDP, GDP/P</td>
<td>No support</td>
</tr>
<tr>
<td>14</td>
<td>Courakis, et al. (1993)</td>
<td>RGDP/P</td>
<td>No support</td>
</tr>
<tr>
<td>15</td>
<td>Henrekson (1993)</td>
<td>RGDP/P, P</td>
<td>No support</td>
</tr>
<tr>
<td>16</td>
<td>Ashworth (1994)</td>
<td>RGDP/P</td>
<td>No support</td>
</tr>
<tr>
<td>17</td>
<td>Hayo (1994)</td>
<td>RGDP/P</td>
<td>No support</td>
</tr>
<tr>
<td>18</td>
<td>Hondoypianiannis and Papapetrou (1995)</td>
<td>RGDP, RGDP/P</td>
<td>No support</td>
</tr>
<tr>
<td>19</td>
<td>Ferris and West (1996)</td>
<td>RGDP/P</td>
<td>No support</td>
</tr>
<tr>
<td>20</td>
<td>Alleyne (1999)</td>
<td>RGDP/P</td>
<td>No support</td>
</tr>
<tr>
<td>21</td>
<td>Halicioglu (2003)</td>
<td>RGDP/P</td>
<td>No support</td>
</tr>
<tr>
<td>22</td>
<td>Man (1980)</td>
<td>RGDP</td>
<td>Support</td>
</tr>
<tr>
<td>23</td>
<td>Ram (1987)</td>
<td>RGDP, RGDP/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>24</td>
<td>Buiram (1992)</td>
<td>RGDP</td>
<td>Mixed results</td>
</tr>
<tr>
<td>25</td>
<td>Koop and Poirier (1995)</td>
<td>RGDP/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>26</td>
<td>Buiram (1995)</td>
<td>RGDP</td>
<td>Mixed results</td>
</tr>
<tr>
<td>27</td>
<td>Payne and Ewing (1996)</td>
<td>RGDP/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>28</td>
<td>Bohl (1996)</td>
<td>RGDP/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>29</td>
<td>Chletsos and Kollias (1997)</td>
<td>RGDP/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>30</td>
<td>Abdel-Rahman and Barry (1997)</td>
<td>RGDP/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>31</td>
<td>Biswal, et al. (1999)</td>
<td>RGDP, RGDP/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>32</td>
<td>Chang (2002)</td>
<td>RGDP, RGDP/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>33</td>
<td>Chang, et al. (2004)</td>
<td>RGDP, RGDP/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>34</td>
<td>Wahab (2004)</td>
<td>RGDP/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>35</td>
<td>Iyare and Lorde (2004)</td>
<td>RGDP, RGDP/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>36</td>
<td>Thorn (1972)</td>
<td>GNP/P</td>
<td>Support</td>
</tr>
<tr>
<td>37</td>
<td>Gyles (1991)</td>
<td>GNP/P</td>
<td>Support</td>
</tr>
<tr>
<td>38</td>
<td>Ram (1992)</td>
<td>RGNP/P, RGDP/P</td>
<td>Support</td>
</tr>
<tr>
<td>39</td>
<td>Thornton (1999)</td>
<td>RGNP</td>
<td>Support</td>
</tr>
<tr>
<td>40</td>
<td>Islam (2001)</td>
<td>RGNP/P</td>
<td>Support</td>
</tr>
<tr>
<td>41</td>
<td>Dritsakis and Adamopoulos (2004)</td>
<td>RGNP/P, GNP</td>
<td>Support</td>
</tr>
<tr>
<td>42</td>
<td>Lall (1969)</td>
<td>GNP/P</td>
<td>No support</td>
</tr>
<tr>
<td>43</td>
<td>Goffman and Mahar (1971)</td>
<td>GNP/P, RGNP/P</td>
<td>No support</td>
</tr>
<tr>
<td>44</td>
<td>Wagner and Weber (1977)</td>
<td>RGNP/P</td>
<td>No support</td>
</tr>
<tr>
<td>45</td>
<td>Singh and Sahni (1984)</td>
<td>GNP/P, RGNP/P</td>
<td>No support</td>
</tr>
<tr>
<td>46</td>
<td>Burney (2002)</td>
<td>RGDP/P, RGDP/R, RGTR/P, NOGDP/P</td>
<td>No support</td>
</tr>
<tr>
<td>47</td>
<td>Ansari, et al. (1997)</td>
<td>RGNP/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>48</td>
<td>Karagianni, et al. (1998)</td>
<td>GDP, GDP/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>49</td>
<td>Asseery, et al. (1999)</td>
<td>GNP/P, RGNP/P</td>
<td>Mixed results</td>
</tr>
<tr>
<td>50</td>
<td>Michas (1974)</td>
<td>GDP/P, other variables</td>
<td>Support</td>
</tr>
<tr>
<td>52</td>
<td>Abizadeh, S. and Gray (1985)</td>
<td>RGDP/P, other variables</td>
<td>Mixed results</td>
</tr>
<tr>
<td>53</td>
<td>Vatter and Walker (1986)</td>
<td>PE</td>
<td>Support</td>
</tr>
</tbody>
</table>

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19 Originally published in 1967
20 Density of population and degree of urbanization
21 Those are Agriculture ratio to GDP, total commercial energy consumption per capita, openness (imports and exports) as a share of GDP and financial intermediaries.
Table 3-7: Types of Measures of Economic Development in Time-series Studies

<table>
<thead>
<tr>
<th>Measure</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Domestic Product</td>
<td>GDP</td>
</tr>
<tr>
<td>Gross National Product</td>
<td>GNP</td>
</tr>
<tr>
<td>Gross National Income</td>
<td>GNI</td>
</tr>
<tr>
<td>Net National Product</td>
<td>NNP</td>
</tr>
<tr>
<td>State gross product</td>
<td>SGP</td>
</tr>
<tr>
<td>Private sector GDP</td>
<td>PGDP</td>
</tr>
<tr>
<td>GDP per capita</td>
<td>GDP/P</td>
</tr>
<tr>
<td>GNP per capita</td>
<td>GNP/P</td>
</tr>
<tr>
<td>Real GDP</td>
<td>RGDP</td>
</tr>
<tr>
<td>Real GDP per capita</td>
<td>RGDP/P</td>
</tr>
<tr>
<td>Real GNP per capita</td>
<td>RGNP/P</td>
</tr>
<tr>
<td>Real GNI per capita</td>
<td>RGNI/P</td>
</tr>
<tr>
<td>Real NNP per capita</td>
<td>RNNP/P</td>
</tr>
<tr>
<td>Public employment</td>
<td>PE</td>
</tr>
<tr>
<td>Non-oil GDP per capita</td>
<td>NOGDP/P</td>
</tr>
<tr>
<td>Real government total revenue per capita</td>
<td>RGTR/P</td>
</tr>
<tr>
<td>Real government disposable revenue per capita</td>
<td>RGDR/P</td>
</tr>
<tr>
<td>Energy consumption per capita</td>
<td>EC/P</td>
</tr>
<tr>
<td>The degree of financial intermediation</td>
<td>FI</td>
</tr>
<tr>
<td>Share of agriculture output in GDP</td>
<td>AR/GDP</td>
</tr>
</tbody>
</table>

It is not possible to be definitive on the efficiency of including additional right hand side variables, because such studies are relatively few in number. However, important points have to be borne in mind at this stage. Firstly, there is little evidence to suggest that additional variables improve the fit of government expenditure to income. It should be noted, for example, that Abizadeh, S. and Yousefi (1988) found asymmetric results using additional variables. Also, the objective is to test the significance of economic development in determining government growth, and not developing a model which tries to explain as much of the variation of fiscal government as possible. That is, models which aim to test the WH should focus on the significance of the fiscal state measure and income growth measure relationship.

The remaining 8% of the studies employed additional measures as well as the growth in income. Additional measures have been implemented mainly in studies that tested
the WH in newly industrializing countries. The additional measures accounted for industrialization and urbanization in the independent variable list. Michas (1974) for instance, employed other variables as well as the growth in income, namely density of population and degree of urbanization. Abizadeh, S. and Gray (1985) related the growth of government expenditure to several measures of economic development other than domestic product per capita (GDP/P) such as urbanization, population growth, and the growth of manufacturing or trends of agricultural production. Abizadeh and Gray measured those factors by employing the following approximations respectively, agricultural production, commercial energy consumption per capita, openness of the economy, and financial intermediaries. Burney (2002) included other factors in testing the WH besides GDP per capita. These factors are the sectoral distribution of the GDP, population age composition, and financial development, which comprises a weighted index of the sectors in the economy, with a weighting in that index toward those sectors which are deemed modern.

The vast majority of empirical studies measure development as aggregate income and use this as a single explanatory variable. This thesis will measure economic development by the growth in income as the single independent variable in modelling the WH. The next section will proceed to discuss the effect of prices on the measurement of the economic variables in the existing studies of the WH.
3.7 Current or Constant Price Data

The WH is based on real relationships where the veil of prices has been lifted. Clearly, time-series studies must use data which is real and not based on current prices. An exception to this is where the ratios of current priced phenomena are used and it is expected that the prices of these different phenomena have experienced similar movements. Thus, for example, if one is using the share of government expenditure in GDP, then this ratio can be calculated from both series in current prices if one assumes that their price indices have followed similar temporal patterns. This assumption is made in this thesis, so that, wherever government expenditure is expressed as a ratio to GDP, that ratio is calculated on current prices in domestic currency units. The issue of constant versus current price data is given a new dimension with the international comparison data. Now the analysis has to resolve the problems of real and nominal in terms of the exchange rates used to produce common numeraire measures.

It is well-known that measures of cross country income based on data derived from ordinary exchange rates and current domestic prices often introduce a systematic bias in cross-country comparisons, because it does not show the differences in country outputs. To overcome this problem in cross country studies, there are some sets of cross-country international data, such as the Summers and Heston data set, known as Penn World Tables, which permit researchers to compile the income cross-section data using purchasing power parity (PPP) exchange rates. Some early studies have recognized the importance of using constant prices in cross-country comparisons. Gupta (1968), for instance, deflated GNP by using per capita GNP in 1961 as a base

---

22 For further discussion on the effect of prices in cross-country comparisons see the methodology chapter
which indicates the purchasing power of GNP compared to United States prices. Gupta recognized that the use of foreign exchange rates might only reflect the relative prices of the goods and services which enter into foreign trade, and so are not typical of relative prices within low-income and high-income countries.

Table 3-6 shows that over 60% of cross-sectional studies such as Lall (1969) and Michas (1974) have compiled income data for the countries included based on data derived from ordinary exchange rates and current domestic prices. These studies may, therefore, produce results that might contain some bias. On the other hand, studies such as Abizadeh, S. and Gray (1985), Koop and Poirier (1995) and Wahab (2004), have considered the importance of using constant prices and compiled the income cross-sectional data using purchasing power parity (PPP) exchange rates. This is the approach applied in the current study, and it is explained in more detail in the methodology chapter. The method to be adopted in this thesis is an empirical test based on pooled cross-sections of data. The pooling is across countries, where each country comprises a short time-series. In terms of international data, the data set comprises a “short, wide” data set. All currency data will be real and converted to a common currency, $US, using a single base PPP exchange rate. The following section examines the different functional forms that have been used in existing empirical studies.
3.8 Models of the Wagner Hypothesis

The different interpretations of the WH discussed in chapter 2 have formed the basis for six different general forms of Wagner’s relationship. These general forms are expressed in the following general equations, where the distinction is in terms of variables used rather than their functional form:

\[
GE = f(GNP) \quad \text{Peacock and Wiseman (1961); (3.1)}
\]

\[
\frac{GE}{P} = f\left(\frac{GNP}{P}\right) \quad \text{Gupta (1967); (3.2)}
\]

\[
\frac{GC}{GNP} = f\left(\frac{GNP}{P}\right) \quad \text{Pryor (1968); (3.3)}
\]

\[
\frac{GE}{GNP} = f\left(\frac{GNP}{P}\right) \quad \text{Musgrave (1969); (3.4)}
\]

\[
GE = f\left(\frac{GNP}{P}\right) \quad \text{Goffman (1968); (3.5)}
\]

and

\[
\frac{GE}{GDP} = f(GDP) \quad \text{Man (1980), (3.6)}
\]

where all of these variables are as defined earlier in tables 3-4 and 3-7. These general specifications have formed the basis for at least 12 testable models of the WH in existing studies:

\[
\begin{align*}
GE &= a + bGDP + e \\
\ln GE &= a + b \ln GDP + e
\end{align*} \quad \text{P&W; (3.7)}
\]

\[
\begin{align*}
\frac{GE}{P} &= a + b\left(\frac{GDP}{P}\right) + e \\
\ln(\frac{GE}{P}) &= a + b \ln \left(\frac{GDP}{P}\right) + e
\end{align*} \quad \text{Gupta; (3.8)}
\]

\[
\begin{align*}
\frac{GC}{GDP} &= a + b\left(\frac{GDP}{P}\right) + e \\
\ln \left(\frac{GC}{GDP}\right) &= a + b \ln \left(\frac{GDP}{P}\right) + e
\end{align*} \quad \text{Pryor; (3.9)}
\]

\[
\begin{align*}
\left(\frac{GE}{GDP}\right) &= a + b\left(\frac{GDP}{P}\right) + e \\
\ln \left(\frac{GE}{GDP}\right) &= a + b \ln \left(\frac{GDP}{P}\right) + e
\end{align*} \quad \text{Musgrave; (3.10)}
\]
\[
\begin{align*}

\begin{aligned}
GE &= a + b \left( \frac{GDP}{P} \right) + e \\
\ln GE &= a + b \ln \left( \frac{GDP}{P} \right) + e
\end{aligned}
\quad \text{Goffman; } (3.11)
\end{align*}
\]

and

\[
\begin{align*}

\begin{aligned}
GE/GDP &= a + b GDP + e \\
\ln(\text{GE/GDP}) &= a + b \ln (GDP) + e
\end{aligned}
\quad \text{Man; } (3.12)
\end{align*}
\]

The first equation in each pair represents a linear model of the WH of that particular version in variables and the second equation represents a log-linear model of the WH of that specific version. Log-linear models are linear in their parameters but not in their variables. It is notable that the left hand side of the equation is either modeled in shares ((3.9),(3.10) and(3.12)) or modeled in levels or per capita figures ((3.7),(3.8) and (3.11)). A clear problem with the linear models of GE in levels as the dependent variable is that the range of GE in principle will be \(-\infty\) to \(+\infty\). Clearly, negative scores of GE are impossible by definition. Table 3-8 classifies the existing studies according to the specifications employed to test for the WH. The table shows the author, version(s), model, and the major result found.

It appears from Table 3-8 that the Musgrave (1969) version seems to be the most frequently tested version of the WH. That is, many studies measure GE as a share of national income and relate this to aggregate income per capita. It will be recalled from the previous chapter that Musgrave’s version is less likely to have an endogeneity problem. This thesis suggests that Musgrave’s version appears to perform relatively better than the other versions of the WH on empirical grounds. On the other hand, it appears that Man’s, Goffman’s and Pryor’s versions are the least used specifications in the existing empirical studies of the WH.
<table>
<thead>
<tr>
<th>No.</th>
<th>Author</th>
<th>Version(s)</th>
<th>Model</th>
<th>Major result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ram (1992)</td>
<td>P&amp;W’s</td>
<td>Log-linear</td>
<td>Support</td>
</tr>
<tr>
<td>5</td>
<td>Courakis, et al. (1993)</td>
<td>P&amp;W’s</td>
<td>Log-linear</td>
<td>No support</td>
</tr>
<tr>
<td>8</td>
<td>Nomura (1995)</td>
<td>Gupta’s</td>
<td>Log-linear</td>
<td>Support</td>
</tr>
<tr>
<td>9</td>
<td>Chletsos and Kollias (1997)</td>
<td>Gupta’s</td>
<td>Log-linear</td>
<td>Mixed results</td>
</tr>
<tr>
<td>10</td>
<td>Thorn (1972)</td>
<td>Musgrave’s</td>
<td>Log-linear</td>
<td>Support</td>
</tr>
<tr>
<td>15</td>
<td>Henrekson (1993)</td>
<td>Musgrave’s</td>
<td>Log-linear</td>
<td>No support</td>
</tr>
<tr>
<td>16</td>
<td>Ashworth (1994)</td>
<td>Musgrave’s</td>
<td>Log-linear</td>
<td>No support</td>
</tr>
<tr>
<td>17</td>
<td>Hayo (1994)</td>
<td>Musgrave’s</td>
<td>Log-linear</td>
<td>No support</td>
</tr>
<tr>
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<td>Burney (2002)</td>
<td>Musgrave’s</td>
<td>Log-linear</td>
<td>No support</td>
</tr>
<tr>
<td>19</td>
<td>Halicioglu (2003)</td>
<td>Musgrave’s</td>
<td>Log-linear</td>
<td>No support</td>
</tr>
<tr>
<td>22</td>
<td>Abdel-Rahman and Barry (1997)</td>
<td>Musgrave’s</td>
<td>Log-linear</td>
<td>Mixed results</td>
</tr>
<tr>
<td>24</td>
<td>Wagner and Weber (1977)</td>
<td>Goffman’s</td>
<td>Log-linear</td>
<td>No support</td>
</tr>
<tr>
<td>26</td>
<td>Man (1980)</td>
<td>Man’s</td>
<td>Log-linear</td>
<td>Mixed results</td>
</tr>
<tr>
<td>27</td>
<td>Oxley (1994)</td>
<td>Goffman’s, Musgrave’s</td>
<td>Log-linear</td>
<td>Support</td>
</tr>
<tr>
<td>29</td>
<td>Ram (1987)</td>
<td>Musgrave’s, P&amp;W</td>
<td>Log-linear</td>
<td>Mixed results</td>
</tr>
<tr>
<td>30</td>
<td>Biswal, et al. (1999)</td>
<td>P&amp;W’s, Goffman’s, Gupta’s</td>
<td>Log-linear</td>
<td>Mixed results</td>
</tr>
<tr>
<td>31</td>
<td>Asseery, et al. (1999)</td>
<td>Goffman’s, Musgrave’s</td>
<td>Log-linear</td>
<td>Mixed results</td>
</tr>
<tr>
<td>33</td>
<td>Dritsakis and Adamopoulos (2004)</td>
<td>P&amp;W, Musgrave’s, Goffman’s, Man’s</td>
<td>Log-linear</td>
<td>Support</td>
</tr>
<tr>
<td>34</td>
<td>Hondroyiannis and Papapetrou (1995)</td>
<td>All versions</td>
<td>Log-linear</td>
<td>No support</td>
</tr>
<tr>
<td>35</td>
<td>Karagianni, et al. (1998)</td>
<td>All versions</td>
<td>Log-linear</td>
<td>Mixed results</td>
</tr>
<tr>
<td>36</td>
<td>Chang (2002)</td>
<td>All versions except Pryor’s</td>
<td>Log-linear</td>
<td>Mixed results</td>
</tr>
<tr>
<td>38</td>
<td>Iyare and Lorde (2004)</td>
<td>All versions</td>
<td>Log-linear</td>
<td>Mixed results</td>
</tr>
<tr>
<td>40</td>
<td>Alleyne (1999)</td>
<td>Musgrave’s</td>
<td>Semi-log linear (log GDP/P)</td>
<td>No support</td>
</tr>
<tr>
<td>41</td>
<td>Ansari, et al. (1997)</td>
<td>Gupta’s</td>
<td>Linear</td>
<td>Mixed results</td>
</tr>
<tr>
<td>42</td>
<td>Lin (1995)</td>
<td>Musgrave’s</td>
<td>Linear</td>
<td>Support</td>
</tr>
<tr>
<td>43</td>
<td>Lall (1969)</td>
<td>Musgrave’s</td>
<td>Linear</td>
<td>No support</td>
</tr>
<tr>
<td>44</td>
<td>Ferris and West (1996)</td>
<td>Musgrave’s</td>
<td>Linear</td>
<td>No support</td>
</tr>
<tr>
<td>45</td>
<td>Abizadeh, S. and Gray (1985)</td>
<td>Musgrave’s</td>
<td>Linear</td>
<td>Mixed results</td>
</tr>
<tr>
<td>46</td>
<td>Al-Faris (2002)</td>
<td>Gupta’s, Musgrave’s</td>
<td>Linear</td>
<td>Support</td>
</tr>
<tr>
<td>47</td>
<td>Legrenzi, G. and Milas (2002)</td>
<td>All versions</td>
<td>Linear</td>
<td>No support</td>
</tr>
<tr>
<td>48</td>
<td>Michas (1974)</td>
<td>Gupta’s</td>
<td>Linear</td>
<td>Support</td>
</tr>
<tr>
<td>49</td>
<td>Singh and Sahni (1984)</td>
<td>Gupta’s</td>
<td>Linear</td>
<td>No support</td>
</tr>
<tr>
<td>50</td>
<td>Koop and Poirier (1995)</td>
<td>Gupta’s</td>
<td>Occurrence framework</td>
<td>Mixed results</td>
</tr>
<tr>
<td>51</td>
<td>Goffman and Mahar (1971)</td>
<td>Goffman’s</td>
<td>Proportional measures</td>
<td>No support</td>
</tr>
<tr>
<td>52</td>
<td>Vatter and Walker (1986)</td>
<td>P&amp;W’s</td>
<td>Proportional measures</td>
<td>Support</td>
</tr>
<tr>
<td>53</td>
<td>Gyles (1991)</td>
<td>Gupta’s</td>
<td>A time-domain transfer function model</td>
<td>Support</td>
</tr>
</tbody>
</table>
Twenty six percent of the existing studies have tested more than one version of the WH, and, of those studies, 26% have supported the WH while 74% either found no support or found mixed results. The models employed to test for the WH have been mainly either linear or log-linear regression based models. However, some studies such as Goffman and Mahar (1971) and Man (1980) relied on simple ratios of percentage changes in government spending and GDP, and interpret the resulting ratios as elasticities. With this method, strong support for the proposition tends to be found where the ratios are greater than unity.

It is notable from Table 3-8 that the majority of the existing studies have employed log-linear (or semi-log-linear) models to test for the WH. These studies have performed 75% of the existing studies, while 25% have employed linear and other models. As discussed earlier in this section for models ((3.7),(3.8) and(3.11)), linear and log-linear forms are not appropriate. Linear forms allow shares to be below zero and rise above one without limit. Log-linear forms may produce an upper limit for the shares which may exceed one, or lead to a fall in shares, which will only stop when the share is zero. Neither of these would be a useful characteristic for the Wagner empirical model. However, some studies such as Ansari, et al. (1997), Al-Faris (2002), Michas (1974), and Singh and Sahni (1984) have modelled the WH in linear form. Even though Table 3-8 shows that some specifications have produced results which are not inconsistent with the WH compared to others, the thesis will not chose to exactly replicate any of the functional forms in that table. This decision is based on theoretical grounds and it is clear that a share equation with a non-linear form producing a share range limited to the interval [0, 1] is more appropriate to test the WH.
3.9 Tests and Estimation Methods

The great majority of the empirical studies of the WH utilize the linear stochastic model, which is the workhorse of econometrics. There are exceptions which use graphical methods or simple ratios of raw data to estimate elasticities; see, for example, Peacock and Wiseman (1961). Those pooled cross-sectional analyses that do stochastic modeling use ordinary least squares (OLS) regression. This section will concentrate on time-series studies of the WH and the two distinct approaches to the stochastic linear model. These studies fall into two groups. On the one hand, there are those studies which apply OLS regression to time-series and ignore the problems of spurious regression, and the outcomes of these studies are associated with non-stationary time-series (Granger and Newbold, 1974) (Engle and Granger, 1987). These contrast with the other group which uses modern time-series techniques to overcome the problem of spurious regression. That is, they test for a long run cointegrating vector and estimate the short run dynamics between government expenditure and aggregate income by way of an error correction model. They can then use Granger causality tests Granger (1969) to establish exogenous and endogenous variables.

Table 3-9 concentrates on time-series studies, classifying them according to author, result and stochastic model used. The table uses a twofold classification of stochastic model, “OLS” denoting those who ignore spurious regression issues and “cointegration” signifying those who use modern time-series methods.
Table 3-9: Econometric Procedures Applied in Existing Studies of the WH

<table>
<thead>
<tr>
<th>No.</th>
<th>Author</th>
<th>Test procedures</th>
<th>Major result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Murthy (1993)</td>
<td>Cointegration</td>
<td>Support</td>
</tr>
<tr>
<td>2</td>
<td>Murthy (1994)</td>
<td>Cointegration</td>
<td>Support</td>
</tr>
<tr>
<td>3</td>
<td>Oxley (1994)</td>
<td>Cointegration</td>
<td>Support</td>
</tr>
<tr>
<td>5</td>
<td>Ahsan, et al. (1996)</td>
<td>Cointegration</td>
<td>Support</td>
</tr>
<tr>
<td>6</td>
<td>Thornton (1999)</td>
<td>Cointegration</td>
<td>Support</td>
</tr>
<tr>
<td>11</td>
<td>Legrenzi, G. and Milas (2002)</td>
<td>Cointegration</td>
<td>No support</td>
</tr>
<tr>
<td>12</td>
<td>Henrekson (1993)</td>
<td>Cointegration</td>
<td>No support</td>
</tr>
<tr>
<td>13</td>
<td>Ashworth (1994)</td>
<td>Cointegration</td>
<td>No support</td>
</tr>
<tr>
<td>14</td>
<td>Hayo (1994)</td>
<td>Cointegration</td>
<td>No support</td>
</tr>
<tr>
<td>15</td>
<td>Hondroyiannis and Papapetrou (1995)</td>
<td>Cointegration</td>
<td>No support</td>
</tr>
<tr>
<td>16</td>
<td>Alleyne (1999)</td>
<td>Cointegration</td>
<td>No support</td>
</tr>
<tr>
<td>17</td>
<td>Burney (2002)</td>
<td>Cointegration</td>
<td>No support</td>
</tr>
<tr>
<td>18</td>
<td>Halicioglu (2003)</td>
<td>Cointegration</td>
<td>No support</td>
</tr>
<tr>
<td>19</td>
<td>Payne and Ewing (1996)</td>
<td>Cointegration</td>
<td>Mixed results</td>
</tr>
<tr>
<td>20</td>
<td>Bohl (1996)</td>
<td>Cointegration</td>
<td>Mixed results</td>
</tr>
<tr>
<td>21</td>
<td>Chletsos and Kollias (1997)</td>
<td>Cointegration</td>
<td>Mixed results</td>
</tr>
<tr>
<td>22</td>
<td>Ansari, et al. (1997)</td>
<td>Cointegration</td>
<td>Mixed results</td>
</tr>
<tr>
<td>24</td>
<td>Abdel-Rahman and Barry (1997)</td>
<td>Cointegration</td>
<td>Mixed results</td>
</tr>
<tr>
<td>26</td>
<td>Asseery, et al. (1999)</td>
<td>Cointegration</td>
<td>Mixed results</td>
</tr>
<tr>
<td>27</td>
<td>Chang (2002)</td>
<td>Cointegration</td>
<td>Mixed results</td>
</tr>
<tr>
<td>31</td>
<td>Vatter and Walker (1986)</td>
<td>OLS</td>
<td>Support</td>
</tr>
<tr>
<td>34</td>
<td>Nagarajan and Spears (1990)</td>
<td>OLS</td>
<td>Support</td>
</tr>
<tr>
<td>35</td>
<td>Gyles (1991)</td>
<td>OLS</td>
<td>Support</td>
</tr>
<tr>
<td>36</td>
<td>Ram (1992)</td>
<td>OLS</td>
<td>Support</td>
</tr>
<tr>
<td>37</td>
<td>Nomura (1995)</td>
<td>OLS</td>
<td>Support</td>
</tr>
<tr>
<td>39</td>
<td>Goffman and Mahar (1971)</td>
<td>OLS</td>
<td>No support</td>
</tr>
<tr>
<td>40</td>
<td>Wagner and Weber (1977)</td>
<td>OLS</td>
<td>No support</td>
</tr>
<tr>
<td>41</td>
<td>Singh and Sahni (1984)</td>
<td>OLS</td>
<td>No support</td>
</tr>
<tr>
<td>42</td>
<td>Courakis, et al. (1993)</td>
<td>OLS</td>
<td>No support</td>
</tr>
<tr>
<td>43</td>
<td>Ferris and West (1996)</td>
<td>OLS</td>
<td>No support</td>
</tr>
<tr>
<td>44</td>
<td>Man (1980)</td>
<td>OLS</td>
<td>Mixed results</td>
</tr>
<tr>
<td>45</td>
<td>Abizadeh, S. and Gray (1985)</td>
<td>OLS</td>
<td>Mixed results</td>
</tr>
<tr>
<td>46</td>
<td>Ram (1987)</td>
<td>OLS</td>
<td>Mixed results</td>
</tr>
<tr>
<td>47</td>
<td>Bairam (1992)</td>
<td>OLS</td>
<td>Mixed results</td>
</tr>
<tr>
<td>48</td>
<td>Bairam (1995)</td>
<td>OLS</td>
<td>Mixed results</td>
</tr>
</tbody>
</table>
As would be expected, Table 3-9 shows that most of the time-series studies conducted during the 1960’s until the 1980s used OLS regression in time-series to test for the WH. After the 1980’s, cointegrating regression appears to be the most important econometric development in the time-series analysis of the WH. A great deal has been written recently on the possibility of spurious regression in the context of testing for the WH using time-series data. Spurious regression results from applying OLS regression to time-series without examining the stationarity characteristics of the time-series data. OLS can only be used with stationary time-series. Non-stationary series of the same order of integration usually established by unit root tests may be cointegrated (Granger and Newbold, 1974) (Engle and Granger, 1987). That is, they may have a long run equilibrium relationship and the cointegration test attempts to identify the existence of a long-run connection among the economic variables.

The cointegration approach is straightforward. Once the degree of integration of the individual variables has been established, it can be determined whether the data series are cointegrated. The cointegration approach shows if a linear combination of the integrated series is stationary, where this linear combination is the cointegrating vector. It is notable from Table 3-9 that over 60% of the existing empirical studies have tested the WH using modern time-series econometric techniques. Those studies include Payne and Ewing (1996), Ansari, et al. (1997), Kolluri, Bharat R., et al. (2000), and Chang (2002). However, Henrekson (1993) is probably the first who raised the issue of spurious regression in testing the WH. Henrekson suggested that the findings of existing time-series studies which supported the WH are likely to suffer from spurious regression, because they used OLS regression on non-stationary
variables. Henrekson tested the data for stationarity by applying Augmented Dicky Fuller (ADF) tests Dickey and Fuller (1981), and tested for cointegration using the Engle and Granger (1987) two-step procedure. He used Granger's (1969) causality test in the second stage error correction model to identify endogenous and exogenous variables in the model. In the absence of standard “t” tests on the long-run relationship, Granger causality tests can be used to test the WH. For example, if it is found that aggregate income Granger causes public expenditure, but public expenditure does not Granger causes aggregate income, then this result is consistent with the WH. Henrekson’s results do not support the WH in Sweden.

Other scholars have followed Henrekson and adopted these new econometric techniques to test for the WH. Chletsos and Kollias (1997) examined the WH in Greece. They applied unit root tests and the cointegration test. They obtained mixed results for the WH in Greece. Ahsan, et al. (1996) tested the WH using time-series data from Canada for the period from 1952 to 1988. The authors adopted the ZA23 unconditional unit root test methodology for establishing the non-stationary characteristics of the series. A subsequent cointegrating test found a cointegrating vector on the long-run dynamics was consistent with the WH. However, some of the existing time-series empirical studies have applied direct estimation methods to time-series data. Some studies have ignored the issue of spurious regression, Courakis, et al. (1993), for instance tested the WH in Greece and Portugal. They have assumed the time-series for those countries were stationary and proceeded to test the WH via the OLS regression method. Therefore the Courakis results might produce spurious regressions and lead to incorrect findings.

23 Zivot and Andrews (1992), hereinafter ZA, and others, have shown that the unit root tests proposed by Dickey and Fuller tend to accept the null too frequently if the time-series contains a structural break. As a result, many 'modified' unit root tests have been suggested to alleviate this problem including ZA’s.
Most pooled cross-sectional time-series analysis employs short time periods. However, those employing long time periods have tested the time-series data for stationarity. Wahab (2004), for instance, used an econometric procedure following three steps. Firstly, stationarity of the respective time-series is assessed using the Augmented Dickey-Fuller (ADF) test. Secondly, long run cointegration between the two series is tested using the ADF test on the residuals from a cointegration regression (CR) involving levels. In the third step, Wahab constructed an error-correction model (ECM). Wahab’s results were asymmetric, and in some cases contradicting, in others not contradicting, the WH.

In terms of the Engle and Granger (1987) two-step procedure, it could be argued that simply establishing a cointegrating vector shows that there is a long-run relationship between, say, government expenditure and GDP per capita. However, whilst this is a necessary condition for the WH, it is not sufficient. The WH proposes a cause from GDP per capita to government expenditure. Cointegration simply identifies a long run relationship and not any direction of determination. Thus, one can only say that the identification of a cointegrating vector is not inconsistent with the WH.

Some studies have contributed to the WH by examining the direction of the relationship between the economic variables. These studies applied the Granger causality tests in the short run error correction model. Granger causality cannot directly test for cause. This is simply a test of the endogeniety versus exogeneity of variables. If, in the error correction model, it can be shown that variable $x$ contains information that can be used to forecast variable $y$, then variable $y$ is endogenous. If $x$ contains no information that can be used to forecast $y$, then $y$ is deemed exogenous.
The test can then be reversed for $y$ containing information on $x$. In terms of the WH with, say, government expenditure and aggregate income, if government expenditure tests as endogenous and aggregate income tests as exogenous, then this result is consistent with the WH.

Singh and Sahni (1984), for instance, attempted to show that government expenditure is an endogenous variable and GDP an exogenous variable. Their results showed that, in some cases, this was not rejected which does not contradict the WH. Afexentiou and Serletis (1996) tested the relationship between government expenditure and economic development for the European Union (EU). The authors employed the relationship between government consumption expenditure and GDP. They also tested for Granger causality between transfer payments and subsidies to GDP. The empirical results found no evidence to suggest the endogeneity of all three classes of expenditure, a result which is consistent with the WH. Karagianni, et al. (1998) examined the WH for the EU countries for the period from 1949 to 1998. They applied the Granger causality test and concluded that, in the majority of the EU countries, there is support for the WH. That is, government expenditure tested as endogenous and aggregate income as exogenous. Payne and Ewing (1996) examined the WH using the error-correction model to examine the presence of Granger causality between the share of real government expenditure and real GDP per capita for a sample of 22 countries. The results obtained supported the WH in terms of endogenous versus exogenous variables. Abizadeh, S. and Yousefi (1998) examined the impact of government spending on South Korea’s economic development. They conducted causality tests. Their findings supported the WH for South Korea. Other scholars have conducted similar causality tests and obtained results consistent with
the WH such as Ram (1986), Ansari, et al. (1997), Biswal, et al. (1999), and Al-Faris (2002). On the other hand, some studies applied causality tests and obtained results which contradict the WH; that is, government expenditures are exogenous and economic development is endogenous. Alleyne (1999), for instance, tested the WH for the Caribbean countries during the period from 1950 to 1997, and the empirical results suggested that the WH does not hold for the selected countries.

Those studies that have used modern time-series techniques have contributed greatly to the empirical tests of the WH. However, Table 3-9 shows that the results of empirical studies that used cointegration are still asymmetric. That is, even with the development of new time-series econometric procedures that have been applied in tests of the WH, mixed results are still obtained. Further, it should be noted that modern time-series techniques are confined to long run linear relationships. In this thesis it is argued that a better empirical specification of the WH would be a non-linear model of the share of government expenditure and aggregate income. Such a model could be accommodated using non-linear least squares on a data set of short time-series pooled across different economies. The next section discusses non-linear modeling in the existing empirical studies of the WH.
3.10 Non-Linear Modelling of the Wagner Hypothesis

The previous chapter has established that Wagner recognized the role of the private sector alongside the increasing role of the public sector in a market economy where the private sector is still active and provides goods and services. Following this rationale, the government share of aggregate income should have a limit. This thesis has established in the previous chapter that Wagner recognized a limit for government growth but he explicitly did not intend to determine this limit as an exogenous value. This study suggests that a proper functional form of Wagner’s relationship is the share model which would allow the data to determine the limit to the share of government expenditure in income.

Some scholars have recognized that the WH is more appropriately modeled as a non-linear, rather than a linear, process. “The generalized nature of the WH makes it tricky to define the correct functional form of the relationship” (Chang, 2002, p.1157). Herber (1975) implemented a theoretical analysis of the WH and suggested that the relationship might be explained by a non-linear “curve”. Herber suggested a less than proportionate rise in government expenditure share of GDP in a ‘pre-industrial’ and ‘post-industrial’ stage of economic growth and a more than proportionate rise in government expenditure share of GDP in an ‘industrialization’ stage of economic development. Herber’s non-linear demonstration curve was not able to explain the economic reasoning for the declining expenditure ratio which cannot be explained by the WH. Further, Herber did not proceed to test the WH using non-linear empirical models.
There are a few studies in the existing literature that have tested the WH as a non-linear process. For instance, as long ago as the late 1960’s, Gupta (1968) tested the WH by employing both linear and polynomial double logarithmic regression functions. Gupta’s results indicated that a non-linear process of the WH would give a better fit for data. Gupta’s work can be regarded as a distinguished paper in the sense that it recognized the limits of modeling the WH in a linear form. Legrenzi, Gabriella (2004) tested for the WH in Italy during the period from 1861 to 1980. Legrenzi adopted the non-linear error correction model. He introduced non-linearity in the short-run dynamics by allowing for government expenditure to respond faster when deviations from equilibrium are large. The empirical results support non-linearities in that government spending adjusts much faster when deviations from its equilibrium with domestic income are larger. “A statistical significance of the non-linear error correction model can provide further useful insights to public choice analysis, as it departs from the standard assumption that the speed of adjustment of government spending is independent of the disequilibrium magnitude” (Legrenzi, 2004, p.202).

Whilst the Legrenzi results are interesting, it must be remembered that the long-run equilibrium model is still a log-linear model. That is, non-linearities in the short-run dynamics do not carry over into the long-run equilibrium relationship. This thesis is interested in the non-linear modeling of the long-run relationships of the WH.

Recently, Florio and Colautti (2003) have attempted to test the WH for the US, the UK, France, Germany, and Italy during the period from 1870 to 1990. They interpreted the WH as a non-linear process captured by the Verhulst’s logistic function. Their estimates showed a pattern of similar curves and suggested that the
logistic view of the growth of government gives a better fit to the observed data\textsuperscript{24}. Their results are based on econometric estimates of the ratio between public expenditures and national income. That is, their dependent variable is government share in GDP. However, the independent variable, unlike the other studies examined so far, has time as the independent variable. That is, theirs is a literal translation of the well-known growth model to a test of the WH. This only works if time is a good proxy for aggregate income, the usual Wagner independent variable. There is no reason why aggregate income should not be used instead of time in such a model and this will be the course taken in this thesis.

Florio and Colautti suggested that the WH ignores the social cost of financing government expenditure by taxation and disregards the Pigouvian effect introduced by Pigou (1947). They took the supply side effect on the WH and assumed that there is only distortionary taxation available to government to finance public expenditures which will act at some stage as a brake to stop the growth of government expenditure. Unlike Florio and Colautti, this thesis is a demand side analysis and no attempt will be made to include supply factors. Furthermore, this thesis recognizes that Wagner had in mind the effect of taxation on the demand of public provision; at the same time, Wagner suggested that any attempt to set an exogenous limit to the relative size of government would fail. The limit to the growth of government share in income in this study will be determined from inside the model and by the data.

\textsuperscript{24} For more details see chapter 2, section 2.3.7
3.11 Justification of Cross-Section Analysis of the Wagner Hypothesis

Section 3.2 shows that there are two main types of analyses followed in testing the WH: time-series, and cross-sectional. It shows also that the majority of the existing empirical studies have used time-series analysis to test for the WH. Time-series analysis focuses on the long-run relationship between the two economic variables in a single country. On the other hand, cross-sectional studies can only infer a long-run relationship from a point in time, which might not be relevant to the WH. Bird (1971), for instance, suggested that cross-sectional data is irrelevant for testing the WH because a postulated change in the public sector happens over time. Henrekson (1993) analyzed Swedish government expenditure and suggested that the growth of the public sector is a process occurring over time in a single country.

Table 3-1 shows that studies using cross-sectional data have been conducted over time spans instead of at single point in time. This is known as pooled cross-section time-series analysis. This type of analysis has received increasing use in testing the WH. Michas (1975), for instance, suggested that tests of the WH by utilizing pooled cross-sectional data could be more effective because the law is generalized for a number of countries, not just for one country as in a time-series econometric analysis. Wahab (2004) also favoured using cross-sectional data pooled over time spans. Wahab suggested that using pooled cross-section data maximizes sample size and increases the power of the test.

Table 3-1 shows that the inclusion of less developed countries is limited in time-series studies. The reason might be related to issues such as measurement and data limitation problems in time-series studies for less developed countries. This
motivated some scholars to test the WH using cross-country data. Ram (1987) suggested that the major reason for focusing on cross-country analysis has been the scarcity of long time-series for less developed countries. The limitation of data for long time spans might be considered an inherent problem that confronts cross-country comparisons. However, recently, cross-country international data has become relatively more available through international sources such as the International Monetary Fund (IMF), International Financial Statistics (IFS), and the Penn World Tables (PWT). These databases are created from the national accounts of different countries at the central level of governments. “The recent availability of a great quantity of comparable cross-country data, due to the work of Robert Summers and Alan Heston25, stimulated a revival of empirical studies on issues such as the determinants of growth” (Slemrod, 1995, p.395). Many recent cross-sectional studies have used data from international sources to test for the WH. Dao (1995), for instance, tested the WH with a sample consisting of 55 middle-income countries for the period 1980-1991. Dao used the data for consumption, output, and price levels from Heston and Summers (1993). Dao’s results supported the WH. Further, a pooled cross-section of short time-series overcomes the problems associated with non-stationary time-series. This facilitates the use of non-linear models which might more truly reflect the WH but, at the same time, overcomes the criticism of using purely static data to test a dynamic process.

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25 Summers, Heston, and Bettina are the key researchers who built the PWT data.
3.12 Summary

The WH has withstood the test of disproof for some empirical studies, or for some of the countries tested, but in others it is either rejected or cannot be confirmed. This chapter has reviewed the testing methodologies followed in the existing studies of the WH and tried to determine whether there are patterns of outcome in terms of the methods used. No clear patterns of results or consistent conclusions emerge from the tests.

This chapter distinguished between the different types of econometric analyses followed in the existing studies of the WH. These types of analyses have varied between time-series and cross-sectional. The time-series studies have formed almost 85% of the existing empirical studies that have tested the WH, and most of these studies have been applied to developed and industrial countries. Cross-sectional studies have included more developing countries in testing the WH. More time-series come up with results consistent with the WH because there are a great many more time-series studies. However, cross-sectional studies have relatively more outcomes which are consistent with the WH. Significant relationships in the statistical models of the WH have been found in developed countries more so than for those in developing countries, but this might reflect the heavy bias toward developed countries rather than be a function of the WH itself.

This chapter also reviewed the types of economy and time spans applied in the existing empirical studies. Most of the empirical studies have tested the WH for developed and industrial countries. However, the majority of the existing studies have tested the WH for data after World War II and they have obtained asymmetric
results. Yet, it seems that testing the WH for countries in their industrialization phase leads to some support for the WH. This thesis suggests that, as economic development continues, the relationship between the two economic variables, public expenditure growth and economic development, may weaken with higher levels of industrialization. However, this is incorporated in the WH, and simply reflects that the relative share of government and private sectors has become stabilised for economies in the post industrialisation phase.

This chapter also reviewed the measures of the economic variables in the WH, namely government size and economic development. The existing empirical studies have followed different paths in measuring government size. However, the majority of these studies have measured government at the aggregate level where all expenditures are included and they have measured economic development by the growth of income. The measures of the economic variables have been affected by the different interpretations of the WH. However, the majority of the empirical studies have adopted the Musgrave version and measured government expenditure as a share in income, and economic development as per capita income.

Despite all of the differences in the methods followed for measuring the economic variables and the variable used, it was established in this chapter that all these measures have been formulated in fiscal terms of the state. However, Wagner recognized also the regulatory aspects of the state; therefore, these studies might be considered only partial ones in terms of the fiscal state and do not represent testing the WH for all state activities.
This thesis suggests that the contradictory results stemming from the empirical tests of the WH might be related to methodological issues other than the type of economy and time span. For example, the existing empirical studies have employed the linear functional form in testing the WH which does not reflect the changing patterns of public spending over time and across countries. This might have contributed to the mixed results from testing the WH since it does not reflect the stages of development in government growth and only provides constant elasticity. Further, the linear models of the share versions of the WH in the existing empirical studies do not allow for a limit to government growth. The majority of the existing empirical studies have not attempted to test for the limit of the share of government expenditure in income. Those that have included functional forms with limits to share (Florio and Colautti) have not tested for it but have imposed a limit from outside the demand model of the WH.

A significant development in time-series tests of the WH is the application of modern cointegration regression. This thesis distinguishes between those studies that used OLS regression and studies that used modern time-series techniques to test for the WH. However, this distinction of modern time-series techniques has not reduced the asymmetric outcomes. Moreover, in this chapter, a clear empirical argument has been made for the use of pooled short time-series, over pure time-series analysis.

The next chapter will present the methodology that will be followed in testing the WH in this thesis. This methodology will give empirical content to the framework suggested so far in chapter 2 and chapter 3. It will suggest that the ratio of government expenditure should be increasing in GDP per capita, but with a limit.
The appropriate functional form is captured by the non-linear logistic and Gompertz functions. These models allow the data to find a limit to the growth of the share of government expenditure in income through the data. It is felt that this methodology will overcome the many issues that have appeared in the existing studies of the WH, and probably, in so doing, will reconcile the differing results and conclusions, so that they will be more appropriate tests of the WH.
CHAPTER FOUR

4 Chapter 4: Models and Estimation Method

4.1 Introduction

The thesis has so far established the theoretical framework of the WH, as well as reviewing the methodologies followed to test the WH in the existing literature. This chapter will proceed to present a methodology that gives an empirical content to the theoretical framework presented in chapter 2 and attempts to avoid the methodological issues which have appeared in the existing empirical studies presented in chapter 3. This methodology will specify a general functional form of the WH and then move on to develop two testable models that fit the theoretical framework based on the recasting of the WH presented in chapter 2. A clear point here is that there is a general functional form, the sigmoid shape, which limits shares to sensible values between zero and one. However, two specific forms are used, the logistics function and the Gompertz equation. The final section of the chapter will deal with the estimation method, non-linear least squares, and the numerical algorithm used to estimate the unknown parameters.

To summarise briefly, Chapter 2 established the development of the theoretical analysis of the WH and pointed out the gaps in the existing literature in the context of the WH. It traced the development of the WH using translations of Wagner’s original writings. Moreover, the development of the different interpretations of the WH was examined in a way that showed how each interpretation has contributed to the WH. This discussion about Wagner’s original statement established the basis for
a more comprehensive understanding of the WH. A possible recasting of the WH was suggested in chapter 2 which proposed that, with the development process in the economy represented mainly by the growth in per capita income, the fiscal state, represented by the share of government expenditures in national income, will increase at a rate higher than that of per capita income. However, this rate will fall until it is at the same rate of expansion as per capita income, so that at some point the share of government expenditure reaches a maximum. Further, this saturation point is determined endogenously from demand side forces.

Chapter 3 reviewed the methodologies followed to test the WH in the existing literature and discussed whether that literature had affected the formulation of the WH and in which ways. Despite all the differences in the methodologies that have been followed in measuring the economic variables used as proxies for the WH, it was established in chapter 3 that all existing studies have tested the WH in the fiscal terms of government. Therefore, all these tests must be considered partial ones, since they have not included the regulatory aspects of the state in measuring government activity. In chapter 3 it was seen that, over the many different tests of the WH, the results have been mixed and often contradictory. For some studies, the empirical outcome was consistent with the WH. For others, test results did not support but contradicted Wagner. Moreover, on some occasions where studies adopted several different tests, the results were mixed, with some being consistent with the WH and others not.

From the review of the existing studies in chapter 3, it is notable that patterns of government expenditure have changed over time and have been different between
countries. However, no clear consensus emerges that these changes and differences were always indicative of the WH. Further, most of the share models of the WH did not allow for a limit to the growth in government share in a market economy, and this may compromise the results that have been produced.

The thesis has so far established that there are a limited number of studies that have interpreted the WH as a non-linear process. However, even those that have tested the WH in a non-linear form have produced contradictory results, and no clear consensus can be discovered here. To the best of the author’s knowledge, the present study is the first to provide a comprehensive interpretation of the WH following a non-linear process, where that process is couched in terms of truly Wagnerian variables. Based on this interpretation, the current chapter attempts to develop a more comprehensive testable research hypothesis and endeavours to deploy a more coherent methodology. In so doing, it develops an empirical model that will be estimated to examine the effect of change in per capita income on the state’s expenditure share of national income across countries and over time. A broad methodology originally developed to analyze the growth of population in the form of sigmoid curves is employed\(^1\). Specifically, two functional forms are used within the test methodology, the symmetric logistics equation and the skew symmetric Gompertz equation. Both functional forms are to be estimated using non-linear regression in which the proportion of government expenditure in income is regressed on per capita aggregate income.

\(^1\) The seminal papers in this area, both of which make use of this basic methodology are described in section 4.3.1
If the growth process is linear, one can measure growth rates as simple average changes in the ratio as a result of changes in real per capita income. This approach has commonly been used to analyze the growth of the ratio over a relatively brief period of time in a single country or for a group of countries at single point in time. As the length of time under investigation is expanded, or the number of states included is expanded, the assumption of linear growth is likely to become considerably less defensible as one now explores share possibilities over its whole range. The growth of government share over a broader time span seems more likely to be non-linear, accelerating first and later slowing to some maximum value or steady state equilibrium, where this equilibrium is determined by the existing demand forces within the economy. This is the intuition that informs the model development and estimation methods produced here.

The remainder of the chapter is structured as follows. Section 4.2 presents a linear statistical model of the WH. First, this section defines the right hand side and left hand side economic variables in the WH. The section then recalls the properties of using linear probability functions for modeling the WH. Two main issues with using linear functional forms will be discussed in more depth than in previous chapters, these being elasticity measures and the endogeneity of regressors in the linear Wagner model. The elasticity of demand for government expenditure with respect to aggregate income is presented. Finally, this section will establish the appropriate measures of the Wagnerian variables specifically in terms of measures as ratios or as absolute levels. Section 4.3 presents the sigmoid curves to capture the process of government growth in the economy. This section presents a brief historical review of the sigmoid curves and places more emphasis on their use in economic research. The
logistic function is then presented in more depth followed by specifying the non-linear Gompertz function to capture the same process of government growth in the economy. Finally, the section discusses the issue of parameterization of the two sigmoid curves. Section 4.4 describes the method of estimation employed to test the logistic and the Gompertz functions of the WH. Section 4.5 summarises the chapter and introduces the next chapter.

4.2 Linear Statistical Model of the Wagner Hypothesis

4.2.1 Selecting the Variables

Wagner proposed that there is a tendency for an expansion of government activity in society as a response to economic progress. Thus, the general form of Wagner’s relationship can be represented as:

\[ G_i = f(I_i) \]  

(4.1)

where the dependent variable \( G \) represents government activity in the \( ith \) state and \( I \) is the independent variable representing the progress of the economy in the \( ith \) state. It should be noted that Wagner did not specify a functional form, but clearly imposed a positive or direct relationship between \( I \) and \( G \), and that change in the former caused change in the latter. However, government activity consists of two main types of activity, fiscal and regulatory, which on the assumption that both can be measured in the same units, can be defined by the following identity:

\[ G_i \equiv GE_i + GR_i \]  

(4.2)
where $GE_i$ represents government fiscal activity in state $i$, and $GR_i$ represents government regulatory activity in state $i$. For $I_i$ in equation (4.1), the economic literature usually focuses on one measure of economic progress, namely the growth in income. Some studies have argued that the growth in income alone might not reflect the development process of a country and have considered exogenous variables in addition to, and other than, income in determining the size of government$^2$. However, in the vast majority of cases, Wagner’s general relationship is of the form:

$$GE_i = f(Y_i)$$ (4.3)

where $GE_i$ represents government activities in terms of the fiscal aspects of the state and $Y_i$ represents the growth in income in that state. Those who wish to test models of the WH tend to measure government activities in fiscal aspects of the state represented by government budgetary expenditure. They are, like this study, clearly forced to do this, because of the difficulty of measuring $GR_i$ in equation (4.2) in some sensible numeraire. The growth in income of a country is often measured by changes in the level of real national incomes. Usually in equation (4.3), $GE_i$ represents government expenditure, and $Y_i$ represents the growth in real GDP, so that:

$$GE_i = f(GDP_i)$$ (4.4)

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$^2$ See chapter 3 for examples of these exogenous variables other than income
Any model based on the general functional form in equation (4.4) will only represent the growth of government expenditure as a response to the growth in real GDP and will not capture the true extent of the state in the economy.

Chapter 3 showed that the existing literature has modeled the WH in the linear statistical form, either in variables or parameters. Some studies have modeled the WH as a simple linear statistical model where the right hand side is GDP in levels with a constant and the left hand side of the equation is GE. Other studies have employed a log-linear model of the WH, that is, a model which is linear in parameters. Thus, equation (4.4) has been modeled so that the left hand side variable GE is a logarithm and on the right hand side of the equation is the logarithm of GDP and that relationship is linear with a constant on the right hand side. The resulting elasticity of GE, given GDP, is constant. These two types of models encounter some problems in their limits. These are explored in some depth in Appendix B. Further, both models have an endogeneity of regressor problem. The chapter now moves on to present a linear model of the WH followed by a discussion of whether the dependent variable in Wagner’s relationship should be specified in absolute levels or as a proportion of government expenditure in income. Importantly, this thesis, even though it will not follow the linear or log-linear form, will use the normally adopted variables of GE and GDP, albeit expressed as ratios. It is stated again that, like other models, the empirical model of this thesis will also be restricted to the fiscal state.
4.2.2 Linear Interpretation of the Wagner Hypothesis and Elasticities

The variables in the general functional form (4.4) are positive and continuous; that is $0 < GE, GDP < \infty$. As a rule of thumb, when the dependent variable is continuous, one can model the relationship using a linear statistical model. Some of the existing studies have followed Peacock and Wiseman (1961) and interpreted the general functional form of the WH in (4.4) as a linear model and regressed government expenditure on GDP. The main difference between P&W and those who follow them is that the former used a deterministic relationship based on graphs and ratios and the latter use the OLS regression estimator in stochastic modeling. The alternative to this is the more common form of the double logarithm relationship. That is, a model which is linear in parameters but not in variables. This is the most common model in empirical economics and finance and has a clear advantage over the linear model whose variables are restricted to being positive. Transforming those variables to logarithms now allows them to range from $-\infty$ to $+\infty$ and reduces the existence of heteroskedasticity in the residuals of the estimated model.

The estimation of elasticities has been an important statistic in applied economics since its early promotion by Marshall (1961). The nature of the linear model in government expenditure and aggregate income has important implications for the share of government expenditure in GDP through the estimated elasticity of government expenditure with respect to GDP. For example, if the quantity of government expenditure demanded increases proportionately more than the increase in aggregate income, then the share of government expenditure in aggregate income must increase. The remainder of this section draws heavily on Appendix B, where there are some reasonable demonstrations of the various elasticity scores and the
implications for the share of government expenditure in income. This section will use these to show that outcomes in linear or log-linear empirical models can contradict the WH in terms of realistic values of the share of GE in GDP. A casual examination of the data also suggests that these models are not appropriate.

Consider a linear model without the state subscripts:

\[ GE = a + bGDP \]  

(4.5)

where \( a \) and \( b \) are the intercept and slope parameter respectively. In equation (4.5), and according to the WH, the parameter \( b \) is greater than 0. However, it is the value of the intercept \( a \) that is important in determining the elasticity score and the limits to the share, GE/GDP. There are three cases:

1. If \( a = 0 \) in (4.5) then the elasticity of GE with respect to GDP is equal to 1. This is constant as GDP increases; see Appendix B equation (B.6), so that GE grows in equi-proportion with GDP. In this case, the share GE/GDP remains unchanged as GDP increases.

2. If \( a > 0 \) then the elasticity of GE with respect to GDP will be less than one, but will approach one from below as GDP limits to infinity. That is, GE grows proportionately less than GDP and approaches equi-proportionate growth with GDP for very large GDP values. This must mean that the ratio GE/GDP falls as GDP grows. However, the rate of decline in GE/GDP will fall so that as GDP becomes very large GE/GDP approaches zero; see Appendix B Figure B-3 and Figure B-4. Whilst GE
grows as GDP grows, the rate of growth and the change in the value of the share GE/GDP seems to contradict Wagner’s observations.

3. If \( a < 0 \) then the elasticity of GE with respect to GDP is greater than 1, but falls toward 1 as GDP approaches infinity. Thus, as GDP grows, GE grows proportionately more, and the share GE/GDP increases as GDP increases. However, as GDP grows, the elasticity approaches equi-proportion so that the rate of increase of GE/GDP slows and reaches an upper limit as GDP becomes large; see Figure (B-6) in Appendix B.

Figure 4-1 gives a scatter plot of GE/GDP against GDP per capita measured in real constant purchasing power parity US dollars. This data is drawn from the IFS and is the data that will be used to estimate the empirical model. An examination of Figure 4-1 shows that the scatter increases from bottom left to top right, ruling out the relationship of (1) and (2). Whilst (3) is asymptotic it could lead to the possibility of a limiting share greater than 1.

**Figure 4-1:** Scatter Graph of GDP per Capita and GE/GDP
Logarithmic models are non-linear in variables and linear in parameters. The model:

\[ \ln GE = a + b \ln GDP \]  \hspace{1cm} (4.6)

where \( \ln \) is log to the base \( e \), has frequently been estimated as a test of the WH. Again, the main test has been the sign and significance of the slope parameter \( b \), where \( b > 0 \) is deemed consistent with Wagner. However, \( b \) is the constant elasticity of GE with respect to GDP; see Appendix B equation (B.9) to (B.13). If \( b > 0 \), it opens up three possibilities for elasticity scores, with clear implications for the share GE/GDP.

1. \( b = 1 \) and GE grows in equi-proportion to GDP so that GE/GDP is constant as GDP grows.
2. \( b < 1 \), so that GE grows proportionately less than GDP, which must mean that GE/GDP declines toward zero as GDP increases.
3. \( b > 1 \) and GE grows proportionately more than GDP as GDP grows, so that GE/GDP increases without limit as GDP increases. This is quite clearly impossible.

One can only conclude that the specification is not an appropriate test for the WH as long-run outcomes are not realistic in terms of government share in the economy and/or are not consistent with Wagner’s views on the share of government in the economy. Finally, whilst casual empiricism must not be over indulged, a cursory examination of Figure (B-1) shows that none of the three elasticity scores apply to the data. The scatter does seem to increase from lower left to top right, but does not seem to follow the improbable explosive case, case (3).
There are two semi-logarithmic forms:

\[ \ln GE = a + bGDP \quad (4.7) \]

and

\[ GE = a + b \ln GDP \quad (4.8) \]

In both equations \( b > 0 \) is consistent with the WH. The elasticity score for (4.7) is \( b \) GDP (see Appendix B equations (B.14) to (B.16)). Thus, the elasticity of GE with respect to GDP grows as GDP grows, leading to the explosive case, case (3) of the logarithmic form, where GE/GDP grows without limit. Alternatively, equation (4.8) gives an elasticity score of \( b/GE \) (see Appendix B equation (B.17) to (B.19)). With \( b > 0 \), and therefore as GDP grows, GE increases so that the elasticity of GE with respect to GDP decreases as GDP increases. This leads to case (2) of the logarithmic and linear in level forms where the ratio GE/GDP falls toward zero. Again an examination of Figure 4-1 suggests that neither of these cases apply to the existing data.

This section has strongly directed the empirical model away from the linear and log-linear forms of modeling GE against GDP. The clear argument here is that the outcomes in terms of the share of GE in aggregate income are not realistic or consistent with shares in the WH. To overcome these problems it is better to model the WH in terms of the relationship of GE/GDP against aggregate income, such as GDP or GDP per capita. Modeling GE/GDP directly allows the choice of functional forms which lead to sensible share outcomes in the limits of GDP. However, there is a good empirical reason why modeling GE/GDP directly will be more appropriate
rather than regressing GE against GDP. This is the issue of the endogeneity of regressors and it is dealt with in the next section.

4.2.3 Endogeneity of Regressors

There is a potential identification problem that arises with the linear specification or the log-linear specification of the WH where those models involve GE as the dependent variable and GDP as the independent variable. This is because GE and GDP may display strong simultaneity (Slemrod, 1995). Simultaneous determination of the left and right hand side variables in an equation leads to a biased estimate if OLS regression is used. A Gauss /Markov condition is that

\[ E\{\varepsilon | x\} = 0 \]

where \( \varepsilon \) is the model disturbance and \( x \) is an independent variable in the OLS model. That is, the regressors in the model are said to be exogenous. If an independent variable is simultaneously determined then the conditional expectation of \( \varepsilon \) is not equal to zero and the covariance between the disturbance and the right hand side variable is not zero. In this case the right hand side variable is deemed an endogenous regressor and the estimate of the slope parameter associated with it by OLS will be a biased estimate (Verbeek, 2000). Recall equation (4.5) of the previous section:

\[ GE = a + bGDP \]

This relationship considers the impact of economic development measured by GDP on the level of government expenditure measured by GE. However, income in the Keynesian model is not exogenously determined and is given as:

\[ GDP = C + I + GE \] (4.9)
In a closed economy, national income results from the summation of consumption $C$ and investment $I$ and government sector $GE$. If equations (4.5) to (4.8) are estimated by OLS then the estimated parameter $b$ if we consider equation (4.9) will be biased. That is, both $GE$ and $GDP$ are endogenous variables, which are jointly determined. Because $GE$ influences $GDP$ in equation (4.9), one can no longer argue that $GDP$ and the error disturbance $\varepsilon$ are uncorrelated in (4.5).

There are several ways of overcoming this problem that are by now well documented in the econometric literature. Firstly, (4.5) could be determined using instrumental variables (Greene, 2003). Alternatively, if (4.5) and (4.9) can be augmented with variables so that the system is identified, then the equations can be estimated simultaneously using full information maximum likelihood or three stage least squares. In time-series, a clear way to overcome this problem would be to introduce a lagged variable income or lagged variable income per capita as an instrumental variable in the two stage least square model (Greene, 2003, p.74). However, the problem is simply not one of time-series, the identification problem arises just as much in cross-sectional models as it does in time-series models.

One way to overcome the problem would be to re-specify the model so that one drives out simultaneity. A potential way to do this would be to replace $GE$ with the share of $GE$ in GDP. Now having the share $GE/GDP$ and $GDP$ or $GDP$ per capita on the right hand side could drive out any simultaneity. In terms of the WH, $GE/GDP$ can be driven by $GDP$ in the long run, but an issue here is; could this relationship be reversed in the long run? If this is the case, then there would be simultaneous determination and therefore endogeneity might well remain.
There is an interesting strand in the literature on long run growth models which seem to suggest that there might well be a reverse relationship. The issue of endogeneity is examined in a series of articles starting with Fölster and Henrekson (1999). In this article the authors model the relationship between economic growth, measured by the log difference in GDP, and the size of the government sector, measured by the share of government in the GDP. Using a panel of international data they estimate a linear growth model in the tradition of endogenous growth theory. Particularly they are testing that there is an inverse relationship between economic growth and government size where an increase in government size is thought to slow down economic growth. In response to this, Agell, J., et al. (1999) argue that this growth model is flawed because causation can be reversed. Further, they use the WH as an indication of this reverse causation, stating that economic growth, through the income elasticity of demand for government services determines the size of government in the economy. Further, Agell, Jonas, et al. (2006)³ re-estimate the Fölster and Henrekson model, but use instrumental variable estimation rather than ordinary least squares regression to drive out potential simultaneity. Interestingly they find contradictory results under different measures of economic growth. For one particular measure of GDP growth they find there is no significant relationship between GDP growth and the size of government expenditure, but for an alternative measure they find a significant relationship. If this is the case, then empirically one would argue that evidence for potential endogeneity in the Wagner model through a long run growth model is contradictory.

³ Thanks are due to an unknown examiner for pointing out this reference.
One can also question the potential for simultaneity in the Wagner variables in terms of questioning the hypothesis tested by Fölster and Henrekson. They argue that there is a (negative) relationship between government size and growth in GDP because of a supply side long run “crowding out”. That is, the public sector deters investment in the supposedly more efficient private sector, leading to lower rates of economic growth. This is reflects a lively debate amongst economic policy analysts in Sweden where there has been an extreme policy shift away from the public to the private sector. It could be argued here that the current empirical literature on this inverse relationship is still too small and contradictory to indicate that those who support this crowding out argument in growth are correct. Again the evidence for simultaneity is not there.

It is also important to examine the two potentially related models in terms of their dependent and independent variables. On the one hand the growth model has the change in the logarithm of GDP as the dependent variable and the share of government in the GDP as an independent variable. The general specification to test the WH suggested in this section places the share of government in the GDP as the dependent variable and the level of GDP per capita as the independent variable, not the growth of GDP measured by the log difference: It is important to make this distinction. The WH is based on the income elasticity of demand for social goods, and therefore the level of social goods and its share in income will be determined by the level of income. This is not the same as the growth in income. In principle economies with high GDP per capita levels may exhibit low or high levels of economic growth. This clearly would not be the case if convergence in growth exists.
However the empirical case for convergence is questionable, see for example Quah (1993)

It is not possible to prove endogeneity. However, one may suspect endogeneity and the existence of simultaneous determination is ground for suspecting endogeneity. The argument here is that, whilst initially the case for endogeneity in the Wagner variable might seem attractive in terms of economic growth models, closer scrutiny reveals significant doubts and that the case for simultaneous determination is not there. Testing the WH through a model with GE/GDP as the dependant variable and GDP per capita as a general specification is therefore currently appropriate. The remaining sections of this chapter will examine the general and specific forms of an empirical model of the WH incorporating measures of government in terms of the share of GE in GDP.

4.2.4 Proportionate Measures of Government Activity in the Wagner Hypothesis

This thesis suggests that Wagner proposes that, as a nation experiences increasing levels of income, an increase must occur in government expenditure which will exceed the rate of increase in income. That is, as GDP increases, the share of government expenditure in GDP, given by GE/GDP must increase. There is enough evidence in the translations of Wagner’s reasoning to indicate that this was the circumstance that he envisaged. Clearly, such a model must scale changes in demand to account for demand pressures on government brought about by demographic change. The thesis uses per capita income figures in real terms to capture the effect of population growth. This is a more comprehensive interpretation of the WH and
recasts that hypothesis in proportionate measures of government expenditure and the
development of the economy. That is, the general form of the WH in equation (4.4) is modified using ratios to yield the following general form of the WH:

\[ \frac{GE}{GDP} = f \left( \frac{GDP}{P} \right) \]  \hspace{1cm} (4.10)

Wagner has often been criticized for not being specific in the nature of the relationship he hypothesized. The thesis argues that the share specification more closely approximates the Wagnerian rationale. Further, the thesis is not alone in this. Since Musgrave (1969) several other researchers have placed the focus of attention on the relative rise of government expenditures rather than or the change in the absolute level of government expenditure. Following Musgrave (1969), the rise in the share of government expenditure in income should lead to an absolute rise in the levels of government expenditure, but it is not necessary that a rise in the levels of government expenditure will lead to a rise in the share of government expenditure in income. Unfortunately, chapter 3 did not arrive at a definitive outcome in terms of prior empirical studies of the WH. Thus, for example, the survey of the empirical literature did not unambiguously show that the share specification is more appropriate by producing relatively more empirical results which did not contradict the WH. However, it can equally be argued that these same results do not provide evidence that the shares specification is inappropriate because it gave mixed to relatively fewer empirical outcomes which contradicted the WH.

It is also more appropriate to measure the growth in income in terms of income per capita. This is well established in the literature of modern growth theory. That
literature invariably measures the growth of the economy with the growth in GDP per capita (see for example Romer (2001)). In existing studies, the share is estimated to a linear function of real GDP per capita

\[ \frac{GE}{GDP} = a + b\left(\frac{GDP}{P}\right) + \epsilon \]  

(4.11)

where all variables and parameters are as previously defined in this chapter. Specification (4.11) avoids the potential endogeneity of regressors’ problem by modeling \( GE \) as a share of \( GDP \). However, a problem with this specification referred to in chapter 3 is that it will allow the predicted values of the proportion to be greater than one or less than zero if the levels of per capita income move far enough on the X-axis. In reality, such share values cannot exist. A key Gauss-Markov condition in the OLS regression is that the model disturbances are independently and identically distributed. That is, the residuals will be homoskedastic. This cannot be when modeling a zero to one variable with a linear model as the squared errors will be a function of the value of the dependent variable (Judge, Hill et al., 1988). Further, the small sample requirement of normally distributed errors for hypothesis testing will be violated in modeling zero to one data using OLS. Both of these issues, therefore, make the standard use of hypothesis tests questionable in linear models of share data, such as that in (4.10). This does not mean that the linear model cannot be used. However it does mean that tests must be made robust to heteroskedasticity.

Wagner explicitly recognized that the economy is a market economy of two sectors, public and private. Further, the share of government expenditure in income cannot
grow forever or more than the economy as a whole. Based on this fact, modeling
government as a share of GDP requires a model based on a function with range (0, 1). That is, the predicted share must fall within the zero to one range and the predicted share of government expenditure in income cannot lie outside the interval (0, 1). A useful general specification in modeling (0, 1) data is the sigmoid curve, which has been used extensively in biological growth models. The remainder of this section will examine sigmoid structures generally.

The sigmoid process implies that the growth rate of the share increases to a maximum before steadily declining to zero. Graphing the share of government expenditure in GDP with respect to changes in GDP per capita result in an “S” shaped curve as depicted in Figure 4-2. As a generalization, at extremely low levels of GDP per capita a true sigmoid shape would give shares approaching zero. Then, as GDP per capita rises, the share value rapidly increases, until it reaches a point of inflection. Beyond this the increase in share value slows, until it approaches a maximum.

**Figure 4-2:** Stylised Sigmoidal Growth Curve
Figure 4-3: Stylised Growth Rate Curve

Figure 4-2 gives the textbook sigmoidal growth curve. The point of inflection in the curve occurs for a zero \( x \) value. For negative values of \( x \), \( y \) increases from a lower limit of zero at an increasing rate. For positive values of \( x \), \( y \) still increases but the rate of increase declines and eventually \( y \) approaches a maximum limit, which in this case is one. Figure 4-3 gives the first derivative \( dy/dx \). This increases to the point of inflection at \( x = 0 \) and then declines symmetrically.

Empirical modeling of the WH requires determining specific functional forms to be estimated. The two sigmoid forms that will be used to model government share will be the logistics and Gompertz functions.
4.3 The Sigmoid Curves

4.3.1 A Brief Historical Review

The S-shaped curves have been used in three general research fields: demographics, biology and economics. In the first two research fields, the sigmoid curves have been used to describe the evolution of populations and in the last field to model, in general, economic growth. The curves are used because of their ability to describe these processes and display their typical phases: developing, inflection and saturation. Without doubt, the expressive power of the curves’ graphic representation has favoured their application.

The Belgian mathematician Verhulst was the first to introduce the logistic function to describe population growth in early nineteenth century. Almost a century later, in 1920, Pearl and Reed rediscovered the logistic curve in the course of their biological study of the evolution of fly populations. Gompertz’s original work was first presented at the Royal Society in London in 1825 (Cramer, 2003). In 1903, the French sociologist Gabriel Trade was perhaps the first to use a sigmoid curve in an economic field to analyze economic growth in relation to innovation. Other scholars have followed up Trade’s ideas during the first half of the twentieth century.

Fisher and Pry (1971) applied a logistic diffusion innovation model based on the analogy between epidemic spread and information circulation. Similar research was also adopted by scholars such as Blackman (1973), and Sharif and Kabir (1976). Further, sigmoid curves have also been applied to model market demand analysis.

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Case (1974), for instance, examines the application of the logistic growth process to modelling market demand functions. The logistic growth process is considered very appropriate for modelling economic growth curves. Meade and Islam (1995) compare the forecasting performance of different growth curve models. They applied seventeen different models to forecasting the development of telecommunication markets, represented by 25 time-series describing telephone access in 15 different countries. Their results showed that the logistic model performed significantly better than any of the other growth models.

The sigmoid curves have also been used in more recent research. Reati (1998) uses both the logistic and the Gompertz curves to model the spread of technological revolutions. Foster and Wild (1999) show how the logistic equation can be used for modeling growth curves in the presence of self-organizational change. The logistic growth process has also been used to model economic relationships that had for a long time been modelled as linear forms. Honda and Suzuki (2000) for instance, estimated an investment function by applying the logistic model. Their results strongly supported the use of a logistic model to estimate investment functions which appeared to follow non-linear relationships rather than linear ones. Aoki and Yoshikawa (2002) capture the role of demand as a mechanism limiting growth itself, using logistic curves to describe the time utility of the new products. Kejak (2003) employed the logistic model to test the relationship between the education sector and human capital formation. The logistic model was able to explain the non-linear relationship between the two variables which were the focus of the model, with the relevant parameters testing as significant.
The extensive use made of sigmoid curves is undeniable. However, no attempt has been made to model the growth of government expenditure using sigmoid curves with one exception. Florio and Colautti (2003) tested the WH in the USA, the UK, France, Germany, and Italy during the period from 1870 to 1990. The Florio and Colautti results confirm a pattern of similar curves in those countries and suggest that the logistic view of growth of government gives a better fit to the observed data. The limited application of the sigmoid models for relationships such as the WH in the existing studies might be due to two main reasons. The existing empirical studies have tested the WH using linear tools either in variables or parameters which did not allow for sensible limits to the ratio of government expenditure in GDP. As well, estimation difficulties of fitting the sigmoid function to actual data in the past have limited its practical application in economic relationships. However, with the development of computational procedures and the availability of software packages, it has become possible to apply the S-shaped growth processes to those economic relationships. The next section will develop the logistic growth process to a more robust model that will represent the recasting of the Wagner’s relationship presented in chapter 2.

4.3.2 The Logistic Function

The prior literature on the logistic process provides a convenient methodology for analyzing the growth trends of government expenditure in the economy. Specifically, this thesis employs a logistic function of market share of government expenditure across a sample of countries and over a short time to describe the dynamic process of

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5 A discussion of their statistical model will be explained later in this section.
government growth in the economy. The model assumes that the proportion of government expenditure in income $GE/GDP$ increases at an exponential rate until it approaches a limit which slows down the growth; this either produces the standard S-shaped curve, or the upper tail of this curve. In other words, it is assumed that the level of real per capita income directly affects the growth rate of the ratio of government expenditure to income across countries. Then, as national real per capita income grows across countries and over time, the proportion of government expenditure in income increases and the distance to the steady state declines.

It seems reasonable to expect the dynamic process of government expenditure growth in the economy to follow this logistic pattern, but with the time index replaced by the government expenditure growth inducing force, GDP per capita. That is, it is a statistical representation of a dynamic process as GDP per capita develops over time. The basic hypothesis is that the share of government expenditure in the economy is positively influenced by the changes in levels of real per capita income. In the following development of the logistics equation to be used in the empirical model, for the sake of notational simplicity, let $y_{it}$ denote the proportion of government expenditure in income (GE/GDP) in country $i$ at time $t$, and $x_{it}$ denote real per capita income (GDP/P) in country $i$ at time $t$.

*The Single Parameter Logistics Function*

The simple single parameter logistics function is:

$$y_{it} = \frac{1}{1 + e^{-x_{it}b}} \quad (4.12)$$
so that $0 \leq y_{it} \leq 1$. Equation (4.12) is a convenient way to represent a proportion dependent variable $y_{it}$, and $x_{it}$ represents the right hand side variable usually modelled as time. The upper limit or the saturation level of the function is set to 1 where the growth of $y_{it}$ stops. The parameter $b$ represents the slope of the function at $x_{it}$, or one can call it the growth rate. Note that the negative of the conditioning variable $x_{it}$ occurs in the equation so that positive values of $b$ indicate that $y_{it}$ is increasing with changes in value of $x_{it}$ and negative values of $b$ are associated with $y_{it}$ decreasing from a maximum of one and approaching a minimum of zero as $x_{it}$ increases. Even though (4.12) has been designated the “single parameter” function, this nomenclature is not strictly true. The combination of the scalar variable and parameter $-x_{it}b$ could be replaced with the vector combination $-\mathbf{x}_{it}'\mathbf{b}$, where $\mathbf{x}_{it}$ is a $1\times k$ column vector of conditioning variables and $\mathbf{b}$ is a $1\times k$ column vector of “slope” parameters. In this way (4.12) can be said to be a k parameter function. The term ’single parameter’ has been adopted for logistics functions with slope parameters only. The empirical model in this thesis only has one slope parameter, because the focus is on the single conditioning variable, GDP per capita. It is important that the logistic function is not confused with the logit regression. The logit regression is in general a result of binomial (two possible outcomes) data. The users of the logit regression set a goal to predict the value of the binomial via the probability of scoring a value of one in the binomial, conditioned on the determining variables. That is, in the logit model the dependent variable can only take on the value 0 or 1, whilst, in the single parameter logistics model, the dependent variable will be limited to the range $0 \leq y_{it} \leq 1$. The single parameter logistics function is represented in Figure 4-4.
Figure 4-4: Stylised Single Parameter Logistics Function

Figure 4-4 is divided into four panels. The upper left panel gives the logistics function estimated by equation (4.12) with the slope parameter \( b = 0.05 \) and \( x \) ranging from -100 to 100. This curve is replicated in the other three panels as the solid line. Note that the curve is symmetric about the \( y \)-axis and that it has a positive slope so that \( y \) is increasing with \( x \) from a lower bound of zero to an upper bound of one. In the upper right panel, the dotted curve has been estimated with \( b = -0.05 \). That is, a negative relationship so that \( y \) decreases from an upper bound of one to a lower bound of zero as \( x \) increases. The lower left panel compares a slope parameter of \( b = 0.1 \), the dotted curve, with \( b = 0.05 \), the solid curve. Note that increasing \( b \) accentuates the increase of \( y \) as \( x \) increases. In other words, the curve becomes steeper and \( y \) moves from its minimum to maximum over a smaller interval of \( x \). The
opposite case is given in the lower right panel where \( b=0.03 \), the dotted curve, is compared with \( b=0.05 \), the solid curve. Decreasing \( b \) reduces the speed of increase in \( y \) over \( x \). The curve then becomes less steep and \( y \) moves from its minimum of zero to the maximum of one over a larger interval of \( x \).

The slope parameter \( b \) will be important in the logistics function as a test of the WH. For the WH to hold the estimated parameter \( b \) must be significant and greater than zero which indicates that as GDP per capita increases (\( x \)), GE/GDP (\( y \)) increases. The slope parameter \( b \) will be referred to as the “Wagner” parameter.

*The Two Parameter Logistics Function*

Equation (4.12) can be augmented with an additional parameter, giving the two-parameter model:

\[
y_{it} = \frac{1}{1 + c e^{-x_{it}b}}
\]

The parameter \( c \) acts to shift the curve around the \( y \) axis and is an important parameter in orienting the Wagner logistics function in the GE/GDP and GDP/P space. Figure 4-5 illustrates how \( c \) undertakes this orientation function.
Figure 4-5: Stylised Two-Parameter Logistic Function

The top left panel of Figure 4-5 replicates the curve of the top left hand panel of Figure 4-4. Please note that in the top left of Figure 4-5, $b=0.05$ and $c=1$. Setting $c=1$ in equation (4.13) gives the single parameter function of (4.12). That is, equation (4.12) is nested in (4.13). The top right hand panel of Figure 4-5 compares the solid curve of $b=0.05$ and $c=1$ with the dotted curve of $b=0.05$ and $c=0.1$. Note that reducing $c$ to less than one shifts the curve up the $y$ axis. That is, it increases the value of the intercept of the curve with the $y$ axis.

Figure 4-5, lower left panel, gives the opposite case and compares $b=0.05$ and $c=1$ with $b=0.05$ and $c=2$, where now an increase in the value of $c$ shifts the curve to the right. That is, the curve shifts down the $y$-axis and the intercept of the curve with the
$y$-axis decreases in value. Finally, the lower right panel compares the solid curve $b=0.05$ and $c=1$ with $b=0.03$ and $c=2$. The difference between both lower panels is that, even though both dotted curves have the same intercept with the $y$-axis, at that intercept the slope of the dotted curve in the right panel is less steep. That is, parameter $b$ does not govern the intercept of the curve with the $y$-axis.

The parameter $c$, the “intercept” parameter, has an important function in modelling the Wagner hypothesis in orientating the curve in the GE/GDP and GDP/P space. It is expected that modern economies, even with low GDP/P, will have some government expenditure and that, because GDP per capita can only be positive, the empirical model will fit a curve which emphasizes the upper right tail of the positive “S” curve and that the intercept with the $y$ axis will be relatively high. That is, the thesis anticipates estimated values of $c$ less than one, such as in the upper right panel in Figure 4-5.

*The Three Parameter Logistics Function*

The logistic function of (4.13) has an upper asymptote of one set as an exogenous value. However, implicit in Wagner’s relationship is the assumption that the economy is a market economy where both the government and the private sectors are active in the economy and provide goods and services. Thus, equation (4.13) is problematic in that it allows $y$ to approach one as $x$ increases without limit. This is not consistent with the WH. If the economy is to be a market economy, then there must be some limit to the share of government in GDP which is less than one. To solve this problem $y$ should be limited to some maximum value less than one which proposes an upper limit to the proportion of government expenditure relative to
income in a market economy. Hill, et al. (2001) added an extra parameter $a$ to equation (4.13) to give:

$$y_{it} = \frac{a}{1 + ce^{-x_{i}b}}$$

(4.14)

Equation (4.14) is a three-parameter logistic function\(^6\) which is a generalisation of the two-parameter model of equation (4.13) where $a$ may take on any value greater than zero. However, in the Wagnerian context, $a$ is the highest proportion of GE in GDP achieved and $a<1$ must prevail. As a generalisation, the curve has asymptotes $y\to0$ as $x\to-\infty$ and $y\to a$ as $x\to\infty$. Setting $a=1$ in (4.14) gives (4.13) which is therefore nested in (4.14). The parameter $a$ is unknown, but can be estimated for empirical tests of Wagner. That is, its value can be determined by the data. This is in keeping with Wagner, who recognised a limit to government in the economy, but was not willing to impose that limit himself. Empirical expectations are that estimates of the parameter $a$ would be positive, but less than one.

**The Four Parameter Logistics Function**

Perron (1997) added a lower asymptote which involves shifting the function vertically:

$$y_{it} = a_{0} + \frac{a}{1 + ce^{-x_{i}b}}$$

(4.15)

Equation (4.14) is nested in (4.15) by restricting the lower asymptote to $a_0 = 0$. That is, as $x \to -\infty$ then $y \to a_0$. However, $a$ is not now the upper asymptote, but rather shows the maximum shift from $a_0$ to the upper asymptote. That is, $y \to a_0 + a$, as $x \to \infty$. Figure 4-6 illustrates the effect of the upper and lower asymptotes on the logistics function.

**Figure 4-6:** Stylised Three and Four Parameter Logistic Function

As in the previous two figures the solid curve in the top left quadrant is the single parameter function where, $b=0.05$ and which is nested in the four parameter function, when $c=1$, $a=1$ and $a_0=0$. This is replicated as the solid curve in the other three panels for purposes of comparison. In the top right hand side of figure 4-6, the upper
asymptote $a$ is set to 0.5. The upper asymptote $a$ can be any number greater than zero and is not restricted to some level less than one. This is the case in the lower left hand panel where $a$=2. The lower right panel illustrates the explicit four parameter case where $a_0$=0.25 and $a$=0.25, giving lower and upper asymptotes of 0.25 and 0.5 respectively.

The Gompertz curve is an alternative to the logistics specification. Prior to dealing with the exact form of the empirical model to estimate and the estimation method, the next section examines the Gompertz specification in some depth.

4.3.3 The Gompertz Function

The Gompertz specification has a four-parameter curve which, with appropriate restrictions, has nested in it the three parameter, two parameter and single parameter specifications.

$$y = a_0 + a \exp\left(-e^{-b(x-c)}\right)$$

(4.16)

Setting $a_0$=0, $a$=1 and $c$=0 in (4.16) gives the one parameter model:

$$y = \exp\left(-e^{-b(x)}\right)$$

(4.17)

Figure 4-7 illustrates the one-parameter case and shows the effect on the curve of changing the slope parameter $b$. The top left hand panel shows the single parameter Gompertz with the slope parameter $b$=0.05. Like the logistics function, reversing the
sign in the Gompertz results in $y$ decreasing from an upper value of one to a lower value of zero as $x$ increases to its limit. This is the dotted curve in the top right panel.

**Figure 4-7:** Stylised Single Parameter Gompertz Function

Again, increasing the parameter $b$ results in a steeper sloped function and from its departure from the lower limiting value of zero, $y$ approaches its upper limit more rapidly. This is shown in the lower left panel with the dotted curve where $b=0.1$ compared to the solid curve where $b=0.05$. The opposite occurs with a reduction in the value of $b$. In the lower right panel, the dotted curve with the value $b=0.03$ approaches the value one less rapidly as $x$ increases. As in the logistics function, in an empirical estimation of a Wagner model with GE/GDP as the dependent variable
and GDP per capita as the independent variable, one would expect positive and significant estimates for $b$.

The parameter $c$ acts to orientate the curve in the $x, y$ space. This two parameter Gompertz function is illustrated in Figure 4-8. As in the one parameter case, the top left panel gives the control curve and the solid line there and elsewhere in Figure 4-8 is the single parameter curve with $c=1$. Decreasing the value of $c$ shifts the curve to the left and increases the value of the intercept of the curve with the $y$-axis. This is seen in the right top panel where for the dotted curve $c=-10$.

**Figure 4-8: Stylised Two-Parameter Gompertz Function**

The lower left panel shows the opposite, an increase in $c$ to $c=10$ shifting the intercept down and the curve to the right in the $x, y$ space. The dotted curve in the
lower right panel is an interesting case if compared to the dotted curve in the lower left. It should be noted that, even though $c$ is the same value, there is a shift in intercept, because the intercept value is determined by the product $cb$ and not $c$ alone. Whilst, the parameter notation has been standardised for the logistics and Gompertz functions it should be noted that, for a comparison of the intercept determinants, the following holds:

$$c_i \approx \exp(c_i b_g);$$

where subscripts $l$ and $g$ refer to logistics and Gompertz functions respectively. Because GDP per capita is non-negative and expecting government share to be of a reasonable size for low GDP per capita countries, one should expect negative values of $c$ in empirical estimation of the Gompertz specification as a test of the WH using pooled time-series across countries.

Finally, the three parameter and four parameter cases resulting in upper and lower asymptotes can be arrived at by setting $a$ to some number other than one, with $a$ now being the upper asymptote for the three parameter case, and $a$ to some number other than one and $a_0$ to some number other than zero, with $a_0$ as the lower asymptote and $a_0 + a$ as the upper asymptote for the 4 parameter case. The three and four parameter cases are illustrated in Figure 4-9.

Following the established convention, the upper left panel is the control, single parameter curve, with $a=1$ and $a_0=0$. This is replicated as the solid figure in the other panels. The three parameter case with a shift down in the upper asymptote from 1 to
0.5 is given in the upper right panel, where \( a=0.5 \). A shift up in the upper asymptote is given in the lower left with the dotted figure having \( a=2 \). The four parameter case is given by the lower right with a lower asymptote of \( a_0=0.25 \) and \( a=0.25 \), giving an upper asymptote of 0.5.

**Figure 4-9:** Stylised Three and Four Parameter Gompertz Functions

An empirical test of the WH using the Gompertz specification would allow the upper asymptote to be freely estimated. That is, one should use the three parameter specification at least. This would allow the data to determine the maximum share of GE in GDP where this share is the \( y \) variable and the estimated parameter would be expected to be positive, but less than one.
4.3.4 Choosing an Exact Functional Form for the Empirical Model

The previous two sections examined in some detail two potential specifications of a Wagner type model of government share in the economy. Both specifications were of the sigmoid type and both, the logistic and Gompertz, could be used to model government share in the economy as a function of some measure of economic development in the economy. The purpose of this section is to distinguish between the two functions and rationalise the exact specification in terms of the various different parameterisations to be used in the empirical analysis.

Whilst the logistics function is symmetric about its upper and lower tails, the Gompertz function is not. Comparing the single parameter logistics and Gompertz functions can easily show this. Figure 4-10 illustrates the logistics and Gompertz curves on the same graph for comparison.

**Figure 4-10: Stylised Logistics and Gompertz Curves**

![Stylised Logistics and Gompertz Curves](image)
The logistics function is drawn with the solid line and the Gompertz is drawn with the dashed line. The curves were drawn as single parameter specifications with the slope parameter set to 0.05 in both cases. The logistics function is symmetric about the upper and lower tails, with the curve intercepting the y axis at $y=0.05$. The Gompertz is skew symmetric about the upper and lower tails, intercepting the y axis at $y=0.35$.

However, this difference does not give an a priori argument for choosing a specific form for the empirical analysis. Therefore, both the logistics and Gompertz functions will be estimated using GDP per capita as the independent variable and the share of government expenditure in GDP, GE/GDP, as the dependent variable. This will allow the data to determine, in terms of, say, goodness of fit statistics, the better specification. The issue of the correct parameterisation remains to be resolved. It could be argued that the three, two and single parameter specifications are nested within the four parameter function and that this should be estimated as a suitably general model. Then redundant parameters could be rejected by suitable significance tests. However, there is a problem with this strategy. There will only be one independent variable, GDP per capita and this could lead to an over-parameterisation of the function relative to the slope parameter.

This can be rationalised in the following way. Even though GDP per capita is the important Wagnerian determinant of government share in the economy, it will not be the only determinant. Whilst these omitted variables may not be related to GDP per capita and, therefore, there will not be omitted variable bias, there will remain a high level of unexplained variance in the data. This unexplained variance would be
“squeezed” between two asymptotes, which may now dominate the specification, importantly overwhelming the slope parameter on GDP per capita, which is the focus of interest. Further, there is no need for a lower asymptote. The lowest share value would be ensured by non-negative GDP per capita and the intercept of the estimated curve with the y-axis; that is, the intercept parameter \( c \) in the logistics function and the interaction of the \( c \) and \( b \) parameters in the Gompertz function. Figure 4-11 is an example of a three-parameter logistic specification with non-negative GDP per capita values.

Figure 4-11: Stylised Three-Parameter Logistic Function with Positive Values

Thus a lower asymptote would be redundant. Therefore, a parsimonious, but suitable test model would be the three parameter specification, where the three remaining parameters would have important Wagner type functions:
i) Parameter $b$ indicates whether there is a smooth increase in government share as GDP per capita increases;

ii) Parameter $c$ in the logistics, along with $b$ in the Gompertz determines the orientation of the government share in GDP per capita. That is, it will determine the minimum share; and

iii) Parameter $a$ determines the maximum share, which should be some positive value, but less than one.

Florio and Colautti (2003) modelled the share of government expenditure using the logistic growth function. This was derived from the WH that the rate of change of the government expenditure over time is proportional to income. However, Florio and Colautti stayed rigidly with the growth concept and they estimated the share of government expenditure in GDP as a function of time. This can only be rationalised if all economies experience similar temporal growth processes in income, because only then can time be used as a proxy for the level of GDP per capita. There is no need to make this rationalisation and within the sigmoid type specifications of not just the logistic but also the Gompertz functions, GDP per capita can be used directly as the independent variable. Further, Florio and Colautti obtained a prior estimate of the upper limit on the proportion of GE/GDP for the countries in the sample. That is, the data was used to give an $a$ priori estimate of the upper share, and this was imposed on the specification as a restriction rather than allowing the simultaneous determination of all parameter values. This approach was rationalised through the Pigouvian effect from the supply side to set the maximum observed value of $GE/GDP$. In this thesis all parameters will be estimated simultaneously, in keeping
with Wagner’s view of an upper limit that would be determined from demand and supply side forces in the economy.

4.4 The Stochastic Model and the Estimation Method

Both the logistic and Gompertz functions of the previous two sections are intrinsically non-linear (Judge, Hill et al., 1988, p.498). That is, they are non-linear in parameters and variables and cannot be transformed into linear in parameters without loss of information. This, combined with the nature of the disturbances in the stochastic model, will determine the method used to estimate the parameters. A suitable stochastic model is given by:

\[ y_{it} = f(x_{it}, \theta) + \epsilon_{it} \]  
\[ (4.18) \]

where:

\( f(\ ) \) gives the non-linear function, which can either be the logistics or Gompertz;

\( \theta \) is a vector of the three parameters, \( a, b \) and \( c \); and

\( \epsilon \) is an additive stochastic disturbance term which is identically and independently distributed.

Recalling that the data are short time-series pooled across countries, subscripts \( i \) and \( t \) refer to country and time. Note that the model is a pooled model and these subscripts reference the dependent and independent variables only and not the parameters of interest. Cameron and Trivedi (2005) show that generally, fixed effects panel models
cannot easily be applied to the non-linear functions of this thesis due to the incidental parameters problem. In principle, a random effects panel model could apply, but as yet tractable estimation algorithms have not been developed. However, this is of no immediate consequence for this thesis, a non-linear pooled model with additive errors is suitable for testing the WH. However, future research could be developed along the lines of a panel model where the country specific effects are represented by a random variable which is not correlated with the right hand regressor. That is a panel random effects specification.

Two estimation methods can be used to estimate the non-linear model with additive errors. Firstly, one could use maximum likelihood methods. However, this would require putting some specific form to the probability density function for the additive errors in (4.18). Usually the additive disturbance is presumed to be identically and independently normally distributed; that is \( \varepsilon_{it} \sim \text{NII}(0, \sigma^2) \). Alternatively, (4.18) can be estimated without specifying the probability density function using non-linear least squares (NLS) regression, Judge, et al. (1988); that is \( \varepsilon_{it} \sim \text{II}(0, \sigma^2) \).

Methods of fitting the logistic model other than NLS are inefficient and might give diverse estimates of the parameters even while accounting for a large part of the variance of the observations Oliver (1964). Therefore, the least squares criterion will be used to estimate the model parameters, with the objective from (4.18) of minimising by choosing:

\[
S(\theta) = \sum_{i=1}^{T} \sum_{t=1}^{T_i} (y_{it} - f(x_{it}, \theta))^2
\]

(4.19)
That is, choose the parameters $\theta = (a, b, c)$ to minimise the sum of squared residuals.\(^7\)

Note: the summation of the squared residuals over the $n$ countries occurs over $T_i$ time periods, where the subscript $i$ on the maximum time period $T$ denotes that $T$ will vary from one country to another. That is, the data comprises a pooling of unbalanced time-series observations.

The first order conditions for a minimum are:

$$\frac{\partial S(\theta)}{\partial \theta} = 0$$  \hspace{1cm} (4.20)

where:

$$\frac{\partial S(\theta)}{\partial \theta} = \left[ \sum_{i=1}^{n} \sum_{t=1}^{T_i} \frac{\partial S(a)}{\partial a} , \sum_{i=1}^{n} \sum_{t=1}^{T_i} \frac{\partial S(b)}{\partial b} , \sum_{i=1}^{n} \sum_{t=1}^{T_i} \frac{\partial S(c)}{\partial c} \right]^T$$

A necessary condition for the asymptotic property of consistency in the NLS estimator is that there exists a unique global minimum for $S(\theta)$. However, in NLS it is usually not possible to obtain the estimated parameters analytically by directly solving the first order conditions because closed-form expressions for the NLS estimators do not exist. This is the case for the two non-linear specifications logistics and Gompertz, and parameter estimation will follow numerical methods. This issue is aptly summarised by:

\(^7\) Bates and Watts (1988) provide a comprehensive reference on non-linear regression and NLS estimation.
“The difficulty of obtaining closed-form expressions for NLS estimators is, perhaps, one of the main reasons why many textbooks have little or no coverage of NLS. As a result, nonlinear regression is often neglected in statistics courses, even though nonlinear regression is widely used in applied research” (Taur and McCulloch, 1999).

A broad estimation strategy can be adopted, as follows. Firstly, choose a suitable numerical procedure to estimate the parameters. Write a procedure in GAUSS V 5.0 and use this procedure to estimate the parameters, standard errors and test for homoskedastic errors. Secondly, use software with “built in”, but different, numerical algorithms to estimate the parameters to validate the estimates. This is particularly important, there is no guarantee in numerical methods that a unique global minimum has been achieved, so that validation by other software goes some way to establish the robustness of the estimates. The two alternative softwares chosen were SHAZAM V. 9.0, Whistler and White (2001) and STATA SE V. 9.0. The non-linear command was used in SHAZAM, which estimates the parameters using likelihood criterion and uses the Broyden, Fletcher, Goldfarb and Shanno (BFGS), quasi-Newton procedure to estimate the parameters. STATA has two specific commands to estimate logistics and Gompertz functions. Both use least squares criterion. Both start with the standard Newton procedure and then switch after initial iterations to the BFGS quasi-Newton method.

The numerical method written in the programming language GAUSS was the Gauss-Newton procedure (Seber and Wild, 2003, Ch. 2&14). Using the notation of Seber and Wild the parameter set is updated using:
\[
\theta_{j+1} = \theta_j + s \left( F'_j F_j \right)^{-1} F'_j \left( y - f(x, \theta_j) \right)
\]

where:

\[
F_j = \frac{\partial f(\theta_j)}{\partial \theta_j}
\]

which is, in this case, a \(3 \times \sum_{i=1}^{T_i} T_i\) matrix of partial derivatives of the logistic and Gompertz functions with respect to the parameters \(\theta = \{a, b, c\}\) for each \(i\) observation. The subscript \(j\) refers to the iteration number and \(\theta_j\) is the parameter vector with value at iteration \(j\). The scalar \(s\) is a manually set step length, which was either set at one or less than one. This is a crude, but effective, approximation to Hartley (1961) and usefully overcame convergence problems because of the scale difference between parameter \(b\) and the other two parameters \(a\) and \(c\). This Gauss-Newton algorithm was a useful counterpoise to the pre-programmed alternatives in SHAZAM and STATA. Finally, writing the numerical procedure in GAUSS facilitated the estimation of heteroskedastic robust standard errors. This and other aspects of the estimation routine, will be returned to in the results chapter.

4.5 Concluding Remarks

This chapter has detailed the empirical method in terms of specifying two empirical models to be used to test the WH, the criterion to be used in estimating the parameters in those models and the softwares and algorithms in fulfilling the criterion. Two non-linear functional forms in the sigmoid family have been
proposed, the logistics and Gompertz functions. The exact specification of these two functions was detailed as the three-parameter specification. Each of these parameters in both specifications was shown to have important implications for the WH and the expected signs and relative values of these parameters were identified in terms of empirical results consistent with the WH.

This chapter has shown that the proposed sigmoid specifications have significant advantages over previous empirical specifications used to test the WH. In this way, the thesis provides further insights into the econometric literature around the WH.

Further, the least squares principle was determined as the main estimation criterion for the empirical model, where the parameters are to be estimated using the Gauss-Newton algorithm. This algorithm will be coded using the programming language GAUSS, but results are to be validated using pre-programmed software using alternative algorithms and, in one case, the maximum likelihood method as an alternative to least squares.

Prior to examining the empirical outcomes and identifying their implications for the WH it is important to examine the nature of the data used in the analysis. This will form, along with a description of data sources, the subject of the next chapter.
5 Chapter 5: Data Source, Transformation and Description

5.1 Introduction

This chapter comprises a relatively small, but nonetheless important, chapter describing the data used in the empirical analysis. The first section deals with the source of the base data from which the data used in the analysis is derived. This section also defines the base data, an important aspect of comparative studies. The following section considers the transformation of the data series into ratios and a common currency. A focus here is on the problem of deriving a sensible common currency unit in constant prices internationally as well as temporally. The final section then gives some important descriptive statistics on the transformed data used in the empirical analysis. This section should give an indication of the relative scales of the data which is important for giving an orientation to the empirical results in Chapter 6.

5.2 Source of Base Data and Base Data Definitions

The data employed in this study is obtained from the International Financial Statistics (IFS) which is produced by the International Monetary Fund (IMF). The IFS database collects data for around 183 countries over time spans starting from 1945 on a variety of subjects including government finance, national accounts and population. The specific data series obtained from the IFS for this study are GDP, the

1 Available online via http://ifs.apdi.net/imf/ifsbrowser.aspx/
GDP deflator (GDPD), GE, and P. The definitions of these series are presented in Table 5-1.

Table 5-1: Data Series Definitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>Nominal Gross Domestic Product in national currency</td>
</tr>
<tr>
<td>GDPD</td>
<td>GDP deflator (base year 1995=100)</td>
</tr>
<tr>
<td>GE</td>
<td>Nominal Government Expenditures: includes all types of government expenditure in national currency</td>
</tr>
<tr>
<td>P</td>
<td>Population</td>
</tr>
</tbody>
</table>

An alternative data source is the Penn World Tables (PWT)\(^2\) which has data on important economic series for a range of different countries. This source was not utilised because, at the time of data collection, the IFS source had more recent observations than the PWT. However, this study uses the 1995 Purchasing Power Parity (PPP) exchange rate from the PWT to compile the income per capita for all countries in 1995 US dollars as a common numeraire. Short time-series from the IFS for the period from 1990 to 1997 for a large number of countries was used\(^3\). Longer series backwards would have led to data attrition, with a loss of many countries. A similar problem arises extending to observations after 1997. Limiting the data to the period 1990-1997 resulted in as wide a sample of countries as possible. Further, not only do short wide panel data sets have the advantage of maximising the country

\(^2\) PWT version 6.1 available online via [http://pwt.econ.upenn.edu/](http://pwt.econ.upenn.edu/)

\(^3\) See appendix D for a list of these countries
sample, but they also have the advantage, through short time-series, of bypassing the issue of non-stationary series.

The following points with regard to the nature and definition of the data series must be borne in mind:

1. For all countries in the current data set, the IFS collect GE data inclusive of transfer payments. There are two exceptions, the United States and the Netherlands, where expenditures and lending minus repayments are used instead.

2. All the data series were obtained in national currency. In the European Union (EU) countries, GDP data series are expressed in national currency through to 1997 and then in Euros from the beginning of 1998. Because of potential sample selection bias in EU attrition in extending the sample from 1998 forwards, this study limited the sample period to 1997.

3. All government finance series in this sample arise from the central government budgetary accounts, and will exclude state and local governments.

4. The IFS collects data series on GDP using unadjusted data except for a small group of countries that use seasonally adjusted data. The reporting country determines this, and it is beyond the control of the IFS. These countries are Australia, Canada, France, Germany, Italy, Japan, Mexico, Netherlands, New Zealand, South Africa, Spain, Switzerland, and the United Kingdom. Further, GDPD prices are also expressed in unadjusted constant prices except for the previously mentioned countries, where the GDPD prices are expressed in seasonally adjusted constant prices. This
adjustment again is determined by the reporting country and is beyond the control of the IFS.

Fortunately this last point has no implications for the empirical analysis. The important thing here is that the seasonal adjustment method for a particular country does not change over the time period. If this happened then the data would be not be comparable over time. However, the data is comparable across countries because the data are annual and, therefore, seasonal adjustment will make no difference in international comparison.

However, points one and three have implications for the empirical analysis. Strictly speaking, following point one, the GE data is now not comparable for the Netherlands and the USA and the rest of the sample. However, rather than lose these two countries they were retained despite the fact that these differences would lead to a higher level of unexplained variance in the data.

Point three affects all countries, and there are two problems. Firstly, the measure of government in this, like other studies, is limited to fiscal government. Further, this fiscal measure is also limited to central government only and, therefore, it generally underestimates the true fiscal government. Secondly, there will be different strengths of centralisation versus decentralisation in government function internationally. Thus, countries that have devolved more government functions to state and local government will appear to have less fiscal public sector than those with highly centralised government. This problem will, therefore, lead to a higher level of unexplained variance in the empirical model, where these differences cannot be
 accounted for. Remarkably, despite these differences, in the empirical model approximately 20% of the variance in the ratio of GE to GDP was explained by the variance in real GDP per capita.

Prior to their use in the empirical model, the data series from the IFS have to be transformed into useful ratio form and adjusted for changes in domestic and international prices to enable international comparison. These data transformations are dealt with in the next section.

### 5.3 The Data Transformations

The measure of fiscal government size used in the study is $GE$, which includes transfer payments. Further, this measure refers to central government expenditure with the exception of Austria and the USA where it is restricted to the federal government only. The data series for the share of $GE$ in $GDP$ denoted by ($GEGDP$) in each country was arrived at by simply expressing the value of $GE$ as a ratio to $GDP$, where both are measured in nominal domestic currency. That is:

\[
GEGDP = \frac{GE}{GDP}
\]  

5.1

$GDP$ per capita in common US purchasing power parity dollars is used as a measure of economic development for each country. That is, real $GDP$ per capita. This was calculated in the following way. Firstly, $GDP$ in real terms ($RGDP$) for each country was obtained by dividing the nominal $GDP$ in national currency value by its $GDPD$ prices with base year 1995. That is, $RGDP$ in 1995 domestic prices:
Clearly this RGDP series is an inappropriate measure of income for use in pooled time-series and international cross-sectional data and the next step is to convert these real series into a common numeraire for comparative purposes.

Simply converting all RGDP to a common currency, such as the US dollar, by using contemporary nominal exchange rates will introduce a bias due to fluctuations in the nominal exchange rate. That is, changes in RGDP, but expressed in US dollars, will not only contain changes in output but will also be due to changes in the demand for, and supply of, the US dollar relative to the domestic currency. For instance, Ward (2004) noted that exchange rates do not behave according to an assumed purchasing-power-parity theory and a uniform currency basis such as U.S. dollars, based on nominal exchange rates, is not comparable across countries. An appropriate comparison of national incomes should be based on the proper purchasing power parities, PPP, where the real or PPP exchange rate at any time \( t \) is:

\[
rex = \frac{pr^*}{pr}
\]  

(5.3)

where \( rex \) is the real exchange rate as the number of foreign currency units to a domestic currency unit and \( pr \) and \( pr^* \) are the domestic and foreign price indices respectively.
A correction for PPP is necessary to give a real GDP measure that is directly comparable across countries as well as over time. One of the generated series in the PWT is GDP at “international” prices. These are the weighted averages of the relative prices of all the countries in the world. They are scaled so that the total GDP of a base country measured in international prices is equal to the base country’s GDP expressed in its own domestic currency. The United States serves as the base country in International Comparison Parities (ICP) so that the US dollar is the numeraire.

There are several studies that have emphasized the importance of using constant international prices in cross-country comparisons. Summers and Ahmad (1974), for instance, provided estimates of relative GDP for 101 countries of the world in 1970 by using a numeraire country through PPP. Heston, Alan and Summers (1993) compared the explanatory power of exchange rate converted GDP per capita versus PPP-converted GDP per capita for a variety of socio-economic variables. They concluded that the PPP converted measure performed better. Another study by Heston, Alan and Summers (1995) analyzed the price parities of a group of low-income countries and provided detailed price information about 35 developing countries. Prices were expressed as price parities of components of GDP at various levels of disaggregation in each country. For each component, the price parity was given as the ratio of the domestic price of the component to the United States price. Heston, Alan and Summers (1997) provided international comparisons derived from the various benchmark studies of the International Comparison Program (ICP). Purchasing power parities (PPP) published in the PWT have been applied by many studies. For instance, the IMF employed PPP converted GDP figures in preparing
estimates of regional and national growth rates for its world economic outlook in 1993.

The application of constant international prices has been recognized in previous cross-sectional studies of the WH. For instance, Gupta (1968) used per capita GNP in 1961 in international purchasing power parity prices. Gupta suggested that the use of nominal exchange rates tends to understate the level of income of low-income countries relative to that of high-income countries because the goods and services produced and consumed domestically in low-income countries are much cheaper, relative to the same goods and services in high-income countries.

Ram (1987) investigated the relationship between GDP per capita and the ratio GE/GDP through time-series and cross-sectional analysis for 115 countries. Ram suggested that comparative studies based on data derived from nominal exchange rates and current domestic prices will often introduce a systematic bias in inter-country comparisons. Therefore, Ram used a data set based on constant international prices. Bairam (1992) tested for the WH by employing a sample of twenty developed O.E.C.D countries during the period from 1950 to 1985. He again used constant international prices. Similarly Dao (1995) also tested the WH with a data set for consumption, output, and price levels at constant international prices.

Several studies have used PPP data directly from the PWT. Slemrod (1995) used GDP per capita data from the PWT to examine the relationship between government spending and the level and growth rate of per capita income. Alleyne (1999) tested

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\[\text{Footnote: For more details see Gupta (1968, p.31)}\]
for the existence of the WH for the Caribbean countries of Jamaica (1953-1991),
He employed data from the PWT expressed in constant international prices.

To avoid the problem of a nominal exchange rate, this study uses the 1995 PPP rate
from the PWT to compile the real US dollar per capita GDP from the real GDP in
domestic currency in 1995 domestic prices. These tables do not report the rate
directly. However, the tables contain the Price Level of GDP in US Dollars and the
nominal exchange rate (local currency to one unit of the US dollar). Following the
data appendix to the PWT, the PPP in Domestic Currency per $US for GDP may be
obtained by dividing the Price Level of GDP in US Dollars in the PWT by 100 and
multiplying by the nominal annual exchange rate given in the PWT. This was done
for each country in the study for the year 1995, so that:

\[
\text{PPP}_{1995} = \frac{\text{PLGDPUS}_{1995}}{100} \times \text{nex}
\]  

where, \( \text{PPP}_{1995} \) is the 1995 purchasing power parity exchange rate, \( \text{PLGDPUS} \) is the
1995 domestic Price Level of GDP in US Dollars and \( \text{nex} \) is the nominal exchange
rate. Note that (5.4) is the reciprocal of (5.3).

A numeraire for \( RGDP \) would be \( RGDP \) expressed in 1995 purchasing power parity
dollars (\( RGDPUS \)), which is obtained by dividing \( RGDP \) of equation (5.2) by
\( \text{PPP}_{1995} \) obtained in equation (5.4):
Finally, income must be scaled to country size and dividing $\text{RGDPUS}$ by country population to give real GDP per capita does this. The resultant series, henceforth designated as $\text{RGDPUS}$, is the independent variable used to condition the dependent variable the ratio $\text{GE/GDP}$, in the empirical model. This latter will, from now on, be referred to as $\text{GEGDP}$.

5.4 Descriptive Statistics

Table 5-2 presents summary statistics of the dependent variable $\text{GEGDP}$ and the independent variable, real GDP per capita, $\text{RGDPUS}$ described in the previous section. The table reports the number of observations, mean, standard deviation, minimum and maximum, and the coefficient of variation (CV)\(^5\) for each of the dependent and the independent variables. There are 677 observations for the $\text{GEGDP}$ variable with 27 missing observations recorded out of the possible 704 potential observations, which is 8 years by 88 countries. On the other hand, there are 694 observations for $\text{RGDPUS}$ with 10 missing observations recorded. Seven of the 10 missing observations for $\text{RGDPUS}$ also appear in the missing years and countries for $\text{GEGDP}$, giving a total of 674 observations for the empirical analysis.

The CV indicates that there is a significant variation in the independent variable $\text{RGDPUS}$, reflecting the gap between rich and poor. The dependent variable $\text{GEGDP}$ has a standard deviation of 0.1053 and mean of 0.2765 which indicates that $\text{GEGDP}$

\[\text{RGDPUS} = \frac{\text{RGDP}}{\text{PPP}_{1995}}\]  

\(^{5}\) The coefficient of variation CV is defined as the standard deviation over the mean.
has a relatively wide spread from its mean given that, by definition, \(0 \leq GEGDP \leq 1\).

The skewness of the distribution of \(GEGDP\) is 0.4593; thus the positive median is less than the mean so that the distribution is skewed to the right if compared with a normal distribution\(^6\). Kurtosis is a measure of the heaviness of the tails of a distribution. A normal distribution has a kurtosis of 3. Heavy tailed distributions (peaked) will have kurtosis greater than 3 and light tailed distributions (flat) will have kurtosis less than 3. The kurtosis score is 2.5718 for GEGDP which is close to, but a little flatter than, a normal distribution.

**Table 5-2:** Summary Descriptive Statistics for the Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
<th>CV</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEGDP</td>
<td>677</td>
<td>0.2765</td>
<td>0.10529</td>
<td>0.05896</td>
<td>0.5846</td>
<td>0.381</td>
<td>0.3803723</td>
<td>2.571884</td>
</tr>
<tr>
<td>RGDPUS</td>
<td>694</td>
<td>8653</td>
<td>7449</td>
<td>648</td>
<td>29872</td>
<td>0.861</td>
<td>0.7974882</td>
<td>2.326175</td>
</tr>
</tbody>
</table>

The relationship between the mean value and the minimum and the maximum values of \(RGDPUS\) reflects the expected skewed international distribution of income. The skewness of the distribution of \(RGDPUS\) at 0.7974882 is larger than that for \(GEGDP\) and, again, the positive median is less than the mean so the distribution is skewed to the right and significantly more so than \(GEGDP\). This indicates that there is not a sample selection bias toward the rich nations.

Table 5-3 gives values for both variables at the different percentiles. This table again points to a right skew in the distribution of both variables, the median values (50%)

---

\(^6\) A normal distribution would have a skewness of zero and a kurtosis of 3.
point) for both variables are less than the mean values. The difference between the median and mean for RGDPUS is large indicating a significant skew. This is also borne out by the relative values for the other percentile points for RGDPUS.

Table 5-3: Data Levels at Various Percentile Points

<table>
<thead>
<tr>
<th>Percentile</th>
<th>GEGDP</th>
<th>RGDPUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.1212</td>
<td>846.55</td>
</tr>
<tr>
<td>10</td>
<td>0.1478</td>
<td>1084.04</td>
</tr>
<tr>
<td>25</td>
<td>0.1923</td>
<td>2493.97</td>
</tr>
<tr>
<td>50</td>
<td>0.2717</td>
<td>5923.915</td>
</tr>
<tr>
<td>75</td>
<td>0.343</td>
<td>14388.23</td>
</tr>
<tr>
<td>90</td>
<td>0.4191</td>
<td>20524.09</td>
</tr>
<tr>
<td>95</td>
<td>0.4621</td>
<td>21835.24</td>
</tr>
</tbody>
</table>

Figure 5-1: Kernel Density Estimate GEGDP
Figure 5-2: Kernel Density Estimate \textit{RGDPUUS}

Figure 5-1 illustrates a kernel density estimate of \textit{GEGDP} confirming that it is slightly right skewed, but also showing evidence of bimodality. Figure 5-2 is a graph of a kernel density estimate of \textit{RGDPUUS}. This confirms significant right skewness. Interestingly, the estimated density also suggests some bimodality giving an indication that in the paired sample data there is a relationship of some form between the two variables.
5.5 Concluding Remarks

The data set for the empirical study is drawn from 88 countries for the years 1990-1997. Of the potential 704 observations for the short wide pooled data set, 674 are available with 30 effective missing country/year observations. The data are compatible over time and across countries. They have been transformed in an explicit fashion to suitable ratio form. Further, the data measured directly in currency have been transformed to reflect real currency and international prices necessary for intertemporal and international comparative studies. Descriptive statistics show that the data is a representative sample and the data distributions are those that would be expected for a representative international sample.
6 Chapter 6: Analysis and Discussion of Results

6.1 Introduction

Chapter 4 presented a methodology that gave an empirical content for the theoretical framework presented in chapter 2 and at the same time built on, but avoided the methodological issues which have appeared in the existing empirical studies presented in chapter 3. Chapter 4 specified a general functional form of the WH, the sigmoid shape, which limits shares to sensible values between zero and one. Further, two sensible forms were identified, the logistics function and the Gompertz equation.

Chapter 5 identified the sources, and described in detail the nature, of a data set of pooled cross-section, short time-series, a “short, wide, unbalanced” annual data set from 88 countries over the years 1990-1997 inclusive. These data comprised important Wagner variables in the form of the dependent variable \( GEGDP \), the share of government expenditure in GDP and \( RGDPUS \), real GDP in 1995 US dollar purchasing power parity prices.

This chapter presents the results of the empirical analysis of the data of Chapter 5 using the functional forms, logistics and Gompertz, identified in Chapter 4. The remainder of this chapter is organized as follows. Section 6.2 gives the results for the estimation of the four parameter logistics and Gompertz functions as a counterpoise to the preferred three parameter models. This section also examines in some detail the issue of over parameterisation in the four-parameter model with one independent
variable. This is followed by section 6.3 which reviews the results of the estimates of
the three parameter model. This section also analyses in some depth the convexity of
the objective function in the least squares estimator in some important parameter
spaces to establish the properties of the parameter estimates. Section 6.4 deals with
the issue of heteroskedastic errors in the model estimates and the computation of
heteroskedastic robust standard errors for this non-linear model. Finally, section 6.5
summarises the main findings of the chapter.

6.2 The Four Parameter Logistics and Gompertz Functions

6.2.1 The Results of the Estimated Four Parameter Models

In chapter 4 it was suggested that estimating the four-parameter versions of both the
Gompertz and logistics functions would over parameterize those functions. The issue
here was that a single independent variable allows for a lot of unexplained variance
and that the four parameter function would be Procrustean, squeezing that
unexplained variance within two very close bounds with the possible crowding out of
the slope parameter. However, for completeness of analysis, the four-parameter
specifications for both functions were estimated. Recall the four parameter logistics
function (4.15) but with an additive disturbance:

\[ GEGDP_{it} = a_0 + \frac{a}{1 + e^{RGDPUS_{it}}} + \varepsilon_{it} + \varepsilon_{it} \sim \text{II} (0, \sigma^2) \quad (6.1) \]

Following the notation of chapter 5, the dependent variable, GEGDP, is the ratio of
government expenditure to GDP and the independent variable, RGDPUS, is real
GDP in 1995 purchasing power parity dollars. For the remainder of this chapter the
stochastic term, $\varepsilon$, will be identically and independently distributed with mean zero and variance $\sigma^2$, unless otherwise stated.

This form proved difficult to converge, but (6.1) can be re-arranged to give a form which proved more tractable to convergence:

$$GEGDP_{it} = a_0 + \frac{a}{1 + e^{-b(RGDPUS_{it} - c)}} + \varepsilon_{it} \quad (6.2)$$

This form was estimated in an attempt to force the function to find a lower asymptote for $GEGDP$, using the pooled sample of 88 countries over the period from 1990 to 1997. The following four-parameter Gompertz function was also estimated over the same pooled sample:

$$GEGDP_{it} = a_0 + ae^{-b(RGDPUS_{it} - c)} + \varepsilon_{it} \quad (6.3)$$

In both cases the non-linear least squares estimator failed to converge using the Gauss Newton routine written in the programming language GAUSS V 5.0. However, for both cases, convergence was achieved using the pre-programmed non-linear least squares estimator of STATA SE V 9.0. This software initiated the search with a standard Newton-Raphson algorithm and then switched to the slower, but more accurate, Broyden, Fletcher, Goldfarb, Shanno (BFGS) quasi-Newton algorithm. Finally, convergence was achieved, to the same estimated parameters, using the pre-programmed maximum likelihood estimator of SHAZAM V 9.0. This
software assumed \( \varepsilon_{it} \sim \text{IIN}(0,\sigma^2) \) in (6.3) and used the BFGS algorithm in maximization.

The estimated parameters are given in Table 6-1. The first column gives the name of the parameters. The second and fourth columns give the estimated parameters for the logistics and Gompertz functions respectively. The third and fifth columns give, for the logistics and Gompertz functions respectively, two sets of standard errors for the estimated parameters, with the heteroskedastic robust standard errors in brackets. The Huber-White sandwich estimator of the heteroskedastic robust standard errors modified for non-linear models will be examined in detail later in the chapter.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Logistic Function</th>
<th>Gompertz Function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>SE</td>
</tr>
<tr>
<td>( a_0 )</td>
<td>0.2362*</td>
<td>0.0073 (0.0076)</td>
</tr>
<tr>
<td>( a )</td>
<td>0.1036*</td>
<td>0.0046 (0.0042)</td>
</tr>
<tr>
<td>( b )</td>
<td>0.0967</td>
<td>1.01 (0.5060)</td>
</tr>
<tr>
<td>( c )</td>
<td>7981.56*</td>
<td>171.02 (83.40)</td>
</tr>
<tr>
<td>( R^2 \pm )</td>
<td>0.23</td>
<td>0.23</td>
</tr>
</tbody>
</table>

*significant p=0.01, \( \pm \) \( R^2 \) between fitted and actual

Both the upper, \( a \), and the lower, \( a_0 \), asymptotes are significant at the 1% level, as is the orientation parameter, \( c \). Further, note that the equivalent parameters for each
model have very similar estimated values. The estimated $R^2$, which is the $R^2$ between the fitted and actual, is higher than the $R^2$ for the three-parameter model. Unfortunately, the slope parameter, $b$, the one designated as the Wagner parameter, is not significant, even at the 10% level. That is, one cannot dismiss the null hypothesis that $b=0$. You will recall from the four-parameter model in Chapter 4 that the upper asymptote is $a_0+a$. As $a>0$, low $RGDPUS$ economies are associated with low $GEGDP$ and high $RGDPUS$ economies are associated with high $GEGDP$ scores. However, because of the uncertainty surrounding the slope parameter, it must mean that at some critical region in $RGDPUS$, the ratio $GEGDP$ suddenly shifts up from the low to the high score. Whilst this is not inconsistent with Wagner who, you will recall, saw a rapid increase in government with economic development, such rapidity seems unlikely.

6.2.2 A Critique of the Results of the Estimated Four Parameter Models

The results of the previous subsection can be explained through the connection of two issues:

a.) Having a single explanatory variable; and

b.) The over parameterization of the model.

A careful examination of the scatter graph of the data and the estimated logistics function in Table 6-1 shows the problem of the coincidence of these two factors. Please note that, because of the similarity of parameter values, the fitted Gompertz would track the fitted logistics and there is no need to include it in Figure 6-1.
Figure 6-1 clearly shows that the four-parameter function splits the sample into two, and that the division between the two sub samples occurs at the approximate mean value of $RGDPUS$. The inclusion of the four parameters forces a lower asymptote for sample values below the mean of $RGDPUS$, and this is brought about at the expense of a smooth transition from lower to upper value. Additional explanatory variables would act to reduce the unexplained variance and the result would be a smoother transition from low to high values of the ratio $GEGDP$.

Alternatively, in the absence of additional explanatory variables, a smooth transition from low to high values might be obtained by allowing for the possibility that both functions intercept the $GEGDP$ axis above the lower asymptote. That is, allow the lower asymptote to occur as if there were negative values of $RGDPUS$. A way to do this is to fix the lower asymptote to zero. That is, estimate three parameter logistics
and Gompertz functions. The results of these estimators are examined in the next section.

6.3 The Three Parameter Logistics and Gompertz Functions

6.3.1 The Parameter Estimates

Recall the three-parameter logistics function of Chapter 4, equation (4.14) but now with an additive disturbance and with Wagnerian dependent and independent variables:

\[ GEGDP_{it} = \frac{a}{1 + e^{-RGDPUS_{it} + b}} + \varepsilon_{it} \tag{6.4} \]

The parameters to this form were estimated directly using the non-linear least squares criterion with the Gauss-Newton algorithm specifically written in GAUSS V 5.0 code. The re-parameterisation:

\[ GEGDP_{it} = \frac{a}{1 + e^{-b(RGDPUS_{it} - c)}} + \varepsilon_{it} \tag{6.5} \]

was estimated using a pre-programmed non-linear function in STATA SE V9.0. As in the four-parameter model this function found those parameters that minimized the sum of the squared residuals using a mixture of the Newton-Raphson and the BFGS algorithm. This was checked against a comparable estimate using the pre-programmed command in SHAZAM V.9.0 which estimated the model parameters by
maximizing the likelihood function based on joint normal disturbances using the BFGS algorithm.

Finally, the three-parameter Gompertz function, Chapter 4, equation (4.16):

\[
GEGDP_{it} = ae^{-(b(RGDPU_{it} - c))} + \varepsilon_{it}
\]  \hspace{1cm} (6.6)

with additive disturbance was estimated using the least squares criterion in a GAUSS V5.0 coded Gauss-Newton algorithm. The results for all three estimates are given in Table 6-2.

**Table 6-2: Results for the Three Parameter Logistic and Gompertz Functions**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Logistics Function Equation (6.5)</th>
<th>Logistics Function Equation (6.6)</th>
<th>Gompertz Function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>SE</td>
<td>Estimate</td>
</tr>
<tr>
<td>(a)</td>
<td>0.3419*</td>
<td>0.0101</td>
<td>0.3418*</td>
</tr>
<tr>
<td></td>
<td>(0.0112)</td>
<td></td>
<td>(0.000)</td>
</tr>
<tr>
<td>(b)</td>
<td>0.00019*</td>
<td>0.00004</td>
<td>0.00019*</td>
</tr>
<tr>
<td></td>
<td>(0.00004)</td>
<td></td>
<td>(0.000)</td>
</tr>
<tr>
<td>(c)</td>
<td>0.7492*</td>
<td>0.0804</td>
<td>-1519.23**</td>
</tr>
<tr>
<td></td>
<td>(0.0740)</td>
<td></td>
<td>(688.75)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.20</td>
<td></td>
<td>0.20</td>
</tr>
</tbody>
</table>

*significant p<0.01, ** significant p<0.05 ± \(R^2\) between fitted and actual

All estimated parameters are significant at the 1% level with the exception of the orientation parameter \(c\) for the reparameterised logistics function, which is significant at the 5% level. These significance levels occur irrespective of the use of the normal or heteroskedastic robust standard errors. Importantly, the slope coefficient \(b\) is significant in all cases, indicating a smooth transition from some
intercept level of $GEGDP$ as $RGDPUS$ increases to a maximum level, which approximates to 0.34 for all three estimates.

**Figure 6-2:** Fitted Against Actual, Three Parameter Logistic Function

Figure 6-2 gives the actual and fitted GEGDP for the three-parameter logistics function. Given the close approximation of the estimated parameters for the logistics and Gompertz functions, the figure does not graph the fitted three-parameter Gompertz. In comparison to the four-parameter model, the $R^2$'s are lower for the three-parameter functions. However, this should be expected, additional parameters can only leave the amount of unexplained variance at the same level, or reduce it. Nor should be this interpreted as a way of choosing between the two different broad parameterizations. Figure 6-2 in comparison to Figure 6-1 shows that the three parameter specifications lead to a smooth evolution of $GEGDP$ with $RGDPUS$ which
tracks the data well and it does not artificially split the sample into a lower and upper sub sample as in the four parameter cases.

These results, particularly the significance and sign of the slope parameter $b$ for all three estimators in Table 6-2, are consistent with the WH. Moreover, even though there is still a high level of unexplained variance, approximately 20 percent of the variance in $GEGDP$ is explained by changes in $RGDPUS$. This is a very high proportion for a single explanatory variable in pooled cross-section short time-series.

6.3.2 The Marginal Effect of $RGDPUS$ on $GEGDP$

Given

$$GEGDP = f(RGDPUS, \hat{\theta})$$  \hspace{1cm} (6.7)

where:

the circumflex, $\hat{}$, denotes estimated;

$f$ is some functional form, in this case the logistics or Gompertz form; and

$\hat{\theta}$ is a vector of estimated parameters, $a, b, c$

then the estimated marginal effect of $RGDPUS$ on $GEGDP$ is:

$$\hat{me} = \frac{dGEGDP}{dRGDPUS} = f'(RGDPUS, \hat{\theta})$$  \hspace{1cm} (6.8)

The estimated marginal effect shows the effect of a unit change in $RGDPUS$ on $GEGDP$. It is the first derivative of the $GEGDP$ with respect to $RGDPUS$ and its
value is not only determined by the value of the estimated parameters \( \hat{\theta} = [\hat{a}, \hat{b}, \hat{c}] \) but also by the level of RGDPUS at which it is evaluated. Table 6-3 is the estimated marginal effect of RGDPUS on GEGDP for the estimated three parameter logistics and Gompertz functions of (6.4) and (6.6) respectively. These marginal effects are scaled to a $100 increase in RGDPUS and are estimated for the mean and various percentile values of RGDPUS in the data set.

**Table 6-3: Estimated Marginal Effects**

<table>
<thead>
<tr>
<th>Equation</th>
<th>GDP/Cap</th>
<th>Logistics</th>
<th>Gompertz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>8770.93</td>
<td>0.00071</td>
<td>0.00068</td>
</tr>
<tr>
<td>25%</td>
<td>2565.95</td>
<td>0.00140</td>
<td>0.00144</td>
</tr>
<tr>
<td>50%</td>
<td>5985.14</td>
<td>0.00102</td>
<td>0.00097</td>
</tr>
<tr>
<td>75%</td>
<td>14576.66</td>
<td>0.00028</td>
<td>0.00028</td>
</tr>
</tbody>
</table>

For example, a $100 increase in RGDPUS will result in an increase of 0.00068 of GEGDP for the estimated Gompertz specification. Given the general sigmoid specification, one would expect the marginal effect to be higher for lower levels of RGDPUS and this is borne out by the data in Table 6-3. Further, because it is skewed in comparison to the logistics, the marginal effects for the Gompertz are higher at the low percentiles and smaller at the median and mean. However, at the upper levels of RGDPUS, because both functions have approximately the same asymptote, the marginal effects are the same. The estimated marginal effects of Table 6-3 are consistent with the WH showing high levels of expansion of government as the economy begins to grow, but with this expansion stabilizing to a fixed share for developed economies.
6.3.3 The Stability of the Parameter Estimates

A problem with the use of gradient methods to estimate parameters, by either minimising a least squares objective or maximising a likelihood objective, is that one is not sure that the estimated parameters reflect a global minimum or maximum if the objective function is not known to be strictly convex or concave. That is, setting some start values might lead to parameter estimates (convergence) which are not at the global minimum or maximum.

There are strategies that can be adopted to reduce this possibility. One is to use different start values to see if the same convergence properties (parameter estimates) are achieved from these different start values. Also, it is useful to use different gradient algorithms and, if possible, to estimate the same parameters using different criterion. All of these strategies were used in estimating the parameters. Different start values were utilised. Different gradient algorithms were employed and alternative softwares were used with least squares criterion and maximum likelihood criterion.

Once the parameters have been estimated, the behaviour of the objective function can be examined by doing a grid search over a reasonable range of values which span the estimated parameters. Further, it is possible to do contour plots of the objective function (least squares or likelihood) in two-parameter space, holding other parameters fixed at their estimated values. In a three-parameter model, this would necessitate drawing three contour plots. Two contour plots are examined in this section, one for the logistics function and the other for the Gompertz function, to
establish the robustness of certain parameter estimates. These are parameters $b$ and $c$ for the three-parameter models, holding parameter $a$ constant at its estimated value. There is a considerable difference in the scale of the two estimated parameters $b$ and $c$. Despite this difference in scale, both parameter estimates seem robust. Figure 6-3 shows that the sum of the squared residuals is convex over the grid with a single maximum centered on the two-parameter estimates. Figure 6-3 gives a contour plot of the sum of the squared residuals for the three-parameter logistics model in parameter $b$ and $c$ space with parameter $a$ fixed at its estimated value of 0.3419. The parameter estimates for equation (6.4) were obtained by minimizing the sum of the squared residuals and the Gauss-Newton algorithm was used to find these parameters. This figure shows the behavior of the sum of the squared residuals over a grid with ranges for parameter $b$ of -0.0006 to 0.0014 and for parameter $c$ of -1 to 5, with both ranges spanning the estimated parameters $b = 0.00019$ and $c = 0.7492$. The lowest contour, 6.05, is the small-unlabeled ellipse centered on these two values. The next highest contour level is 6.1. Contour levels increase at irregular intervals outside of this level passing from levels 7 to 11.

There is a considerable difference in the scale of the two estimated parameters $b$ and $c$. Despite this difference in scale, both parameter estimates seem robust. Figure 6-3 shows that the sum of the squared residuals is convex over the grid with a single maximum centred on the two-parameter estimates.
Figure 6-3: Contour Plot of the Sum of the Squared Residuals, Three Parameter Logistics Function, Over Parameters $b$ and $c$, with $a = 0.3419$

The Gompertz function is not quite as regular in the same parameter space. Figure 6-4 gives a contour plot of the sum of the squared residuals for the three parameter Gompertz model in parameter $b$ and $c$ space with parameter $a$ fixed at its estimated value of 0.3431. This figure is a grid with ranges for parameter $b$ of -0.0002 to 0.0008 and for parameter $c$ of -40 000 to 80 000, with both ranges spanning the estimated parameters $b = 0.00016$ and $c = -3442.46$. The lowest contour level is 6.1, marked as six in the figure with the maximum contour of 30, with intermediate contours at irregular levels of 6.5, 7, 8, 9, 10, and 20.
Figure 6-4: Contour Plot of the Sum of the Squared Residuals, Three Parameter Gompertz Function, Over Parameters $b$ and $c$, with $a = 0.3431$

There is a clear difference between Figure 6-3 and Figure 6-4, with the latter showing non-convex areas over the grid. Particularly, there are two minima, marked at A and B in the figure. The minimum at B is local, and does not fall below the contour level 8. The minimum at A is lower than B reaching the lowest contour of level 6.1, marked in the figure as 6. Minimum A is centred on the estimated parameter values of $b = 0.00016$ and $c = -3442.46$. Thus, the grid search of Figure 6-4 validates the parameter estimates of Table 6-2 for two parameters where their scales are quite different.

The grid searches of Figure 6-3 and Figure 6-4 indicate that the estimated parameters $b$ and $c$ for the logistics and Gompertz modes using least squares criterion are fairly robust. But this finding is subject to two considerations:
(a) The extent of the grid, and
(b) The “fineness” of the grid.

Similar grid searches are given in Appendix E for the remaining four parameter spaces and these searches indicate a similar robustness to the estimates. However, even though the estimated parameters are good estimates of the population parameters, the estimated errors proved not to be homoskedastic. Fortunately, all estimated parameters are significant using heteroskedastic robust standard errors. The test results for homoskedasticity are dealt with in the next section, as is the robust standard error estimation method.

6.4 Tests for the Null of Homoskedastic Errors and Robust Standard Error Estimates

6.4.1 Homoskedasticity Tests

The Lagrange Multiplier test devised independently by Breusch and Pagan (1979) and Godfrey (1978) is employed to test for heteroskedasticity. In this test, the estimated errors are retrieved from the main regression. Then an auxiliary regression of the square of the estimated errors from the model is tested against an $m \times 1$ vector of variables, $z_t$, including a constant:

$$\hat{\varepsilon}_t^2 = z_t' \hat{b} + v_t \quad v_t \sim \operatorname{II}(0, \sigma_v^2) \quad \{i=1,2,\ldots,n\}, \{t=1,2,\ldots,T_i\} \quad (6.9)$$

where,

\(\hat{e}\) is the estimated errors from the model under test; and
$i$ and $t$ are cross-section unit (in this case country) and time indices in a pooled sample.

The squared errors are used as sample counterparts to the unknown disturbance variance which may be a linear function of the variables $z_i$. These usually comprise some, or all, of the right hand side variables in the original model. The test statistic commonly used is $\sum_{i=1}^{n} T_iR^2 \sim \chi^2(m)$, under the null hypothesis of homoskedastic errors, where the summation of $T$ over $I$ from the interval 1 to $n$ is the pooled (unbalanced) sample size $R^2$ is the coefficient of determination of the auxiliary regression and the degrees of freedom $m$ is the number of regressors in the auxiliary (test) regression (Mittelhammer, Judge et al., 2000, p.536).

Even though the auxiliary regression (6.9) is linear, it need not be confined to testing for heteroskedasticity in linear models, but can be used to test for heteroskedasticity in all stochastic models that contain additive disturbances, and which are necessarily independently and identically distributed. That is, the test can be used to test for heteroskedasticity in the general stochastic model:

$$y_{it} = f(x_{it}, \beta) + \varepsilon_{it} \sim \mathcal{N}(0, \sigma^2) \quad \forall_{i} \{i = 1, 2, \ldots, n\} \quad \forall_{t} \{t = 1, 2, \ldots, T_t\} \quad (6.10)$$

This general model contains in it non-linear pooled models such as the logistic and Gompertz functions.
Within the context of the models within this thesis, to test for the null of homoskedasticity, the estimated errors were first retrieved from the main three-parameter logistics model of (6.4) and Gompertz model of (6.6). The square of these errors was then regressed against $RGDPUS$ with a constant.

An alternative to this dependent variable heteroskedastic test is one involving the estimated dependent variable. Here the square of the errors from the main equation is regressed against the fitted $GEGDP$ values with a constant. Again, the estimated test statistic is $\sum \frac{n}{T_{i}R^2} \sim \chi^2(m)$.

In both tests, a critical test statistic was obtained by simply taking the product of the sample size and the $R^2$ from each regression. The estimated test scores and probability values are given in Table 6-4. Columns two and four give the test scores and columns three and five give the $\chi^2$ probabilities for the logistics and Gompertz functions. Column one gives the form of the test regression.

### Table 6-4: Test for the Null of Homoskedasticity

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Logistics Model (6.4)</th>
<th>Probability (p)</th>
<th>Gompertz Model (6.6)</th>
<th>Probability (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RGDPUS$</td>
<td>$\sum_{i=1}^{n}T_{i}R^2$</td>
<td>9.211</td>
<td>0.0024</td>
<td>9.327</td>
</tr>
<tr>
<td>$GEGDP$</td>
<td>$\sum_{i=1}^{n}T_{i}R^2$</td>
<td>8.700</td>
<td>0.0032</td>
<td>9.008</td>
</tr>
</tbody>
</table>

Note that the test scores for the alternative parameterisation of the logistics model, equation(6.5), are not given as these simply replicate those for the model (6.4). In all
cases, $p<0.05$, so that the null of homoskedasticity can be rejected at the 5% level. That is, the estimated errors for both specifications are heteroskedastic and, under these conditions, it was determined to use heteroskedastic robust standard errors to test the significance of the estimated parameters in the three parameter logistics and Gompertz functions. The estimation of the robust standard errors are described in the next subsection.

### 6.4.2 Estimated Heteroskedastic Robust Standard Errors

The objective of this sub-section is to briefly describe the estimation of the robust standard errors used in hypothesis tests in the main model and which were necessitated by the presence of heteroskedastic errors. The estimation method is an augmentation of the standard Huber-White sandwich estimator.

Recall the $\sum_{i=1}^{n} T_i \times 3$ matrix of partial derivatives of the logistic and Gompertz functions with respect to the parameters $\theta = \{a, b, c\}$ for each $i$ observation of Chapter 4, section 4.4:

\[
F_j = \frac{\partial f(\theta_j)}{\partial \theta_j}
\]  

(6.11)

The usual standard errors are estimated as:

\[
se_\theta = \sqrt{\text{diag} \left( \hat{\sigma}^2 \left( \hat{F}' \hat{F} \right)^{-1} \right)}
\]  

(6.12)
where:

\[ \hat{F}_j \] is the matrix of partial derivatives of the logistic and Gompertz functions with respect to the parameters \( \theta = \{a, b, c\} \) for each \( it \) observation evaluated at the estimated parameters \( \hat{\theta} = \hat{a}, \hat{b}, \hat{c} \); and

\( \hat{\sigma}^2 \), the estimated variance, is the sum of the squared residuals divided by the sample size adjusted for the number of estimated parameters 3. That is, using the notation of Chapter 4:

\[
\hat{\sigma}^2 = \frac{S(\hat{\theta})}{\sum_{i=1}^{n} T_i - 3}
\]

For these standard estimates see, Judge, et al. (1988) and Seber and Wild (2003).

The Huber-White estimator of the robust standard errors, see Cameron (2005, P.155):

\[
se_{\mu_{HW}} = \sqrt{\text{diag} \left( \hat{\sigma}^2 \hat{V}_{HW} \right)}
\]  
(6.13)

where

\[ \hat{V}_{HW} = \left[ \hat{F} \hat{F}_j \right]^{-1} \hat{F}_j \text{diag} \left( \hat{\epsilon} \hat{\epsilon}^T \right) \hat{F}_j \left[ \hat{F} \hat{F}_j \right]^{-1} \]

and where \( \hat{\epsilon} \) is the vector of estimated errors in the model.
The usual standard errors estimated by (6.12) are reported in columns three, five and seven of Table 6-2. The heteroskedastic robust standard errors estimated by (6.13) are reported in parentheses in the same columns of that table. Moreover, all parameters are significantly different from zero at the 1% or 5% levels, so that outcomes are not inconsistent with the WH for the three-parameter models.

6.5 Summary

This chapter has presented the results of two sigmoid functions estimated with four and three parameters. The four parameter estimates, both for the logistics and Gompertz functions, were not inconsistent with the WH with the share of government expenditure in GDP, \( GEGDP \), increasing in real GDP per capita, \( RGDPUS \). However, that increase occurred over a range of very small incomes around the mean sample \( RGDPUS \). It is unlikely that Wagner envisaged such a sharp increase from a lower to a higher asymptote and the result is caused by the overparametrisation of the estimated equation in the face of a single explanatory variable.

The estimated three-parameter model provided for a smooth increase in \( GEGDP \) with increases in \( RGDPUS \) which are consistent with the WH. The overall results obtained using the two three parameter sigmoid functions were very encouraging. These functions appear to fit the data well. The t-values of the slope coefficients were significant and the overall fit \( (R^2) \) was relatively high, considering the large cross-section sample with one explanatory variable. Given these results, it is suggested that the logistic and the Gompertz functions perform quiet well in describing the observed S-shaped path of government growth in the economy across
countries. This, in turn, supports the use of sigmoid curves to model cross-country government growth over relatively short periods.

Underpinning this is the fact that the non-linear estimates appear to be robust. Post estimation grid searches confirmed that the estimates converged at sensible minima for the non-linear least squares estimates. Further, the results for the three-parameter models were replicated using different pre-programmed software with different objective function. Even though the estimated errors in the logistics and Gompertz models were subject to heteroskedasticity, correct hypotheses tests were made based on heteroskedastic robust standard errors.

Finally, no attempt was made to choose between the three-parameter logistic and three-parameter Gompertz specifications. Both have performed well. The estimated parameters were similar for both models and both performed equally well in describing a Wagnerian data generating process where the development of the economy significantly determines the relative size of government, albeit measured fiscally, in the economy.
CHAPTER SEVEN

7 Chapter 7: Summary and Concluding Remarks

7.1 Introduction

The growth of government expenditure is one of the most lasting issues in public economics literature. The main objective of this thesis has been to examine the effect of economic development on the growth of government expenditure share in the economy. The German economist Adolph Wagner was perhaps the first to propose a direct explanation that the expansion of government activity share in the economy responds positively to changes in economic development, so that as a country’s income rises, the size of that country’s public sector relative to the whole economy rises too. The attempt to test the WH in past years has, in turn, spawned a vast amount of empirical literature. The WH has not been contradicted in some of these studies, or for some of the countries tested, but in others it is either rejected or subject to inconsistent test outcomes. No clear pattern of results or consistent conclusions emerges from the tests. A range of key elements have been proposed as potentially important in determining Wagner’s intent. The empirical work on the WH has, however, generally been focused on developed countries and seems to prefer modeling the WH in linear forms either in variables or parameters.

This thesis has suggested looking at the variation of government growth in cross-country levels of income over short time periods. This framework is more comprehensive than other popular empirical approaches that have tested the WH. This cross-country approach to the WH, combined with the existence of a limit to the
growth of public expenditure was simply captured by sigmoid curves, namely, the logistic growth curve and the Gompertz function. This study has successfully applied empirical tests of the WH in non-linear form using a pooled cross-section of data over 88 countries from 1990 to 1997.

The analysis of the WH in non-linear form has, until recently, been limited. This provided a central motivation for the present study with the objective of adding knowledge to the existing literature in this field through non-linear empirical analysis. The thesis provided a detailed analysis of the non-linear growth process of government expenditure in market economies. Further, the findings of this cross-section study could form the basis for future research by using disaggregated components of government expenditure, and by looking at the secular growth of government expenditure in time-series analysis using the non-linear process such as the logistic and the Gompertz functions.

The aim of this chapter is to summarise the main findings of the study, to discuss some of their possible implications for public policy research and to identify areas for further research. The remainder of the chapter is structured as follows. Section 7.2 presents a brief review of the chapters produced in the thesis. Section 7.3 provides a discussion of the results and some of their possible implications. This is followed in section 7.4 by a discussion of the limitations of the study. Finally, section 7.5 offers some recommendations for further research.
7.2 Overview of the Thesis

Chapter 1 provided a general introduction to the topic of the WH and the motivation for the thesis. It established that the thesis was motivated by (1) the importance of, and the ongoing debate about, the WH within economic and political research, (2) a lack of detailed evidence about, and analysis of, the WH especially in cross-sectional studies of market economies, and (3) the particular scarcity of studies on the WH using non-linear methodology. The chapter also discussed the objectives and possible significance of the study.

Chapter 2 gave a consolidation of the WH based on a comprehensive review of Wagner’s work. Such a review was necessary and has been lacking in the extant literature. It also reviewed the existing interpretations of the WH and pointed out the key elements of differences in those interpretations.

Chapter 2 provided a detailed review of the WH using translations of Wagner’s writings and, in so doing, established the critical key elements of the WH as consistent as possible with his writings. Wagner’s original statement suggested that economic development in industrializing countries, such as Germany, Britain and the United States, in the late nineteenth century, was accompanied by a growing role of the state in society. Wagner based his hypothesis on several underlying assumptions, an organic view of the state being one of them. Wagner’s organic view considers the public economy represented by public activity as an organic part of the social system which will tend to grow, proportionally, with the growth of the whole economy. For Wagner, the growing role of the state in the late nineteenth century was consistent
with his organic view of the state, a view he shared with a number of other German intellectuals of the time.

Wagner identified three main forces that lead to government involvement in an industrializing economy. First, the demand for the enforcement of law and order, both internally and externally, increases as economies grow. Second, the demand for culture and welfare services such as health, education and other services increases government involvement as a collective producer of those services. Third, the participation of public ownership in material production increases because of new technical processes which often resulted in natural monopolies.

Whilst Wagner identified these three forces driving the expansion of the state in the economy, he explicitly recognized that, in market economies, private institutions would still be active in the economy and provide goods and services. Wagner applied his hypothesis to market economies where some activities are provided in the private sector and other activities are provided in the public sector. Both sectors, according to Wagner, function in a complementary relationship. In his original writings, he recognized that there is a limit to government growth. He recognized that the size of state activity in the economy would not grow without limit or more than the whole economy. Therefore, Wagner recognized that there is a maximum share of government in the economy that, at the least, will result from the taxpayer’s resistance to government growth. Wagner did not attempt to specify this limit \textit{a priori} and suggested that any attempt to set a limit to state expansion exogenously will fail.
Recent translations of the WH offered new insights in the area of government activity. Wagner was referring not only to state expenditures (and taxes), but also to the state as a regulatory institution. In other words, even though he tended to measure the state as a fiscal activity represented by state expenditure and taxes, Wagner also recognized the effect of regulation as another type of state activity. He did not include regulatory activities in the measurement of the state activity mainly because of the difficulty of measuring regulatory activity. The translations indicated also that Wagner did not specify the functional form of the relationship between the two economic variables. However, a sensible functional form representing the WH would need to be able to express a limit to the growth of the share of government expenditure in a market economy in the form of a maximum share of government in the economy.

Chapter 2 also reviewed the different interpretations of the WH that have been produced in the existing literature. All of these interpretations have related the growth in public expenditure to economic development which was seen to determine that expenditure. All existing interpretations of the WH have measured the state as a fiscal entity and none have considered the regulatory aspects of the state in their analysis. The existing interpretations of the WH have also not recognized the limits of government growth in the WH with the very notable exception of Florio and Colautti (2003). However, it could be argued here that Florio and Colautti have imposed a restriction to government growth from outside the demand model of the WH in the form of taxpayer resistance and that a careful examination of the WH leads one to conclude that Wagner envisaged an ‘organic’ limit to government offered by several forces in the economy. The recasting of the WH in this thesis
proposes that with the development process in the economy, represented mainly by the growth in per capita income, the fiscal state, represented by the share of government expenditures in national income, will increase at a rate higher than that of per capita income. However, this rate will fall until it is at the same rate of expansion as per capita income, so that the share of government expenditure reaches a maximum. Further, this saturation point is determined endogenously from demand side forces in the economy.

Chapter 3 reviewed the empirical test methods followed in the existing studies of the WH and tried to determine whether there were patterns of outcome in terms of the methods used. The chapter distinguished between the different types of econometric analyses followed in the existing studies of the WH. Those types of analyses have varied between time-series and cross-section. The time-series studies formed almost 85% of the existing empirical studies that tested the WH. Most of these studies were applied to developed and industrial countries. Cross-sectional studies naturally included more developing countries in testing the WH. Significant relationships in the statistical models of the WH were found in developed countries more than in developing countries, but this might reflect the heavy bias toward developed countries rather than be a function of the WH.

This chapter also reviewed the types of economy and time spans applied in the existing empirical studies. It was found that most of the empirical studies have tested the WH for developed and industrial countries. However, the majority of the existing studies have tested the WH for data after World War II and obtained contradictory results. Yet, it seems that testing the WH for countries in their industrialization phase
leads to considerable support for the WH. This thesis suggests that as economic
development continues, the relationship between the two economic variables, public
expenditure growth and economic development, may weaken with higher levels of
industrialization. However, this is incorporated in the WH, and simply reflects that
the relative shares of government and private sectors have been established.

Chapter 3 also reviewed the measures of the economic variables in the WH,
government size and economic development. The existing empirical studies followed
different methodologies in measuring government size. However, the majority of
these studies measured government at the aggregate level, where all expenditures are
included, and measured economic development by the growth of national income.
The measures of the economic variables have been affected by the different
interpretations of the WH. However, the majority of the empirical studies have
adopted the Musgrave version and measured government expenditure as a share in
income and economic development as per capita income.

Despite all the differences in the methods used in measuring the economic variables
and the variables used, it was established in chapter 3 that all of these measures have
been formulated in fiscal terms of the state. However, Wagner recognized also the
regulatory aspects of the state. Therefore, these studies might be considered partial
ones in the sense of the fiscal state and do not represent testing the WH for all state
activities.

This thesis suggests that the contradictory results of testing the WH might be related
to methodological issues other than the type of economy and time span. For example,
the existing empirical studies employed linear functional forms of the WH which do not reflect the changing patterns of public spending over time and across countries. This might have contributed to the mixed results of testing the WH since it does not reflect the stages of development in government growth and only provides constant elasticity in the log-linear form. Further, the linear models of the share versions of the WH in the existing empirical studies did not allow for a limit to the share of government expenditure in the economy. The majority of the existing empirical studies did not attempt to test for the limit to the share of government expenditure in income. Those that did attempt to test for it, such as (Florio and Colautti, 2003), tended to impose a limit from outside the model of the WH.

Chapter 3 showed that the use of modern cointegrating regression was a significant development in time-series tests of the WH. The thesis distinguished between those studies that used OLS regression and studies that used modern time-series techniques to test for the WH. However, this distinction of modern time-series techniques did not reduce asymmetric outcomes. That is, different studies using tests for cointegrating vectors found contradictory results, some rejecting the WH, others not being able to. Moreover, in this chapter a clear empirical argument was made for the use of pooled short time-series, as much as pure time-series analysis. This was because short time-series overcome the problem of non-stationary series, thus allowing non-linear models to be estimated by the least square criterion.

Chapter 4 detailed the empirical method in terms of specifying two empirical models that were used to test the WH, the criterion used in estimating the parameters in those models, and the software and algorithms used in fulfilling the criterion. Two non-
linear functional forms in the sigmoid family were proposed, the logistics and Gompertz functions. The exact specification of these two functions was detailed as the three-parameter specification. Each of these parameters in both specifications was shown to have important implications for the WH and the expected signs and relative size of these parameters were identified in terms of empirical results consistent with the WH. This chapter also showed that the proposed sigmoid specifications had significant advantages over previous empirical specifications used to test the WH. In this way, the thesis provides new insights into the economic literature around the WH.

Further, the least squares principle was determined as the main estimation criterion for the empirical model, where the parameters were estimated using the Gauss-Newton algorithm. This algorithm was coded using the programming language GAUSS, and the results were validated using pre-programmed software with alternative gradient algorithms, and, in one case, the maximum likelihood criterion as an alternative to least squares.

Chapter 5 provided a detailed description of the data used in the study. The data set for the empirical study was drawn from 88 countries for the period from 1990 to 1997. Of the potential 704 observations for the short wide pooled data set, 674 were available with 30 effective missing country/year observations. The data were transformed in an explicit fashion to suitable ratio form. Further, the data measured directly in currency were transformed to reflect real currency and international prices necessary for intertemporal and international comparative studies. Descriptive
statistics showed that the data was a representative sample and the data distributions were those that would be expected for a representative international sample.

Chapter 6 presented the results for two sigmoid functions estimated with four, and then three, parameters. The four parameter estimates, both for the logistics and Gompertz functions, were inconsistent with the WH with the share of government expenditure in GDP, $GEGDP$ increasing in real GDP per capita, $RGDPUS$. However, that increase occurred over a very small range of incomes around the mean sample $RGDPUS$. It is unlikely that Wagner envisaged such a sharp increase from a lower to a higher asymptote and the result was quite likely caused by the overparametrisation of the estimated equation in the face of a single explanatory variable.

The estimated three-parameter model provided for a smooth increase in $GEGDP$ with increases in $RGDPUS$, which is consistent with the WH. The overall results obtained using the three-parameter sigmoid functions were very encouraging. These functions appeared to fit the data well. The t-values of the slope coefficients were significant and the overall fit ($R^2$) was relatively high, considering the large cross-section sample with one explanatory variable. Given these results, it is suggested that the logistic and the Gompertz functions perform quiet well in describing the observed S-shaped path of government growth across countries. This, in turn, supports the use of sigmoid curves to model cross-country government growth over relatively short periods.
Underpinning this is the fact that the non-linear estimates appeared to be robust. Post estimation grid searches confirmed that the estimates converged to sensible minima for the non-linear least squares estimates. Further, the results for the three-parameter models were replicated using different pre-programmed software with different starting values. Even though the estimated errors in the logistics and Gompertz models were subject to heteroskedasticity, correct hypotheses tests were made based on heteroskedastic robust standard errors.

No attempt was made in chapter 6 to choose between the three-parameter logistic and three parameter Gompertz specifications. Both performed well. The estimated parameters were similar for both models and both performed equally well in describing a Wagnerian data generating process, where the development of the economy significantly determines the relative size of government, albeit measured fiscally, in the economy.

The results of chapter 6 tended to confirm a general convergence of state expenditure share in GDP relative to economic development. That is, the economic development process explained a considerable share of the observed variation in state expenditure growth. The sigmoid functions performed quite well in describing the observed path of government growth in the economy across countries. As the model performed well and the countries in the sample seemed to follow a non-linear process, the result can be interpreted as consistent with the suggested framework of the WH. This, in turn, supports the use of a non-linear, sigmoid approach to model state government growth over relatively short periods.
Finally, the empirical results showed that the independent variable was important in determining the growth of government share in income. This result was generally consistent with the WH and lends support to the logistic or Gompertz process of government growth. Following the discussion above, the principal conclusion was that the rise of the share of government expenditure in income in the cross-section sample of 88 countries could be explained, in significant part, by the fluctuations in the levels of real GDP per capita for those countries.

7.3 Implications of the Results

Based on the analysis of the theoretical discourse and findings presented in this study, several implications can be drawn about the growth of government expenditure size in market economies. First, this thesis developed a framework of the WH that provided an interpretation of the WH that is consistent with observation. The thesis reviewed the literature dealing with the evidence of the influence of economic growth on public activity in the context of the WH. The results of the existing literature have generally been mixed. An important preliminary theme here is that none of the existing empirical studies either positively supported or contradicted the WH. This thesis develops the argument that the way that the WH has been interpreted previously has been incomplete both on theoretical and empirical grounds. In this way the thesis has contributed to the literature on the WH.

Second, the current framework suggested that the size of public expenditure relative to the economy could not grow forever or more than the economy as a whole. Thus, this thesis modeled the WH to allow for a limit to the share of government expenditure in the economy and this limit has been determined endogenously from
inside the demand model of the WH by allowing existing data to determine this limit. This non-linear process of government growth has been captured by two sigmoid functions, the logistics and the Gompertz functions. In this process, the share of government expenditure in the economy is increasing at an increasing rate during earlier stages of economic development where the first part of the sigmoid curve is convex and reflects the growing role of government in the economy.

Finally, the evidence derived from the literature suggests that the WH holds in developed economies. In this cross-sectional study of 88 developed and developing economies, the WH is still provides an important explanation for developing economies. After all, real per capita GDP in developed economies reflects economic development more than that in the developing economies. Further, one would expect there to be a direct relation between the income and the size of government expenditure in the economy as Wagner hypothesized. The findings are broadly consistent with other studies of the WH in developed economies, but the study is robust enough to capture this relationship for developing economies as well.

7.4 Limitations of the Study

Like all other empirical studies of the WH, this study has its limitations. An obvious limitation is the measurement of government activity with fiscal measures only. Wagner had a twofold view of government in the economy, the regulatory government as well as fiscal government. However, the former cannot be included in this, or any other empirical study, until it is capable of a transparent, unambiguous measure which can be incorporated into empirical modeling.
It is important to note that, despite this imprecision in measuring the true extent of government in the economy, this empirical study found significant relationships. Any possibilities of incorporating regulatory government in empirical models in the future can only reduce the current level of unexplained variance or leave it unchanged under normal conditions. That is, it can only improve, or leave unchanged, existing significant results.

However it could be argued that the current model is an inconsistent estimator of the Wagner relationship if the omitted variables are correlated with GDP per capita as well as GE/GDP. If this is indeed the case, inclusion of these variables would clean the equation of this potential endogeneity problem as it removes any covariance between GDP per capita and the additive stochastic term in the non-linear models estimated here. If this omitted variable problem exists, then a solution would be to utilise panel fixed effects, which would incorporate these omitted variables as country specific effects. Unfortunately, such a non-linear panel estimator is not presently feasible. This is revisited in the next section on panel models and further research. At present the literature is silent on any potential inconsistency in the estimation of models limited to the single explanatory variable GDP per capita. Nevertheless this issue should not be dismissed lightly and it is an avenue to be explored in future research. This is relevant in terms of augmenting alternative non-linear estimators with additional variables.

The study is based on a data set that is constructed from the publications of the IFS. The reliability and accuracy of that data will, therefore, affect the robustness of the results of the present study. All efforts have been made to ensure the accuracy of the
data, but this potential data problem remains. Some data was not available, which
prevented the researcher from incorporating other potential determinants of the
growth of government activity in the economy. These would be variables which
would account for the cultural and political determinants of government
expenditures. However, following remarks in the previous paragraph, exclusion of
these variables does not negate the current empirical study as a test of the WH, but
simply increases the amount of unexplained variance in the model. Thus, their
inclusion can only improve a model which already has significant outcomes in terms
of the maintained hypothesis. This issue will be considered again in the next section.

7.5 Further Research

There are some suggestions for further research. The framework that this study has
proposed seems convenient to model short-term trajectories of the ratio of
government expenditure in income. However, it is based on some assumptions that
should be removed in a more elaborate framework. First, demographic changes in
population are not necessarily captured by per capita figures, and it would be
interesting to include in the analysis a model of cross-country demographic
processes. The logistic function was invented to solve the problem of population
growth, but this thesis has not tried to integrate demographic factors into the
framework other than population. Thus, other demographics should be included, such
as the age, gender profile and household characteristics of the population. Moreover,
growth in income as a long-term process should also be modeled, and there may be
interactions as well amongst growth in income, population, and public expenditure.
Thus, a possible extension of this research is a system of simultaneous differential
equations to try and capture these trends. A clear problem here is that econometric
theory has not yet developed to the extent that simultaneously determined endogenous variables, which are non-stationary, can be modeled within a system of equations.

A related theme is a specific study of the non-linear process. It should be investigated in time-series analysis for both developed and developing countries to check whether the single countries follow the same process as the cross-sectional sample of countries. Again, the problem here is that modern time-series techniques have not yet been developed to incorporate modelling non-linear combinations of non-stationary variables. An interesting development which might be more immediately operational is the application of non-linear panel models to the pooled cross-sectional data. This data is a short wide panel and would be suitable for estimation methods which capture the richness of panel data. Cameron and Trivedi (2005) set up a non-linear random effects panel model which, in principle, could apply to the logistics or Gompertz functions. The main problem here is identifying the necessary likelihood function for estimation. This function has not yet appeared in the theoretical literature, but recent developments in non-linear panel models suggest that this development may be imminent; if, indeed, it is, then this a way of incorporating country specific effects, albeit in a random component which could significantly reduce the unexplained variance in the model. This could address the limitation of the current study in the form of the cultural and political factors alluded to in the previous section. Further, improvement would be gained if this specification could be modified to include some index of the size and extent of regulatory government.
This study is the first attempt to empirically examine the WH using non-linear functions where the right hand side variable is real GDP per capita. It has employed a well-defended testing methodology and produced a set of results which are of interest in themselves. The results have provided a number of insights which could form the basis of both further research in disaggregate components of government expenditure, and comparative research in market economies. The literature on the growing role and size of government is extensive, and it remains a core issue of modern public economics. In examining this issue, therefore, the thesis has helped to reveal much about the nature of the process of government growth in market economies, and potentially opened up wider issues of the role of government in expanding its capacity in the economy.
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APPENDICES

A. Pigouvian Taxation Theorem

Following the discussion in section 2.4.7, the following review is intended to expand further the discussion of the Pigouvian taxation as a way of imposing a limit on state fiscal activity. However, no attempt will be made here to discuss in detail each contribution, and even less to examine all those studies; instead, some of the major issues about the optimal supply of public goods may be noted since it is of primary interest in this study of the WH.

The literature on the Pigouvian taxation theorem is extensive; Kaplow (1996) re-examined the relationship between the optimal supply of public goods and the distortionary cost of income taxation. Kaplow demonstrated that it is possible to finance a public good in a manner that results in no additional distortion. The method of finance involves adjusting the income tax so that, at each income level, tax increases just offset the benefit from the public good, and the net effect is distribution-neutral. If public goods are not financed in the manner described above, Kaplow suggested that the distortionary cost identified in the previous literature will arise, but will be accompanied by a countervailing change in redistributive benefits that has been ignored. Kaplow noted that:

*Policy analysis should have two components: assessing the merits of the public good according to cost-benefit analysis, and determining whether the change in redistribution is desirable. This later step does not involve an adjustment for the marginal cost of funds because*
distortion is only half of the story. Instead, distortionary costs must be balanced against redistributive benefits” (Kaplow, 1996, p.525).

Kaplow concludes that it is optimal to supply the public good whenever the simple cost-benefit test is satisfied which leads to a greater provision of public goods.

Unlike Kaplow, Feldstein (1997) attempted to evaluate the welfare cost or marginal excess burden of an increase in the size of government. He noted that it depends on both the magnitude of the tax rate change that is required to raise the needed revenue, and the dead weight loss associated with changes in tax revenue. Feldstein suggested that the desirability of rising government spending depends on comparing the benefit of that spending to its total cost, including the deadweight loss of raising the revenue. He also suggested that the dead weight loss of increased tax revenue is likely, in many cases, to be larger than the direct revenue-cost ratio. Feldstein concluded that:

Past experience suggests that financing additional government spending by across-the-board tax rate changes would involve a total cost that exceeds two dollars for every additional dollar of government spending” (Feldstein, 1997, p.212).

Feldstein considers three types of behavioral effects on tax revenue and, without these behavioral effects, a proportional increase in all income tax rates would raise revenue by the same proportion. However the behavioral effects lead to less revenue being raised, he suggests.
Browning and Liu (1998) commented on Kaplow (1996), as they searched for the appropriate definition of distortion, and suggested that the definition has much to do with the distortionary cost of income taxation. They define distortion to mean there is no alternative method of financing the provision of a public good that is better (measured by higher utility) for the taxpayers and, if there exists a different source of revenue other than the increase in income tax rates which will finance the public good and leave all taxpayers better off, then the income tax financing of that public good is distortionary. They show that taxpayers will be better off, and will prefer a larger quantity of the public good, if lump sum taxes are used to finance the public good rather than an increase in the income tax rate. Explicitly, Browning and Liu, in their analysis, agree that an income tax may be non-distorting according to some other definitions. Unlike Kaplow’s conclusion, Browning and Liu’s main conclusion is that the income tax financing of a public good is always distortionary.

Kaplow (1998) replied to Browning and Liu (1998). He suggested that most of their comment is concentrated on how distortion should be defined. Kaplow agreed that the substitution of a lump sum tax for labor income taxation, as a method of finance presented by Browning and Liu, involves less distortion because there is less redistribution which matches his conclusion in the previous work. Finally, Kaplow suggested, “Browning and Liu do not dispute his argument about how policy analysis should be conducted in the present context. Instead, they focus entirely on the distortionary costs of the income tax. By doing so they ignore that the reason for redistributive income taxation is to redistribute income” (Kaplow, 1998, p.124).
The support for Pigou’s optimal level of government spending remains uncertain because of the difficulty of measuring the social welfare costs of taxation. Sandmo (1976) considers the theory of optimal taxation in the presence of externalities. Sandmo’s analysis demonstrated that there is a case for having taxes or subsidies on related goods in addition to a tax on the externality-creating commodity itself. The analysis also suggests that the implementation of the Pigouvian tax principle may call for more than seems to be recognized in much of the previous literature, since there have been many difficulties associated with estimating marginal social damage. Esq (1999) revisited Pigou’s argument and examined whether public expenditures are crowded out by distortionary taxation as Pigou suggested. Esq aimed at examining Pigou’s claim in an economy with heterogeneous households without the poll tax. Esq analyzed the effect of distortionary taxes on the optimal provision of a public good for the single consumer economy and found that Pigou’s claim is right. The same result holds for heterogeneous households under the condition that equity considerations are assumed to be negligible. “If equity considerations are relevant, an increase in public expenditures may help to achieve distributional objectives. Then, distributional effects act against the efficiency effects discussed by Pigou and can lead to the over-provision of a public good” (Esq, 1999, pp.15-26).

It might be observed that there are mixed results regarding the effect of the distortionary cost of income taxation on the optimal supply of public goods in the previous literature. A study by Ng (2000) reconciles the studies of Kaplow (1996) and Feldstein (1997) concerning the optimal supply of public goods. Also, Ng reviewed the environmental disruption effects and the relative-income effect, both of which favor more public spending. Ng’s reconciliation of Kaplow and Feldstein
suggested “A public good can be financed with higher income tax rates without affecting labor supply” (Ng, 2000, p.254). Ng’s suggestion proposes that combining the Kaplow and Feldstein arguments result in some intermediate level of public spending. Ng’s suggestion is consistent with Browning and Liu (1998).

Ng realized that, since the Kaplow principle is concerned with the size of the required benefit/cost ratio for public goods rather than the level of public spending directly, it is not affected by the counteracting effect through complementarity on the marginal valuation of public goods. Moreover, he noticed that most of the recent contributions to the theory of optimal taxation/expenditure, using a single representative consumer model, ignore the countervailing change in redistributive benefits for cases where the distortion costs are not offset by complementarity to become zero. Ng noted that:

“However, before reading Kaplow I thought that requiring the benefit cost ratio for a public good to exceed one by a sufficient margin was based on the efficiency principal, as a dollar of public revenue costs the economy more than one dollar. In other words, I accepted the conventional position here completely. Thus, I find Kaplow’s paper extremely important and hope that it will be given the attention it deserves” (Ng, 2000, pp.256-57).

Ng agrees with Feldstein (1997, p.155) that, given the various ways individuals adjust to higher tax rates, raising an additional dollar of public revenue on its own does impose, in general, significant distortion costs over and above the revenue
collected. However, Ng disagrees with Feldstein that the benefit cost ratio for the public project is as high as more than two, as Feldstein suggested, because there are offsetting benefits on the spending side. “As shown by Kaplow, if the valuation of higher public spending is the same as the extent of the higher taxes at each and every income level, then the higher disincentive effects of higher taxes will be exactly offset to give no net increase in disincentive effects” (Ng, 2000, p.259).

The Florio and Colautti (2003) framework suggested that the distortionary taxations will act as a break for the growth of government spending relying on Pigou’s effect. However, from the previous discussion, this study might suggest that there is no consensus to support the Pigou effect, and there is a considerable amount of literature that suggests that the distortionary cost of income taxation might lead to a greater provision of public goods instead.
B. Elasticity of Government Expenditure with Respect to Aggregate Income

In its broadest sense the WH sees a direct relationship between government and economic development in the economy, and this is determined by the fact that the social demand for public goods will increase as income increases. Thus, the elasticity of government expenditure, which is the traditional measure of government in the Wagner economy, with respect to aggregate income, which is the traditional measure of development in the economy, will be important in any empirical test of the WH. This is referred to directly in Chapters two, three and four and the aim of this appendix is to support the conjectures on elasticity made in those chapters. That is, in simple terms, this appendix assists in the examination of the nature of elasticity scores which are sensible in terms of testing the WH. The standard first year economic definition of elasticity measures the proportionate change in the quantity of one variable brought about by the change in the quantity of another variable. Thus, calculating the scores of, say, income elasticity of demand follows a basic formula, the percentage change in quantity demanded divided by the percentage change in income, as depicted in (B.1):

\[
\eta = \frac{\%\Delta Q_d}{\%\Delta Y} = \frac{(\frac{\Delta Q_d}{Q_d})}{(\frac{\Delta Y}{Y})} = \frac{\Delta Q_d}{\Delta Y} \cdot \frac{Y}{Q_d} \Rightarrow \lim_{\Delta Y \to 0} \frac{dQ_d}{dY} \cdot \frac{Y}{Q_d} \quad \text{(B.1)}
\]

Here \( \eta \) represents the coefficient of income elasticity, \( Q_d \) represents the quantity demanded, and \( Y \) represents income or budget in the consumer model of microeconomics.
In the context of the WH, government expenditure is that which is determined and income is that which is determining. If government expenditure is denoted as GE and income as GDP, then one can talk of the elasticity of GE with respect to GDP. That is,

\[ \eta_{GE,GDP} = \frac{\Delta GE/GE}{\Delta GDP/GDP} = \frac{dGE}{dGDP} \times \frac{GDP}{GE} \]  

(B.2)

Note that (B.2) is different from the marginal effect (ME) which is defined in Appendix C for the shares version of the WH and would, in this case, simply be the first derivative of (B.2).

This appendix uses (B.2) to examine the implications of various elasticity scores of GE with respect to GDP for the share of government expenditure in GDP, that is GE/GDP, where the relationship between GE and GDP may be linear, log-linear, or semi log-linear. The two former are specifications which have been used in previous empirical studies of the WH. The focus is on the implications for the share of government expenditure in GDP because this thesis, following the tradition of Musgrave (1969) regards that share as the important long run Wagnerian variable.

*Elasticity in the Linear Function Form of the WH*

A linear function form of the WH with variables in levels could follow three types: a relationship that has an intercept at the origin, that is, a line emanating from the origin; a relationship with a positive intercept; or one with a negative intercept. Consider the first case, the linear function form of the WH with slope parameter \( b \):
This is illustrated in Figure B-1 and this figure will be used to identify the elasticity score of GE with respect to GDP in a simple geometric fashion and illustrates the fact that the marginal and average effects of a change in GDP on GE in (B.3) are the same.

**Figure B-1:** A Linear Function of the WH in Levels with zero Intercept

Any straight line down through the origin with variables measured in levels will have unit elasticity. The following holds for Figure B-1

\[
\frac{GE}{GDP} = \frac{\Delta GE}{\Delta GDP} \quad \text{(B.4)}
\]

Further the elasticity of GE with respect to GDP \( (\eta_{GE,GDP}) \) is:
Now substitute (B.4) into (B.5) to get:

\[ \eta_{GE,GDP} = \frac{\Delta GE}{\Delta GDP} \cdot \frac{GDP}{GE} = 1 \]  

(B.6)

from this, one can see a clear and important implication for the relationship of the share \( GE/GDP \) with respect to GDP. That is, with unit elasticity, \( GE \) increases in equi-proportion with GDP so that \( GE/GDP \) remains unchanged as GDP grows. This clearly violates the views of Wagner who saw the relative share of government as increasing in the economy.

Figure B-2: GE/GDP with Unit Elasticity of GE with Respect to GDP

\[
\eta_{GE,GDP}
\]
Now consider the second and third cases, a linear function in levels with intercept:

\[ GE = a + b \text{GDP} \]  

(B.7)

Using the general expression for elasticity given by the right hand side of (B.2)

\[
\eta = \frac{dGE}{d\text{GDP}} \frac{\text{GDP}}{GE} = \frac{b\text{GDP}}{a + b\text{GDP}}
\]

(B.8)

If the intercept is greater than zero, as illustrated in figure B-3, then, for all levels of GDP, the elasticity score will be positive but less than one. However, this elasticity score will approach unity from below as GDP increases. This is because the average effect is larger than the marginal effect at low GDP, but approaches the marginal effect as GDP increases so that the elasticity of GE with respect to GDP approaches unity from below. The implications are, as GDP increases, the share, GE/GDP, declines until it reaches some stable level as illustrated in Figure B-4. This is a circumstance which was not considered in the WH.

**Figure B-3**: Linear Function of the WH in Levels with Positive Intercept

\[ \eta_{GE,\text{GDP}} < 1 \]

But approaches 1 as \( \text{GDP} \to +\infty \)
**Figure B-4:** GE/GDP with Elasticity of GE with Respect to GDP Approaching Unity From Above

However, if $a$ is less than zero in (B.7) and (B.8), as in Figure B-5, then the elasticity score will be greater than one but will approach one from above as GDP increases without limit. This is because the average effect is smaller than the marginal effect, but approaches the marginal effect as GDP increases.

**Figure B-5:** Linear Function of the WH in Levels with Negative Intercept
Thus, the share GE/GDP must increase as GDP increases, but this rate of increase declines until it approaches an unknown asymptote. This limiting share value could be greater than one.

**Figure B-6:** GE/GDP with Elasticity of GE with Respect to GDP Approaching Unity From Below

\[
\text{GE/GDP}
\]

\[
\text{GDP}
\]

*Elasticity and the Log-Linear Functional Form of the WH*

Consider a log-linear relationship of the WH:

\[
\ln GE = a + b \ln GDP \quad (B.9)
\]

Any positive log-linear relationship has a positive elasticity of GE with respect to GDP. Further, it has a constant elasticity, which is the slope of the log linear relationship as illustrated in Figure B-7.
Figure B-7: The Log-Linear Form of the WH

The marginal effect of the function (B.9) is

\[
ME = \frac{dGE}{dGDP} = \frac{dGE}{dGDP} \frac{d\ln GE}{d\ln GDP} \frac{d\ln GDP}{dGDP}
\] (B.10)

which can be simplified to:

\[
ME = b \frac{GE}{GDP}
\] (B.11)

The elasticity is:

\[
\eta_{GE,GDP} = \frac{dGE}{dGDP} \frac{GDP}{GE}
\] (B.12)

Substituting the marginal effect (B.11) into the elasticity of (B.12) produces:

\[
\eta_{GE,GDP} = b \frac{GE}{GDP} \frac{GDP}{GE} = b
\] (B.13)
The elasticity, $b$, is constant and positive. If $b$ is greater than zero and less than 1 ($0 < b < 1$), then, as GDP grows, GE also grows but this growth is proportionally less, so that as GDP increases, GE/GDP declines toward zero as illustrated in Figure B-8.

**Figure B-8:** GE/GDP, with Elasticity of GE, with Respect to GDP, Positive, Constant, but less than One

**Figure B-9:** GE/GDP, with Elasticity of GE, with Respect to GDP, Constant and Greater than One
If the elasticity scores are greater than one and less than infinity \((1 < b < \infty)\), then as GDP grows, GE grows but proportionately more, so that as GDP increases, GE/GDP increases without limit as depicted in figure B-9.

**Elasticity of Semi Log-Linear Function Form of the WH**

A semi log-linear function form could measure either the dependent or the independent variables in logarithms. Any positive semi log-linear function form will have a variable elasticity dependent on the value of GDP or GE. If the dependent variable GE is measured in logarithms:

\[
\ln GE = a + bGDP
\]  
(B.14)

Which is illustrated in Figure B-10.

**Figure B-10**: The Semi-Log Linear Form of the WH

![Figure B-10: The Semi-Log Linear Form of the WH](image)
To obtain the elasticity scores for this function form, define the marginal effect as:

$$\frac{dGE}{dGDP} = \frac{dGE}{d\ln GE} \frac{d\ln GE}{dGDP} = bGE$$  \hspace{1cm} (B.15)

Reacall the definition of elasticity from equation (B.5):

$$\eta_{GE,GDP} = \frac{dGE}{dGDP} \frac{GDP}{GE}$$

and using the marginal effect from (B.11), then

$$\eta_{GE,GDP} = bGE \frac{GDP}{GE} = bGDP$$  \hspace{1cm} (B.16)

Thus, elasticity is positive and growing as GDP increases. Therefore, the share of GE/GDP grows without limit as GDP increases.

Alternatively, the independent variable GDP may be measured in logarithms:

$$GE = a + b \ln(GDP)$$  \hspace{1cm} (B.17)

which is illustrated in Figure B-11.
The marginal effect is:

\[
\frac{dGE}{dY} = \frac{dGE}{d \ln GDP} \frac{d \ln GDP}{d GDP} = \frac{b}{GDP}
\]  \hspace{1cm} (B.18)

Which, on substitution into the general expression for elasticity, gives:

\[
\eta_{GE,GDP} = \frac{b}{GDP} \frac{GDP}{GE} = \frac{b}{GE}
\]  \hspace{1cm} (B.19)

IF GE increases as GDP increases then the elasticity score is positive but declines as GDP increases and approaches zero. Thus, the share measure GE/GDP declines and approaches zero as GDP increases.
C. The Marginal Effect in Sigmoid Functions

Consider the three parameter logistics function:

\[ y = \frac{a}{1 + ce^{-xb}} \]  

(3.1)

and the three parameter Gompertz function:

\[ y = a \exp\left(-e^{b(x-c)}\right) \]  

(3.2)

The marginal effect for the logistics function is:

\[ \frac{dy}{dx} = \frac{abce^{-xb}}{(1+ce^{-xb})^2} \]  

(3.3)

and the marginal effect for the Gompertz function is:

\[ \frac{dy}{dx} = b ye^{(-b(x-c))} \]  

(3.4)

Figure C-1 illustrates the relationships between the logistics and Gompertz functions and their respective first derivatives, the marginal effect of \( x \) on \( y \). In this four panel figure the first row comprises the logistics function to the left and Gompertz functions to the right. Both have been estimated over the interval \( x=[-100,100] \) with parameters \( a=1, \ b=0.05 \) and \( c=1 \). The lower row gives the respective first
derivatives. That is, the marginal effect of $x$ on $y$ for the logistics is to the lower left, with the Gompertz to the lower right.

Figure C-1 The Logistics and Gompertz Functions and the First Derivatives

This figure clearly illustrates the relationship between the marginal effect and the function. In both cases, the functions have their points of inflection located on the $y$ axis. Thus, points on the logistics and Gompertz curves to the left of the $y$ axis, that is, determined by $x < 0$, have $y$ increasing from zero, and at an increasing rate toward the $y$ axis. Scanning to the lower graphs in the figure, the first derivatives over this interval of $x$ are positive and increasing from zero toward a peak on the $y$ axis.

Returning to the first row of the figure, points on the logistics and Gompertz curves to the right of the $y$ axis, that is, determined by $x > 0$, have $y$ increasing from a point of inflection on the $y$ axis, but increasing at a decreasing rate toward an asymptote of one in this case. Scanning to the lower graphs in the figure, the first derivatives
over this interval of $x$ are positive but decreasing from a peak on the $y$ axis toward zero as the parent function approaches its asymptote.

Finally, the figure shows that the difference between the two functions evaluated over the same interval of $x$, and using the same parameters, is that the Gompertz is asymmetric about the $y$ axis in comparison to the logistics function. Thus, in comparison to the first derivative of the logistics function, the first derivative of the Gompertz function shows a much steeper increase to the left of the $y$ axis and reaches a higher maximum.
### D. Regional Grouping of 88 Countries*

<table>
<thead>
<tr>
<th>Region</th>
<th>Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Europe</td>
<td>Switzerland, Egypt, Asia, North America, Korea, Republic of</td>
</tr>
<tr>
<td>Austria</td>
<td>Africa, Canada, Indonesia</td>
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<tr>
<td>Belgium</td>
<td>Tunisia, Costa Rica, Malaysia</td>
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<td>Denmark</td>
<td>Morocco, El Salvador, Singapore</td>
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<td>Finland</td>
<td>Burkina Faso, Guatemala, Thailand</td>
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<td>France</td>
<td>Cameroon, Honduras, India</td>
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<td>Germany</td>
<td>Ghana, Mexico, Iran</td>
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<td>Greece</td>
<td>Burundi, Panama, Nepal</td>
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<td>Hungary</td>
<td>Rwanda, The USA, Pakistan</td>
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<td>Iceland</td>
<td>Zambia, South America &amp; Caribbean, Syria</td>
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<td>Ireland</td>
<td>Kenya, Chile, Turkey</td>
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<td>Italy</td>
<td>Madagascar, Paraguay, Israel</td>
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<td>Malta</td>
<td>Mauritius, Uruguay, Jordan</td>
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<td>Netherlands</td>
<td>Botswana, Belize, Yemen</td>
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<td>Norway</td>
<td>Lesotho, St.Vincent, Philippines</td>
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<td>Spain</td>
<td>South Africa, Bolivia, Vietnam</td>
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<td>Romania</td>
<td>Zimbabwe, Brazil, Sri Lanka</td>
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<td>Croatia</td>
<td>Mali, Colombia, Ociena</td>
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<td>Czech Republic</td>
<td>Nigeria, Paraguay, Australia</td>
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<td>Estonia</td>
<td>Sierra Leone, Venezuela, Fiji</td>
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<td>Portugal</td>
<td>Congo Republic, Barbados, New Zealand</td>
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<td>Slovenia</td>
<td>Seychelles, Dominican Republic, Papua New Guinea</td>
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<tr>
<td>Sweden</td>
<td>Uganda, Haiti</td>
</tr>
</tbody>
</table>

*The grouping of the countries has been done according to PWT classification.
E. The Convexity of the Objective Function

Two contour graphs are referred to in the results chapter, Chapter 6. These show level sets of the least squares objective for the parameter estimates for the logistics and Gompertz functions. These level sets were estimated across a grid for the three parameter function, with two of the parameters variable and the third parameter fixed at its optimal estimate. Specifically, parameter $a$ was fixed and the parameters $b$ and $c$ were varied across the two-dimensional grid. These two parameters were chosen because $b$ shows a smooth accumulation of GE/GDP as GDP increases and was designated the “Wagner parameter” and this parameter was relatively small compared to the parameter $c$. In Chapter 6, it was argued that exploring the convex properties of the objective function in this parameter space was a way of identifying the robustness of the estimate to local minima.

There are two clear limits to this:

1.) the extent of the grid; and

2.) the grid tolerance.

However, given these two limitations, the level plots indicated that even though, for the Gompertz function, the objective function displayed non-convex sections, the estimates appeared to converge to the correct minimum. This appendix completes this robustness check by plotting the four level curves in the remaining two sets of parameter spaces for each function.

Figure E-1 gives the level curves for the objective function in parameters $a$ and $c$ with $b$ as the fixed parameter for the logistics function. Figure E-2 plots the level
curves for the sum of the squared residuals in parameters $a$ and $b$ space with $c$ fixed for the logistics estimate. Both grids seem to give similar elongated valley structures, but with a single identifiable minimum within the grid range and tolerance.

Figure E-1: Logistics $b$ fixed at 0.00019, Contours {6.05,7,8,9,10,15,20,25,40,60,100,500}

Figure E-3 gives the level curves for the objective function in parameters $a$ and $c$ with $b$ as the fixed parameter for the Gompertz function. Figure E-4 plots the level curves for the sum of the squared residuals in parameters $a$ and $b$ space with $c$ fixed for the Gompertz estimate. As in the logistics estimate, both figures show an elongated valley structure. Again, both have a single identifiable minimum within the grid range and tolerance.
Figure E-2: Logistics, $c$ fixed at 0.74924, Contours \{6.1, 7, 8, 10, 15, 20, 30, 40, 50, 100, 500\}

![Logistics Contours](image)

Figure E-3: Gompertz $b$ fixed at 0.00016, Contours \{6.1, 7, 8, 9, 10, 20, 30, 50, 80, 100, 500, 1000\}

![Gompertz Contours](image)
Figure E-4: Gompertz c fixed at -3442.46, Contours \{6.2,7,8,9,10,20,30,50,80,100,500,1000\}