Yesterday, Today and Tomorrow:
Promoting Conceptual Understanding in Mathematics using a Five Question Approach

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Statement of Authentication

The work presented in this thesis is, to the best of my knowledge and belief, original except as acknowledged in the text. I hereby declare that I have not submitted this material, either in full or in part, for a degree at this or any other institution.

John Ley
# Table of Contents

1  Introduction ........................................................................................................... 12  
1.1 The Current Context of Mathematics Teaching in Australia ....................... 13  
1.2 Australian Teaching in the International Context ........................................ 18  
1.3 The Significance of the Problem .................................................................... 21  
1.4 Five Question Approach (FQA) .................................................................... 27  
1.5 Aims of the Research .................................................................................... 28  
1.6 Overview of Methodology ............................................................................. 30  
1.7 Thesis Overview ........................................................................................... 32  

2  Review of Literature ........................................................................................... 34  
2.1 Quality Mathematics Teaching ........................................................................ 37  
2.1.1 Quality teaching ....................................................................................... 37  
2.1.2 The student perspective ........................................................................... 42  
2.1.3 Student engagement ................................................................................ 44  
2.1.4 Framework for quality mathematics teaching in the Australian context .. 46  
2.2 Mathematics Curriculum in NSW, Australia ................................................ 47  
2.3 Knowledge and Understanding for Quality Teaching ................................. 50  
2.3.1 Procedural and conceptual knowledge ..................................................... 51  
2.3.2 The procedure or the concept, which comes first? .................................. 55  
2.3.3 Constructivist approaches to teaching ..................................................... 58
4.1.2 Using a focus group approach .................................................. 122
4.2 The Research Design .................................................................. 123
  4.2.1 The school context ................................................................. 124
4.3 The Participants ........................................................................ 127
  4.3.1 Teacher participants .............................................................. 127
  4.3.2 Student participants .............................................................. 129
4.4 Data Collection ........................................................................ 131
  4.4.1 Teacher interviews ................................................................. 132
  4.4.2 Student focus group discussions ............................................. 136
  4.4.3 Classroom observations ......................................................... 140
  4.4.4 Field notes ........................................................................... 142
4.5 Data Analysis ........................................................................... 142
4.6 Ethical Considerations ............................................................... 145
  4.6.1 Informed consent ................................................................. 145
  4.6.2 Anonymity and confidentiality .............................................. 146
4.7 Validity and Reliability ............................................................. 148
4.8 Assumptions and Limitations .................................................... 148
4.9 Conclusion .............................................................................. 150
5 Analysis of Data St Patricia’s Year 8 ............................................ 151
  5.1 Year 8: Setting the Scene ......................................................... 151
    5.1.1 The typical lesson versus the FQA ..................................... 154
5.1.2 Engagement in mathematics ................................................................. 155
5.1.3 Timing of learning .............................................................................. 158
5.1.4 Links and themes arising from phase 1 .............................................. 160
5.2 Enjoyment of the FQA .......................................................................... 165
  5.2.1 Success and positive attitude ............................................................. 167
  5.2.2 Non-Linear knowledge development .................................................. 170
  5.2.3 Adjusting the questions .................................................................... 171
  5.2.4 Updated table of themes .................................................................. 173
5.3 The Examinations .................................................................................. 177
  5.3.1 Half yearly examination preparation ................................................. 178
  5.3.2 Half yearly examination performance .............................................. 179
  5.3.3 The yearly examination ..................................................................... 182
5.4 Conclusion .............................................................................................. 186

6 Analysis of Data St Charles’ Year 9 .......................................................... 191
  6.1 St Charles’ Year 9: Setting the Scene .................................................... 192
    6.1.1 The typical lesson and the FQA ....................................................... 195
    6.1.2 Success and engagement ............................................................... 196
    6.1.3 Time taken to learn ....................................................................... 198
    6.1.4 Time and understanding ............................................................... 200
  6.2 Implementation of the FQA ................................................................. 201
    6.2.1 Success builds confidence ............................................................. 202
7.3.4 After the half yearly examination .............................................. 233
7.4 Harriette Final Interview ............................................................ 234
7.5 Conclusions .............................................................................. 237

8 FQA Cross Case Analysis................................................................. 239
8.1 Teaching Styles .......................................................................... 239
8.2 Implementation of the FQA .......................................................... 240
8.3 Engagement .............................................................................. 244
8.4 Academic Performance ............................................................... 246
8.5 A Change in Mindset ................................................................. 246
8.6 A Change in Class Brings a Change in Attitude................................. 248
8.7 Conclusion ............................................................................ 251

9 Conclusion .................................................................................... 253
9.1 Summary of Results ................................................................... 254
9.2 Research Question 1 .................................................................. 257
9.3 Research Question 2 .................................................................. 259
9.4 Limitations ................................................................................ 263
9.5 Additional Findings ................................................................... 265
9.6 Suggestions for Future Research ................................................... 266
9.7 Conclusion ............................................................................ 268

References ...................................................................................... 271

Appendix A ...................................................................................... 295
List of Figures

Figure 2.1 Quality Mathematics Teaching .......................................................... 35
Figure 2.2 Links between Schema and Concepts ................................................. 55
Figure 2.3 Demonstration of the Learning Process ................................................. 69

Figure 5.1 Year 7 Final Examination Rankings vs Year 8 Half Yearly Examination
Rankings .............................................................................................................. 181

Figure 5.2 Year 7 Final Examination Rankings vs Year 8 Yearly Examination
Rankings .............................................................................................................. 184

Figure 6.1 Year 8 Final Examination Rankings vs Year 9 Half Yearly Examination
Rankings .............................................................................................................. 212

Figure 6.2 Year 8 vs Year 9 End of Year Examination Rankings ......................... 215

Figure 7.1 Year 9 Final Examination Rankings vs Year 10 Half Yearly Examination
Rankings .............................................................................................................. 232

Figure 7.2 Year 9 Final Examination Rankings vs Year 10 Yearly Examination
Rankings .............................................................................................................. 236
List of Tables

Table 2.1 Division of Fractions ................................................................. 52
Table 3.1 FQA: Examples of five questions sequence. ............................ 103
Table 3.2 Sample Continuum of Percentage Questions Asked in the First Four Questions .................................................................................. 105
Table 4.1 The Timing of Qualitative and Quantitative Data Collection .......... 119
Table 4.2 Description of Teacher Participants ........................................ 128
Table 4.3 Focus Group Student Name and Rank Details Based on Examination Results from the Previous Year .......................................................... 129
Table 4.4 Student Focus Group Participants St Patricia’s Year 8 .................. 137
Table 4.5 Student Focus Group Participants Saint Charles Year 9 ............... 137
Table 4.6 Student Focus Group Participants Saint Charles Year 10 .............. 137
Table 5.1 Timing of Data Collection for St Patricia’s Year 8 ....................... 151
Table 5.2 Five Questions from the First Observed 8M2 Lesson .................. 152
Table 5.3 Ranking and Engagement Level of Year 8 Focus Group Students .... 153
Table 5.4 Themes Emerging after the First Data Phase .............................. 161
Table 5.5 Five Questions for the 8M2 Second Observed Lesson ................. 166
Table 5.6 Themes Emerging after the Second Data Phase ......................... 173
Table 5.7 Five Questions for the 8M2 Third Observed Lesson ................... 177
Table 5.8 Five Questions for the 8M2 Fourth Observed Lesson .................. 183
Table 6.1 The Timing of each Data Collection Point for St Charles’ Year 9 .... 191
Table 6.2 Five Questions from the First Observed 9M5 Lesson ................... 193
Table 6.3 Ranking and Engagement Level of Year 9 Focus Group Students .... 193
Table 6.4 Five Questions from the Second Observed 9M5 Lesson ............... 202
Table 6.5 Five Questions from the Third Observed 9M5 Lesson ................. 208
Table 7.1 Timing of each data Collection Point for St Charles’ Year 10 ..........220

Table 7.2 Ranking and Engagement Level of St Charles’ Year 10 Focus Group

Students ........................................................................................................................................221

Table 7.3 Five Questions from the Second Observed 10M5 Lesson ..................226

Table 7.4 Five Questions from the Third Observed 10M5 Lesson ......................230

Table 8.1 Half Yearly and Yearly Year 8 Rankings of Students who Entered and Left the FQA Classroom During the Study .................................................................249
Abstract

During recent decades in Australia and internationally, academics and policy makers have expressed concern about decreasing levels of engagement with mathematics, with many students choosing not to pursue the study of mathematics at higher levels. With fewer students taking up tertiary studies that require high-level mathematics the result is a shortage of mathematics teachers in Australian schools. This thesis is the culmination of a thirty-year teaching career in which I developed the Five Question Approach (FQA) to teaching mathematics. It reports on the influence of the FQA on student engagement and academic performance in secondary mathematics classes (Years 7 to 12) in Australia.

Student engagement is influenced by the degree of success that is experienced in the mathematics classroom. In turn, success is often determined by the depth of understanding that students gain during predetermined time frames. The FQA removes the predetermined time frames using the yesterday, today and tomorrow approach where questions are provided on previously learnt material for revision and fluency, current material for consolidation, and future material for preparation allows greater flexibility in content delivery, pacing and consolidation of content. This mixed-methods case study draws on data collected in three Australian secondary classrooms. The findings indicated that the FQA increased student engagement with mathematics in the classroom, with students feeling that they were better at mathematics and more able to answer questions and solve problems. There was a change from a fixed to a growth mindset and significant improvement in academic performance. The findings further indicated that the FQA led to increased student engagement, academic improvement and a significant decrease in examination anxiety.
1 Introduction

The Five Question Approach (FQA) to teaching mathematics emerged from my experience of secondary mathematics teaching in five large New South Wales (NSW) Western Sydney Catholic Year 7 to 12 high schools, engaging with thousands of students and hundreds of teachers over 30 years. The mathematics teaching praxis I developed and refined positioned the FQA as a central element. Over my high school teaching career, I was passionate about the teaching and learning of mathematics and encouraged positive attitudes and experiences in the mathematics classroom for all students. I held positions of Head of Mathematics in three schools, Assistant Principal in three schools, and Acting Principal in one school. As a member of a school system audit team I conducted a review of mathematics departments in over thirty schools and experienced many instances of ‘traditional’ teaching and a lack of student engagement in these mathematics classes.

I developed the FQA as part of an overall teaching approach designed to improve student understanding, encourage student engagement, cognitive, operative, and affective, and promote success and mathematical discussion in the classroom. The FQA is much more than an instructional activity. It is a pedagogical approach that is designed to provide a strategy for teachers using a ‘traditional’ teaching approach to begin to incorporate open ended questions with multiple solution pathways into their praxis. While I have presented this approach at many Australian conferences and some international conferences, there had been no prior empirical research undertaken on this type of approach. Without this research it could be acknowledged that it was myself as a teacher and not the FQA that resulted in increased academic performance and engagement. However, it is envisaged that the approach stands alone, independent of the teacher. This thesis reports on a study of the FQA teaching
approach with four teachers and three classes investigating its influence on engagement and academic outcomes. The study will examine the theoretical basis for this approach and the influence that this approach has on the academic performance (perceived and actual) and engagement of high school students. This first chapter of the thesis presents the background focus and significance of the study, an overview of the methodology and the structure of this thesis document.

1.1 The Current Context of Mathematics Teaching in Australia

According to Anderson, White, and Sullivan (2005), Anthony and Walshaw (2009), Simon and Tzur (2004), and Steffe (2010), mathematical learning takes place when students are engaged in the solution of problem style tasks involving open-ended, investigative style questions where various solution paths are discussed. These tasks need to be engaging and specifically selected by the teacher to achieve the desired educational goals. Such an approach contrasts with the well-established ‘traditional’ teaching approach in which students follow step-by-step teacher-led procedures where students are provided with examples, repetitive exercises are completed, and answers are given without discussion (Givvin, Hiebert, Jacobs, Hollingsworth, & Gallimore, 2005; Sullivan, 2011b; Vincent & Stacey, 2008).

In his review of mathematics teaching in Australia based on the Australian Council for Educational Research (ACER) conference held in Melbourne in 2010 (Sullivan, 2011b) opines that it is this traditional method of teaching that is the most common mathematics teaching practice currently used in Australia and has resulted in a decrease in engagement for many students. This approach is often referred to as teaching that results in the development of instrumental understanding only as
opposed to relational understanding (Skemp, 1987), and is arguably better for the
development of procedural knowledge, not conceptual knowledge, which is an
understanding of the actual concept not a list of steps to be followed for a solution
(Star, 2005). This issue is not restricted to Australia as the National Council of
Teachers of Mathematics NCTM in their 2014 review titled ‘Principles to actions:
ensuring mathematical success for all’ discuss the unproductive realities of too many
classrooms that focus on learning procedures without application or understanding
and a lack of sufficient attention to problem solving and reasoning (NCTM, 2014).
Hattie, Fischer & Frey (2017) describe ‘traditional’ teaching as the teaching of
procedures first followed by repetitious practice which they describe as “show and
tell” or “drill and kill” (p3).

Ideally, students should be provided with opportunities to think about and apply
mathematics learnt rather than simply complete and practice the question types they
have been shown. It is when students are thinking about mathematical questions,
particularly open-ended questions, and connecting their ideas from different areas of
mathematics, (that is, thinking for themselves), that mathematics becomes interesting
and enjoyable (Anthony & Walshaw, 2009). NCTM (2014) noted that ‘Student
learning is greatest in classrooms where the tasks consistently encourage high-level
student thinking and reasoning and least in classrooms where the tasks are routinely
procedural in nature (p.17). The traditional teaching approach has a focus on
completing almost identical questions to the given example and does not necessarily
allow students to think about mathematics, which possibly results in a decrease in
engagement. The traditional teaching approach with a focus on instrumental
understanding and a skills-based approach to the teaching and learning of mathematics
has caused concern to many scholars, including Ball, Sleep, Boerst, and Bass (2009),
Clements (2003), and White (2013).

The traditional approach centres on the presentation of material that generally follows
a scope and sequence document, based on the syllabus and developed by the school,
which outlines the order of topics and the amount of time to be spent teaching each
topic. The time limits applied through the scope and sequence document restrict the
time that a teacher has available to teach each topic. The linear nature of teaching one
topic then another without making connections between the mathematical ideas in
different topics may hinder the development of conceptual understanding. Conceptual
understanding occurs when mathematical ideas from different areas are linked
(Anthony and Walshaw 2009).

The typical traditional lesson usually involves the teacher presenting examples of
skills or procedures to be followed and copied by the students, who then complete
many questions directly similar to the examples presented. These exercises are marked
right or wrong and there may be no discussion of different solution paths as usually
only one method is demonstrated, and all questions are solved using this method. This
traditional teaching approach is sometimes referred to as a rote learning approach
when the teacher makes little attempt to help the students to construct meaning—it
relies on memorisation and repetition only. This instrumental understanding has been
described as 'rules without reasons' (Skemp 1987). For many students and their
teachers, the possession of such a rule and the ability to use it is what constituted
'understanding'. Why the rule works is not a significant consideration.

According to Skemp (1987), the main reason that teachers continue to use a teaching
approach that has a focus on instrumental understanding is that the mathematical
content is initially easier for students to understand. The teacher provides examples
and the students complete questions directly related to those examples, resulting in correct solutions to questions, without necessarily understanding the concept related to the topic. The benefit of this approach is the immediate and apparent rewards as questions are answered correctly but the downside is that there is often a lack of development of conceptual knowledge.

In contrast to instrumental understanding, relational understanding is concerned with meaning and developing connected understanding or knowledge. Relational understanding or conceptual knowledge means 'knowing both what to do and why' and the development of schemas is evidence of the construction of relational understanding (Skemp 1987). Teaching for relational understanding and the development of conceptual knowledge is often associated with a constructivist approach to teaching that uses open-ended questions and tasks to allow students to build their mathematical understanding and develop links between different areas of mathematics. In a teaching approach focused on the development of relational understanding the teacher sometimes takes the role of a facilitator of learning rather than being the ‘font of knowledge’ where facts are simply transmitted to students with the expectation that it can be regurgitated at the appropriate time.

Sullivan (1992) completed research that compared the results of students taught in a traditional manner with those taught using open-ended questions only. The research found that teachers using the open-ended question approach still needed to provide some guidance to the students for effective teaching to take place. To achieve this, teachers found a compromise between constructivism and direct teaching by varying the level and timing of scaffolding provided by teachers when students were attempting open-ended questions. These findings were confirmed in other studies that
found at least some direct teaching was required for students to learn (Bell & Pape, 2014; Sweller, Kirschner, & Clark, 2007; Tait-McCutcheon, Drake, & Sherley, 2011). While it appears that there is a dichotomy between instrumental and relational approaches to developing students’ mathematical understanding, there is evidence that regardless of the approach taken, that teacher guidance is required for learning to occur. The way in which the guidance is presented is a challenge for teachers and Brousseau’s (1984) didactical contract construct could assist in examining the level and method of teacher guidance. Students often focus on ‘not failing’ when doing a task and try to obtain assistance to reduce the complexity, and to increase the explicitness of the task, thereby reducing the cognitive level of the task. The challenge for the teacher is to ensure that any guidance given to the students does not dilute the task or that future task complexity is not reduced as a consequence. On this, Sullivan (1992) wrote: “teachers for their part, tend to react to the response of pupils by selecting tasks that are familiar and easy” (p. 511). In order to set tasks of an appropriate level of challenge, teachers need to be aware of the amount of background knowledge that the students may have from previous years and use that background knowledge to assist them in designing appropriate tasks based on the needs of the students (Clarke & Clarke, 2012; Sullivan 1992).

This section has focused on the issue of ‘traditional’ teaching with a focus on the development of instrumental understanding and procedural knowledge and the influence that this approach has on student learning. Next, the performance of Australian students in an international context will be examined to determine Australia’s standing in comparison with other countries, providing a contextual basis for the site of this study.
1.2 Australian Teaching in the International Context.

This section will examine information from the Teacher Education Ministerial Advisory Group (TEMAG) Action Now: Classroom Ready document (TEMAG, 2014), Programme for International Student Assessment (PISA) (Organisation for Economic Co-operation & Development, 2010) and The Trends in International Mathematics and Science Study (TIMSS) (TIMSS, 2013) to compare Australian student mathematics achievement levels with those of other countries.

In a report on the challenges to education in Australia, Masters (2016) wrote about the decline of Australian students in mathematics in PISA testing from 2000 to 2012, compared with the improvement in performance of countries such as Germany and South Korea. He stated that “an immediate national objective should be to reverse the current trend as reflected in PISA” (Masters, 2016, p. 12). A significant factor in the decline was the lack of ability of students to apply the mathematics learnt to everyday problems. Masters was also concerned with the steady decline, over the past twenty years, in Australian students participating in higher levels of mathematics. In Australia, progression to the next school year is not dependent on performance and students who have not developed their mathematical understanding are promoted to the next school year as a matter of course. As a result, some students performing below expectations in one year can become further behind in the next year. The continued lack of success in mathematics may then result in a further decline in engagement. There needs to be a change in strategy to address the issue of the increasing numbers of underperforming students. “By the middle years of schooling many of these students have become disenchanted and disengaged” (Masters, 2016, p. 17).
While Australia was performing above average in the 2009 PISA (fifteenth position), the overall ranking had declined. This decline was an issue that Dinham (2013) felt needed to be addressed. According to Dinham (2013) the top five countries: Shanghai, Singapore, Hong Kong, South Korea and Chinese Taipei had a focus on rote learning, cramming and testing as a strategy for improving PISA test results. As a result, Dinham (2013) stated that a comparison with these countries, or in most cases parts of a country, was invalid. Australia was performing well in comparison with like countries, for example Canada was 10th, New Zealand 13th, Germany 16th, France 22nd, UK 28th and USA 31st. However, he stated “despite our overall performance as a nation on international and national measures of student performance, we can and need to improve” (Dinham, 2013, p. 98). Dinham (2013), further noted that the method of improvement in teaching and learning needed to be informed by research into the way in which mathematics is taught, teachers are prepared, and teacher professional development is conducted. Subsequently the TEMAG report, Action Now: Classroom Ready document (2014), focused on the issue of improvement of teacher quality.

Prior to 2014, Australian governments had recognised the need for teacher standards and as a result set up the Australian Institute for Teaching and School Leadership (AITSL) in 2010. Teaching standards were developed at Graduate, Proficient and Highly Accomplished levels with seven aspects for consideration:

- Know students and how they learn
- Know the content and how to teach it
- Plan for and implement effective teaching and learning
- Create and maintain supportive and safe learning environments
- Assess, provide feedback and report on student learning
• Engage in professional learning
• Engage professionally with colleagues, parent/carers and the community

(AITSL, 2017a, p. 5).

The teaching standards developed by AITSL in conjunction with the TEMAG findings will be used as a lens to describe quality teaching in this research.

The Action Now: Classroom Ready document (2014) focused on the quality of teaching and the training of teachers. The concern was raised that many teaching practices used in Australian schools were not informed by research and that some teacher education programs were not up-to-date with current research. The report confirmed the declining participation in higher-level mathematics courses and in part attributed this decline to the lack of trained mathematics teachers. According to the Australian Council for Educational Research (ACER, 2014) in 2010 39% of teachers of Years 7 to 10 and 24% of teachers of Years 11 and 12 mathematics were unqualified in mathematics.

In 1999, an extensive video study of classroom practice was conducted in conjunction with and additional to TIMSS testing. This, to date, is the most recent data available in this format via the TIMSS study. In a study of the TIMSS video results from 1999 Hiebert (2003) found that Australian eighth grade students used a textbook or worksheets in 91% of lessons. Interestingly, this was the lowest of the seven countries analysed. The six countries included in the mathematics teaching analysis were Australia, the Czech Republic, Hong Kong, the Netherlands, Switzerland and the United States. The seventh participating country, Japan, did not participate in the mathematics section of the study and so their results were excluded. Hiebert (2003) stated that 75% of Australian lessons involved repetition of similar problems that were of low complexity and procedural in nature. This was the highest
percentage of the six countries and highlighted the issue that is referred to in this thesis as the ‘traditional’ teaching approach. Hiebert (2003) completed the study with the view to improve the quality of teaching and adds strength to Clements’ (2003) assertion that “to better understand and ultimately improve student’s learning, one must examine what happens in the classroom” (Hiebert, 2003, p. 2).

In an analysis of the same TIMSS video data, Vincent and Stacey (2008) found that Australian lessons consisted of questions of low complexity, meaning they were procedural in nature, with significant repetition and a lack of reasoning. They then commented that the textbooks, used as a resource in class and for homework tasks, generally focused on routine procedural questions which they described as being representative of ‘shallow teaching syndrome’ (p. 83). This description aligns with the traditional teaching approach which was the concern for Clements (2003).

The current context of mathematics teaching with respect to the traditional teaching methods discussed, needs to change to arrest the decline in performance of Australian students in the international context, and to address the issue of disengagement and the large number of underperforming students. As discussed by Hiebert (2003), a traditional teaching approach was used in 75% of mathematics lessons in Australian schools. The problem of traditional teaching as the main teaching style in Australian mathematics classes is clearly significant as discussed in the following section.

1.3 The Significance of the Problem

The current context of mathematics teaching within Australia and in the international context was briefly outlined in the previous section with emphasis on the need to
move from ‘traditional’ teaching, as described, to teaching that promotes relational understanding leading to gains in conceptual knowledge. The significance of the problems inherent in the traditional teaching approach will be discussed in this section.

In a report for the Australian Council for Educational Research (ACER), Sullivan (2011b) engaged in an extensive review of the teaching of mathematics drawing on his own extensive research and the findings of the National Research Council’s review. The research showed that the methods of teaching required to develop procedural fluency and conceptual understanding still hold challenges for teachers using a traditional approach. According to Sullivan (2011b) and Vincent and Stacey (2008) the problems inherent in traditional teaching have three main elements. First, the students follow step-by-step teacher-led procedures where students are provided with examples, and then repetitive, similar exercises are completed in class following the algorithm provided and usually finished for homework. Often when marking the exercises, the answers are given without any discussion and there is no concluding summary of the concepts from the lesson. Second, the students’ level of engagement with mathematics decreases due to the low level and repetitive nature of the mathematical tasks. Third, the over simplification or excessive scaffolding of tasks reduces their complexity to a level that results in a task that has a single answer and promotes procedural understanding only. The reduction in complexity of tasks resulted in lessons that were mainly repetition-based and the practicing of algorithms resulting in what has been described as traditional teaching is also a concern for Anthony and Walshaw (2009).
These issues of traditional teaching were raised over a decade ago when Clements (2003) considered the need for a change in the practice of mathematics teaching to be urgent, and that need is still an issue today.

There is an urgent need to change school mathematics and mathematics teacher education in fundamental ways… Old patterns and methods are so deeply entrenched in many schools and teacher education institutions, and particularly in the minds of teachers, lecturers and students, that there is an urgent need to problematise existing practice and equip and empower practitioners to achieve change. (Clements, 2003, p. 638)

The need to move away from traditional teaching was urgent in 2003 and remains urgent (Anthony & Walshaw, 2009; Clements, 2003; Dinham, 2013; Loewenberg et al., 2009; Masters, 2016; Sullivan, 2011b; White, 2013). The problem is significant as there is a need to develop conceptual knowledge through relational understanding developed by a student-centred approach to teaching. This needs to be considered as a replacement for or an additional facet to the traditional teaching that has a focus on instrumental understanding and procedural knowledge only.

The traditional mindset of many teachers in their practice and pedagogy, is central to this research as there is a need for investigation into methods of teaching which allow students to form links with material learnt and to develop deeper understanding of concepts and their applications. In a ten-year study of the development of resources for the teaching of K-8 students and the training given to teachers, Ball, Sleep, Boerst and Bass (2009) stated that improvement in the learning outcomes within the classroom come from attention to the practice of teaching. As
previously stated the need for improvement in teacher training was confirmed in the Action Now: Classroom Ready 2014 report.

As Clements (2003) and more recently White (2013) have stated, while there has been further research, it has not influenced the common practices of ‘traditional’ teaching. These included a lack of development of relational understanding and conceptual knowledge and an emphasis on instrumental understanding and procedural knowledge. This is problematic and requires research to provide direction for overcoming these problems. The traditional teacher has the textbook as the central focus of the teaching in the classroom. Vincent and Stacey (2008) agree with Sullivan (2013) when they defend the use of textbooks as a resource and suggest that teachers provide learning opportunities to their classes outside of the context of the textbook. Pehkonen (2004), in a study of textbook use in Finland, states “teachers want the mathematics textbooks to concentrate on the basics, since they believe the basics constitute good and proper mathematics teaching” (p. 519). It is this definition of ‘good and proper’ mathematics teaching that is the concern of Clements (2003) and others, for example, Boaler (2013), Ball et al. (2009), Dinham (2013), Hiebert (2007), Sullivan Warren & White (2000), White (2013), who proport the use of open-ended questions and investigations as part of the student-centred teaching paradigm.

However, while many school mathematics teachers work very hard and are very dedicated, using classroom strategies that are nourishing for young minds, Clements (2004) was deeply disturbed by the practices of some teachers exhibiting what he regarded as serious examples of cognitive undernourishment. He made use of Brousseau’s (1984) didactical contract construct to illustrate the issue. This construct examined the teacher’s role in the investigation of a higher-level mathematics task
and Brousseau observed that the opportunity for the students to develop a deep understanding of the task was significantly diluted by the teacher. This occurred when the teacher asked several simple questions that lead the students to the solution, or one solution, of the task without the challenge of the investigation of the task and this was an emptying of the cognitive challenge for the students (Clements 2004). Sullivan (2011a) noted that when students were having trouble the teacher provided easier tasks rather than have the students persist. He saw this as teaching that rewarded low level success and raised the concern that this was “…a recipe for anger, bitterness and self-doubt” (p. 17).

Brousseau’s (1984) didactical contract construct, as described by Clements, was further developed by Lim (2000, as cited in Clements 2004) who, following the investigation of the teaching and learning in a Form 4 class, wrote a seven-step contract. The contract gave a set of processes that was followed by the classroom teacher and that ultimately resulted in cognitive undernourishment of the students. This seven-step contract stated the way the teacher and student interacted, how the teacher taught and the expectation of student response. The contract had links to the use of scaffolding and the impact of scaffolding on working memory and cognitive emptying. The level of support can and should be different for individual students and too much scaffolding can remove all the cognitive aspects of the solution to a problem and is defined by Clements (2004) as cognitive emptying. When a complex task was over-scaffolded, the result was a task with little challenge that was an example of cognitive emptying, thus promoting procedural understanding only.

The level of task scaffolding must be considered with care to avoid cognitive emptying. The issue of excessive scaffolding in the solution of open-ended questions was problematic for both the teacher and the student. When interviewing upper
secondary teachers, Sullivan (2011a), was informed that in the teachers’ opinion if the solution to a problem is not immediately obvious then the students wanted to be told what to do rather than attempt to solve the problem. This attitude was a concern for Sullivan and is a concern for teachers as they decide on the amount and method of scaffolding or direction to provide in the completion of tasks. The students could be requesting additional scaffolding because of the ‘traditional’ teaching approach where the students are normally given examples and the questions that follow are directly related to the examples. Stacey (2010) in a study interviewing over 20 educators curriculum specialists and teachers concluded that Australian Mathematics teaching was generally repetitious, lacking complexity and rarely involved reasoning. The students are not required to think, or problem solve as they simply follow the example exactly. Therefore, the potential for cognitive emptying of the task needs to be foremost in the planning by the teacher. Effective teachers, prompt students, suggest alternative methods or discuss the question with others and have the students consider the problem further (Anthony & Walshaw, 2009).

However, according to Stacey (2010): teachers commonly report experiencing difficulties in incorporating problem solving and reasoning into their mathematics classrooms. The need for urgent change in mathematics teaching methods raised by Clements (2003) has been shown to still persist today. The issues with the traditional teaching approach and the cognitive emptying of tasks has been outlined and will be expanded on in Chapter 2. The significance and need for change in teaching practice as raised by Clements (2003) with support from White (2013), Dinham (2013), and Ball et al. (2009) has been outlined and the way in which the FQA may contribute will be briefly addressed in the next section and developed further in Chapter 3.
The FQA was developed to address the lack of development of conceptual knowledge and relational understanding in the ‘traditional’ teaching method used in many of today’s mathematics classrooms as outlined in the previous section. This section briefly describes the FQA, which will be expanded on in detail in Chapter 3. The FQA is a pedagogical approach that uses five carefully chosen questions given at the start of every lesson. The first four questions are designed to develop instrumental understanding and procedural fluency. The fifth question is designed to expand a students’ mathematical knowledge and understanding, providing possible opportunities for developing a relational understanding designed to result in an increase in conceptual knowledge. The types of questions used in the fifth question are open-ended and investigational in style and, in most cases, have more than one correct solution and multiple solution paths allowing students to construct or build their mathematical knowledge. Hints and scaffolding given while the five questions are being answered allow students who are having difficulty with one or more of the questions to have greater opportunities to answer correctly. The use of scaffolding and hints is closely monitored by the teacher so that the cognitive demands of the questions are not significantly decreased. While the students are attempting the questions, the teacher moves about the room and provides individual attention, positive suggestions and diagnostic feedback to the students.

The FQA was developed in response to the two main issues with the traditional teaching approach. First, the focus on teaching, which promotes instrumental understanding and procedural knowledge, rather than relational understanding and conceptual knowledge and; second, the following of the linear scope and sequence document, which restricts teachers to a fixed time frame for each topic. The fixed
time frame discourages teachers from spending additional time developing students’ understanding or returning to prior topics due to the lack of available time. The FQA provides a means of consolidating previously taught material, developing procedural fluency and then promoting the development of conceptual knowledge and schemas through investigative problems. The FQA was designed to provide students with the opportunity to develop their mathematical understanding of concepts over an extended period. Examples of the FQA are given in Chapter 3 and in Chapters 5, 6, and 7 where the results of the individual classes involved in the research are discussed. Although this study is focused on a pedagogical approach that uses five questions at the commencement of the teaching lesson, and not the entire teaching process, the FQA could provide some solutions that may be useful in addressing the issues raised in the previous sections, which have formed the aims of the research.

1.5 Aims of the Research

The overall aim of this study is to investigate the influence of implementing the FQA at the commencement of each mathematics lesson on the academic performance of students, perceived and actual, and the engagement of students. As previously mentioned the FQA may potentially provide a strategy that could be useful in addressing part of the need for change expressed by Ball et al. (2009), Clements (2003), Dinham (2013), and White (2013).

There is a wealth of research on the constructivist approach to teaching and learning in mathematics education with a view to developing conceptual understanding. See, for example, Barrett and Long (2011), Baxter and Williams (2010), Confrey and Kazak (2006), Dieterle (2010), Ernest (1991), and Fast and Hankes (2010). There is
some research on problem-based learning (Woods, 1985), particularly on the one problem per day method of learning mathematics (Grady, Yew, Goh, & Schmidt, 2014) and the use of focus questions at the start of a lesson (Gerver, 2011). However, there is no research on the constructivist approach of using a combination of instrumental understanding and procedural knowledge in conjunction with relational understanding and conceptual knowledge questions at the commencement of every lesson as in the FQA.

This study examines any change in student engagement and academic performance resulting from using the FQA as part of an overall teaching approach. It is anticipated that this research may have an influence on the traditional mathematics teaching paradigm by providing mathematics teachers with an alternate approach that may improve student learning and engagement. This could be actioned as the FQA provides a strategy that encourages the development of conceptual knowledge and relational understanding and this may lead in part, to a solution of the issues previously raised by Clements (2003).

This research aims to provide some direction for improvement in the teaching and learning of mathematics and mathematics pedagogy in general. The central research questions are:

1. What influence does the FQA have on perceived and actual academic performance of students in mathematics?

2. What influence does the FQA have on student engagement in mathematics?

The questions are informed by the following sub-questions:

- Are there specific aspects of the FQA that contribute to student engagement?
What influence do teachers perceive the FQA has on learning in the classroom?

The aims of the research are to measure any changes in perceived or actual academic performance, and any changes in student engagement. The next section explains the methodological approach.

1.6 Overview of Methodology

The research questions from the previous section were explained through a mixed-methods approach through trials in two coeducational schools. A Year 8 class, with two teachers in a job-share situation, was chosen from a 7 to 12 school and two classes one from Year 9 and one from Year 10 were chosen from a Years 7 to 10 school. Both schools graded the mathematics classes from Years 7 to 10 and had seven streams in each year. The data was collected on four occasions over an entire school year.

The research involved student focus group discussions, teacher interviews and classroom observations to collect the qualitative data on engagement and perceived academic performance. School test results in the half yearly and yearly examinations were used to collect quantitative data on actual academic performance. The student rankings in these examinations were compared to the previous year’s final examination ranking to determine any change in student academic performance. The rankings were determined from the examination marks. The range of marks in the examinations varied between 45 and 60 indicating that a difference in rankings may be the result of more than one mark. The data was collected, coded, and collated then
analysed according to the statistical methods outlined in Chapter 4, where the methodology is discussed in detail.
This thesis consists of nine chapters that present the rationale, method and outcomes of the study. A brief description of each chapter is provided below.

Chapter 2 provides a review of the current literature pertaining to teaching, learning, pedagogical practice and engagement within the context of the mathematics classroom.

Chapter 3 outlines the specifics of the Five Question Approach linking the FQA to the literature review in Chapter 2.

Chapter 4 details and justifies the methodological approach undertaken in this study and poses the research questions forming the basis of the study.

Chapter 5 is an analysis of the data from the St Patricia’s College Year 8 class. It includes the qualitative data collected through teacher interviews, student focus group discussions, classroom observation and the quantitative data from the half yearly and yearly examinations.

Chapter 6 is an analysis of the St Charles’ College Year 9 class data as described above for St Patricia’s College.

Chapter 7 is an analysis of the St Charles’ College Year 10 class data as described above for St Patricia’s College.

Chapter 8 is a comparison of the implementation of the FQA in the three classes. This chapter discusses the unique case of the focus group students who changed classes after the half yearly examination.
The thesis concludes with Chapter 9, which provides a summary of the results and a discussion of the implications for mathematics teaching and learning with respect to student academic performance and engagement. Suggestions for further research relating to the FQA in the teaching and learning of mathematics are presented.
In December 2008, the ‘Melbourne Declaration’ (MCEETYA 2008) was signed by all State, Territory and Commonwealth Education Ministers. This declaration ratified educational directions for Australia for the 21st century. The future approach to mathematics education in Australia and the two goals of the Melbourne Declaration were focused on young Australians. The first goal was that “Australian schooling promotes excellence and equity” (p. 7). Schools and teaching were central in the promotion of excellence and equity with high-quality schooling and quality teaching and leadership at the forefront. Mathematics is a key discipline area in schools. As discussed in Chapter 1 of this thesis, the quality of mathematics teaching and student engagement is an ongoing concern. To achieve the first goal of the Melbourne Declaration there needs to be a change in the teaching of mathematics in Australia. As this study will be conducted in New South Wales (NSW) the curriculum focus for this study will be on the NSW mathematics curriculum in conjunction with the Australian curriculum.

This literature review will examine the question of what is considered to be quality mathematics teaching. Quality mathematics teaching requires effective teachers and aspects of quality mathematics teaching from the teacher and student perspectives will be linked with the mathematics curriculum. Teaching and learning approaches and their links to student engagement and attitude will then be presented. Figure 2.1 shows the progression through this chapter and the flow of the literature review.
Figure 2.1 *Quality Mathematics Teaching*

- **Quality Mathematics Teaching**
  - PCK, MKT, MCK, Explanations, Examples

- **Curriculum**
  - NSW K-10 Syllabus, Australian Curriculum

- **Teaching and Learning Approaches**
  - Instrumental and Relational understanding,
    Procedural and Conceptual knowledge,
    Constructivist Teaching
  - Spacing, Interleaving, Fluency,
    Competence, Repetition with Variation

- **Engagement and Attitude**
  - Feedback, Discussion, Growth Mindset,
    Confidence, Understanding, Reduced Anxiety

- **Engaged High Achieving Learners**

- **Effective Mathematics Teaching**
Sections 2.1 and 2.2 will discuss quality mathematics teaching and the links between quality mathematics teaching and the curriculum, in this instance the NSW K-10 Syllabus (2012). As Figure 2.1 indicates, there is a relationship between quality teaching and the curriculum. This discussion then informs the teaching and learning approaches, which are the focus of sections 2.3 and 2.4. These in turn link to section 2.5, which focuses on student engagement and attitude. Section 2.3 examines instrumental and relational understanding in mathematics, linking these to procedural and conceptual knowledge. The constructivist approach to teaching and the place of direct instruction as a teaching approach are discussed in sections 2.3.3 and 2.3.4. The ‘traditional’ teaching approach, which usually relies on textbook or worksheet exercises that are massed and promote overlearning, will be contrasted in section 2.4 with the more effective teaching and learning strategies of spacing, interleaving and repetition with variation in the development of both fluency and conceptual understanding. The engagement and attitude section of the diagram encompasses aspects of sections 2.4 and 2.5 as it links the teaching and learning approaches with student engagement and attitude. Aspects of feedback, discussion, student confidence, growth mindset and understanding that results in reduced mathematical and examination anxiety will be discussed in sections 2.4 and 2.5. The final section, section 2.6, links with the end result of the diagram—engaged high achieving learners resulting from quality mathematics teaching. Thus, this literature review will focus on the requirements for quality mathematics teaching which requires effective mathematics teachers and engaged high achieving learners. To begin, a definition of quality mathematics teaching is provided in the following section.
2.1 Quality Mathematics Teaching

There are two aspects to the development of mathematical knowledge: the teaching process and the learning process. In this section I will outline that which constitutes quality or ‘good’ mathematics teaching and subsequently the methods by which students learn mathematics. Quality mathematics teaching requires effective teachers and engaged students. To enable quality teaching and learning students need to be engaged in the learning process. In determining if the Five Question Approach (FQA) to teaching mathematics can influence the academic performance and engagement of students in mathematics classes, it is necessary to determine what constitutes quality, effective or ‘good’ teaching and particularly quality, effective or ‘good’ mathematics teaching. The teaching process is discussed in the following section on quality teaching. Elements of quality teaching and student engagement in the secondary classroom will also be discussed in this next section.

2.1.1 Quality teaching

There is a body of research that has examined the characteristics of effective mathematics teachers, resulting in broad consensus on many of the requirements. Sullivan (2011b) compiles a list of six principles for quality teaching of mathematics. They are:

- Articulating goals
- Making connections
- Fostering engagement
- Differentiating challenges
• Structuring lessons, and
• Promoting fluency and transfer (p vii).

NCTM (2014) lists eight mathematics teaching practices, they are:

• Establish mathematical goals to focus learning
• Implement tasks that promote reasoning and problem solving
• Use and connect mathematical representations
• Facilitate meaningful mathematical discourse
• Build procedural fluency from conceptual understanding
• Support productive struggle in learning mathematics
• Elicit and use evidence of student thinking (p.10).

Both lists contain the same aspects reflected slightly differently but both supporting the meaningful development of conceptual understanding through rich and meaningful mathematical problems and investigations. This is in contrast to the ‘traditional’ teaching paradigm. These principles are also reflected in the list of the attributes of effective teachers compiled by Ediger (2012), which is similar to those of Anthony and Walshaw (2009), Ball and Rowan (2004), Bliss, Askew, and Macrae (1996), Ernest (1989), and Goe (2007). The research by Ediger (2012) found characteristics of effective teachers that are have been synthesised into the following list:

1. Promote student interaction through group work and class discussions through the way in which the content was presented, enabling students to attach meaning to the material being presented;

2. Provide students the opportunity to interact with each other which enabled them to build on the thinking of their peers;
3. Provide a comfortable and safe learning environment with a structured and organised classroom; and

4. Possess strong pedagogical content knowledge with multiple methods of instruction and knowledge of the typical or common areas of difficulty in student understanding that facilitated the learning.

To teach the content, an effective teacher must understand the mathematical skills and knowledge of their students and be able to constantly monitor the progress of students and adjust the pace and scaffolding of the lesson to ensure that student understanding is promoted. Pacing and scaffolding as the knowledge is constructed is seen as crucial by Bell and Pape (2014), Ediger (2012), Bliss et al. (1996), and Wass and Golding (2014), as it ideally allows students to operate within their Zone of Proximal Development (ZPD) (Vygotsky, 1978). The ZPD is the place of transition between that which a student is able to complete without assistance compared with possible achievement with assistance from teaching and learning activities and the teacher. Students working within their ZPD feel comfortable but to expand their knowledge they need to move beyond this, which usually requires assistance from their teacher often in the form of additional scaffolding. Teacher decisions regarding the level and amount of scaffolding provided is important as under-scaffolding of the lesson may reduce the success of the students, while over-scaffolded lessons may result in very superficial learning or ‘cognitive emptying’ taking place. Vygotsky (1978) found that when a complex task was over-scaffolded, the result was a task with little challenge and hence, an example of cognitive emptying; promoting procedural understanding only. A lesson that is too fast paced may result in students being unable to keep up with the content and concepts while a lesson presented too slowly may result in student boredom and disengagement.
Teacher decision making regarding the amount of scaffolding required is often dependant on their level of Pedagogical Content Knowledge (PCK), which Shulman (1987) describes as a combination of content knowledge and pedagogy. Shulman states that content knowledge is the teacher’s understanding of the concepts or skills that must be taught to the students, but that PCK combines that content knowledge with the appropriate teaching method, with an expert teacher usually having greater PCK than a novice teacher (Shulman 1987).

Building on the research of Shulman (1987) on PCK, to teach effectively, a teacher needs a high level of Mathematical Knowledge for Teaching (MKT) which, as described by Ball and Forzani (2010), and Hill et al. (2008), is a construct that contains both mathematical content knowledge (MCK) itself, and the knowledge of mathematics teaching or PCK. Both Ball and Forzani (2010), and Hill et al. (2008) conclude that there is a strong relationship between the two factors. The more a teacher knows and the greater their pedagogical understanding, the better they can enact effective instruction in mathematics. An effective mathematics teacher not only has a strong and extensive knowledge of content but a depth and variety of teaching strategies that can be implemented in the classroom (Hill et al., 2008).

These concepts are further developed by Goos (2013) who examined the links between MCK and PCK within MKT. In this Australian study, the conclusion was that both are required for quality teaching. Interestingly, a high level of MCK does not necessarily make an effective teacher and both MCK and PCK are needed. While PCK cannot exist without MCK, a high level of MCK does not necessarily imply a high level of PCK. Effective teachers according to the Australian Institute for Teaching and School Leadership teaching standards (AITSL 2017b), need to know their subject matter (MCK) and how to teach it (PCK). Included in the PCK
construct is the ability to build on the current level of students’ understanding to develop new understanding which links to Vygotsky’s ZPD. Using the ZPD effectively allows teachers to use their PCK to decide which tasks will allow students to learn, by not using an activity that they can already do or a task that is too difficult (Wass & Golding, 2014). Learning takes place when a student moves beyond their ZPD to solve a problem that they would not have been able to solve without assistance from a peer or teacher (Levykh, 2008).

In conclusion, quality teaching of mathematics requires that the effective teacher has sufficient MCK, PCK and MKT. It is generally accepted that the purpose of teaching is to develop students’ understanding and knowledge. According to Anthony and Walshaw (2009), Attard (2011), Kaur (2009), Murray (2011), and Shimizu (2009) students described teachers who helped them to understand as ‘good’ teachers. Effective teachers understand the need to develop conceptual understanding and the fact that ‘traditional’ teaching focused on procedural knowledge rather than conceptual knowledge does not allow the development of conceptual understanding.

Presenting the research on quality mathematics teaching has shown that effective mathematics teachers need strong mathematical content knowledge, a high level of PCK, an awareness of students’ current level of understanding and the ability to promote cognitive, operative, and affective engagement. The next section examines students’ perspectives on quality mathematics teaching and effective teachers.
2.1.2 The student perspective

The previous section examined the research on quality mathematics teaching. While the research involved teachers and students it was the teaching process that was the focus. The definition of an effective teacher from students’ perspectives will now be considered.

In a longitudinal study involving 20 middle years students across three school years, Attard (2011) documented students’ responses when asked about what makes a ‘good’ mathematics teacher. The responses indicated that a ‘good’ teacher is one with a positive attitude and passion for their subject; who uses scaffolding to assist problem solution and gives clear explanations. The ‘good’ teacher is aware of students’ prior knowledge and is responsive to students’ individual needs.

This view was supported by Murray (2011) in a study of 90 students in two schools who found that according to the students, ‘good’ teachers gave clear patient explanations, provided individual assistance, and had a classroom that was a safe, positive and a controlled learning environment where explanations were clear. In terms of the previous section which detailed teaching requirements, the students were describing teachers with strong PCK. This conclusion was supported by Shimizu (2009) in a study of 60 Japanese students where approximately half of the students in the study stated that understanding was the most important aspect of a ‘good’ mathematics lesson. They said it was the explanation given by the teacher along with class discussion that promoted student understanding and contributed to a ‘good’ lesson.

A study of Singaporean students by Kaur (2009) drew similar conclusions to Attard (2011), Murray (2011), and Shimizu (2009) concerning the importance of teacher
explanation. The student interview data highlighted three main aspects of a ‘good’ lesson. These aspects were: the explanations were given by the teacher in a clear and simple manner that reviewed past knowledge and introduced new knowledge; the students had an opportunity to work on interesting material either individually or in groups; and the lesson included review and feedback. The students preferred lessons presented using a problem solving constructivist approach to the traditional example and repetitive question format.

The student perspective and that of the teachers lead to the conclusion that ‘good’ or effective teachers know their content and can explain it well. However, to have the opportunity to explain content it is necessary to have engaged learners (Bonnett, Yuill, & Carr, 2017). Student engagement and the issue of student disengagement is the topic of the next section.
2.1.3 Student engagement

There is significant research into student engagement including studies by Fredericks (2011), Kiemer, Gröschner, Pehmer, and Seidel (2015), and Munns and Woodward (2006), which aligns with the definition according to Attard (2014). She states that engagement is related to the depth at which the students relate to their classroom work and outlines three dimensions of engagement: behavioural, cognitive, and affective. These three dimensions are manifested in active participation, valuing of learning and the willingness to be involved.

Engagement with mathematics is an issue and according to Plenty and Heubeck (2013) there has been a decline in participation in mathematics over the past 20 years. There are reduced numbers in enrolments in higher levels of mathematics in schools and universities and this trend is not restricted to Australia, as the continued decline of enrolments in higher levels of mathematics continues in the UK and other developed countries (Brown, Brown, & Bibby, 2008). The results of the study conducted over a year by Plenty and Heubeck (2013) showed disengagement to be higher in half of the groups tested with the level of disengagement shown in Year 7 continuing into Year 9. The study used a modification of the Motivation and Engagement Scale-High School (MES-HS) (Martin, 2007) a tool with 11 measures of academic motivation and engagement. When engagement in mathematics was compared with other subjects, it was found to be less positive in mathematics (Plenty & Heubeck, 2013).

The second goal of the Melbourne Declaration (MCEETYA 2008) states that “all young Australians should become successful learners, confident and creative individuals and active and informed citizens” (p. 7). In the detail of that goal the
significant risk of students’ lack of engagement in the middle years of schooling was raised as a major concern. The engagement of students, particularly in the middle years of schooling, Years 5 to 8, has been the focus of considerable research. The level of engagement of students in mathematics generally declines from the end of primary school, typically Year 6 through the first year in secondary and into Year 8 (Brown et al., 2008, Martin, Way, Bobis & Anderson, 2015, Plenty & Heubeck, 2013).

To achieve the second goal of the Melbourne Declaration (2008) and enhance development in the middle years, specific teaching approaches need to be put forward to counteract the great risk of disengagement during this time in a student's schooling. The Melbourne Declaration states that “focusing on student engagement and converting this into learning can have a significant impact on student outcomes” (p. 12). When discussing the promotion of a world-class curriculum, the Melbourne Declaration (2008) notes that “English and Mathematics are of fundamental importance in all years and are the primary focus of learning in the early years” (p. 14). Clearly, quality teaching and particularly the teaching of mathematics is vital to the successful performance of young Australians in the world market. The declaration flags engagement in the middle years of schooling as a significant concern. As such, engagement in mathematics will be further investigated in section 2.5. With quality teaching, effective teachers and engaged students, learning can take place and the next section examines a framework for quality teaching of mathematics.
2.1.4 Framework for quality mathematics teaching in the Australian context

Engaged students result from quality teaching. Quality teaching from both the teacher and student perspective was discussed in sections 2.1.1 and 2.1.2 and this section sets out a framework for the examination of quality mathematics teaching. It is the work of effective teachers working with the curriculum and engaging students that results in quality mathematics teaching.

A result of the signing of the Melbourne Declaration (2008) was the formation in 2010 of the Australian Institute for Teaching and School Leadership. AITSL is funded by the Australian Government and operates as a government agency. It acts on the behalf of all education ministers of Australia. The work of AITSL is shaped by the two goals of the Melbourne Declaration. In its 2017 – 2020 Strategic Plan, the mission of AITSL is: “promoting excellence so that teachers and leaders have the maximum impact on student learning in all Australian schools and early childhood settings” (AITSL, 2017b p2). The AITSL goals are designed to guide teachers towards quality teaching by having a focus on the aspects of quality teaching raised in section 2.1.1. i.e. making connections, fostering engagement, differentiating challenges, structuring lessons, and, promoting fluency and transfer.

In its mission statement AITSL:

- defines and maintains national standards for teachers and principals
- leads and influences improvement in teaching and school leadership
- supports and recognises high-quality professional practice.

AITSL has developed the National Professional Standards for teachers, which is a document outlining quality teaching in six statements, with subsections, of quality
standards. These standards are within three domains of teaching: professional knowledge, professional practice, and professional engagement. These three domains of teaching developed by AITSL, are in alignment with the aspects of quality mathematics teaching and effective teachers discussed in previous sections of this thesis. As such, it is the areas of professional practice, professional knowledge, and professional engagement that will be the focus of this research. Teachers teach from a set curriculum and need to develop pedagogy suitable for that curriculum. They can use their MKT, MCK and PCK to assist them in finding methods of explanation and teaching that engage students. The set curriculum is the vehicle used by the teachers to develop quality teaching and learning activities and is the focus of the next section.

2.2 Mathematics Curriculum in NSW, Australia

Across the world, each country and in some cases jurisdictions within countries develop their own unique mathematics curriculum. Although these documents have much in common there are also differences in content and form. This section will begin with a brief history of the mathematics curriculum in NSW, Australia where the research was conducted.

Prior to 2014, Australia did not have a national curriculum. Instead, each state determined and implemented their own curriculum and the complexity and expectations of each state’s mathematics syllabus varied considerably. The development of a national curriculum commenced with the formation of the independent statutory authority, the Australian Curriculum and Assessment Reporting Authority (ACARA) in 2009. The responsibilities of ACARA included
the development of the national curriculum, evaluation of that curriculum, data
collection and accountability, and reporting on schools. The development of the
Australian National Curriculum Mathematics F-10 commenced in 2009 and was
implemented in 2014.

Australia was not alone in the development of a national curriculum. At a similar
time, the United Kingdom, Singapore and Hong Kong were implementing new
national curricula. Commencing in 2000 New Zealand underwent a review of their
outcome-focused curriculum that had been in use since 1992. The review process
produced a draft document in 2006 and the new national standards were adopted in
2010. The United States of America does not have a national curriculum but has an
optional set of standards, in place since 2000, which have been taken up by many
school districts and states. There is a great commonality amongst the mathematics
curricula of Australia, the United Kingdom, Singapore, Hong Kong and the USA.
All have the aims of concept development in a problem-solving mathematical
context. All countries except Singapore have aims associated with the appreciation
of mathematics and these aims will be investigated under the heading of engagement
in section 2.5.

The curriculum in NSW has undergone many changes over the past 20 years and in
contrast to the other states, has had different levels or pathways in Years 9 and 10. In
the NSW, the Board of Studies NSW was renamed the Board of Studies Teaching
and Educational Standards (BOSTES) in 2014 and combined the curriculum,
teaching and assessment functions of the Board of Studies NSW with the registration
and policy functions of the NSW Institute of Teachers. The NSW Institute of
Teachers oversees a system of accreditation and recognition of a teacher's
professional capacity against professional teaching standards. The standards describe
what teachers need to know, understand and be able to do (AITSL, 2017b). The NSW K-10 Mathematics Syllabus (2012) document was developed from the ACARA national curriculum at the same time as the national curriculum. The NSW K-10 Mathematics Syllabus (2012) contains all of the national curriculum material and additional curriculum items specific to NSW. The NSW Education Standards Authority (NESA) (2017), replaced BOSTES on 1 January 2017 and included all BOSTES responsibilities including the monitoring of teaching standards, the support of teachers and the improvement of student achievement.

The rationale behind the development of the NSW BOSTES K-10 Mathematics Syllabus (2012) was:

“Mathematics in K-10 provides students with knowledge, skills and understanding in Number and Algebra, Measurement and Geometry, and Statistics and Probability. It focuses on developing increasingly sophisticated and refined mathematical understanding, fluency, communication, logical reasoning, analytical thought and problem-solving skills. These capabilities enable students to respond to familiar and unfamiliar situations by employing strategies to make informed decisions and solve problems relevant to their further education and everyday lives” (p. 10).

The rationale outlines the important aspects of the teaching and learning involved in the implementation of the NSW curriculum according to the authors of the curriculum document. The capabilities outlined in the rationale closely link many of the attributes of the effective teacher from section 2.1.1 and the students’ perspective of an effective teacher in section 2.1.2. Effective teachers develop their students’
mathematical concepts, fluency, communication, reasoning and problem solving. This requires particular knowledge and understanding on the part of the teacher and the next section explains what constitutes the knowledge and understanding for quality teaching.

2.3 Knowledge and Understanding for Quality Teaching

The NSW K-10 Syllabus for mathematics, under the heading of knowledge, skills and understanding, states that when students are working mathematically they: “develop understanding and fluency in mathematics through inquiry, exploring and connecting mathematical concepts, choosing and applying problem-solving skills and mathematical techniques, communication and reasoning” (p. 14). Therefore, the intent of the curriculum is to develop confident mathematical problem solvers, who can apply their mathematical skills beyond routine applications and communicate their solutions and the reasoning used to solve mathematical problems. Section 2.3 will discuss the knowledge and understanding required by teachers to be effective in the implementation of the mathematics curriculum. The development of procedural and conceptual knowledge and the importance of each type of knowledge will be discussed along with links to instrumental and relational understanding. The constructivist approach to teaching and the timing and point of direct instruction within the constructivist learning environment will also be considered in section 2.3.

In section 2.1.1 it was shown that effective teachers develop students’ understanding and knowledge by building on their current knowledge. Quality teaching requires teachers to develop conceptual knowledge and relational understanding using their MKT, MCK and PCK applied to the relevant curriculum. In contrast, the
‘traditional’ teaching approach as described in Chapter 1, has a focus on procedural knowledge and instrumental understanding. This section describes the types of knowledge and understanding required for quality teaching.

2.3.1 Procedural and conceptual knowledge

In earlier sections it has been established that it is important to develop conceptual knowledge and relational understanding rather than only procedural knowledge and instrumental understanding. In the examination of procedural and conceptual knowledge, a link will be made with instrumental and relational understanding.

According to Skemp, *understanding* involves the formation of schema, which is a linking of concepts to form a conceptual structure that can be communicated to another person, whereas *knowledge* is developed by individuals as they connect newly developed schema and their existing schema (Skemp, 1987). It is the links between the concepts that takes relational understanding to conceptual knowledge, or the development of schemas.

The combining of concepts into schemas increases a person’s understanding and ability to solve more complex problems. Skemp (1987) opines that the greater the number of concepts and resulting schema the more equipped a person is to solve complex or unfamiliar problems. It is important that schemas are accurate as the building of knowledge is based upon previous schema, and the concepts within that schema. If the schema is flawed, then either the new concepts will not be understood, or the new concepts will be assimilated incorrectly.

An example will be taken from the fraction topic area to illustrate the development of a schema (Skemp, 1987). The procedural knowledge and instrumental understanding of the division of fractions is the learnt rule that states to turn the
second fraction upside down and change division to multiplication. By applying this rule correctly, the division of fractions can be successfully completed. In contrast, a schema for the division of fractions would indicate understanding of how the process works and why this process is used. This can be developed in several ways. For example, by writing the two fractions with a common denominator and then multiplying both fractions by that denominator to turn the fraction division into a standard algorithmic division question. Alternatively, a pattern of division starting with eight divided by four showing that as the divisor is halved the quotient is doubled leading to dividing by a half is equivalent to multiplying by two. The methods are displayed in Table 2.1.

Table 2.1 *Division of Fractions*

<table>
<thead>
<tr>
<th>Invert and Multiply Strategy</th>
<th>Common Denominator Strategy</th>
<th>Pattern Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{2}{3} \div \frac{4}{5} ] = [ \frac{2 \times 5}{3 \times 4} ] = [ \frac{10}{12} ] = [ \frac{5}{6} ]</td>
<td>[ \frac{2}{3} \div \frac{4}{5} ] = [ \frac{10}{15} \div \frac{12}{15} ] = [ \frac{10}{12} ] = [ \frac{5}{6} ]</td>
<td>[ 8 \div 4 = 2 ] [ 8 \div 2 = 4 ] [ 8 \div 1 = 8 ] [ 8 \div \frac{1}{2} = 16 ] [ 8 \div \frac{1}{4} = 32 ]</td>
</tr>
</tbody>
</table>

These alternative processes of fraction division allow students to develop a relational understanding of division of fractions resulting in a schema and the development of conceptual knowledge. This relational understanding could then lead to the
development by the student of the invert and multiply technique as a solution for fraction division. Students would then be able to apply their understanding to the solution of a fraction division with more than two fractions whereas students with an instrumental understanding of fraction division would not be able to extend their understanding to the solution of fraction division with more than two fractions.

To proceed, a definition for procedural and conceptual knowledge is required. There are many studies on procedural and conceptual knowledge including those by Baroody, Feil, and Johnson (2007); Engelbrecht, Bergsten, and Kågesten (2009); LeFevre et al. (2006); Rittle-Johnson, Star, and Durkin (2009); Schneider, Rittle-Johnson, and Star (2011); Star and Rittle-Johnson (2009) and Wearne and Hiebert (1988). Based on the definitions from these studies the following definitions will be used. Procedural knowledge is the application of a learnt process to solve a problem by following rules and manipulating symbols without necessarily understanding how that process works. Conceptual knowledge is an understanding of how and why a procedure works and consists of links between schema to promote further understanding.

An analogy would equate procedural knowledge to a tradesperson’s tool box with the necessary tools to complete a task and the knowledge to operate each tool, constituting instrumental understanding. Conceptual knowledge then is the tradesperson’s understanding of how to complete the task required, which tool to use and the order in which the task is to be attempted, and the relational understanding that allows modification of the procedure in the case of an unexpected issue.

Similarly, in mathematics, a student may have the tool that enables them to solve linear equations (procedural understanding) but is not able to interpret a written question to obtain the equation (conceptual understanding), that needs to be solved.
When writing on mathematics curricula, Ollerton (2009) states that concept development and conceptual knowledge occurs at different rates and to different depths for different individuals. This is because conceptual knowledge for individuals is more of a map or network rather than a linear progression. Unfortunately for school mathematics teachers and students, most syllabus documents are written in a form that is often interpreted in a linear progression. The time available to students, as determined by the scope and sequence, is predetermined and may not allow sufficient time for some students to develop the understanding required before moving on to the next topic.

Conceptual knowledge and relational understanding are required at some level to develop deep procedural knowledge and fluency in the solution of mathematical problems in familiar and unfamiliar contexts. Skemp (1987) highlights that it is the addition of meaning to memorisation that increases the effectiveness of retaining the information over a process that is simply rote learnt. However, it is possible to develop conceptual knowledge from procedural knowledge providing that the procedural schema is linked together resulting in the formation of a concept (Skemp, 1987). These ideas have been synthesised into Figure 2.2 which shows the links and processes resulting in the formation of a concept.
Procedural and conceptual knowledge can be developed through teaching activities, examples and problem-solving activities resulting in the development of a schema. This schema is confirmed through input from further teaching and learning activities. The linking of schema through the development of conceptual knowledge results in the formation of a concept. The issue of whether to teach conceptually prior to introducing procedures or vice versa, will be addressed in the next section.

2.3.2 The procedure or the concept, which comes first?

According to Skemp (1976), students with relational understanding and conceptual knowledge are better able to learn new procedures and to retain their understanding and skills for a longer period. Hiebert and Wearne (1996) found that the link between conceptual understanding and procedural understanding is still unclear but the research indicates that conceptual understanding does influence the way in which procedural knowledge develops in the classroom. They found that the students
would develop their own procedures from their conceptual understanding and in most cases, develop their own procedures in conjunction with teacher guidance.

Where the teaching style aims to develop relational understanding and conceptual knowledge, a means is provided for students to either develop their own procedures to solve a problem or modify existing procedures for the same purpose. The students are better able to interpret procedures presented by other students. “Conceptual understanding makes possible the construction and deployment of meaningful solution procedures” (Hiebert & Wearne, 1996, p. 280). In contrast, instrumental understanding and procedural knowledge allows only a previously learnt solution path to be applied to the problem.

Procedural knowledge as defined by McCormick (1997) is the ‘doing’ aspect of completing a mathematical task and conceptual knowledge is the explanation of why the procedure is correct, reflecting that the student has understanding. Until an approach to solve a problem is developed, it is not possible to implement the correct procedure. McCormick (1997) links the developing of relational understanding with conceptual knowledge and the development of schema in alignment with the views of Skemp (1976).

A problem in a familiar context can be solved using instrumental understanding and procedural knowledge but relational understanding and conceptual knowledge is required to solve a problem in an unfamiliar context. There is significant research examining the development of procedural and conceptual knowledge (Byrnes, 1992; Canobi & Bethune, 2008; Engelbrecht et al., 2009; LeFevre et al., 2006; Panasuk, 2011; Schneider et al., 2011; Schneider & Stern, 2010; Star & Rittle-Johnson, 2009; Star & Seifert, 2006). These studies conclude that some problems can be solved with
only procedural knowledge providing the problem type is recognised and that type has been solved before and that the student remembers the procedure correctly. However, conceptual knowledge is required to determine the appropriate procedure for the solution of an unfamiliar problem and to determine the most efficient method of solution. Conceptual knowledge is required to solve other problems where the procedure is not immediately obvious, or the most efficient method of solution is required.

For example, in the NSW 2-unit mathematics Higher School Certificate (HSC) course, the questions that cause the most difficulty to students are the ones that combine concepts from multiple topics of the course. Examples of these types of questions are discussed in the examiners comments available on the NESA website (www.educationstandards.nsw.edu.au). In many cases, each concept or group of concepts, is taught separately in a discrete chapter from a text and therefore only problems related directly to that concept are investigated and this is repeated for each topic and chapter. Exponentials and logarithms, calculus of trigonometric functions and applications of calculus to the physical world are all taught separately. An HSC question combining these topics is therefore perceived as difficult due to the students and teachers not combining the concepts into a useful schema or a joining of schema during their class instruction.

In conclusion, conceptual and procedural knowledge are related and improvement in either or both one can result in improvement in student performance. Thus, the greater prior knowledge, whether procedural or conceptual, the more potential that exists to solve problems in unfamiliar contexts. The implication for teachers is that both approaches need to be developed in students. The constructivist teaching
approach develops both procedural and conceptual knowledge and is the content presented in the next section.

2.3.3 Constructivist approaches to teaching

The previous section concluded that procedural and conceptual knowledge along with instrumental and relational understanding are the outcomes of effective teaching drawing on teachers’ MCK, PCK and MKT. The constructivist approach to teaching is designed to build on students’ current knowledge to promote conceptual knowledge and relational understanding. The NSW K-10 Mathematics syllabus (2012) in the outcomes section states that “students need appropriate time to explore, experiment, and engage with the underpinning concepts and principles of what they are to learn” (p. 15). This section examines the constructivist method of developing mathematical concepts.

Constructivism is the creation of mathematical knowledge where students construct their own knowledge and build on their current knowledge (Dieterle, 2010; Ernest, 1991). Teaching and learning using a constructivist approach positions the learner as building new knowledge for themselves because of the teaching and learning activities in the classroom. This new knowledge must be based on previous knowledge and attached to that previous knowledge and be constructed through engagement in mathematical activities and discussions in a learning environment, typically a classroom (Baxter & Williams, 2010). Concepts are developed through the construction of learning which occurs in context within the environment as opposed to the transmission of knowledge from teacher to student approach, ‘traditional’ teaching (Piaget, 1954). The development of knowledge as described in
this section directly relates to the theories of knowledge development proposed by Skemp (1976) and discussed previously.

Constructivism or constructivist teaching approaches allow students to develop conceptual understanding and skills in communicating learned ideas by encouraging independent thinking through open ended problem based activities that facilitate shared understanding that is communicated and discussed in the classroom (Ross 2012). In their study of 16 videos lessons Ross (2012) concluded that students learn effectively in a classroom where the focus is on constructivist strategies, problem solving and the discussion of mathematical ideas. In contrast to ‘traditional’ teaching constructivism allows students to construct their own understanding and knowledge as they engage in the problem solving, open ended activities with the teacher providing the activities that allow knowledge construction (Fast 2010). “A constructivist teacher must have an extensive mastery of the content and the ability to follow a student's logic and guide appropriately when erroneous conjectures arise” (Barrett, 2011 p. 77). To teach in a constructivist manner, it is the role of the teacher to use his or her extensive MKT, MCK and PCK to determine the individual learner’s starting point. The teacher then analyses the development of errors or student misunderstandings during the learning process, aligning with the ZPD of Vygotsky (1978). It is through this process that the teacher as a facilitator can build student knowledge and skills. The constructivist approach to teaching is sometimes aligned with discovery learning and perceived to be a case of the ‘blind leading the blind’ in the investigation of mathematical concepts. This inaccurate understanding stems from a belief that while the teacher is the facilitator of learning that the learning is unstructured, and the students receive no guidance at all as in some examples of discovery learning (Alfieri, Brooks, Aldrich, &Tenenbaum, 2011;
Kirschner et al., 2006). Whereas in a constructivist orientated learning environment links to prior learning, scaffolding, direct teaching and discovery learning are all important aspects (Barrett & Long, 2011).

What then is the direct instructional approach? Using a constructivist approach to learning does not preclude the presentation of examples, demonstrations or lectures. It is the appropriate decision making by the teacher that determines the approach. Learning does not take place without some elements of instruction, particularly in the provision of a solid base on which to construct mathematical learning (Barrett & Long, 2011). The amount of guidance and structure that a teacher provides to students in their learning is problematic. Too much assistance will dilute the learning and insufficient scaffolding may not allow the learning to progress as desired by the teacher (Baxter & Williams, 2010). It is often referred to as the transmission of knowledge with the teacher being the font of knowledge, transferring this knowledge to the student and resulting in the development of a procedural approach to problem solving (Fast & Hankes, 2010; Kirschner, Sweller, & Clark, 2006; Snel, Terwel, Aarnoutse, & Van Leeuwe, 2012).

A teaching approach in Mexico was described by Garcia and Pacheco (2013) in terms of direct instruction, where the learning was not designed for individual differences of the students in the class but was directed at one level to the entire class at the same time. In discussion of the transition of the teaching from an instructional to a constructivist approach, Garcia and Pacheco (2013) define the change that is necessary and in doing so provides a definition for teaching using a constructivist approach. “It is important that Mexican elementary school teachers understand that knowledge acquisition occurs from knowledge that the students already possess, and this differs from student to student” (Garcia & Pacheco, 2013 p. 27). The Mexican
research has links to the Australian and NSW context as it proports the use of a constructivist approach and a move from the instructional or procedural approach that is the basis for the ‘traditional’ teaching approach previously described.

Teachers who take a constructivist approach operate on a continuum, moving from one approach to another based on the mathematics being studied and the needs of the students (Barrett & Long, 2011). The current reform movement in mathematics education urges teachers to support students as they make sense of mathematics. Teachers must ensure that the students gain specific mathematical skills and knowledge and maintain a balance between leaving students to work on their own and providing some support in the mathematical concept building process (Baxter & Williams, 2010).

The importance of a teacher being an analyser of student errors and misconceptions rather than just an instructor of mathematical skills was emphasised by Fast and Hankes (2010). The key to quality teaching is for a teacher to be able to analyse student errors and use their PCK to provide clear explanations and assist students to be successful. Teachers should scaffold work by asking appropriate questions or providing hints to enable students to construct their learning rather than being told what to do. According to Fast and Hankes (2010), it is the skill of knowing what question to ask that comes from the PCK of the teacher. Hints, provided by the teacher, allow students to move forward in the solution of the problem and proceed in a direction to arrive at the required learning. The level of scaffolding, hints or support through clear explanations is reduced as the student develops in their understanding of the concept being investigated. Therefore, an effective teacher can adjust the level of scaffolding using their PCK to enable every student to successfully learn in the classroom.
Learning in the classroom involves scaffolding and hints but also involves the presentation of teacher-led examples. Intentional teaching (direct instruction) through teacher provided examples increased content knowledge and had a significant positive influence on student confidence and engagement (Fast & Hankes 2010). Garcia and Pacheco (2013) also recognised the importance of a solid basis of clear and correct examples that can be used to further construct new mathematical skills and understanding. They write: “The constructivist model recognises the benefits of students participating in tasks that enable the active construction of their own knowledge domain” (Garcia & Pacheco, 2013 p. 28).

The constructivist approach ideally is a combination of direct instruction, discovery learning, and problem solving, but has been criticised by proponents of direct instruction. ‘Traditional’ teachers state that the constructivist approach takes too much time and that direct instruction is more efficient. In the analysis of their research Fast and Hankes (2010) state: “Detractors of an integrated approach often mention time availability as an obstacle. This study, however, showed that this was not an issue” (p. 338). Thus, the time argument used against the constructivist approach to validate a more direct form of instruction did not hold in that study. The purpose of using a constructivist approach is to aid students in their development of concepts and schema. In the NSW K-10 Mathematics syllabus (2012) the development of mathematical concepts is clearly stated as one of the aims. Direct instruction through examples is a component of a constructivist teaching approach along with discovery learning and problem solving. However, proponents of the ‘traditional’ approach use direct instruction as the prime or only method of student learning. Direct instruction will be examined in more detail in the next section.
2.3.4 Direct instruction

The previous section discussed the constructivist approach to teaching. The ‘traditional’ model of teaching uses direct instruction as the main teaching method. Direct instruction is a teacher-centred approach to learning and is based on the transmission of mainly procedural knowledge and the development of instrumental understanding. It is also possible to use a direct instruction method to develop conceptual knowledge and relational understanding. It is the level of scaffolding that is the key. This section investigates the direct instruction model of teaching.

Explicit instruction consists of tasks that could be completed by repeating a demonstrated procedure without any real thinking. Teachers using explicit instruction only provide little opportunity for students to participate or have time to develop understanding or discuss their reasoning (Sullivan 2015). Explicit and direct instruction that is used solely to demonstrate examples that are used to complete repetitive exercises reflecting the exact example as in the ‘traditional’ teaching model described earlier will be considered together when further discussing direct instruction in this section.

Direct instruction is a method of teaching and learning that centres on the teacher transmitting their knowledge directly to the student using a lecture style of presentation of examples that directly relate to the questions that are presented for consolidation. The examples provide the student with an exact method for the solution of a specific question. In mathematics this is usually through a worked example specific to a particular question. A definition of direct instruction as proposed by Kirschner et al. (2006) is:
Direct instruction guidance is defined as providing information that fully explains the concepts and procedures that students are required to learn as well as learning strategy support that is compatible with human cognitive architecture. Learning in turn, is defined as a change in long-term memory (p. 75).

Snel et al. (2012) define direct instruction as “highly structured and describes or even scripts classroom activities in considerable detail” (p. 356). The scripting of the classroom activities is achieved through examples which are presented to the class by the teacher. The students then follow that process to solve similar questions. Hattie’s (2009) view on direct instruction is not focused on the traditional teaching method. He describes direct instruction as teachers being clear in their teaching intentions and outcomes for the students. The teachers then plan learning experiences that result in student achievement of those planned outcomes.

Using worked teaching examples is a method of reducing cognitive load and providing students with procedures for the solution of problems. If there is no worked example provided then the teaching approach would be more aligned with a discovery learning approach to problem solving (Kirschner et al., 2006). Tuovinen and Sweller (1999) state that when engaged in problem solving in unfamiliar contexts students could achieve better outcomes by using worked examples as a base of knowledge. Pawley, Ayres, Cooper and Sweller (2005) observe: “The success of worked examples adds to a growing body of research which has demonstrated that they are a powerful instructional tool” (p. 93). For students working in a familiar context or one in which they at least had some level of experience the worked examples were not as important to them in the solving of the problem (Tuovinen & Sweller, 1999). The value of examples to the students’ learning demonstrates some
overlap between the direct instruction approach and the constructivist approach indicating that constructivism and direct instruction may not be mutually exclusive.

Not all researchers agree on the value of worked examples as a teaching and learning strategy. Kuhn (2007) makes the case that students need to engage in problem-solving activities without worked examples, even though they are risking cognitive overload, as the world outside of school requires just that, the solution of problems for which there has been no worked example. Here cognitive load refers to the amount of working memory that is used in the solution of a problem therefore cognitive overload occurs when the mental requirements of solving a problem exceed the available working memory (Hiebert, 1988; Sweller, 1988). Thus, teaching using a constructivist learning approach requires the teacher to use their MKT and knowledge of the ability level of individual students to decide on the appropriateness of particular teaching examples to promote learning without cognitive overload.

It is proposed that in the case of students learning mathematics, it is the mathematical toolbox and procedural fluency that they take into any problem-solving situation that reduces cognitive load and provides the precedents and skills that allow the successful solution to the problem task being investigated. As Pawley et al. (2005) note “Schemas, held in long-term memory, reduce working memory load and enable us to engage in activities that otherwise would be impossible” (p. 76). This is summed up by stating that there needs to be not one method or another but a combination of teaching and learning methods that achieve the outcome of generating mathematically skilled students. The design of the task and the method of completing the task, whether through a direct instruction or a constructivist approach or even a discovery learning approach, is the key to effective student learning: “… it
is an argument for the need to contemplate instructional methods within the broader context of instructional goals” (Kuhn, 2007 p. 112).

A combination of teaching and learning methods, whether direct instruction or a constructivist approach, are required to improve students’ understanding of mathematical concepts. This is in contrast to the ‘traditional’ teaching approach that uses direct instruction through examples as the primary method of teaching and learning. The next section examines student learning through different types of activities, fixed and growth mindsets, working memory and examination anxiety as they relate to student learning.

2.4 Factors Influencing Student Learning, Engagement and Attitude

The ‘traditional’ teaching paradigm as first described in Chapter 1 uses a similar approach for most lessons. Often it takes five or more minutes for the students to be settled and prepared before the lesson commences. The lesson begins with a review of homework, new teaching examples are discussed, and a worksheet or textbook exercise is given to consolidate the material from the examples. The exercise is usually repetitive with many similar questions, the remainder of which is set for homework. The focus of the lesson is mostly on the development of an instrumental understanding or procedural knowledge through repetition of problems emphasising finding the one correct answer (Sullivan, 2011b; Vincent & Stacey, 2008).

As previously mentioned, in the typical Australian classroom the content is taught according to a scope and sequence document that provides the order of topics and the duration of time spent on that topic. Each school develops their scope and sequence
document from the syllabus curriculum documents and the class teacher follows this document in their lesson planning. Sample scope and sequence documents are provided in Appendices C, H, and L. The teaching and learning structure is usually linear with a fixed time for each topic and little linking between topic areas. In the week preceding the half yearly or yearly examinations there is usually a period of dense, sometimes rushed revision where all topic areas are revisited. This approach can lead to a high level of examination stress (Plenty & Heubeck, 2013).

Thus, there are a number of aspects of the ‘traditional’ teaching approach that negatively influence student learning and engagement. They include:

- a linear fixed time progression through content (Perso, 2005)
- dense repetitive worksheets and textbook exercises (Nichol & Robinson, 2000)
- disengagement (Sullivan, 2011b)
- lost teaching time at the start of the lesson (Gerver, 2011)
- fixed mindset (Boaler, 2013)
- examination stress (Plenty & Heubeck, 2013).

In the study of the TIMSS video data, Vincent and Stacey (2008) analysed Australian mathematics lessons and this is the most recent study of this type of data. The results indicated that the majority of classrooms displayed many ‘traditional’ teaching aspects.

This section will examine these issues with the ‘traditional’ teaching paradigm and alternate teaching and learning strategies that have the potential to enhance the student learning experience. The textbook or worksheet focus on repetitive questions based on the principle of overlearning will be contrasted with the spacing and mixing
of questions. The fixed mindset to student mathematical ability will be challenged in section 2.4.3. The importance of procedural fluency in the reduction of the cognitive load on working memory was previously mentioned and will be further developed in section 2.4.4. The issue of examination anxiety is the focus of section 2.4.5.

Figure 2.3 describes the links between the content of this section that examines the factors influencing student learning, engagement and attitude. This section outlines how teaching and learning activities that use spacing and mixing (section 2.4.2), rather than massing and overlearning (section 2.4.1), enable better understanding. The focus on quality teaching and learning activities allows the development of procedural and conceptual knowledge thus developing schema in long-term memory through automatisation (section 2.4.4). Long-term memory contains schema which are developed through procedural knowledge, fluency and flexibility, and conceptual knowledge. The greater the number of schema in long-term memory, the greater the automisation (section 2.4.4), and the capacity to decrease cognitive load by using knowledge from the long-term memory to enable the solution of more complex problems in working memory (section 2.4.4). As a result, learning takes place, which then increases the number of schema in long-term memory. The teaching and learning activities contribute to the development of procedural and conceptual knowledge and schema in long-term memory and as the capacity of schema available for the working memory increases the complexity of problems that can be solved using working memory also increases. The FQA with a focus on both procedural and conceptual knowledge attempts to foster learning in this way. With the increase in knowledge comes a growth mindset (section 2.4.3), and the fluency and confidence through spacing, mixing, and repetition with variation acts to reduce examination anxiety (section 2.4.5).
Figure 2.3 Demonstration of the Learning Process

LONG TERM MEMORY

SCHEMA

Reduced Cognitive Load

Increased Problem Solving Capacity

SHORT TERM MEMORY

Teaching and Learning Activities using Mixing, Spacing, and Repetition with Variation

KNOWLEDGE

Conceptual Procedural: Fluency and Flexibility.

Understanding
While the development of conceptual knowledge and relational understanding should be the focus of teaching, it is important to also develop students’ procedural fluency in mathematical processes (Sullivan, 2011b). The traditional teaching method of developing fluency is to use repetitive worksheets that follow the teaching examples precisely utilising the process of overlearning, which is the topic of the next section.

2.4.1 Overlearning

Originally the term overlearning was associated with early research on language acquisition and refers to training that results in improved retention. Overlearning as a teaching strategy has significant support from many early studies that were particularly interested in language acquisition or the development of practical skills (Dougherty & Johnston, 1996; Postman, 1962). Overlearning as defined by Driskell, Willis, and Copper (1992) is the “deliberate training of a task past a set criterion” (p. 615). They state that overlearning as a teaching strategy has been known to provide enhanced performance in many types of training and this has been acknowledged by many researchers. The focus of their studies, however, were practical activities such as the disassembly and cleaning of rifles, and as such may not pertain to the learning of mathematics.

In the traditional teaching paradigm, extensive repetitive worksheets or textbook exercises are used to consolidate student understanding through the process of overlearning where the learner continues to practice after they have achieved success. Most mathematics textbooks have exercises that only relate to each day’s lesson following the specific examples and this practice is referred to by Rohrer,
Taylor, Pashler, Wixted, and Cepeda (2005) as massing. Overlearning in a mathematical context was examined by Rohrer and Taylor (2006), in a study where students learnt a new strategy to solve a mathematical problem and either had three or nine practice problems directly following instruction. They found no difference between either of the groups on test scores after one or four weeks. They concluded that if the process of overlearning is not effective then many students are spending time doing multiple repetitive questions that offer no benefit.

In a further study Rohrer (2009a) concluded that overlearning is an inefficient use of study time and that after a certain amount of overlearning the extra study time allocated is unlikely to result in a greater recall of material later. While overlearning was shown to improve test scores there was a significant cost in the additional study time required (Pashler, Rohrer, Cepeda, & Carpenter, 2007; Rohrer, 2009a). Additionally, while the overlearning group had better initial test results, after the first week the difference decreased significantly over time (Rohrer et al., 2005).

Interestingly, a recent study of overlearning and the implications on further learning indicated that overlearning made neurochemical changes to the brain that resulted in the inhibition of subsequent learning that followed immediately afterwards (Kazuhsa et al., 2017). As a result of this research, a move away from overlearning may improve learning outcomes resulting in similar or better recall without the significant extra time input and this approach is examined in the next section.
2.4.2 Spacing, mixing and learning

When considering the massed practice of typical textbooks, which Rohrer and Taylor (2007) refer to as promoting ‘overlearning’, the assertion is that the practice is a result of the belief that the longer the practice directly following the new learning the longer the material will be retained. Most textbooks in NSW are of a similar format. They include examples with a set of practice problems directly related to the examples and all examples usually pertain to the same topic area. Rohrer and Taylor (2007) describe this aspect of textbooks as ‘massing’ as opposed to ‘spacing’ where the question types are dispersed over many exercises. They describe the practice of including only questions of a single topic as ‘blocking’ with the opposite practice as ‘mixing’. Their description of educational curriculum documents aligns with the NSW K–10 Syllabus (2012) when they describe the rarity of spacing of mathematics problems as an indication of the popularity of overlearning and that the time allocated to overlearning results in a reduction of the time available if the problems are spaced.

As previously stated, most of the overlearning research has investigated the learning of languages. In a study of second language vocabulary retention Pashler et al. (2007) found that spacing had strong positive effects on retention and that the benefits of spacing seemed to apply to mathematical skills as well. According to Rohrer and Taylor (2007) teaching practices that use spacing result in improved examination performance, and they describe this as the ‘spacing effect’. In a later study they show that ‘mixed review’, which is a combination of spacing and mixing, results in a significant improvement in mathematics test results (Rohrer, 2009b).
Mixed review involves the provision of questions on multiple topic areas over a period and is related to repetition with variation. Superficially there is confusion between rote learning and repetition with variation, but the latter is quite different to rote learning (Fuson, 2009). Repetition with variation is likened to the repeated exposure to a complex idea that results, over time, in a greater and deeper understanding. The repetition with variation approach differs from rote learning which has a focus on identical question types. In repetition with variation the slight variations to the questions over time allow the learner to develop a greater understanding and has the potential to develop into schema acquisition in a way that rote learning does not allow. Mathematics teaching, following a rote learning approach, may involve a teacher giving some examples of a type of problem to be solved, minimal instruction, and then questions very like the examples. Students would be required to replicate the steps from the example without necessarily having any real understanding and has been defined as a rote learning approach to teaching or “teaching without learning” (Handa 2012, p. 344). Repetition with variation as opposed to rote learning can result in the automation of schema thus decreasing the load on working memory (Handa, 2012). Repetition with variation is like the spiral curriculum, first explained by Bruner (1966) which advocates the revisiting of a curriculum item many times over a period thus building an understanding. Curriculum and content build over time rather than being delivered all at one time (Gibbs, 2014; Paley et al., 2004). Repetition with variation is an application of variation theory where further tasks are constructed to consolidate the learning of the initial task by leaving some elements the same and varying others so that the concept can be developed (Sullivan, P., Borcek, C., Walker, N., & Rennie, M. 2016).
Watson, A., & Mason, J. (2006) propose the identification of which aspects to vary and the correct sequence of variation as crucial aspects in variation theory.

Repetition with variation can lead to a mastery learning teaching approach (Bonnett, Yuill, & Carr, 2017). The mastery learning approach is related to the work of Bloom (1956) in the mid-seventies where he investigated the varied learning rates of students concluding that all students are capable of learning, given the right conditions and sufficient time (Guskey & Jung, 2011). Students who are in a classroom with a mastery learning approach are more likely to seek conceptual understanding and be engaged in learning (Bonnett et al., 2017; Munns & Woodward, 2006). A class orientated to mastery learning can select tasks that are more challenging as the students are focused on the nature of the task and are more able to cope with challenge as they do not focus on their perceived ability, attributing failure to the difficulty of the work. Due to the positive reinforcement provided in a mastery learning environment, the student learning experience can be enhanced through a growth mindset (Skaalvik & Federici, 2016), and this is the topic of the next section.

2.4.3 Growth mindset

Some students and teachers believe that their mathematical ability or intelligence is unchangeable (fixed mindset) while others believe that intelligence is not fixed and can change (growth mindset) (Dweck, 2015). Students with a growth mindset believe that ability can change and that working hard on challenging activities can result in a growth in capability. It is the struggle that provides the opportunity for growth and not that the struggle indicates a lack of intelligence (Hochanadel &
Finamore, 2015; Paunesku et al., 2015). Students with the opposite, fixed, mindset may try to avoid any activity that results in a struggle as failure may indicate they are less intelligent than previously thought (Claro, Paunesku, & Dweck, 2016; Murphy & Dweck, 2016).

Students who believe that they are poor at mathematics behave in a manner to consolidate this view and see failure as consolidation of their lack of ability rather than a challenge that could result in an increase in intelligence (Rattan, Good, & Dweck, 2012; Yeager, Romero, Paunesku, Hulleman, Schneider, Hinojosa, & Dweck, 2016). For students to improve in mathematics they need to move from a fixed mindset, also called the entity theory of intelligence, to a growth mindset where effort and challenge can result in improvement (Dweck, 2015; Good, Rattan, & Dweck, 2012).

The ‘traditional’ secondary classroom often has the students selected into classes based on their performance in the previous year’s major examination. This practice of grouping students based on their abilities is referred to as streaming. By allocating students in this way schools are encouraging a fixed mindset, and this may be one of the issues related to the performance of the students (Williams, 2004). Streaming is not generally beneficial for students who are less able (Forgasz, 2010). Earlier research by Boaler (1997) indicated that some students allocated to the higher streamed classes were also not benefited as they suffered from the pace and expectations of the higher classes. The issue of streaming is significant but for the purposes of this research the link between streamed classes and the fixed mindset is the relevant aspect. Teachers who have a fixed mindset may focus on the current ability of their students rather than working towards developing their students’ ability and focusing on improvement (Dweck, 2015).
The ‘traditional’ teachers’ interpretation of the syllabus documents may actually provide a rationalisation for streaming and lead to a fixed mindset. The NESA Syllabus, for Years 9 and 10, describes three ‘pathways’ for students to follow (NSW Education Standards Authority, 2017). It is intended that teachers follow a pathway and select topics from other pathways and develop the curriculum that best suits their students’ interests and needs. Unfortunately, these pathways are sometimes incorrectly interpreted to mean three separate, distinct levels and this results in the classes in Year 9 and 10 being streamed. This practice can flow back to Years 7 and 8, as they can be streamed with the rationale that this is preparation for Years 9 and 10. The fixed mindset is then propagated through this process during the secondary school years.

A growth mindset enables students to realise that they can learn and improve with effort. To learn, students need to increase the number of schema and automised processes in long-term memory thereby releasing working memory to focus on solving mathematical problems resulting in further learning. One aspect that impedes the growth of students’ mathematical performance is the limitations of working memory and a lack of automised processes that are instilled into long-term memory (Pawley et al., 2005) and this is the focus of the next section.

2.4.4 Working memory and automisation

The previous section discussed the growth mindset and the need to increase the number of schema and automised processes in long-term memory. When engaged in mathematical learning activities, working memory is used in the development of the solution. If all the working memory is used in the solution of the problem then there
is none remaining to develop schema (Sweller, 1988). This section examines working memory, cognitive load and the influence of automisation.

Working memory is only able to function effectively with between two and four new elements. However, schema may be brought from long-term memory to short-term memory to assist the cognitive processing. This is because long-term memory does not have the storage limitations of short-term memory. A complex schema brought from long-term memory counts as just one process in short-term memory. This is despite the level of complexity of that schema exceeding the normal limitations of short-term memory. The greater the number of automated schema available in long-term memory, the greater the capacity to solve complex problems successfully (Paas, Renkl, & Sweller, 2003).

Working memory is made up of three types of cognitive load: intrinsic, extraneous, and germane (Paas, Renkl, & Sweller, 2003). Intrinsic cognitive load is fixed as it is related to the demands of the problem being solved and can only be reduced by utilising automated schema from long-term memory. Otherwise, intrinsic cognitive load is reduced by either providing a simpler task or scaffolding the task to omit some of the elements of the problem. Extraneous cognitive load is the cognitive load resulting from unnecessary information, including mathematics anxiety, that inhibits the solution of the problem and the acquisition of schema. If the intrinsic cognitive load is high, then the extraneous cognitive load inhibits the solution of the problem but if the intrinsic cognitive load is low then the extraneous factors do not play a significant part. Germane cognitive load enhances learning and is the cognitive load resulting from schema acquisition and the automation of existing schema. However, if the total cognitive load, intrinsic, extraneous and germane, exceeds the resources of working memory then concept development cannot take place and, while the
mathematical problem may be solved, learning may not occur (Paas & Ayres, 2014; Paas et al., 2003; Sweller, 1988).

Unsworth and Engle (2007) came to the same conclusion. They refer to short-term memory as ‘primary memory’ and long-term memory as ‘secondary memory’. Working memory capacity is influenced by the transfer of schema from secondary memory to primary memory and the efficiency by which this occurs. The system is dynamic with information moving in both directions. This equates to the use of long-term memory schema in short-term memory and the development of schema from short-term memory to long-term memory equating to learning.

The collection of mathematical processes and problem-solving skills that have been acquired by the learner and installed in the long-term memory, in other words, the students’ mathematical toolbox of automised schema, is material that has been learnt (Kirschner et al., 2006). Working memory is then used to understand, address and complete the task. Working memory is the area where the actual processing involved in the completion of the task takes place. “We are only conscious of the information currently being processed in working memory and are more or less oblivious to the far larger amount of information stored in long-term memory” (Kirschner et al., 2006 p. 77).

The mathematical toolbox then allows students to explore and examine mathematical problems without overloading their working memory. For example, consider the scenario where students are learning to add and subtract fractions with unequal denominators. A student with a sound knowledge of the multiplication tables will find the calculation of a common denominator simple. Therefore, there is less strain on their working memory than for the student who does not have multiplication
tables in their mathematical toolbox (long-term memory). When students are involved in learning which includes new information, the working memory has limitations on the amount of new material that can be processed. The more information that is utilised from the long-term memory in the application to the new learning the more working memory that is available to be applied to the learning situation (Kirschner et al., 2006). Therefore, having a greater number of tools available in the mathematical toolbox (long-term memory) will reduce the cognitive load when using working memory in the solution of mathematical investigations.

When discussing cognitive load Pawley et al. (2005) state “If the materials require working memory resources to be expended by completing tasks irrelevant to learning, such as understanding the instructions, then learning is inhibited” (p. 76). The knowledge of multiplication tables with respect to the example of addition and subtraction of fractions is an example of the inhibition of learning. The lack of multiplication facts and automisation of multiplication tables requires working memory to calculate the product. The working memory used to calculate the common denominator is then unavailable to be used in the actual solution of the fraction problem. Learning takes place when aspects required to solve the problem from short-term memory are moved to long-term memory, therefore freeing working memory available to continue with problem solution (Pawley et al., 2005).

Students who have automised rules or processes, which are available in long-term memory, are more able to solve more complex problems and develop new schema without overloading working memory (Hiebert, 1988). “If we are to make progress in Mathematics, it is essential that the elementary processes become automatic, thus freeing attention to concentrate on the new ideas which are being learnt which, in their turn, must also become automatic” (Skemp, 1987, p. 83). The greater number of
automatic processes that are in a student’s long-term memory the more working memory that is available to solve the problem. As a result of learning from problem solving the student may be able to shift further processes to long-term memory. Students who have incorporated elementary processes into long-term working memory will therefore have a reduced cognitive load when using working memory to solve problems.

The greatest strain on working memory occurs in major examinations where students are required to answer questions on many topics that are increasing in difficulty. As described in section 2.4.3, the examination then has an influence on the class in which the student is placed, which, in turn, may influence their future learning potential (Mavilidi, Hoogerheide, & Paas, 2014). The stress associated with examinations is described in the next section.

2.4.5 Examination anxiety

Working memory is limited and mathematics anxiety can impact on the amount of working memory (extraneous cognitive load) available to answer questions in an examination and hence a student’s performance in the examination. Anxiety factors may be related to self-esteem and self-confidence, feelings of failure, being unprepared, worries, and negative comparison with peers (Beilock & Willingham, 2014; McDonald, 2001; Zatz & Chassin, 1985). While there is evidence that mathematics performance is negatively influenced by mathematics anxiety, it is not clear which comes first; the anxiety or the poor performance (Foley, Herts, Borgonovi, Guerriero, Levine, & Beilock, 2017).
To perform in an examination, information from the long-term memory needs to be retrieved and used in the working memory to answer the questions. As discussed in the previous section, working memory is limited, and any other process, including concerns about poor performance, compete for working memory. Thus, the anxiety factors mentioned can compete with the processes required to answer questions and hence affect examination performance, and the more complex the examination the greater the effect of the stress factors (Lang, Lang, Beilock, & Ramirez, 2011; Mavilidi et al., 2014; Zatz & Chassin, 1985)

One strategy suggested to assist students suffering from examination anxiety is to read through the examination before commencing, thus preparing the students by having them know what to expect (Mavilidi et al., 2014). Another is to have students write about their emotions or feelings before the examination which was purported to settle the students but has had a negative influence on students who did not suffer from examination anxiety (Lyons & Sian, 2012). A third is to ensure that students have the basic skills required to answer questions (Beilock & Willingham, 2014). In their research involving over 400 students, Lavasani, Hejazi, and Varzaneh (2011) state “excluding the role of environment, any action that can increase the person’s sense of efficacy is the most powerful action in avoiding math anxiety” (p. 561).

Research by Necka, Sokolowski, and Lyons (2015) involving the degree to which students identified themselves with mathematics indicated that students who identified themselves strongly with mathematics had less mathematics anxiety than those who identified themselves less with mathematics. As previously discussed, streamed classes may also impact on students’ identification with mathematics as they may feel that their class defines their ability. Therefore, students in lower-
graded classes may feel that their ability is low and as a consequence mathematics anxiety may increase.

This section outlined aspects that influence student learning in mathematics. The issue of overlearning through dense worksheets and exercises was contrasted with spacing, mixing and repetition with variation and the automisation of processes. The importance of developing a growth mindset was highlighted and the practice of streamed classes, a common practice for mathematics classes in NSW schools, was raised as an issue that may hinder the growth mindset. Competence and confidence were linked as factors that may improve performance and engagement and the aspect of engagement is expanded on in the next section.

2.5 Engagement

In section 2.1.3 student engagement was raised as a serious issue with the traditional model of teaching, particularly for those students in the middle years of schooling. The NSW K-10 Mathematics syllabus in the rationale states that “The study of the subject enables students to develop a positive self-concept as learners of mathematics, obtain enjoyment from mathematics, and become self-motivated learners through inquiry and active participation in challenging and engaging experiences” (p. 16). Therefore, it is important that the mathematics classroom has a stimulating and engaging atmosphere with a focus on student enjoyment of mathematics which is relevant and engaging.

There are many definitions of engagement ranging from a simplistic view of behaviour and participation, to multidimensional definitions including cognitive,
emotional and behavioural aspects (Fredricks, 2011; Martin, et al., 2015; Munns & Woodward, 2006). Therefore, student engagement occurs when the three previously mentioned cognitive, emotional and behavioural aspects are closely aligned.

Students are engaged when they are:

- Reflectively involved in deep understanding and expertise (high cognition)
- Genuinely valuing what they are doing (high emotion)
- Actively participating in school and classroom activities (high behaviour)

(Munns & Woodward, 2006, p. 194).

This definition then delineates between the procedural form of student engagement, where students are following teacher directions and are compliant, and true engagement, which involves high cognition, emotion and behaviour aspects.

According to Attard (2012) to be engaged students need to be doing more than just answering questions. Good behaviour in the classroom is not engagement. The students need to be engaged on three levels i.e., the affective, cognitive and operative domains. “It is only when students are experiencing cognitive challenge, are actively involved, and are enjoying and appreciating mathematics that they are truly engaged” (Attard, 2012, p. 23).

In NSW, the Fair Go Project was an action research project examining the engagement of students from disadvantaged areas. This project linked the three facets of engagement and concluded that amongst other aspects student self-assessment was vital to student engagement (Munns & Woodward, 2006; Munns, Zammit, & Woodward, 2008). However, it was noted that the classroom with a focus on procedural engagement did not require student self-assessment, as the aim of the lesson is to have the students complete the tasks set by the teacher without further
consideration. Student self-assessment occurs in an engaging classroom and enables students to:

- link the content to their lives
- evaluate and improve their performance
- be a partner with the teacher and classmates in their learning
- have a place and be valued
- have a voice in the learning (Munns & Woodward, 2006).

An analytical tool is required to assess the level of engagement or assess if engagement is increasing or decreasing. While there is significant research into engagement and motivation in mathematics there is only one specific framework for exploring engagement in mathematics. The Framework for Engagement in Mathematics (FEM) Attard (2014) was developed from a study on influences of student engagement and will be used as an analytical tool. It is described in the next section.

### 2.5.1 Framework for engagement in mathematics (FEM).

Student engagement was raised as a serious issue in section 2.1. To examine engagement in the classroom a framework for engagement is helpful. The FEM as proposed by Attard (2014) has been selected as the lens through which to investigate engagement in this study. The FEM is split into pedagogical relationships and pedagogical repertoires which are the elements necessary for engagement to occur. The FEM states:
In an engaging mathematics classroom, positive pedagogical relationships exist where:

- students’ backgrounds and pre-existing knowledge are acknowledged and contribute to the learning of others
- the teacher is aware of each student's mathematical abilities and learning needs
- interaction amongst students and between teacher and students is continuous
- the teacher models enthusiasm and an enjoyment of mathematics and has a strong pedagogical content knowledge
- feedback to students is constructive, purposeful and timely

In an engaging mathematics classroom, engaging pedagogical repertoires mean:

- there is substantive conversation about mathematical concepts and their applications to life
- tasks are positive, provide opportunity for all students to achieve a level of success and are challenging for all
- students are provided an element of choice
- technology is embedded and used to enhance mathematical understanding through a student-centred approach to learning
- the relevance of the mathematics curriculum is explicitly linked to students’ lives outside the classroom and empowers students with the capacity to transform and reform their lives
- mathematics lessons regularly include a variety of tasks that cater to the diverse needs of learners
Students are engaged with mathematics when:

- mathematics is a subject they enjoy learning
- they value mathematics learning and see its relevance in their current and future lives
- they see connections between the mathematics learnt at school and the mathematics used beyond the classroom (p. 23).

The self assessment factors of Munns (2006) are all found in the FEM. The students link the content to their lives through the engaging pedagogical repertoires when there is substantive conversation about mathematical concepts and their applications to life and the relevance of the mathematics curriculum is explicitly linked to students' lives outside the classroom. The students evaluate and improve their performance in the positive pedagogies section through constructive purposeful and timely feedback. They are a partner with the teacher and classmates in their learning in both aspects of the FEM and they have a place and are valued when the teacher is aware of each student's mathematical abilities and learning needs and when interaction amongst students and between teacher and students is continuous. Finally, the students have a choice in their learning as discussed in the engaging pedagogical repertoires section.

While the FEM will not be used in this study to determine whether engagement increases or decreases, it will be used to examine which elements of engagement are present in each observed lesson.

This section has outlined the FEM which will be used to provide information on engagement. For students to evaluate their performance and to have clear strategies
to improve their performance they need to be provided with diagnostic feedback, which is the subject of the next section.

2.5.2 Feedback

The previous section focused on engagement and indicated that students were more engaged when they received information that enabled them to improve their performance in mathematics. There is significant evidence that feedback has a major influence on learning and achievement and that the influence can be positive or negative depending on the type and timing. Ideally, feedback needs to be diagnostic, targeting errors rather than mistakes, and timely in its delivery. (Harks, Rakoczy, Hattie, Besser, & Klieme, 2014; Hattie & Timperley, 2007; Powell & Kusuma-Powell, 2015).

The research on formative assessment, feedback and the teaching of mathematics by Black and Williams (2010). Harks et al. (2014) and demonstrated the importance of feedback in the teaching and learning process. They indicated that elaborated feedback, where specific information is given pertaining to the correctness or incorrectness of mathematical processes and/or problem-solving activities was particularly successful. Grade specific feedback, where students were provided with a mark or grade as feedback, was not seen as useful. Elaborated feedback was developed with respect to secondary mathematics specifically. Their findings indicated that by providing elaborated feedback students showed improvement in interest, engagement, and achievement.
Hattie and Timperley (2007) researched feedback and the influence on primary school students’ achievement. The study was based on a synthesis of over 500 meta analyses examining over 100 influences on student achievement. The study found that feedback was in the top ten influences on student learning along with direct instruction, reciprocal teaching and students’ prior cognitive ability. Clearly feedback can have a significant influence on student learning. However, the type of feedback was important. Diagnostic feedback on a task and how to improve the solution method of that task was far more effective than feedback that simply included praise.

The timing of the feedback was important and the closer the feedback to the time of the error the greater the effect. Pashler et al. (2007) found that diagnostic feedback given at the time a student made an error resulted in a five-fold increase in test performance compared with simply telling the student whether the response was correct or not. This contrasts with the ‘usual’ feedback received by students involved in the traditional teaching paradigm that comprised right or wrong responses to answers or feedback in the form of a mark or grade from a test given after a topic has been completed.

Feedback allows the student to improve their understanding and hence build their confidence in mathematics. As discussed in section 2.4.5 this confidence may assist in the alleviation of some mathematics anxiety. In a classroom with a focus on student engagement, which includes student self-assessment, students need to be a partner in the learning and have an active voice. This is elaborated on in the next section.
2.5.3 Student voice

At the start of section 2.5, the definition for engagement used the multidimensional combination of the three elements pertaining to high cognition, emotion, and behaviour proposed by Fredricks (2011). Then the five engagement factors were listed that related to student self-assessment. The first, link the content to student lives, is clearly outlined in the NSW K-10 Syllabus as an outcome and is clearly important. The second, evaluate and improve their performance was the content of the previous section on feedback. This section relates the final three aspects of being a partner with the teacher and classmates in their learning, having a place and being valued, and having a voice in the learning together as student voice (Munns & Woodward, 2006).

Student voice provides an opportunity for teachers to examine students’ cognitive activities in the mathematics classroom (Anthony, 2013). The definition of student voice means involving the students in all aspects of their learning and that their opinions are valued in determining their experiences in school (Robinson & Taylor, 2013). According to the definition of student pedagogic voice put forward by Baroutsis, McGregor, and Mills (2015), student pedagogic voice includes the three separate aspects relating to self-assessment and engagement mentioned separately in the previous paragraph. Based on these definitions, the focus of this section is on student voice as it provides opportunities for rich discourse and shared learning in the classroom.

A classroom that values student voice encourages dialogue between the teacher and students which allows the students to feel part of the learning. This shared aspect allows the students to feel that they belong which results in a positive feeling about
the class resulting in increased engagement. Again, as with feedback, it is possible that a non-genuine manipulated attempt at students’ voice may result in the opposite feeling (Baroutsis et al., 2015; Smyth, Czerniawski, & Kidd, 2012). Students have worthwhile opinions on what and how they are taught and can articulate these to teachers providing there is the opportunity to do so. The student voice although implying one voice is actually the voice of each individual whether it is expressed verbally, written or in other ways (Kane & Chimwayange, 2014).

Smith & Stein (2011) outline five practices that allow teachers to use students’ responses to advance the mathematical knowledge of the students. The five practices are:

1. **Anticipating** likely student responses to challenging mathematical tasks;

2. Monitoring students actual responses to the tasks while the students are working on the tasks;

3. Selecting particular students to present their mathematical work during the whole class discussion;

4. Sequencing the student responses that will be displayed in a specific order; and

5. Connecting different students’ responses and connecting the responses to key mathematical ideas. (p. 8)

The FQA uses all five of these elements in the discussion phase and in particular the discussion of the conceptual understanding as a result of the investigation of the fifth question. This is expanded in chapter 3.
The issue with the ‘traditional’ classroom is that there is no avenue for a student voice as the teacher is the sole decision maker with respect to the teaching and learning within the classroom. This could be a determining factor in declining engagement in that style of classroom. The next section synthesises the information in this chapter and determines a strategic plan for moving forward.

2.6 Teaching and Learning

This review of literature has examined quality mathematics teaching from the viewpoint of the teacher and the student and linked the teaching to the curriculum. The development of conceptual and procedural knowledge and instrumental and relational understanding was linked to the constructivist approach to teaching and direct instruction. Influencing factors on student learning such as overlearning, spacing, mixing and the spiral curriculum along with the development of a growth mindset, automisation, working memory and mathematical anxiety were discussed. Finally, engagement, feedback and the student voice were considered.

As a result, the literature suggests that for quality teaching with effective teachers, student learning and engagement, teachers should:

- Teach effectively for understanding (Ball & Forzani, 2010; Ediger, 2012)
- Teach using a spiral curriculum (Cowan, Morrison, & McBride, 1998; Gibbs, 2014; Rohrer, 2015)
- Teach concepts using a constructivist approach (Barrett & Long, 2011; Confrey & Kazak, 2006; O’Shea & Leavy, 2013; Tynjälä, 1999)
• Avoid overlearning, instead space and mix content areas (Carpenter, Cepeda, Rohrer, Kang, & Pashler, 2012; Rohrer, 2009a, 2009b)

• Instil a growth mindset in students and teachers (Dweck, 2015; Hochanadel & Finamore, 2015; Rattan, Savani, Chugh, & Dweck, 2015)

• Use automisation to reduce cognitive load and the strain on working memory (Cowan, 2014; Paas & Ayres, 2014)

• Adopt procedures and processes to reduce mathematics and examination anxiety (Beilock & Willingham, 2014; Foley et al., 2017)

• Increase engagement (Fast & Hankes, 2010)

• Provide meaningful diagnostic feedback (Harks et al., 2014; Hattie & Timperley, 2007; Powell & Kusuma-Powell, 2015; Rakoczy, Harks, Klieme, Blum, & Hochweber, 2013)

• Allow students to have a voice (Baroutsis et al., 2015; Smyth et al., 2012).

Unfortunately, the ‘traditional’ teaching method as explained in section 2.1 allows few of these aspects to occur. The FQA focuses on all of these aspects of teaching, learning and engagement.

This chapter has outlined mathematics teaching, curriculum, knowledge and understanding, student learning and engagement as displayed in Figure 2.1. The list for quality teaching, learning and engagement complied at the end of this chapter will now be compared with the FQA approach. The next chapter will detail the FQA and its application in the classroom and outline how the FQA may address the aspects of teaching, learning and engagement detailed in this literature review.
As outlined in Chapter 1, the Five Question Approach (FQA) to teaching mathematics was developed during my 30-year secondary mathematics teaching career in NSW. I first used the FQA in the early 1980s with a quick five question approach to the start of each mathematics lesson, which was an approach used by many teachers at that time. The ‘quick question’ approach was designed to settle the class and focus the students’ minds on mathematics (Gerver, 2011; Jones & Couchman, 2001). The questions were simple procedural questions that were randomly chosen from previously learnt material.

At that time, I would have been described as a ‘traditional’ teacher in accordance with the definition given in previous chapters, teaching the way in which I was taught. Throughout my teaching career, I adapted both the five questions at the start of the lesson and my teaching approach. I realised that the students needed more opportunities to develop conceptual understanding, as procedural understanding was not sufficient to develop the required mathematical skills. I found that the students were more engaged when working on problem-solving questions with multiple layers of understanding and solution pathways. However, the students still needed fluency in procedural skills and so the FQA needed to develop conceptual knowledge and procedural fluency. Over time the FQA evolved from five quick randomly selected questions, usually taking five minutes, to five carefully considered and well-planned questions that could take from ten minutes to the entire one hour lesson. Every set of five questions comprised four procedural questions followed by a fifth question which was a conceptual question. Students were required to complete the questions in order, from one to five, to ensure that they did not omit the questions.
that they were less confident about or did not want to do. The conceptual question was always last so that the students who completed the first four questions could then spend more time and delve deeply into the concepts involved in the fifth question. I would add another layer of complexity to the fifth question if any students completed the fifth question with time to spare and provide scaffolding through a hint for those having difficulty starting the fifth question.

The FQA was designed to allow traditional teachers to take small steps towards a change in practice without completely changing their entire approach to teaching. According to Stacey (2010) teachers comment that they have difficulty in incorporating problem solving and reasoning into their mathematics lessons. The FQA was designed to help alleviate that issue without completely changing the teachers approach to teaching. The FQA does not replace the teacher’s normal lesson but is meant to be one of the tools at the teacher’s disposal to potentially increase positive learning outcomes. In response to the problem of ‘traditional’ teaching focusing solely on procedural knowledge, the FQA evolved as a means of not just consolidating procedural knowledge and developing fluency but also developing conceptual knowledge. The first four questions in the FQA were used to develop procedural fluency and consolidate mathematical skills. The fifth question was designed to promote the development of conceptual knowledge and schemas through investigations and more complex problems.

The historical development of the FQA through the move from five randomly selected procedural questions to planned procedural and conceptual questions has been outlined in this section. More detail about the operation of the FQA is detailed in the next section.
3.1 The Five Question Approach

The FQA was designed to provide students with the opportunity to develop their mathematical understanding over time through repetition with variation (questions one to four), conceptual mathematical investigations (question 5) and positive diagnostic feedback.

The first four questions in the FQA focus on instrumental understanding, designed to develop procedural knowledge and fluency (Skemp, 1976, Sullivan, 2011b). The questions are chosen to build student understanding and knowledge using a constructivist approach to the development of student learning over time. The constructivist approach requires the questions to increase in difficulty and complexity as students show that they understand the method of solution or solutions. The use of variation theory to modify some aspects of the question while leaving other aspects unchanged provide greater opportunity for conceptual understanding (Sullivan, Borcek, Walker, & Rennie, M. 2016, Watson, & Mason,. 2006). The questions are not chosen to foster rote learning, but to provide an opportunity to use memorisation, repetition with variation, and spacing to consolidate the mathematical skills required by, and to connect with, other areas of knowledge (Rohrer, Dedrick, & Stershic, 2015). The teacher must make decisions about how often a question is repeated before the level of difficulty is increased. It is the level of knowledge of the individual student and the class group that must be considered by the teacher in the progress of question development (Pawley et al., 2005).

The FQA first four questions require students to use memorised procedures in routine ways to promote and strengthen student understanding and competence in basic mathematical skills and develop student procedural fluency (Stein & Smith,
These questions are designed to be relatively straightforward so the majority of, if not all, students complete them successfully within a reasonable time frame. Student success then builds confidence, and as a result may increase student engagement (Skemp, 1989; Winheller, Hattie, & Brown, 2013). The repetition of questions and the development of greater complexity at a rate tailored to student need, provides students with success and positive feedback every lesson and as such, the FQA provides opportunities for both confidence and competence to grow. According to Skemp (1989), when students are working within their region of competence (domain), they feel confident and secure. When working outside of this region they may feel frustration and anxiety. The FQA allows students to gradually move to the limits of their region of competence and beyond by using a careful selection of questions that gradually extend students’ mathematical knowledge and competence beyond their Zone of Proximal Development (ZPD) (Fuson, 2009; Vygotsky, 1978).

The FQA uses repetition with variation to promote learning by gradually building the students’ skills and slowly increasing the complexity of the questions. It is not sufficient for the students to be able to successfully complete a question, they must also understand why they carry out specific steps to correctly complete that solution. Casual observers of the FQA may liken the first four question types to rote learning but that is not accurate. It is closer to the practice and memorisation style using repetition with variation as previously mentioned. The students develop their understanding of the concepts involved and use repetition with variation to consolidate that learning.

In the development of conceptual knowledge, it is important to consider the cognitive level of the question and not provide support that simplifies the question to
a degree that results in cognitive undernourishment or cognitive emptying (Bell & Pape, 2014). Cognitive undernourishment or cognitive emptying occurs when the scaffolding or hints given to the students during the completion of the task reduce the cognitive aspects of the task to a level where the purpose of the task is lost (Clements, 2004). The structure and use of scaffolding and hints in the FQA is designed to reduce the cognitive undernourishment of the students by only allowing hints, as a form of scaffolding, to be given after all students have at least made some attempt at answering the question (Ediger, 2012; Wass & Golding, 2014). From my personal experience one effective scaffolding strategy is to provide the ‘answer’ that occurs when the most common misconception is applied to the problem as the hint, by stating that this answer is not correct. This is similar to the approach discussed in NCTM (2014) where the teacher focusses on their favourite mistake to highlight conceptual misunderstandings. According to Hattie, Fisher & Frey (2017) cues should be used to enable students to think about why their solution may be correct or incorrect and asking students to consider the origin of their answer whether correct or incorrect promotes thinking. Stating the answer that is the result of the most common misconception is an example of this type of cue. Teachers have control over the assistance given to each student in the FQA, as the teacher is the only person who provides hints, scaffolding or assistance while the questions are being attempted. Students can then share their ideas and suggestions when the solutions are discussed at the end of the process. The use of students’ voice and discussion allow the students to develop their communication skills, an aspect of working mathematically in the NSW K-10 Syllabus, by listening to each other and the teacher thus allowing the student voice. Student and teacher discussion is designed to assist in moving student learning from an instrumental understanding of procedural
knowledge to a relational understanding of conceptual knowledge through the presentation of different solution processes and the resulting discussion (Skemp, 1976).

The basis for the first four questions being of a procedural nature is threefold. First, they are designed to develop procedural fluency, thus decreasing the load on working memory when problem solving. Second, they are developed from previously encountered material as direct revision and consolidation. Third, they are used to prepare students for future content areas by providing the procedural basis for future topics. This structure is referred to as the ‘Yesterday, Today and Tomorrow’ aspects of the FQA.

For example, in Year 9 mathematics, number and algebra (yesterday) is studied before simple and compound interest (tomorrow). In the FQA there would be a question on the current material (today) and in preparation for the section on simple and compound interest (tomorrow) there would be questions involving substitution into the simple and compound interest formulas. By using the formula and completing subsequent calculations, students are provided with a revision of algebra (yesterday) and associated calculations, along with preparation for simple and compound interest using the relevant formulae (tomorrow). Later, when the section on simple and compound interest is studied (tomorrow), more time is available to investigate the conceptual understanding of simple and compound interest as the students are already competent in the manipulation of the formula and subsequent calculations. Due to the time limitations on the scope and sequence, the emphasis in the traditional classroom would often be on the correct substitution into and evaluation of the formula first without consideration of the concepts involved in simple and compound interest.
The development of conceptual knowledge is the focus of the fifth question. It is designed to expand students’ conceptual mathematical knowledge and understanding and to provide opportunities for developing a relational understanding which may lead to an increase in conceptual knowledge. The questions are generally open-ended, targeting a specific concept and in most cases, have multiple solution paths and may be set in an unfamiliar context. The fifth question provides an avenue for allowing students to construct their mathematical knowledge through investigative conceptual questions.

The design of the fifth question was the most challenging aspect of the teachers’ implementation of the FQA and required significant assistance as was the observation of the research by Stacey (2010). The fifth question is designed to provide opportunities for students to think conceptually and encourage students to make connections (Stein & Smith, 1998). However, the teachers deviated from this design and needed further assistance to return to the intended design. One explanation for this could be the lack of experience with this style of task design for the teachers and students’ lack of experience in solving this type of question as students were more disposed to waiting to be told what to do by the teacher (Stein & Smith, 1988).

An example of the fifth question could be a question investigating a Year 8 student’s understanding of the difference between area and perimeter. For example; ‘A rectangle has a perimeter of 24cm. What could the area be and what are the dimensions that give the greatest area?’ The question varies from the normal procedural questions that require students to find area or perimeter given the dimensions of a rectangle. There are many solutions to this question as it is designed to ensure students really understand the difference between perimeter and area. The
students investigate possible areas when the perimeter is constant. The solutions to the question and subsequent discussion ensures that the students can do more than just complete procedural perimeter and area calculations.

The FQA is much more than the simple provision of five questions for students to solve at the commencement of a lesson. As part of an overall teaching approach the purpose or intention of the FQA is to:

- Use mixed review which combines repetition with variation, spacing and mixing to revise and consolidate previously learnt material and develop procedural fluency.
- Increase working memory by reducing cognitive load through the automisation of processes.
- Provide a conceptual platform for upcoming new topics and concepts.
- Develop students’ problem-solving skills in familiar and unfamiliar contexts.
- Build concepts through a constructivist approach where the degree and depth of complexity is increased over time.
- Provide opportunities for concept linking and the acquisition of schemas.
- Enable teachers to gather data on students’ background knowledge to use in lesson planning and use individual student data to design the learning experiences, thus personalising the teaching and learning for all students.
- Provide opportunity for individual diagnostic student feedback.
- Cater for individual student differences in learning style, level of support required, and time needed for understanding.
- Allow students to discuss their strategies for question solution and learn from each other.
• Provide success and encouragement that could result in an increase in student cognitive, operative, and affective engagement.
• Provide opportunities for students that are absent from school to catch up.
• Provide opportunities for student voice.
• Reduce mathematics anxiety, particularly examination anxiety, through the development of confidence and understanding by allowing sufficient time for student understanding.
• By providing success and building confidence have students develop a growth mindset with respect to their mathematical ability.

The FQA is designed to take the best aspects of skill based procedural teaching with a focus on instrumental understanding and combine it with conceptual approach that is designed to build relational understanding and conceptual knowledge, by implementing constructivist learning theories to promote positive student learning outcomes.

This section has outlined the structure and aims of the FQA, revealing that the FQA is much more than five random quick questions. The various aspects that make up the FQA have been outlined. The next section gives specific examples of the questions used in the FQA.
3.2 What do the Questions Look Like?

The previous section outlined the purpose and basis of the FQA. This section gives specific examples of the questions and the development of understanding of a topic through the sequence of questions over time. Some specific examples from the FQA will be provided and the reasoning behind their selection is discussed.

The first FQA example is for a Year 9 class following the 5.2 pathway in the middle of Term 3. The scope and sequence would have ratio and percentages completed in Term 1, equations in Term 2 and statistics in Term 3. The class would be working on the trigonometry section at the time as these sample questions. Thus questions 1, 2, and 3 are revising and consolidating previously learnt material (yesterday). Question 4 is relevant to the current topic (today) and questions of that type may have been the focus of the previous lesson. Question 5 is exploring a conceptual understanding of the statistics topic completed a week or two ago and expanding the students’ understanding further in preparation for statistics in Term 4 (yesterday and tomorrow). The sample five questions are displayed in Table 3.1 on the next page.
Table 3.1 *FQA: Examples of five questions sequence.*

<table>
<thead>
<tr>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Write 35: 20 as a ratio in simplest form.</td>
</tr>
<tr>
<td>2. Find 17 ½ % of $540</td>
</tr>
<tr>
<td>3. Solve: 5 - 3x = 2x – 10</td>
</tr>
<tr>
<td>4. Triangle ABC has side AB = 8.3cm and angle ACB = 42°. Calculate the length of the hypotenuse BC</td>
</tr>
<tr>
<td>5. Write 5 scores that have a mean of 4, a median of 3 and the mode is not 3. How many solutions are possible? How do you know you have all the solutions? Extension: Add another aspect that reduces the number of possible solutions.</td>
</tr>
</tbody>
</table>

The first four questions are procedural in nature focusing on skill development, procedural fluency, and automisation. The fifth question explores statistical understanding and is quite different to the typical procedural questions usually associated with this topic area. It requires a sophisticated understanding of the measures of central tendency along with problem-solving skills. The question links the content to the working mathematically outcomes from the K-10 NSW Syllabus. When designing the five questions the teacher needs to examine the curriculum for the entire year so there is an understanding of which aspects of content and mathematical skills are required for later content areas. The teacher needs to consider the content areas that contain the most common student misconceptions and plan the sequence of questions to develop the appropriate conceptual knowledge. The
questions then need to be carefully built from the students’ current knowledge to
develop the skills and understanding required.

While the teachers all admitted that their lessons were mainly procedural in nature
there is still a difference between the ‘normal’ procedural lesson and the procedural
aspects of the FQA. In the ‘normal’ procedural lesson there is a single focus on a
single procedure, for example, finding the percentage of a quantity. The FQA has
four questions from different content areas that reinforce the procedures required to
solve them rather than having an example of a procedure followed by repetitious
questions.

To illustrate the sequence of questions over the course of a school term, an example
for the percentages content area in Year 8 is used. The percentages topic would be in
Term 1 of the scope and sequence and the financial mathematics topic would be later
in the year. A key percentage in financial calculations is 17 ½ % which is used to
calculate holiday loading. So, the planning for the FQA would involve questions that
would have the students familiar with calculating 17 ½ % of a dollar amount. The
syllabus outcomes pertaining to the percentage section include mental calculations
and calculations using technology. The FQA would use mental calculation only to
develop the students’ percentages skill. By using a mental calculation method, the
often-misunderstood concept of place value is consolidated through the calculations
of 10% and 1% of a quantity. The following would be a sequence of questions that
would appear individually within questions 1 to 4 over time. None of these questions
would be used in the fifth question. They would not all be done at the same time and
each type of question would be repeated until the students indicated they were ready
for question types of the next level of difficulty. A sample continuum of the type of
question asked in the first four questions is displayed in Table 3.2.
Questions like question 1 would require students to use the division by 10 strategy using their understanding of place value and decimals. Once this strategy of finding 10% is consolidated then the second question type is given. The students move the number one place to the right, equivalent to the decimal point moving one place to the left to find 10% and this acts as a review and consolidation of students’ understanding of place value. Once the students were proficient in finding 10% of a quantity question type, questions like question 3 would be given. Note, that the hint to find 10% first is not given until students have attempted the question as many will realise that they need to find 10% first and then halve the value to calculate 5%. The teacher would continue in developing the skills of the students until the students were proficient in all percentage question types, which as previously stated could be spread over an entire school term. During the rest of the year a percentage question
would be given at regular intervals in questions 1 to 4 to ensure students maintain the skills they have developed.

This section has provided examples of the five questions and the progression of a topic area through multiple sets of five questions. The next section will describe what the FQA looks like in the classroom.

3.3 FQA in the Classroom

The five questions are completed at the start of each lesson and are usually drawn from the different content areas of number and algebra, measurement and geometry, and statistics and probability. The first four procedural questions usually contain aspects from the previous lesson, the current lesson and future topic areas previously described as the ‘yesterday, today and tomorrow’ aspect of the FQA.

The class would commence with the teacher writing the five questions on the whiteboard or projecting them onto a screen. As the students enter the classroom the questions would be available immediately. The students take their seats and commence answering the questions. In practice, once the students are familiar with the process they come in and start immediately without the need for any teacher instruction. The students complete the questions in order and do not move to the next question unless they have completed the previous question or if the teacher has allowed them to omit a question.

While the students are completing the questions, the teacher moves about the room. It is usually possible to speak with every student during the time taken for the students to complete all five questions. Understandably, the conversations with
individual students are different. They are usually quite short and very specific to a need of the student. Moving around the room and speaking to each student is an important aspect of the FQA. There are five main reasons to move about the room. The first is to immediately target students that the teacher knows will have difficulties with specific questions. The difficulties could arise from a lack of knowledge about the process from previous classes or confusion with the application of a specific procedure. It is good practice to make the early question ones that you know some students do not have a procedure for solution or are still developing a solution strategy for and assist them right from the start. If necessary, they can be given a similar question to solve to consolidate their understanding. If, because of the extra question, the student is then unable to complete all questions that is acceptable but, they should always try to attempt the fifth question. The next day a similar question can be included to ensure the student has developed that understanding. Second, it is important to speak with each student to provide diagnostic feedback and encouragement. If the students have answered a question that previously caused difficulty for them then praise is given to the student for their persistence and success. If a student has made an error in the solution of a question then the teacher will provide them with positive diagnostic feedback, encouragement, and possibly another question. Third, the teacher will assist students that may have been absent from class with the material they missed. The FQA typically includes a question from the previous lesson and any student who was absent from that lesson can then be identified to assist them to understand and learn the material they missed. The student can be provided with further questions to answer on that topic while the other students
continue with the five questions. Again, the student may omit the other questions, but should still attempt the fifth question.

Fourth, the teacher will respond to individual student questions and decide if scaffolding for the class is required. If more than two students have the same issue with the same question, then a hint or some scaffolding to the entire class is required. Otherwise the teacher could spend all of the time allocated to visiting each student answering the same question multiple times. Sometimes the hint may be the ‘answer’ obtained by using a common misconception and this is written on the board and identified as an incorrect answer. Students with this answer realise that it is incorrect and can rethink their solution strategy. Hints and scaffolding given while the five questions are being answered allows students who are having difficulty with one or more of the questions to have an opportunity to answer correctly. However, the hints and scaffolding must be chosen very carefully by the teacher so as not to contribute to the cognitive emptying of the task.

Fifth and finally, the teacher will view student responses to questions to enable the teacher to select appropriate students to discuss their solutions with the class. One aspect of the FQA is to build student confidence through success. Asking a student to present a solution to the class that they are not confident with may result in a decrease in their confidence and negatively affect their engagement in the future. Viewing and discussing a student’s response to a question before asking them to present to the class is a strategy that enables the teacher to have quiet or less confident students speak in front of the class knowing that he or she will be successful. Sullivan et al. (2013) suggest that by observing the students, teachers could select those students, who used a solution or strategy that made an important contribution to the learning in the class to provide feedback.
Sullivan (2013) makes the point that when selecting students to report back and explain their method of solution that it is important to select the students and the order in which they present with care. A good practice would be to commence with a student who had partially completed the task but had difficulty with an area or perhaps a common misconception. The student should then be followed by a student who had solved the problem in a typical manner, resolving the misconception from the previous student. Finally, a student who had solved the problem using a non-typical method is chosen which allows discussion leading to greater understanding.

By using this method, it helps to ensure that over time every student gives a presentation and has a voice in the learning of the class.

In summary, teacher movement around the room and interaction with students is important for a variety of reasons such as:

- monitoring student progress and tailoring future five questions to their needs
- providing appropriate hints or scaffolding
- viewing student solutions
- selecting students to demonstrate solutions
- providing diagnostic feedback and encouragement.

After an appropriate time has been given for the students to answer the questions, the teacher then begins the solution phase of the FQA. The first four questions may simply be answered with little discussion unless there is a question that has either caused a problem for some students, or if the teacher wants to emphasise a point or there are multiple solution pathways. The main area for discussion is around the fifth
question. This discussion would involve different solution strategies used by students, along with deviations from the original question or a particular solution strategy to enhance the development of conceptual understanding. As previously described, the teacher would select some students and ask for student volunteers to discuss their solutions with the class. During the class discussion the teacher can highlight important mathematical aspects as the solutions are presented. After the discussion phase the teacher would continue with their ‘normal’ lesson. The process is then repeated in the next lesson. All teachers indicated that this type of discussion phase was not part of their normal lesson particularly as they did not use questions similar to the question 5 type in class and therefore there was nothing to discuss.

The history, development, detail and implementation of the FQA has been outlined so far in this chapter. The next section draws on the literature on effective teaching and learning from Chapter 2 and aligns each point with the FQA.

3.4 Comparison of FQA with the Literature Review Conclusion

The previous sections outlined the development, structure and implementation of the FQA, and this section links the FQA with aspects of teaching and learning identified in the literature review. In the final section of Chapter 2 a list of aspects related to quality teaching, effective teachers, student learning and engagement was prepared and is repeated here and discussed with regards to how each point relates to the FQA:

- *Teach effectively for understanding* (*Ball & Forzani, 2010; Ediger, 2012*).

The types of question used for the fifth question are open-ended targeting a concept and, in most cases, have multiple solution paths with an unfamiliar
context allowing students to construct or build their mathematical knowledge.

- **Teach using a spiral curriculum** (Cowan et al., 1998; Gibbs, 2014; Rohrer et al., 2015). Each day, the five questions use multiple content areas and subsequent questions build over time on the previous questions in a spiral nature.

- **Teach concepts using a constructivist approach** (Barrett & Long, 2011; Confrey & Kazak, 2006; O’Shea & Leavy, 2013; Tynjälä, 1999). The FQA builds on students’ current knowledge and allows for the acquisition of new knowledge to promote relational understanding and conceptual knowledge.

- **Avoid overlearning, instead space and mix content areas** (Carpenter et al., 2012; Rohrer, 2009a, 2009b). Every iteration of the FQA spaces and mixes content.

- **Instil a growth mindset in students and teachers** (Dweck, 2015; Hochanadel & Finamore, 2015; Rattan et al., 2015). The first four questions are designed to provide students with multiple opportunities to be successful and build their confidence thus encouraging a growth mindset.

- **Use automisation to reduce cognitive load and the strain on working memory** (Cowan, 2014; Paas & Ayres, 2014). The building and consolidation of the procedural skills through the first four questions allows automisation of tasks resulting in a reduction in cognitive load and releasing working memory to solve problems.

- **Adopt procedures and processes to reduce mathematics anxiety** (Beilock & Willingham, 2014; Foley et al., 2017). The increased confidence from success with the first four questions, the additional time to develop
understanding and the conceptual development through question 5 provide scaffolding to potentially reduce anxiety of mathematics, and particularly examination.

- **Increase engagement** (*Fast & Hankes, 2010*). The success achieved in successfully answering the five questions, the move to a growth mindset, along with questions that are tailored to the needs of individual students may promote an increase in engagement.

- **Provide meaningful diagnostic feedback** (*Harks et al., 2014; Hattie & Timperley, 2007; Hochanadel & Finamore, 2015; Powell & Kusuma-Powell, 2015; Rakoczy et al., 2013*). Diagnostic teacher feedback is an integral part of the FQA and occurs every lesson for every student.

- **Allow students to have a voice** (*Baroutsis et al., 2015; Smyth et al., 2012*). Through individual conversations, student presentation of solutions, and classroom discussion, students have a voice in their learning.

The list at the conclusion to Chapter 2, the literature review, has been linked to aspects of the FQA that may assist with quality teaching, effective teachers, student learning and increased engagement. It is important to note that the FQA is only part of what occurs in the classroom and the individual teacher has further input into the student learning. The FQA has been linked to all the facets of engagement, teaching and learning from the literature review and the next section draws these aspects together.

### 3.5 Conclusion

The previous sections outlining the FQA in theory, the FQA in the classroom and a link between the quality teaching, effective teachers, student learning and increased
engagement list from Chapter 2 provided evidence that the FQA comprises structures and processes that addresses each of these issues.

The next step is to conduct a study to answer the research questions:

1. What influence does the FQA have on perceived and actual academic performance of students in mathematics?

2. What influence does the FQA have on student engagement in mathematics?

The FQA has been explained in detail and aligned with the review of the literature presented in Chapter 2. The next chapter describes the methodology used to collect the data to answer the research questions.
4 Methodology

The purpose of the research was to investigate the influence of the Five Question Approach (FQA) on student engagement and on academic performance both perceived and actual. For this research, as examined in Chapter 2, the definition of engagement used is based on Attard (2014); the deep student relationship with classroom work that is a combination of behaviour, emotion and cognition. The hypothesis is that students who are engaged with mathematics see it as worthwhile and valuable in class and in the world outside of school, and as a result feel positive towards mathematics; they also think deeply and actively while fully participating in the classroom learning tasks. These aspects of engagement are explicit in the pedagogical relationships and repertoires as described in the FEM (Attard, 2014).

This chapter provides the rationale for the methodology used to investigate the following central research questions:

1. What influence does the Five Question Approach have on perceived and actual academic performance of students in mathematics?

2. What influence does the Five Question Approach have on student engagement in mathematics?

This chapter begins with a review of the research methodology used in this study and the reasons for the choice of that methodology. The procedures for data collection and analysis are described in detail and the conclusion to the chapter summarises the methods used, ethical considerations, assumptions, and limitations of the research.
4.1 Research Approach

Both qualitative and quantitative data were collected from two school sites to explore these research questions. As the collection of data to answer the research questions involved both qualitative and quantitative methods, initially a mixed-method approach was considered for this study. Creswell (2014) and Johnson (2008) propose that mixed-methods research lies somewhere on the continuum between pure qualitative and pure quantitative research methods as it involves aspects of both research methods. Pure quantitative research relies on the collection of quantitative data (i.e., numerical data). The data collected can be analysed in a statistical manner so that it can be used to answer the research questions. Quantitative research questions (i.e. non numerical data) can be very specific to enable the collection of measurable data.

The quantitative academic performance data was collected from the half year and end of year examinations held at both schools. This was used to answer part of the first research question. The data on perceived academic improvement and engagement were collected qualitatively through student focus group discussions, teacher interviews and classroom observations.

Both research questions are broad in their scope with many factors contributing to academic performance and engagement. With this as a consideration, qualitative methods were providing more data than the quantitative methods for some aspects of the study. Wilson (2013) describes a qualitative research approach as the asking of questions that are answered in the form of words.

As mixed-methods research uses qualitative and quantitative data collection methods several decisions on the use, importance and timing of each data collection method
were made. Johnson (2008) describes these decisions on mixed method design as:

“… (1) time orientation of the qualitative and qualitative components (concurrent vs sequential) and (2) paradigm emphasis (equal status vs dominant status)” (p. 446).

The first decision was whether the collection of the data using each method should be conducted concurrently or sequentially. The second decision involved consideration of whether or not the qualitative and quantitative aspects of the research were of equal importance or if one aspect was of greater importance or dominant of the other.

Further consideration was required to make the final decision on these issues. Creswell (2012) outlines four questions that were reflected upon in the consideration of the type of mixed-methods research to be undertaken. They were:

1. What priority or weight does the researcher give to the quantitative and qualitative data collection?

2. What is the sequence of collecting the quantitative and qualitative data?

   Relating to the aspects of timing and importance as outlined by Johnson (2008).

3. How does the researcher actually analyse the data?

4. Where in the study does the researcher ‘mix’ the data? (pp 539 - 540)

In response to the four questions posed by Creswell (2012), it was decided that a mixed methods approach was not an appropriate approach for this study. More appropriately a comparative case study approach would be used. This decision was made as the first research question was answered using qualitative and quantitative methods while the second was answered using qualitative methods only. There would be no real mixing of the data but the data on each case would be used in a
comparative case study that would contribute to the answers to each research question.

Comparative case method is used to determine the similarities and differences on some specific attributes of each case with a view to making an overall comparison using information from all cases. Each case is reported separately with the purpose of making comparisons based on the chosen attributes (Stake 2006). In this research the specific cases are each of the three classes and their teachers and so are similar in nature with their own specific differences. All cases have internal and external factors of influence and it is usually impossible to consider every aspect in any research project. Decisions must be made on which factors are considered in the research with respect to the two focus questions (Stake 2006). The main factor is the influence of the FQA on student engagement and academic performance, but in a classroom there are many aspects that contribute to student learning.

Finally, the comparative case method was chosen as it would support the recognition that emergent outcomes are the result of ‘configurations of causes’ that interact in any direction, similarities and differences, and incorporate many elements of the complex nature of the classroom learning environment. In particular acknowledging that learning is not caused by teaching, but rather occasioned by teaching and the many facets of teaching and learning in the classroom (Davis & Sumara 2006).

Qualitative methods were chosen to obtain data on both perceived academic performance and engagement through student focus group discussions, teacher interviews and classroom observations. Quantitative methods were applied to the examination results to collect data on the actual performance of the students, at the end of Term 2 and the end of the year (Term 4). The qualitative data was then
compared with the perceived academic performance qualitative data to determine the answer to the first research question.

The two types of data were collected at different times, and in this research, qualitative data was the main data source used. Qualitative data collection, according to Flick (2009), is used “…to clearly isolate causes and effects, to properly operationalise theoretical relations, to measure and to quantify phenomena, to create research designs allowing the generalisation of findings, and to formulate general laws” (p. 5). The information from each of the three types of qualitative data were analysed to develop themes. The themes developed from each type of data collection were compared to determine similarities and identify effects and phenomena that could then be synthesised into general statements relating to all areas.

The first collection of qualitative data on engagement and perceived academic performance was undertaken during weeks 1, 2, 3 and 4 of Term 1. The data was collected through student focus group discussions, teacher interviews and classroom observations. This qualitative data collection occurred on three further occasions. The quantitative data used to investigate actual academic performance, required for the second aspect of the first research question, came from the half yearly and yearly examination results. The final quantitative data collection occurred after all qualitative data had been collected. The timing of the data collection is displayed in Table 4.1.
Table 4.1 The Timing of Qualitative and Quantitative Data Collection

<table>
<thead>
<tr>
<th>Term</th>
<th>Weeks</th>
<th>Data Collection Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 to 4</td>
<td>Focus groups, teacher interviews, classroom observations</td>
</tr>
<tr>
<td>1</td>
<td>9 and 10</td>
<td>Focus groups, teacher interviews, classroom observations</td>
</tr>
<tr>
<td>2</td>
<td>9 and 10</td>
<td>Focus groups, teacher interviews, classroom observations</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>Half Yearly examination results</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>Focus groups, teacher interviews, classroom observations</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>End of year examination results</td>
</tr>
</tbody>
</table>

The first and second research questions relied on different sources of data. The first research question relied on both quantitative and qualitative data. The second research question required the analysis of qualitative data only. To answer the first research question and make a statement about the academic performance of the students experiencing FQA, the results of the qualitative and quantitative data collected were compared. The details of the comparisons are given in each of the data analysis chapters. Further details of the specifics of data collection are provided in the next section.

### 4.1.1 Reflexive Account

All qualitative researchers need to acknowledge that their writing is shaped by their experiences and viewpoints. As Goldberg and Allen (2015) point out, “… authors may wish to consider and discuss their own positionality in relation to the subject matter in general and the participants specifically” (p. 63). My background as described in Chapter 1, led to the development of the Five Question Approach (FQA) to teaching mathematics during my 30 years of secondary mathematics teaching. I believed that the FQA was an effective approach to increase student academic performance and cognitive, operative, and affective engagement. Given
this, care needed to be taken to eliminate bias in the collection of qualitative data as much as possible. Researchers face challenges when using qualitative interviewing and must consider instrumentation rigour and the management of bias (Chenail, 2011).

The methodology and research questions were developed and chosen with consideration to my background as the developer of the FQA. The student focus group and teacher interview question prompts were designed to be broad and as open as possible. Participants were informed that I developed the approach but were told to be honest and open in their responses as I wanted to hear the positives and negatives of the approach. The participants were informed that their responses may enable me to improve the approach. As any qualitative research involves human beings and interpretations it was the intention of the researcher to be objective, but it must be acknowledged that this is not always completely possible.

To reduce bias, it is necessary to investigate the potential sources. Roulston and Shelton (2015) split bias into four categories. Each of the categories of bias: selection, experimenter, observer and confirmation; will be defined and placed within the context of this research and my position on the FQA.

Selection bias occurs when the researcher influences the selection of subjects for interview (Roulston & Shelton, 2015). There were three aspects of selection in this research. First, the schools were selected by the Catholic Schools Office of Newcastle without input from the researcher. The Diocese of Newcastle was chosen as I, the researcher, was not well known in that area, as I had previously worked in the diocese of Parramatta and Wollongong. Second, the teachers and classes were selected by the leadership team of each school without input from me. Third, 100%
of the students returned their consent forms and so the pool for selection of focus
groups consisted of all students in each class. As the researcher, I, had no input into
the selection of students. The participating teachers selected a cross section of
students to provide a mixture of ability and engagement levels for the focus groups
and provided the list to me. This method of selection was chosen to reduce selection
bias.

The student focus group discussions and teacher interview questions were set before
the interviews and I only asked additional questions to elicit further responses. This
particular design was chosen in an attempt to eliminate researcher experimenter bias.
According to Best and Kahn (2003, pp. 168–169), as cited in Roulston and Shelton
(2015), “Experimenter bias occurs when the researcher contaminates the data
collection by affecting subjects’ responses or reactions to an experiment” (p. 344).
During student focus group discussions and teacher interviews the participants
answered each interview question first before I asked for any other questions. This
was to reduce experimental bias.

Classroom observations took place at all four data collection stages with me seated
in the room mostly taking a passive role. At times, I walked around the room to
observe the students to determine if they were actually completing the questions and
not doing something else. As the observations were influenced by my perspectives,
the possibility of observational bias was considered in the analysis of the classroom
observations. Finally, according to Nickerson (1998), as cited in Roulston and
Shelton (2015), confirmation bias occurs when the researcher is selective in the data
collected and interprets the data so that it supports the researcher’s position. This was
considered in the reporting of the data. The perceived qualitative academic
performance data was triangulated with the actual quantitative academic
performance data in an attempt to address possible confirmation bias.

The potential sources of bias have been considered and steps taken to minimise any
bias from the research. The collection of data through focus group discussions is
described in the next section.

4.1.2 Using a focus group approach

Following discussion of any researcher bias and the strategies for reduction in bias,
the next step was to select the appropriate qualitative method for the study. It was
decided to use individual teacher interviews coupled with classroom observations
and student focus group discussions to collect the qualitative data. Interviews are an
effective way to gather qualitative data. As the interviewer asks questions and has
the interviewee explain themselves the cultural and personal meanings that cannot be
assessed through quantitative methods come through resulting in richer responses
(Magnusson, 2015).

Six students from each class were chosen by the classroom teacher to participate in
focus group discussions. The decision to use groups of students rather than
individual students was made as students’ responses within the group may stimulate
thinking and result in more extensive responses. “The interaction between focus
group participants has the potential to create a dynamic synergy that is absent in
individual interviews” (George, 2013, p. 257).

When participating in the focus group discussions the students responded in a cyclic
order, in the first instance Student 1 answered the first question and then the others
responded in order of their seating arrangement. The second question was answered
first by Student 2 and the pattern continued. This structure was chosen so that every student had an opportunity to be the first to respond to a question. The cyclic order allowed all students equal opportunity. However, students were encouraged and supported to make additional comments after all others had given their viewpoint and this occurred often. This section has explained the process in general terms and the specific context of the schools and participants is outlined in the following section.

4.2 The Research Design

In order to examine the influence of the FQA on academic performance and engagement it was decided to collect data from two schools. Contact was made with a systemic Catholic school diocese to request permission to use two schools within their diocese. The Catholic system was chosen due to time constraints on Department of Education schools being ready for the start of the school year. There was no requirement of selection imposed except that the schools had to be of sufficient size so that comparisons could be made between classes in the same academic year. The detailed context of each school follows.
4.2.1 The school context

Two secondary schools were selected for the research and contacted by the Diocesan office. Their details were provided to me and I contacted each school. The names of the schools and the names of individual teachers and students were fabricated to maintain privacy but all other information pertaining to the school including socio-economic data and NAPLAN scores, etc are real. The leadership teams of both schools were enthusiastic about participating and were hoping for the project to have a positive influence on their students. In order to be able to collect meaningful quantitative data concerning academic performance a single class was chosen from a year level in each school. It was then possible to compare the performance of the students in that class with the performance of all the students in the other classes in that particular year group as they all completed the same half yearly and yearly examinations.

The first school, referred to as Saint Charles’ College, is a Year 7 to 10 coeducational school with a student population of approximately 720. The school was established in 1916 by the Dominican order of nuns as a girls’ day and boarding school. In 1976 it became the first school in that diocese to be staffed entirely by lay staff. In 1983 boys were enrolled in Year 7. The school motto is ‘Fostering individuality and potential.’ In 2015 the school had an Index of Community Socio-Educational Advantage (ICSEA) value of 1015 compared with the average ICSEA value of 1000 indicating that the school population was slightly above the average with respect to the socio-educational background of the students. The ICSEA does not include staff, school facilities or curriculum. The MySchool website (https://myschool.edu.au/) indicates that the mean 2015 National Assessment Program—Literacy and Numeracy (NAPLAN) Year 9 numeracy result of 570 was
below average when compared to the mean of similar schools, 586, and with all schools, 592. Approximately 47% of students were female and 53% male, with 6% of the students identified as Indigenous and 12% were from a language background other than English.

All mathematics classes in each year are graded. Initially a Year 9 class was selected as the teacher of that class was looking for a different teaching approach to use with those students. The class was graded as the fifth class out of the seven Year 9 mathematics classes. The teacher, Jenny, (a pseudonym) taught the class nine days each fortnight with another teacher taking the Friday class every second week. As that teacher only saw the class once per fortnight it was decided that the first teacher would set the five questions for that day.

The mathematics curriculum structure in NSW has three pathways in stage 5, i.e. Years 9 and 10. These are the 5.1 pathway, which was designed for students who are still working towards the completion of stage 4 (Years 7 and 8) outcomes; and the 5.2 pathway that builds on the content of stage 5.1 and was designed for students who completed the stage 4 outcomes by the end of Year 8. The 5.3 pathway builds on the content of the 5.2 pathway and was designed for students who achieved the stage 4 outcomes before the end of Year 8.

This class is following a 5.2 pathway and is the third of the three classes following this pathway and consequently, it contains a number of students that may be more suited to the less demanding 5.1 pathway.

Following initial conversations between myself (the researcher), the Assistant Principal (Curriculum), Head of Mathematics and participating teacher, Jenny, it was decided that the Head of Mathematics, Harriette, would also participate in the
research. The class selected was a Year 10 class taught by Harriette. This class was following the 5.2 pathway and was graded as the fifth class out of seven in Year 10 and the third of three following the 5.2 pathway. It is graded into the same rank fifth out of seven as Jenny’s Year 9 class.

The second school, referred to as Saint Patricia’s College, is a Year 7 to 12 coeducational school with a student population of approximately 980. The school was established in 1984 and underwent a major building program in 2014. The school has a focus on academic excellence. In 2015 the school had an ICSEA value of 1025 compared with the average ICSEA value of 1000 which rated it higher than St Charles. The MySchool website indicates that the mean 2015 NAPLAN Year 9 Numeracy result of 586 was close to the mean for similar schools, 594, and all schools, 592. Approximately 49% of students were female and 51% male; 3% of the students identified as Indigenous and 2% were from a language background other than English.

All mathematics classes were graded, and a Year 8 class was selected for the research, by the Head of Mathematics, for two main reasons. The first was that one of the teachers in that class had attended a presentation on the FQA at the Mathematical Association of New South Wales (MANSW) annual conference in the previous year and had attempted some of the strategies in the last few weeks of Term 4, 2015. The second is that the school was trying to have more students attempt higher levels of mathematics in Year 9 and were hoping the FQA would help to increase the numbers. The teacher, Anne, is part of a job-share situation where she teaches Monday, Tuesday and Wednesday and the other teacher, Karen, teaches Thursday and Friday. The class is graded as the second class out of the seven Year 8 mathematics classes. All classes in Year 8 follow the same curriculum.
With classes in Years 8, 9, and 10 there is a broad range of year groups for the research with a mix of graded abilities. The details of the actual participants are in the next section.

4.3 The Participants

The research involved three secondary mathematics classes. The Year 8 class was taught in a job-share situation, the Year 9 class in a partial job-share situation and the Year 10 class by a full-time teacher. All students in the selected classes were invited to be part of the research and were provided with an information sheet and consent form, copies of which are in Appendix A. Every student in all three classes returned the consent forms and a focus group of six students from each class was selected by the class teacher or teachers. The focus group met four times as previously described in section 4.1. The teacher details for the three classes in the research are in the next section.

4.3.1 Teacher participants

As described the Year 8 class was taught by job-sharing teachers and the details of both teachers are provided. The Year 9 class had only one lesson per fortnight taught by the second teacher who had both the five questions and the entire lesson prepared and provided by the main teacher. Therefore, only the details of the main teacher are provided. The Year 10 class was taught entirely by the Head of the Mathematics Department, Harriette. Details regarding the qualifications and experience of the teachers are displayed in Table 4.2.
Table 4.2 Description of Teacher Participants

<table>
<thead>
<tr>
<th>Teacher Pseudonym</th>
<th>Qualifications</th>
<th>Experience</th>
<th>Full-Time Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jenny</td>
<td>Bachelor of Teaching, Bachelor of Science, Master of Education</td>
<td>More than 10 years</td>
<td>0.95</td>
</tr>
<tr>
<td>Harriette</td>
<td>Bachelor of Teaching</td>
<td>More than 10 years</td>
<td>1.0</td>
</tr>
<tr>
<td>Anne</td>
<td>Bachelor of Teaching, Bachelor of Mathematics</td>
<td>3 years</td>
<td>0.6</td>
</tr>
<tr>
<td>Karen</td>
<td>Bachelor of Teaching, Bachelor of Mathematics</td>
<td>4 years</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Educational research deals with human beings and it is the researcher’s responsibility to protect the rights and welfare of the participants (McMillan, 2010). Participants need to understand that they may choose not to answer any question or even end an interview or choose not to participate in the study at all (Magnusson, 2015). All participants in this study were assured of their anonymity and the confidentiality of any data gathered. Each student and teacher were allocated a pseudonym. All participants received an information sheet and signed a consent form. Examples of the participant information sheets and consent forms are in Appendix A. Participants were asked verbally to consent to all interviews before each interview started.

Given that the teachers knew that the interviewer was the creator of the FQA there was the potential for bias in their responses. Bias was discussed in section 4.1.1 and other assumptions and limitations are discussed in section 4.8 later in this chapter.

The next section provides the details of the student participants.
4.3.2 Student participants

For each participating class the teacher selected a sample of six students from the class and they participated in the focus group discussions. The sample was stratified by selecting female and male students across a broad range of academic performance and engagement levels based on the teacher’s knowledge of the students. The academic performance was based on the end of year final examination in the previous year which determined the streamed class for the year of the study. In order to determine the engagement level of each student selected for the focus group the teachers were provided with the FEM and pedagogical relationships and repertoires were discussed and the level of engagement based on the student’s level of enjoyment, value and relevance in the learning of mathematics, and the connections with mathematics in the classroom and outside world. All teachers had been at their current school for at least two years and had general knowledge of most students.

The two teachers at Saint Charles had taught the majority of the students in their chosen classes in the previous year which reduces the teacher effect from the results. The student focus group sample for each class along with their rankings in the final examination of the previous year is shown in Table 4.3.

Table 4.3 Focus Group Student Name and Rank Details Based on Examination Results from the Previous Year

<table>
<thead>
<tr>
<th>Year 8</th>
<th>Yr. 7 Rank (out of 177)</th>
<th>Year 9</th>
<th>Yr. 8 Rank (out of 180)</th>
<th>Year 10</th>
<th>Yr. 9 Rank (out of 180)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lewis</td>
<td>25</td>
<td>Gina</td>
<td>102</td>
<td>Joe</td>
<td>92</td>
</tr>
<tr>
<td>Ella</td>
<td>37</td>
<td>Jim</td>
<td>121</td>
<td>Sara</td>
<td>106</td>
</tr>
<tr>
<td>Sue</td>
<td>39</td>
<td>Steven</td>
<td>133</td>
<td>Allan</td>
<td>107</td>
</tr>
<tr>
<td>Corey</td>
<td>46</td>
<td>Allie</td>
<td>137</td>
<td>Alison</td>
<td>115</td>
</tr>
<tr>
<td>Brenda</td>
<td>53</td>
<td>Shane</td>
<td>148</td>
<td>Sonia</td>
<td>136</td>
</tr>
<tr>
<td>Gemma</td>
<td>73</td>
<td>David</td>
<td>164</td>
<td>Evan</td>
<td>139</td>
</tr>
</tbody>
</table>
Having outlined the selection of teachers’ classes and focus group students in this section the methods of data collection and the justification for each are now explained.
4.4 Data Collection

This research aims to provide some directions for improvement in the teaching and learning of mathematics and mathematics pedagogy in general through the use of the FQA. This research addresses two central questions, as described previously in section 1.5 and repeated here:

1. What influence does the FQA have on perceived and actual academic performance of students in mathematics?

2. What influence does the FQA have on student engagement in mathematics?

The questions were informed by the following sub-questions:

- Are there specific aspects of the FQA that contribute to any fluctuations of student engagement?
- What influence do teachers perceive the FQA has on learning in the classroom?

Marks from the half yearly and yearly examinations were used to rank the students based on their academic performance in that examination. The rankings were used to show the relative movement of the selected students from the sample class compared with the other students in the year group. As the content in the examination was directly related to the content of the FQA this analysis gave an indication of the influence of the FQA. The rank for each student was compared with the initial rank that was used to form the class. The ranking was then compared to the previous year’s final examination rank and the data displayed on a scatter plot.

The timeline for data collection was outlined in section 4.1 and the collection of data through teacher interviews is described in the following section.
4.4.1 Teacher interviews

The initial teacher interviews were the first data collected and this section details the teacher interview process.

The teachers were interviewed on four occasions. As the initial interview was conducted before the FQA was implemented, the teachers did not have any initial understanding of the FQA except for Anne who was familiar with it from a conference presentation and some very limited classroom implementation. The question prompts for the first interview were used to collect baseline data from the teachers with respect to their views on teaching and learning, the level of engagement of their students, and their classroom contexts. The teacher prompts for the next three interviews had a greater focus on the influence of the FQA as the teachers had an increasing level of experience of the FQA. Therefore, the interview prompts varied across all four interviews with the greatest difference between the first and all other interviews. The teacher and student focus group interviews took less than one hour each.

The prompts for the teacher interviews were as follows:

**Teachers’ initial interview prompts**

- Describe a typical mathematics lesson.
- Tell me about your students and their engagement with mathematics.
- What things do your students enjoy or dislike about mathematics?
- Describe your students’ mathematical ability.
- Are you familiar with the five question approach?
- What do you think is the purpose of the five question approach?
• What do you like/dislike about the five question approach?

• Describe your teaching style/philosophy of teaching?

• How do you think the five question approach will change your teaching style?

• How do you think the five question approach will help, or not help, your students’ academic performance?

• How do you think the five question approach will influence your students’ engagement with mathematics?

Second teacher interview prompts

• I would like to find out what you think of the five question approach after using it in your classroom.

• Lately you have been using five questions at the start of each mathematics lesson. Can you tell me about that?

• Tell me about any challenges associated with the five question approach.

• How have you dealt with these challenges?

• Can you describe any notable changes in the academic performance of the students as a result of using the five question approach?

• Can you describe any notable changes in the engagement of the students as a result of using the five questions?

• The first four questions are designed to use repetition with variation to promote automation in procedural aspects allowing more working memory for problem solving. Has this occurred? Explain?

• The fifth question is designed to promote conceptual understanding. Has this occurred? Explain?
Please comment on the influence of the five question approach on student engagement and academic performance.

**Third teacher interview prompts**

I would like to find out more about what you think of the five question approach after using it in your classroom.

- This year you have used five questions at the start of every lesson. Can you tell me about that?
- What was the best thing or worst thing about using the five questions at the start of each lesson?
- Has the five question approach enabled you to have more in depth mathematical conversations in your classroom? Please comment with any specific examples.
- How have your students responded to the five question approach?
- Please talk about how the five questions affected your students’ academic performance.
- Please talk about any influence that the five questions had on student engagement in completing mathematics questions.
- Initially the five question approach takes time from the normal scope and sequence, but this is supposed to be regained due to greater student conceptual understanding. Please discuss the effect the five question approach has had on your completion of the scope and sequence.
Final teacher interview prompts

I would like to find out more about what you think of the five question approach after using it in your classroom.

- This year you have used five questions at the start of every lesson. Can you tell me about that?
- Have you made any changes to the five question approach?
- How have your students responded to the five question approach?
- Please talk about how the five questions affected your students’ academic performance particularly with respect to the final examination.
- Please talk about any influence that the five question approach had on your students’ preparation for the final examination.
- Please talk about any influence that the five questions had on student engagement in completing mathematics questions.
- The five question approach takes time from each lesson. Please discuss any effect this has had on completing the scope and sequence this year? Compare this class with your other classes and comment. Compare this class with the other classes in the same year group and comment.
- Thinking ahead to next year’s mathematics class(es), explain why you would want or not want to continue this approach.
- Are there any variations to the five question approach that you would like to discuss?

Teacher interviews were conducted during the school day in the teachers’ unallocated lessons. All interviews were recorded using an electronic device and verbatim transcriptions were made by a transcribing company. The teacher interview
and student focus group discussions had some similarity in question prompts and the student focus group process is discussed in the next section.

4.4.2 Student focus group discussions

Following the information on the teacher interviews this section describes the student focus group discussion process.

Focus groups discussions were used as the method for the student data collection rather than individual interviews as used for the teachers. According to Hess (1968) as cited in Vaughn (1996), the five distinct advantages to using focus group discussions over individual interviews are synergism, snowballing, stimulation, security and spontaneity. Synergism refers to group interactions. Snowballing is when one response initiates other responses. Stimulation is when the discussion elicits excitement. Security involves the group providing comfort to participants. Spontaneity fosters genuine comments.

Focus group discussions were used for the students in preference to individual interviews to reduce the total interview time and to promote richer responses. McLafferty (2004) indicates that the focus groups’ interaction and dynamics result in rich data. The students were interviewed in groups of usually six students from each participating class. Sometimes the number was less than six due to student absence on the interview day. The actual students interviewed at each stage are listed in Tables 4.4, 4.5 and 4.6.
<table>
<thead>
<tr>
<th>Interview 1</th>
<th>Interview 2</th>
<th>Interview 3</th>
<th>Interview 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brenda</td>
<td>Brenda</td>
<td>Brenda</td>
<td>Brenda</td>
</tr>
<tr>
<td>Sue</td>
<td>Sue</td>
<td>Sue</td>
<td>Sue</td>
</tr>
<tr>
<td>Corey</td>
<td>Corey</td>
<td>Corey</td>
<td>Corey</td>
</tr>
<tr>
<td>Gemma</td>
<td>Gemma</td>
<td>Gemma</td>
<td>Gemma</td>
</tr>
<tr>
<td>Lewis</td>
<td>Lewis</td>
<td>Lewis</td>
<td>Lewis</td>
</tr>
</tbody>
</table>

Table 4.5 Student Focus Group Participants Saint Charles Year 9

<table>
<thead>
<tr>
<th>Saint Charles Year 9 student focus group participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interview 1</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>Jim</td>
</tr>
<tr>
<td>Allie</td>
</tr>
<tr>
<td>Shane</td>
</tr>
<tr>
<td>Gina</td>
</tr>
<tr>
<td>David</td>
</tr>
<tr>
<td>Steven</td>
</tr>
</tbody>
</table>

Table 4.6 Student Focus Group Participants Saint Charles Year 10

<table>
<thead>
<tr>
<th>Saint Charles Year 10 student focus group participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interview 1</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>Allan</td>
</tr>
<tr>
<td>Joe</td>
</tr>
<tr>
<td>Sonia</td>
</tr>
<tr>
<td>Alison</td>
</tr>
<tr>
<td>Sara</td>
</tr>
<tr>
<td>Absent</td>
</tr>
</tbody>
</table>

Due to the early exit of Year 10 from St Charles’ College there was no fourth interview. The student focus group interview prompts follow:

**Students’ initial focus group prompts**

I would like to find out what you think about learning mathematics.

- Tell me about school and your mathematics lessons.
• What things do you enjoy or dislike about mathematics?

• Describe your mathematical ability.

• Describe what happens in a typical mathematics lesson.

• How do you feel when you need to complete mathematics questions?

**Student second focus group prompts**

I would like to find out what you think of the focus question approach used in your classroom.

• Think back to your mathematics class last year. What was different about mathematics class then, and what is different about mathematics class now?

• Lately you have been doing five questions at the start of each mathematics lesson. Please tell me about that.

• Have the five questions helped you to better understand mathematics?

• Do you think you are better able to answer mathematics questions after having the five questions?

• Do you think you are better at mathematics after having the five questions every lesson? Why?

• As a result of having five questions at the start of each lesson are you more likely to attempt maths questions now than you would last year? If so why?

• Has the five question approach changed the way you feel about maths? Explain.

**Third students focus group prompts**

I would like to find out more about what you think of the five question approach used in your classroom.

• What was the best thing or worst thing about the five questions at the start of each lesson?
• Have the five questions influenced or changed how you learn?
• Do you think you have a better understanding of mathematics as a result of
  the five question approach?
• Do you think you are better at answering mathematics questions after having
  the five questions every lesson? Why?
• As a result of having five questions at the start of each lesson are you more
  likely to attempt mathematics questions now than you would last year? If so
  why?
• What influence did the five questions have on your engagement in
  completing mathematics questions?
• If you had to give some advice to your teacher about the five question
  approach, what would it be?

Final students’ focus group prompts

I would like to find out more about what you think of the five question approach
used in your classroom.

• Tell me about your mathematics lessons this year.
• Have the five questions influenced or changed how you learn?
• What influence did the five questions have on your engagement in
  completing mathematics questions?
• Do you think that the five question approach had any effect on your
  preparation for and performance in the end of year examination? Explain.
• Thinking ahead to next year’s mathematics class. You have the choice of a
  class that uses the five question approach and one that does not. Which
  would you choose? Why?
• If you had to give some advice to your teacher about the five question approach, what would it be?

The student focus group discussions were conducted during the students’ normal school day and all took less than an hour usually about 45 minutes. The students were withdrawn from class and interviewed in the conference room in the school administration block. Students were returned to class following the interview. The classroom observations were conducted during the students’ mathematics lesson. The structure of the classroom observations is detailed in the next section.

4.4.3 Classroom observations

This section discusses classroom observations, which were the final aspect of the qualitative data collection. Direct observation is the most fundamental aspect of qualitative research and is a form of qualitative data collection that is frequently used. Creswell (2012) states that there are many advantages to this form of data collection particularly the study of actual behaviour in the setting normally used by the participants. This direct observation is different to a second-hand account through interviews and discussion and the observations can be triangulated against the data collected from the interviews.

The most significant disadvantage of observations is that it may be difficult to develop the necessary rapport between the teacher and the researcher that is very important in the collection of valid observational data (Creswell 2012). The observed must be comfortable in the observation so that they are natural and do not behave in the manner that they perceive the observer may be expecting. According to Johnson (2008) “Trust and rapport with the group being studied are essential if valid data are
going to be obtained” (p. 215). My experience in secondary schools allowed the quick and easy development of rapport and a high level of trust between the teacher and observer. I discussed the classroom observations with the teachers before and after each observation. The teachers all stated that they felt comfortable with me in their classroom. The dates and times for the classroom observations were negotiated with the teachers following the teachers’ consent. The students in the class were provided with letters explaining the purpose of the observations and my details so that they could ask any relevant questions.

I was introduced to the class at the commencement of each classroom observation. My role as the researcher was outlined so that students were aware of the purpose of the observation. An observation protocol was developed, and a table of headings and observational questions based in engagement was developed from the research Framework of Engagement (FEM) (Attard, 2014). I used this prepared table for the observations and a copy is provided in Appendix B.

In schools, teachers are not always experienced in research and with having observers in their classroom. The presence of an observer may result in the teacher acting in the manner that they perceive the researcher expects rather than the way they act normally. Students may act in a different manner as their normal classroom situation has changed, thus affecting the validity of the data. Johnson (2008) labels this as ‘front stage behaviour’ where the teacher and students only allow the researcher to see the aspects that they want to show. This is different to what Johnson (2008) labels as ‘backstage behaviour’ where there is little acting, and the teacher and students would perform as normal in the context without an observer. The teachers and students appeared to be comfortable with me in their classroom. As a result, the observations were considered to be realistic and a true example of a
typical lesson. During all school visits the I wrote field notes and that process is described in the next section.

4.4.4 Field notes

Field notes were recorded during the study to assist in the interpretation of data and the recalling of thoughts during the collection of data and immediately following interviews. According to Ritchie, Lewis, Nicholls, and Ormston (2013) field notes are an avenue to collect information that is seen or heard outside of the interview. The field notes were kept in a notebook and used to record my thoughts with respect to the data collection and initial analysis. These field notes were later used as the basis for more detailed, lengthier, complex and conceptual writing into memos (Corbin, 2008). This concludes discussion on the data collection and a detailed account of the data analysis process forms the following section.

4.5 Data Analysis

This section expands on the analysis of the qualitative data collected from the teacher interviews, student focus group discussions and classroom observations and quantitative data collected from examination results.

The analysis of the data occurred across three categories:

- qualitative engagement data
- qualitative perceived academic performance data
- quantitative academic performance data.
The quantitative data consisted of the half yearly and yearly examination marks. In both schools the half yearly and yearly examinations were independent as the topics tested in the half yearly were not repeated in the yearly examination, so they were two standalone examinations. The marks from the examinations were used to determine the ranking of each student within the entire year group. These rankings were then compared with the students’ yearly examination rank from the previous year. Students who did not have a previous year ranking were removed from the data set. The rankings were displayed in a scatter plot with a straight line at 45 degrees indicating students whose rank did not change. Rankings above the line indicate improvement in ranking while those below the line indicate a decline in ranking.

It needs to be noted that many factors contribute to examination performance and hence a particular student’s ranking. The ranking data was taken at face value as the research had no input into the examinations. On reviewing the examinations, all had procedural and conceptual questions with a mix of direct application of theory questions combined with problem solving style questions. As previously indicated there are multiple factors that contribute to student learning and one of the major factors is the teacher effect. The year 9 and year 10 teachers both taught the majority of the students in the previous year and as such contributed to the initial rankings along with the final rankings. The year 8 job share teachers had not taught the class previously so the teacher effect may be more significant.

Comparisons with the quantitative examination academic performance data were made with the qualitative perceived academic performance data collected through the teacher interviews and student focus group discussions. The analysis of the qualitative data was conducted using coding themes. The participants were interviewed and their responses to the interview questions were transcribed. The
emerging themes from the interviews and review of the transcripts were used to code the data. Following the initial coding all the data was read again and the process reapplied with consideration given to any other emerging themes and a modification of the codes if required. The data were organised and interpreted. The interpretations and findings of the perceived academic performance data was triangulated using the quantitative data of the actual academic performance collected through the processes outlined previously.

This concludes the explanation of the process of data collection and analysis; the next section provides information on ethical considerations.
4.6 Ethical Considerations

This study was conducted using the ethical guidelines implemented by Western Sydney University and was approved by the University’s Ethics Committee (Approval number H11276). The method of addressing ethical concerns in this study is outlined in the following sections.

4.6.1 Informed consent

Prior to the commencement of data collection, all students and their caregivers were provided with an information sheet and consent form. Sample copies are in Appendix A. The information sheet provided specifics about the students’ role in the collection of data for the research. Each student and his or her caregiver were required to sign a consent form which, in conjunction with the information sheet, provided full details of the aims and focus of the research. Participants were informed that they could withdraw their consent at any time without any prejudice.

Prior to the commencement of focus group discussions, all participants were required to provide verbal confirmation of their continued participation in the data collection. Students were then provided with a copy of the interview question prompts as described in section 4.4.2. The focus group discussions were recorded to allow me to review the contents and prepare a verbatim transcript. Students and teachers were assured of the confidentiality of their responses and that process is detailed in the next section.
4.6.2 Anonymity and confidentiality

Students were assured before focus group discussions that anything they said would remain confidential and that teachers at the schools would not have access to any transcripts. Students and teachers participating in the study were provided with a pseudonym. I was the only person with access to audio recordings and transcripts and all records were stored securely in a locked filing cabinet at my home. Transcripts, recordings and other related computer files will be retained in secure, password protected, storage for five years. All collected data was checked for validity and reliability as outlined in the next section.

4.7 Validity and Reliability

Once data is collected and analysed it must be checked for validity and reliability. The qualitative data on the perceived academic performance was compared with the quantitative data on actual academic performance from the half yearly and yearly examinations. It was expected that this data would converge but the possibility of divergence was considered. The comparisons of student performance within all classes in the year group were displayed in a scatter plot to give a visual demonstration of the interrelation.

The most significant issue with respect to the validity of combining data sets in this research is the variation in sample size. The quantitative data will involve every student in the year group, the qualitative classroom observation will include all students in the class, while the student focus group qualitative data will involve six of the students in each class. The students in the focus groups were selected by the teacher to be representative of the general class population with a range of abilities.
not just high achieving students. The student ranking from the previous yearly examination was used to determine that the students in the focus groups were in fact representative of the general class population.

The use of rankings from the examination results may affect the validity and reliability. The examinations are set by members of the mathematics faculty and not necessarily the teachers from the classes involved in the FQA research. As such it is not possible to align the examination with the combination of procedural and conceptual questions that would be most appropriate for the investigation particularly of the influence of the fifth question. Additionally, just a few marks may make a difference of many ranking values and so care needs to be taken with the before and after comparisons of the student rankings. The 8M2 class ranking comparison is limited by the fact that there are only 30 places available for the students to improve and so may restricts the number of ranking values above those in the class.

A positive aspect is that the half yearly examination only uses content from terms 1 and 2 and the yearly examination only content from terms 3 and 4. This makes the examinations independent and so any students changing classes following the half yearly examination, a practice in both schools, would provide the opportunity to compare the FQA and non FQA classes in their influence on particular students. The validity and reliability of the data has been discussed but all research has assumptions and limitations, and these are the content of the next section.
Following data collection and analysis, any assumptions and limitations need to be considered before and after data collection. The research was providing an understanding of the influence of the FQA on students’ engagement and academic performance. While the data was collected by myself, the researcher, as an outsider, my previous and extensive experience as a secondary mathematics teacher provided an insider perspective as well. My familiarity with the context of the secondary school was an advantage as it allowed the quick development of a high level of trust with the teachers. My classroom teaching experience allowed me to quickly develop a rapport with the students making them comfortable to discuss their viewpoints in the focus group discussions.

The main disadvantage in the collection of qualitative data was that the teachers and students were aware that I was the developer of the FQA. This provided the potential for bias in the data collection (Roulston & Shelton, 2015). In addition to the steps for combating bias outlined in section 4.1.1 I began every interview by clearly stating that it was important for the interviewees to be honest in their opinions and that the teachers would not have access to the focus group data. It was noted that there was always the chance that students adapt their responses to please the researcher but considering these were teenagers this was not as likely as with younger children.

The school context had a number of limitations. Both schools were Catholic Systemic schools. This means they were part of a supervising diocese of schools with a basis in the Catholic tradition. Both schools graded their classes and as a result the students had a strong focus on their examination performance as that was the basis for their class allocation. In fact, all secondary schools in that diocese
graded their mathematics classes and some other classes as well. However, grading would not be a limitation if all other schools graded their classes. If the other schools grade their classes, then there is an assumption that the results of this study would be applicable to other school systems beyond the systemic Catholic system. If not, then grading could be a limitation.

The Year 8 class had a teacher job share situation, meaning that two teachers had to develop questions and communicate well with each other in the application of the FQA. At this first stage of research, a single teacher class may have been more appropriate, but I did not have the option of choosing the class as it was allocated by the school leadership team.

This research does have the potential to be bigger in terms of involving more school systems, teachers and students; in particular, non-denominational schools and schools using a mixed ability approach to mathematics classes. Although the collection of data from two schools and three different year levels provided some insight into the value of the FQA, extension of the study into more schools may provide a much better understanding of the potential of the FQA.
4.9 Conclusion

This chapter has described the research approach including data collection, ethical considerations, validity and reliability.

The qualitative and quantitative data was analysed on a school by school basis. Chapter 5 is based on the analysis of St Patricia’s Year 8 data, Chapter 6 on analyses St Charles’ Year 9 data, and Chapter 7 on St Charles’ Year 10 data. Chapter 9 will synthesise the findings and directly address the study’s two research questions. The next chapter examines the Year 8 St Patricia’s data in detail.
This chapter focuses on the analysis of data collected from the St Patricia’s College School Year 8 class. At each of the four data collection phases, the three aspects: teacher interviews, student focus group discussions, and classroom observations, are discussed. The timing of the data collection (2017) is displayed in Table 5.1.

Table 5.1 Timing of Data Collection for St Patricia’s Year 8

<table>
<thead>
<tr>
<th>Phase</th>
<th>Term</th>
<th>Week</th>
<th>Date</th>
<th>Data Collection Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4/2</td>
<td>Anne and Karen teacher interview 1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>11/2</td>
<td></td>
<td>Year 8 focus group 1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>11/2</td>
<td></td>
<td>Year 8 classroom observation 1 Karen</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>10</td>
<td>31/3</td>
<td>Karen teacher interview 2</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>31/3</td>
<td></td>
<td>Year 8 focus group 2</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>31/3</td>
<td></td>
<td>Year 8 classroom observation 2 Karen</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>10</td>
<td>27/6</td>
<td>Anne teacher interview 3</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>27/6</td>
<td></td>
<td>Karen teacher interview 3</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>27/6</td>
<td></td>
<td>Year 8 focus group 3</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>27/6</td>
<td></td>
<td>Year 8 classroom observation 3 Anne</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>7</td>
<td>23/11</td>
<td>Year 8 classroom observation 4 Anne</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>23/11</td>
<td></td>
<td>Year 8 focus group 4</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>23/11</td>
<td></td>
<td>Anne teacher interview 4</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>24/11</td>
<td></td>
<td>Karen teacher interview 4</td>
</tr>
</tbody>
</table>

The analysis begins with the first phase of data collection.

5.1 Year 8: Setting the Scene.

The Year 8 class at St Patricia’s College was taught by two teachers in a job share arrangement. Anne taught Monday, Tuesday and Wednesday while Karen taught Thursday and Friday each week. These two teachers had not taught together before and Karen had recently returned from maternity leave. The class was 8M2, the second graded Year 8 mathematics class out of seven classes in Year 8. All classes in
Year 8 followed the same scope and sequence, which determined the order and time
duration for each topic. A copy of the Year 8 scope and sequence is provided in
Appendix C. The class comprised thirty students of which eighteen were girls and
twelve were boys. The students were in a seating plan determined by the teacher, and
it was varied every five or six weeks.

For the first interview, the teachers were interviewed together. This enabled both
teachers to be clear about each other’s attitude to teaching and as a result, the
teachers and I were all starting with a shared understanding of the baseline
conditions. The first classroom observation took place nine days after the initial
teacher interview on a Thursday in the third lesson, immediately following the
morning tea break. The class had been using the FQA for the past three lessons. The
lesson commenced with the teacher writing the five questions on the whiteboard.
Table 5.2 shows the five questions from that lesson.

<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10÷2=</td>
<td>-10÷2=</td>
</tr>
<tr>
<td></td>
<td>10÷2=</td>
<td>-10÷2=</td>
</tr>
<tr>
<td>2</td>
<td>Simplify</td>
<td>12ab – 7ba</td>
</tr>
<tr>
<td>3</td>
<td>Calculate 10% of 75</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Between which 2 whole numbers does √90 lie?</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Michael said the answer to 9 – 3 x m is 6m. What did he do wrong? What is the correct answer?</td>
<td></td>
</tr>
</tbody>
</table>

The full detail of the classroom observation is provided in Appendix D.

The student focus group, selected by the teachers, represented students with a range
of academic abilities and levels of engagement. As the class consisted of eighteen
girls and twelve boys, the focus group consisted of six Year 8 students, four girls and two boys. The focus group met with me to develop an understanding of their classroom learning experience before the FQA. This took place immediately following the classroom observation. Table 5.3 displays the ranking and engagement level of each student. The rankings were taken from the students marks in the final examination from the previous year and shows the results in the half yearly and yearly examinations for the current year. The engagement level was based on the discussion of the FEM as described in section 4.3.2. and the teachers were to base the level of engagement of each focus group student on a scale of Low (L), Medium (M) or High (H) when referenced to the FEM.

Table 5.3 Ranking and Engagement Level of Year 8 Focus Group Students

<table>
<thead>
<tr>
<th>Year 8 Focus Group Students</th>
<th>Year 7 (out of 177)</th>
<th>Engagement</th>
<th>Year 8 half (out of 180)</th>
<th>Year 8 final (out of 176)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lewis</td>
<td>25</td>
<td>M</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>Ella</td>
<td>37</td>
<td>M</td>
<td>43</td>
<td>69</td>
</tr>
<tr>
<td>Sue</td>
<td>39</td>
<td>H</td>
<td>7</td>
<td>67</td>
</tr>
<tr>
<td>Corey</td>
<td>46</td>
<td>M</td>
<td>46</td>
<td>52</td>
</tr>
<tr>
<td>Brenda</td>
<td>53</td>
<td>M</td>
<td>27</td>
<td>15</td>
</tr>
<tr>
<td>Gemma</td>
<td>73</td>
<td>H</td>
<td>7</td>
<td>52</td>
</tr>
</tbody>
</table>

Note: Gemma and Sue were promoted to 8M1 based on their half yearly examination results.

The details of the class structure and selection of the focus group students has been discussed and the next section examines the typical lesson versus the FQA.
5.1.1 The typical lesson versus the FQA

When asked to describe a typical lesson before implementing the FQA, both teachers followed what has been previously described in Chapter 1 as a ‘traditional’ mathematics lesson. Karen described her teaching as: “I am probably more traditional now than I was five years ago to be honest with you which is not the right way to be going”. Their lessons typically began with a discussion of the homework from the previous lesson followed by an introduction to the topic for the day. Some examples were given by the teacher with the students copying these examples into their books and then attempting some questions like the examples either from the textbook or a teacher prepared worksheet. The remaining questions from the exercise were set for homework.

Both teachers used the phrase ‘chalk and talk’ in a description of their lessons. Anne described her typical lesson as “I would often do chalk and talk about the concept and once I am happy with where they are I move on to discussion, theory, and exercises”. The lessons described were almost completely teacher-centred. The students’ description of a typical mathematics lesson aligned with that of the teachers. “We’d just write an exercise sheet and all the answers in our book and then we’d go over it and mark it, the teacher would explain to us and we finish for homework” (Ella). The students expressed that the quality of the explanation from the teacher was an important aspect of their learning, aligning with findings from the literature (Attard, 2012; Murray, 2011; Shimizu, 2009). Corey said, “In my class our math teacher would give us work and explain it really well”.

The observed lesson was different to the traditional lessons normally presented by the teachers as it began with the FQA. Following the five questions the ‘new’
content was centred on Pythagoras’ theorem and right-angled triangles. This was presented in a traditional manner and began with three examples of calculating the hypotenuse of a triangle. The students were told to copy the examples into their books and were given a worksheet with 20 questions that required the calculation of the hypotenuse of a triangle. There was very little discussion during this ‘repeated practice’ phase of the lesson in direct contrast to the FQA phase. The FQA phase was observed to have teacher to student, student to teacher and student to student substantive conversations, showing the existence of positive pedagogical relationships and repertoires which are indicators of an engaging classroom (Attard, 2014). The level of engagement changed between the two phases of the lesson and is further described in the next section.

5.1.2 Engagement in mathematics

When asked about the engagement of their students in mathematics lessons, both teachers linked engagement to ability. “Obviously kids with more ability are more engaged and like learning…if it is a higher streamed class they stay engaged” (Karen). Anne agreed with Karen but added that students in lower ability classes may be more engaged if they are successful in class. “If you’re setting the work in the lower classes at the right level you’re more likely to get engagement” (Anne).

The teachers expressed the idea that greater success in the classroom usually resulted in increased engagement in class which aligns with current research (Attard, 2012; Winheller, Hattie, & Brown, 2013).

Both teachers displayed a fixed mindset to the students’ mathematical ability (Claro et al., 2016; Dweck, 2015; Rattan et al., 2012). The teachers believed that the
students were unable to learn certain topics: “…once they don’t do well with a topic from then on they hate that topic” (Anne). This negativity may have occurred in the past, and as a result both teachers believed that the students would not engage at all with that topic when it is revisited. The students expressed how they felt frustration and anxiety, which influenced their attitude to the topic. Both teachers stated that they needed to break the cycle of student self-handicapping and saw that competence leading to confidence through the FQA might be a successful strategy (Martin et al., 2015; Plenty & Heubeck, 2013). The importance of breaking this cycle is reflected in the second goal of the Melbourne Declaration (2008) that states “… all young Australians should become successful learners, confident and creative individuals and active and informed citizens” (p. 7). Karen reflected that as a teacher she needs to “… break that cycle of the negativity”.

During the classroom observation, it became evident that the FQA provided opportunities for positive reinforcement and encouragement for the students. Teacher feedback supports the learning process and engagement of the students, particularly when it is diagnostic (Hattie & Timperley, 2007; Kiemer, Gröschner, Pehmer, & Seidel, 2015). Karen provided positive reinforcement to the students as she went around the room during the solution phase of the five questions. Some of the comments she made were “well done”, “you have good working out”, “that is a good way of doing that question”, “you are nearly there just think about it a bit more”. The students were engaged in the solution of the questions, asking the teacher for help and discussing their solutions with the students seated around them. The continuous interaction between teacher and students, and students and students was an element of the positive pedagogical relationships of an engaging classroom. The teacher then
drew the attention of the class to the front and they discussed the solutions to the questions. Some students were asked to present their solutions to the class.

The classroom observation revealed a high level of affective and cognitive engagement as evidenced by the large number of students who raised their hands to offer ideas and solution methods. The solution to question 5 led to further discussions on square numbers and surds. The teacher stated that these would form the basis of a question in tomorrow’s FQA. One student provided a novel solution to the question and explained the method to the class. Another student presented a further modification of that method for discussion. The observation showed a very high level of conceptual discussion taking place with respect to the fourth and fifth questions which indicated high cognitive engagement (Attard, 2014). The teacher admitted after the class that she would never have come up with one of the solutions given by a student and that the students were only exposed to that discussion and solution because of the fifth question.

The students stated that they liked the topic areas in which they performed well. Initially, Ella didn’t like or understand the algebra topic. She commented that she did not have sufficient time to develop an understanding of algebra as the teacher moved too quickly through that topic area. She later commented that the extra time and repetition of the FQA had provided her with the opportunity to develop an understanding of algebra. Other students made similar comments about insufficient time to learn and the benefit of the repetition of the FQA. The pacing of the lessons was important as a lesson that was too fast paced may not have allowed the students to understand what was taught (Ediger, 2012). Further elaboration on the aspect of timing follows.
5.1.3 Timing of learning

In the traditional classroom, the scope and sequence document usually allocates two to three weeks to complete a topic, for example, Pythagoras’ theorem. After the allocated time the teacher leaves the current topic and commences the next. The teacher does not usually revisit any previous topics until just before the half yearly or yearly examinations. The FQA allowed the teachers to include a question, for example on Pythagoras’ theorem, or other previously completed topics after they had been completed. More opportunities were provided for the students to develop a deeper, conceptual, rather than procedural, understanding of previously completed topics. In the focus group discussion, the students commented that sometimes there was not sufficient time spent in class to develop an understanding of a concept or topic before they moved on to the next. Rather than having a topic started and finished over three weeks, the students preferred to continually revisit the topic area which gave them time to understand it better. Concept areas that were spaced or spread over a longer period resulted in better retention of knowledge (Bird, 2011; Dunlosky, Rawson, Marsh, Nathan, & Willingham, 2013; Rohrer, 2015).

As the FQA allowed revisiting of past topics the two teachers felt that the FQA might result in an increase in academic performance and engagement. “I’m hoping there will be more lightbulb moments for the kids” (Anne). The teachers felt that a change in focus from procedural to conceptual understanding, particularly through question 5, may improve the students’ academic performance, retention of knowledge and ability to successfully solve questions (Engelbrecht et al., 2009). The teachers felt that the FQA might allow them to continue to develop the students’ understanding after the allocated time for the topic was exhausted.
The FQA allows teachers to move away from the traditional programming of mathematics topic by topic with a set time for completion, allowing more time for students to develop their conceptual understanding. The teachers hoped that the success in the classroom would equate to an increase in academic performance and student engagement (Attard, 2012). However, the teachers and students expressed the concern that the time taken out of the ‘normal’ lesson by the FQA might result in them falling behind the other classes in scope and sequence. This concern was shown to be unfounded in later data collection phases.

The teachers saw the linking of areas of mathematics as a positive aspect of the FQA. Karen stated, “I like the fact that it’s constantly requiring them to utilise all the concepts and areas of mathematics”. The FQA used repetition with variation, mixing, spacing and interleaving, which provided opportunities to revisit areas the students found difficult in previous topics (Handa, 2012; Rohrer, 2015). “I like the idea that if we strike an issue today with our five questions we readdress that tomorrow” (Anne). Repetition with variation is likened to the repeated exposure to a complex idea that results, over time, in a greater and deeper understanding similar to Bruner’s spiral curriculum (Bruner, 1966; Handa, 2012). This contrasts with ‘traditional’ teaching with a teacher giving some examples of a type of problem to be solved, providing minimal instruction, and then students complete questions very similar to the examples as a rote learning approach to teaching or “teaching without learning” (Fuson, 2009, p. 344). Spacing questions rather than massing, as in the traditional worksheet, allowed multiple opportunities to promote understanding and more time to process the concept through the varied types of questions over time. (Bird, 2011; Dunlosky et al., 2013; Kapler, Weston, & Wiseheart, 2015; Rohrer et al., 2015). The teachers commented that the change in focus from worksheets to the
FQA approach resulted in greater student participation, engagement and success. Students were more likely to attempt the FQA and focus on the completion of the questions.

During the focus group discussion, the students made comparisons with the FQA and the normal lesson containing repetitious worksheets. The students preferred the FQA approach of a mixture of topic areas and one question on each rather than a worksheet full of similar questions. The students made many comments about the repetition of questions in pre FQA mathematics classes and the amount of time spent doing the questions. They were concerned that the worksheets or exercises from the text book contained too many questions that were very like the examples given and were very time consuming. The students felt that they understood what was required without needing to complete all the questions. They only completed them as they would get in trouble from the teacher which demonstrates compliance and operative engagement only. Gemma summed up the feeling of the group by saying “But most of the time I didn’t really need to do as many questions”. The issue of the number of questions required and other themes from the first data collection phase are expanded on in the next section.

5.1.4 Links and themes arising from phase 1.

A comparison of the teacher interview, student focus group discussion, and the classroom observation revealed many similarities and several themes. First, the students seemed to have a naturally high level of operative engagement and enthusiasm for the learning of mathematics. This aligns with the initial teacher comments that the “higher” level mathematics students are more engaged than the
“lower” level mathematics students. This will be contrasted in later chapters where the classes are the fifth of seven within a year level as opposed to the second of seven.

The students agreed with the desire of the teachers to move away from the ‘traditional’ teaching approach. They commented that previously, the lessons felt rushed with insufficient time to understand the concepts before moving to another concept in the next lesson. The students expressed that too much time had been spent completing repetitive worksheets where the questions were very like the examples. The worksheets used the process of overlearning where many more questions than necessary are provided, which was shown by Rohrer and Taylor (2006), not to be beneficial in a mathematical environment.

Success in the answering of questions and understanding of concepts is clearly important to the students and teachers. The teachers in their interviews commented that the students performed better and were more engaged when they were achieving success. The students made the same comment in their interviews. However, in the classroom observation of the ‘traditional’ part of the lesson there were very few opportunities for the students to achieve success as it was completely teacher centred. This was in direct contrast to the FQA section of the lesson that had high cognitive and effective engagement as evidenced by the student-led solutions and discussions. These and the other emergent themes noted at this beginning stage are outlined in Table 5.4.

Table 5.4 Themes Emerging after the First Data Phase

<table>
<thead>
<tr>
<th>Theme</th>
<th>Sub-Themes</th>
<th>Detail</th>
</tr>
</thead>
</table>

161
<table>
<thead>
<tr>
<th>Engagement, success and positive attitude</th>
<th>Students who do not achieve success develop a dislike for that topic</th>
<th>Students enjoyed and were more engaged with topics that they understood and were confident in solving questions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students have multiple opportunities for positive reinforcement and success</td>
<td>Engagement with the FQA</td>
<td>Teachers found that the students engage and participate more in the FQA than they do in the ‘normal’ class lesson.</td>
</tr>
<tr>
<td>Non-linear knowledge development</td>
<td>Multiple Opportunities</td>
<td>Repetition with variation and diagnostic, positive feedback from the teacher provides multiple opportunities for success.</td>
</tr>
<tr>
<td>Non-Linear Approach</td>
<td>The use of five questions on different topics at the same time enhances student learning more than a worksheet with multiple repetitive questions that is given once for a lesson. Students can make links between mathematical concepts by working</td>
<td></td>
</tr>
<tr>
<td>Questions, discussions, and understanding</td>
<td>Student Discussion of Solutions</td>
<td>on multiple topics at the same time which promotes deeper cognitive engagement.</td>
</tr>
<tr>
<td>------------------------------------------</td>
<td>---------------------------------</td>
<td>--------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Topics are revisited</td>
<td>The scope and sequence allocated a set time for each topic and after that time elapsed the teacher moved to the next topic whether the students are competent or not. The FQA allows continuation of topics until students develop understanding and competence, thus promoting deeper cognitive engagement.</td>
<td></td>
</tr>
</tbody>
</table>
This summary table concludes the first data collection phase for the Year 8 class at St Patricia’s College. The main themes have been developed and they will be further analysed in the next data collection section.
5.2 Enjoyment of the FQA

The second phase of data collection took place in week ten of Term 1, seven weeks after the first data collection phase. Karen was interviewed during lesson 1, with the classroom observation in the next lesson and the student focus group discussion immediately following the classroom observation. The student focus group comprised the same students as the initial phase.

The teacher and students entered the classroom and the observation began with Karen writing the five questions on the board. The students took out their books and immediately started working on the questions. There were four students who arrived five minutes after the class had started. They apologised to the teacher for being late, sat in their seat and began the five questions immediately without any teacher direction. This is an indication of high behavioural (operative) engagement (Attard 2014). After writing the questions on the board the teacher moved about the room and answered questions as required, looked at student work, and provided constructive positive reinforcement. The complete classroom observation is provided in Appendix E and the five questions for the observed lesson are listed in Table 5.5.
<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Find the value of x.</td>
</tr>
<tr>
<td>2</td>
<td>Simplify $10m \times 2 + 8m \div 4$</td>
</tr>
<tr>
<td>3</td>
<td>Decrease 1600g by 14%</td>
</tr>
<tr>
<td>4</td>
<td>How many mm in 2.6 metres?</td>
</tr>
<tr>
<td>5</td>
<td>What was the base price, before GST was added, to give a retail price of $21.45?</td>
</tr>
</tbody>
</table>

The three main themes and sub-themes identified in the first phase of data collection are re-examined along with any other emerging themes from the second phase of data collection.
5.2.1 Success and positive attitude

The first phase of data collection indicated that this class were engaged in their learning. This was confirmed in the second phase with Karen stating: “…they’re certainly engaged in the FQA. I’m finding that they are enjoying doing the five questions”. Discussions with Karen indicated that the fifth question had allowed deeper learning and conceptual development promoting greater cognitive engagement and learning and that is leading the students to seek a conceptual rather than procedural understanding. “They actually want to have the conceptual understanding rather than the procedural” (Karen).

Because of her enjoyment using the FQA and the positive influence it had on her students’ engagement and their enjoyment of mathematics, Karen partially implemented the approach in her other classes. She said: “…it’s sort of starting to filter through my way of thinking so it’s really interesting to see that happening”.

The students said that the FQA helped them to feel happier about going to mathematics class. Sue summed up the feelings of the group when she said: “… in my old maths class last year I wasn’t comfortable, but now, I feel confident at the start of every lesson”.

The class had completed two single topic tests and these tests indicated areas where some students had not understood concepts. The analysis of the results was supposed to influence teaching to remedy the issues identified. However, due to the time restrictions on topic completion there was no time available to implement changes in the scope and sequence and unfortunately the students continued with some gaps in their knowledge and understanding which is a significant issue with the traditional model of teaching. “We’re supposed to do assessment for learning, but time
restrictions make it impossible – you do the task and you go, this is what mark you got and move on” (Karen). In this case the tests could contribute to the students’ lack of engagement with the teacher moving on to the next topic without remediating the issues with the previous topic.

The FQA provided an opportunity to address the problem areas indicated in the test. The teacher included questions in the FQA that allowed the students opportunities to develop the conceptual understanding that the assessment indicated they lacked.

“…you guys didn’t do so well in this particular concept in the exam. So, I’m going to use those in the FQA over the next few weeks and see if we can fix that up” (Karen). The further learning opportunities afforded by the FQA would not have existed in the normal teaching paradigm as the new topic would have been started immediately following the topic assessed in the test.

With the half yearly examination approaching, the focus turned to examination performance. The constant revision component of the FQA reduced the need for cramming preceding the examination. “…we are not cramming the week before, we’re revising all the time. Hopefully that will make it easier for them when it comes to exams” (Karen).

The focus group students commented that the FQA with its constant revision, repetition with variation (Handa, 2012), consolidation over time, and interleaving (Rohrer et al., 2015), had helped in the reduction of examination stress. The students had developed an understanding of concepts over time which provided the opportunity for them to develop a more complicated map or network in their long-term memory rather than a simple linear progression (Ollerton, 2009). The students felt better prepared and more able to answer questions because they were less
anxious and could think back to the five questions from previous lessons (Mavilidi et al., 2014). In the focus group interview the students indicated the following: Brenda no longer felt scared about exams, Ella was more comfortable, Gemma, Lewis, and Corey were more confident. Sue, who previously commented that the five questions took too much of the lesson, believed the FQA really helped her in the examinations.

*I reckon the five questions have given me a massive change, because if I went to an exam, like last year, maths exam, I'd be like oh this is a question from ages ago and I have no idea how to do this and I just get the question wrong., But with the five questions it gives you a chance to come back, understand what you've done and, you know relearn it so that you’ve got it for the exams. Because it's not possible to learn something once and then know it for ages away, you need to come back and revisit it because practice makes perfect* (Sue).

While a practice makes perfect approach can be linked to the traditional teaching approach using worksheets with many repetitive questions there is a difference in the FQA. The point of the first four questions in the FQA is to promote fluency and this is done through repetition with variation, the major difference to the traditional teaching approach is that the questions are repeated over time and not massed (Rohrer & Taylor 2007). If the FQA comprised just the first four questions then it would be promoting procedural fluency and knowledge, but it is the combination with the fifth question that promotes conceptual knowledge and relational understanding. Specifically, in this instance, the non-linear approach of the FQA as opposed to the massing of the traditional classroom assisted the students’ learning and examination preparation and this is expanded on in the next section.
5.2.2 Non-Linear knowledge development

Karen found that the FQA was working well and emphasised the fact that having questions on already covered topics (yesterday), the current topic (today), and future topics (tomorrow), had a positive influence on the students’ learning. This yesterday, today and tomorrow aspect of the FQA also allowed students absent from school to catch up and was utilised by Lewis: “The FQA really helped me because I wasn’t here for two weeks. And I came back, and I saw five questions on the board, I asked for help on a couple and then like a couple of days after I got the hang of it” (Lewis).

The continuous review was also a positive aspect. ‘I’m really liking the fact that it's keeping concepts in their working memory that normally they would just forget’ (Karen). The FQA was used to fill the gaps in student knowledge from the previous year so that all students were prepared for the upcoming topics. “I give them questions from work that they should have done last year, and they’ve forgotten… I can cover that background knowledge prior to when we actually need it” (Karen).

The students agreed and added that the challenge of the fifth question appealed to them. “I think our five questions are a good idea because it is like jogging back our memory and we do get to revise things. But our teacher will put in one question that’s sort of like new and a bit different, so like you have that one question and its cool and then each week it gets harder” (Brenda).

Karen provided questions on topics that the students should have covered in their Year 7 class last year. She found that there were gaps in some students’ knowledge that she did not expect. As a result, she could fill in those gaps using a selection of questions within the FQA.
The concern about the amount of time taken from their textbook work by the FQA was again raised by the students. However, they said that they learnt from the FQA. “I still feel like I'm learning heaps, I think it's a good idea, I just think that sometimes it takes too long” (Sue). The classroom observation indicated that the teacher had allowed too much time for the students to work on the questions before commencing the solution phase of the lesson. The timing is related to the difficulty of the questions and is expanded on in the following section.

5.2.3 Adjusting the questions

The teachers found that it was difficult to get to every student who required assistance and was a function of the degree of difficulty of the first four questions. I read through the FQA from the past four weeks and noticed a pattern. Some of the first four questions were too hard and the development of the questions occurred too rapidly over time. Question 1, $5(x+3) – 2(x-7)$ from early March was too difficult as they had not had easier questions of this type to develop their skills. It should have been question 5 with some hints. The question $35f ÷ 7$ was given as question 3 and then the next question of that type two lessons later was $45xy^2 ÷ 3xy$. The increase in difficulty progressed too quickly.

This was confirmed by Karen’s comment: “So I’m wondering if we should be making them a bit easier, so they need less help”. I agreed, and the questions were made easier with the development of difficulty appropriate to the needs of the class which resulted in more teacher/student interaction. The students liked it when the teacher moved around the room and spoke to them one to one (Murray, 2011). The teacher and students interaction is an integral aspect of the FQA and enables the
development of positive pedagogical relationships (Attard, 2014). “I like how our teacher walks around the class and checks if everyone is doing it right” (Ella). When compared with her class last year Ella said: “sometimes when teachers are just sitting at their desks and you’ll have your hand up for like 5 minutes and they won't see you”.

The fifth question needed no adjustment as the level of difficulty was appropriate for the class. The students described this class as better because of the FQA and the discussion generated in the solution phase, particularly around the fifth question, rather than an entire lesson focused on the textbook. “In my class last year, we did like a lot of textbook work. Now it is better, the teacher’s more involved” (Corey).

Gemma commented that she understood mathematics better because the discussions around the fifth question allowed her to see that the areas of mathematics are all related. The students expressed many of the aspects of working mathematically as described in the NSW K-10 Mathematics syllabus. “I can understand maths better…because everything in maths is connected” (Gemma). Ella and Sue repeated that they could answer questions better, Brenda said that it warms up your brain, Lewis repeated that he has learnt more than he has before, and Corey said it helps him remember. These opportunities for working mathematically may be linked to improved student cognitive engagement. The next section links these ideas together along with the themes raised in the first data collection phase.
5.2.4 Updated table of themes

The second phase of data collection confirmed the themes and sub-themes that emerged from phase one and raised some additional sub-themes, which were:

- examination stress reduction
- yesterday, today and tomorrow aspect of the questions
- time taken in class to complete the five questions
- quality and variety of questions.

Table 5.5 has been expanded to Table 5.6 to include the additional sub-themes from the second data collection phase. The additional sub-themes have been shaded for easy recognition.

Table 5.6 Themes Emerging after the Second Data Phase

<table>
<thead>
<tr>
<th>Theme</th>
<th>Sub-theme</th>
<th>Detail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engagement, success and positive</td>
<td>Students who do not achieve success</td>
<td>Students enjoyed and were more engaged with topics that they understood and were confident in solving questions. Students are considered to have achieved success by increasing their understanding of mathematical concepts.</td>
</tr>
<tr>
<td>attitude</td>
<td>develop a dislike for that topic</td>
<td></td>
</tr>
<tr>
<td>Focusing aspect of FQA</td>
<td></td>
<td>Teachers and students liked the settling and systematic aspects of the FQA with students entering class and starting immediately.</td>
</tr>
<tr>
<td>Students have multiple opportunities for successful understanding of mathematical concepts.</td>
<td>Repetition with variation and diagnostic, positive feedback from the teacher provides multiple opportunities for the development of mathematical understanding.</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>Positive student attitude and improved performance</td>
<td>The level of proficiency in answering the five questions contributed increased enjoyment in mathematics.</td>
<td></td>
</tr>
<tr>
<td>Engagement with the FQA</td>
<td>Teachers found that the students engaged and participated in the FQA more than they did in the ‘normal’ class lesson.</td>
<td></td>
</tr>
<tr>
<td>Examination Stress Reduction</td>
<td>Students felt well prepared for examinations and that the mathematics was ‘imprinted’ on their brains. That enabled them to attempt more questions in the examination. The students felt this reduced stress and increased confidence.</td>
<td></td>
</tr>
<tr>
<td>Non-linear knowledge development</td>
<td>Topics are revisited The scope and sequence allocated a set time for each topic and after that time elapses the teacher moves to the next topic whether the students are competent or not. The FQA allows continuation of topics until students</td>
<td></td>
</tr>
<tr>
<td>Non-Linear Approach</td>
<td>The use of five questions on different topics at the same time enhanced student learning more than a worksheet with multiple repetitive questions. Students could make links between mathematical concepts because of working on multiple topics at the same time.</td>
<td></td>
</tr>
<tr>
<td>---------------------</td>
<td>--------------------------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Multiple Opportunities</td>
<td>The aspect of repetition with variation allowed student engagement with mathematics and developed confidence in answering questions and understanding. The multiple opportunities came through as a very strong positive aspect of the FQA.</td>
<td></td>
</tr>
<tr>
<td>Yesterday, today and tomorrow</td>
<td>The FQA revised past topics, consolidated current topics and prepared students for future topics. Students could catch up after absences from school.</td>
<td></td>
</tr>
<tr>
<td>Time in class</td>
<td>The time taken by the FQA in class was an issue for both teachers and students. There was concern that the FQA took too much time from the ‘real learning’ from the</td>
<td></td>
</tr>
<tr>
<td>Questions, discussions, and understanding</td>
<td>Conceptual understanding</td>
<td>The repetition with variation combined with the conceptual question 5 and the class discussions was resulting in increased conceptual understanding.</td>
</tr>
<tr>
<td>-------------------------------------------</td>
<td>-------------------------</td>
<td>----------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Student Discussion of Solutions</td>
<td></td>
<td>The discussion of solutions and student demonstration of their solution methods was seen to be valuable in learning and the development of understanding. The variety of solution methods was important along with the move from teacher-centred discussion. Students commented that they learnt from each other.</td>
</tr>
<tr>
<td>Quality and variety of questions</td>
<td></td>
<td>The variety of questions was a positive aspect of the FQA in contrast to the repetition of the same questions in the textbook. The accessing of higher order questions because of the fifth question and subsequent discussion of multiple solution methods allowed a more conceptual rather than</td>
</tr>
</tbody>
</table>
This section has utilised the themes developed from the first data collections and examined them in relation to the second data collection. The themes were then modified and will be further examined next in the third phase of data collection.

### 5.3 The Examinations

The third phase of data collection took place in week ten of Term 2, following the half yearly examination, ten school weeks after the second collection of data. The students had been using the FQA for seventeen school weeks. Both teachers were available and were interviewed separately. The student focus group discussion immediately followed the classroom observation. The five questions from the classroom observation are shown in Table 5.7.

**Table 5.7 Five Questions for the 8M2 Third Observed Lesson**

<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$4x + 2 = -10$</td>
</tr>
<tr>
<td>2</td>
<td>Harry rides his bike at 24km/h a) How far will he ride in 10 minutes? b) How far will he ride in 50 minutes?</td>
</tr>
<tr>
<td>3</td>
<td>4hr 10min – 2hr 40 min</td>
</tr>
<tr>
<td>4</td>
<td>Calculate mentally 30 x 99</td>
</tr>
</tbody>
</table>
The detailed classroom observation is provided in Appendix F. The three themes and sub-themes from the second phase of data collection appeared again in the analysis of the third phase data which confirmed them as valid themes. The half yearly examination was completed, and the next section focuses on examination preparation.

5.3.1 Half yearly examination preparation

The students were asked about the influence of the FQA on their preparation and performance in the half yearly examination and they all stated that their level of stress and anxiousness was drastically reduced. The reduction in examination stress because of the FQA allowed the students to reduce the extraneous cognitive load that anxiety causes, resulting in greater working memory availability for the solution of questions (Mavilidi et al., 2014). The teachers agreed that the students were more engaged, better prepared, more confident and less anxious with the half yearly examination due to their successful development of mathematical procedures and concepts with FAQ (Attard, 2012; Winheller, Hattie, & Brown, 2013).

Lewis and Corey said that the FQA jogged their memory in the exam and when they attempted exam questions realised that they had completed similar questions in class,
and it helped them remember. Brenda said, “In the exam if I have a question I can think back, I’ve done something like this, in the FQA”. Ella, Gemma, and Brenda used the five questions as study revision at home and found the fact that the questions were different really helped in the exam. Brenda said “…so I decided to use it as study material, all of the questions are different, and it’s good to use to get the question right”. The FQA mirrored the half yearly examination by providing questions from all topic areas and the students felt as a result they were well prepared for the half yearly examination.

When asked if she felt the FQA had increased the student academic performance in the half yearly examination Anne replied: “Has the five questions helped? I think it must have. I just don’t know if I can answer how much”. Karen added: “I certainly don't think it has made them go backwards, I don't believe that for a second”. The actual academic performance of the class is analysed in the next section.

5.3.2 Half yearly examination performance

The half yearly examination contained both procedural and conceptual types of questions and was not set by either of the teachers of this class. There was a much greater emphasis on procedural, approximately 90% of questions, than conceptual question types in the examination. Both teachers felt that the students had performed well in the examination but did not have the grade comparison to determine the level of improvement. The results indicated that three of the students were in the top ten and ten were in the top thirty in the year group which showed strong performance considering the students were initially graded to range from positions thirty to sixty.
Figure 5.1 shows a scatter plot of the rankings of the end of Year 7 final examination, before the FQA with the half yearly examination rankings for Year 8 after two terms of the FQA. The straight line indicates equal rankings so points above the line show an improvement in ranking. Students that did not have a Year 7 ranking, who were not at the school in Year 7 or did not sit the Year 8 examination, were omitted.
Superficially the scatter plot indicates that approximately half the students improved their rankings and about half did not. However, the class was initially designed with the students ranking between 30 and 60. In the examination 9 students were ranked above 25 thereby pushing all other students down by 9 placings. Therefore, students who performed at the same level would be reported as 9 places below their previous ranking. Examining the data with that lens, realising that there are limited places above the current student rankings the following conclusions are reasonable. Nine
students showed significant improvement, seventeen performed within their expected range and four performed below expectations.

The students expressed that the FQA had enhanced their confidence and examination performance and had reduced their level of stress in the half yearly examination. Anne stated: “We’ve just had our exam results, reports are being written and some kids have done exceptionally with three students going up to the 8M1 class for Semester 2 and two others were on the borderline to go up”. Two of the students moved to 8M1 were from the focus group and they were interviewed after the yearly examination to compare their individual performance over the two classes. The fourth data collection followed the yearly examination and is discussed in the next section.

5.3.3 The yearly examination

The fourth and final phase of data collection took place after the yearly examinations in week seven of Term 4. Anne was interviewed on the same day as the class observation and student focus group discussion. Karen was interviewed the following day.

Two of the focus group students, Gemma and Sue, had been moved up to the 8M1 class based on their half yearly results and were able to make a comparison between the two classes. One student, Chloe, had moved down from the 8M1 class and she was added to the focus group to provide a different perspective to the two who went up. This meant the focus group contained the original students plus Chloe. The focus group discussion directly followed the observation of Anne’s lesson. The five
questions from the lesson are in Table 5.8. The full detail of the classroom observation is provided in Appendix G.

Table 5.8 *Five Questions for the 8M2 Fourth Observed Lesson*

<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Factorise fully $24m^3n^2 + 60m^2n^4$</td>
</tr>
<tr>
<td>2</td>
<td>The price of a shirt is increased by 20%. If the new price is $48, what was the original price?</td>
</tr>
<tr>
<td>3</td>
<td>Find the value of $\Theta$. Give a reason.</td>
</tr>
</tbody>
</table>

![Triangle with angles](image)

| 4      | Find the most common score of 4, 4, 5, 6, 6, 6, 7                        |
|        | The “mode” is _____?                                                    |
|        | The median is _____?                                                    |
|        | The range is _____?                                                     |
| 5      | Two circles are the same size. What other parts of the circles are the same? |
The focus group students commented that they liked having the FQA at the start of each lesson and that it helped them prepare for the yearly examination. Gemma and Sue had moved up to the 8M1 class and both said they missed the FQA and found the new environment difficult. Sue liked the learning environment of 8M2 much better than 8M1. Chloe came down from 8M1 and she liked the review aspect of the five questions and felt it made exam preparation easier. The individual cases of Gemma, Sue and Chloe are the subject of additional analysis in Chapter 8.

Figure 5.2 shows a scatter plot of the rankings of the end of Year 7 final examination, before the FQA with the yearly examination rankings for Year 8 after four terms of the FQA.

**Figure 5.2 Year 7 Final Examination Rankings vs Year 8 Yearly Examination Rankings**
The yearly examination reflected the same ratio of procedural to conceptual questions, and again neither of the two class teachers set the examination. Both teachers felt that the students had done well in the yearly examination but as the results had just been released, neither had an opportunity to analyse the class performance compared to the other classes. Anne had spoken to some of the students who were very pleased with their results when they had compared them to their friends in other classes and Anne said: “some were really happy with their mark, there’s one girl who to me has always been just very middle of the road, really hard worker and she got an exceptional mark and she’s thrilled”. Karen noticed there was a lot of overlap with the 8M1 class which was not expected as there had been some minor adjustments to the classes based on student performance in the half yearly examination. However, analysing Figure 5.2 in the same manner as the half yearly examination showed that eight improved significantly, nineteen as expected, and 5 performed below expectation. These are good results, particularly as the three best performing students from the half yearly were no longer in the class and not in the comparison. It needs to be noted that a variation in rankings would be expected with any class and the changes may be simply typical for the group. However, the students felt that they had performed better than they would have if not using the FQA and it appears that the results are atypical. While the quantitative test results may not be conclusive, overwhelmingly the qualitative teacher and student data indicated atypical positive performance. The deepened teacher level of PCK through interaction with the FQA may also be a factor in the improved results. The yearly examination performance and data from the other phases will be synthesised in the conclusion section.
5.4 Conclusion

The themes and the sub-themes from Table 5.6 have been confirmed and consolidated by subsequent collections of data. The three main themes, supported by the sub-themes as listed in Table 5.6, resulting from the collection of data were:

- Engagement, successful development of mathematical understanding and positive attitude
- Non-linear knowledge development
- Questions, discussions and understanding.

The focusing aspect of the FQA was a positive, although neither teacher could explain why it happened. Teachers and students acknowledged that they started working on the FQA immediately after entering the classroom. This was confirmed by the classroom observations and the students who arrived late to class went to their seats, took out their books, and started working on the questions without any instruction from the teacher. Karen said “I love the way that it just settles them, …they walk in … sit down and … get their books out… I like the routine aspect of it”. Anne said, “I think the kids have enjoyed the routine and the rhythm”.

Anne felt the students’ overall level of confidence had increased as the students willingly acknowledged when they made an error and wanted to discuss the solution so that they developed the correct understanding. “But people are quite happy to say what they got wrong and not be embarrassed about it” (Anne). Brenda, a shy student said that confidence was a problem for her and in the past, she was afraid to ask for help but the FQA enabled her to ask for help. “When you get help, it’s a really good kick” (Brenda). Anne worked through the solutions to the examination and felt that her solution was convoluted, “and a girl put her hand up and said, ‘I got the same
answer and I did it this way’… a girl could tell me a really simple way to get the same answer” (Anne). The students enjoyed the solution discussion and learning from each other. The observations indicated that there was a shared learning with the student voice as important as the teachers.

The review aspect of the FQA gave students more confidence in completing questions during class and in examinations. It enhanced students’ enjoyment of mathematics class and as a result their affective engagement level was high. The positive attitude through successful completion of questions and development of mathematical understanding in the FQA increased student perceptions that their academic performance was improving.

Working Mathematically is an important aspect of the NSW K-10 Syllabus and throughout this data collection various aspects of working mathematically have been emphasised. For example, both Karen and Anne expressed that the students’ fluency and communication using mathematical language had improved through the FQA. Anne said: “… with the fluency I think that is gaining some momentum”. Karen was enjoying the use of mathematical language along with the promotion of reasoning and said: “I like the fact that because we are discussing which requires them to communicate using their mathematical language”.

The students identified the yesterday, today and tomorrow aspect of the FQA when they commented that there were questions that are related to upcoming topics and that this preparation for upcoming topics was very useful. Ella said, “I never realised there is a question for a future topic, when we get to the topic, we already have a little bit of knowledge on the topic... that that’s helped me a lot”.

187
The FQA section of each observed lesson had active and involved discussion that indicated high operative engagement of the students, which contrasted with the low level of engagement during the teacher-centred section of the lesson (Attard, 2012). “I like the fact that FQA enables a lot of discussion and I think otherwise that may not happen” (Karen). In Chapter 1, ‘traditional’ mathematics teaching was raised as a major concern. Having been exposed to the FQA the teachers and students experienced a different style of teaching to the ‘traditional’ methods that were described by the teachers and students in the first phase of data collection.

The focus group students preferred the variety of the FQA to a repetitive worksheet or textbook exercise. Brenda represented the opinions of the group when she said “…if it’s just the same question all the way down, you get lost and think about other things. But with FQA you must focus, it makes you think a lot harder and engage in the lesson”. The teachers agreed that the students enjoyed the challenge and variety of the different topics of the FQA which enabled them to deeply investigate the concepts and the subsequent discussion promoted understanding.

The time taken by the FQA was an issue for both teachers and students. In my opinion the first four questions were still not easy enough and increased in difficulty too quickly. Karen said “…I think we’ve done big jumps instead of little steps”. The greater requirement for teacher assistance resulted in more time before the solution phase as every student having difficulty received assistance before starting the solutions. These time aspects were addressed with the teachers by me and rectified by the teachers over time.

The discussion of solutions and student demonstration of their solution methods was seen to be valuable in the learning aspect and the development of understanding. The
accessing of higher order questions because of the fifth question and subsequent
discussion of multiple solution methods allowed a more conceptual rather than
procedural understanding of mathematical concepts. The linking between concepts
through the FQA increased student understanding. The lessons involving the FQA
were less teacher centred and there was more cooperative learning with student input
and student to student discussion seen as valuable and important.

There were strong views expressed that the FQA using different topics enhanced
student learning more than a worksheet with multiple repetitive questions. The
students clearly articulated that they did not enjoy the repetitive worksheet.

_I remember that when I did the FQA I felt really motivated to do stuff and
now I’m 8M1 I kind of get handed a sheet and I look down at it and I'm
like I’ve got no clue what I'm doing ... it’s just kind of, it's very boring,
it’s repeating the same thing over and over again, so it just kind of goes
in one ear and out the other (Sue)._  

The difference between the way questions were asked in the FQA and in a
‘traditional’ repetitive worksheet have been raised at every data collection phase.

The concluding teacher interview question concerned the continuation of the FQA in
the next year. Both teachers replied that they would with some caveats. Karen
intended to use the approach with two of her classes so that she could fine tune her
understanding of the method and then move to all her classes in the future. She liked
the linking of topics aspect, and the higher order thinking questions typical of the
fifth question. Anne wanted to use the approach in all classes but once or twice a
fortnight have just a ‘teaching’ lesson without the five questions. She acknowledged
that this may defeat the purpose and perhaps just have a short time on the five
questions in that lesson instead. Anne wanted to continue with the FQA as she saw the benefits for the students, as they liked it and the structure and constant revision helped them to learn.

If given a choice, every student in the focus group wanted to be in a class doing the FQA. They mentioned advantages such as the repetition over time helping them to understand, the reduction in examination stress, the quality of the discussion around the solutions and the variety of solution methods.

_I’ve probably said in previous interviews that it’s a bit annoying that they’re slow, but I would definitely go back to them, I miss them more than ever because they did help me a lot. I didn’t realise that until I moved up and it was kind of like eye opening to me to see that I didn’t do as well without them (Gemma)._  

The repetition with variation and discussion enabled students to understand with Gemma and Sue commenting that the mathematics was engraved in their minds so that they did not need to revise for the examination, they just understood how to do the questions. This indicated a conceptual understanding. Gemma said: “I understood it and without realising it’s been imprinted in my mind and it didn’t feel like hard study or anything. It was enjoyable”.

The fifth question provided the opportunity for students to access higher order questions which then promoted high levels of cognitive engagement. Karen liked that at least one higher order question was attempted every lesson using the FQA and compared the FQA to the traditional lesson.
I think probably a lot of maths classes, you set the questions in class and they’re obviously in order from easiest to hardest, and by the time they churn through the easy ones the bell goes and they’re left to do the hard ones at home. My class is particularly bad at coming back and asking help on the questions they were supposed to do. (Karen).

The FQA attempts to rectify the lack of opportunity for conceptual understanding through the fifth question. This is the aspect that delineates the FQA from a review or quiz only starter question approach or a quick question approach as suggested by Gerver (2011).

This concludes the analysis of the St Patricia’s data and the next chapter analyses the data collected from the St Charles’ Year 9 class.

6 Analysis of Data St Charles’ Year 9

This chapter focuses on data collection from the St Charles’ Year 9 class taught by Jenny. It follows the data collection through the four phases. Each of the three data sources, teacher interviews, student focus group discussions, and classroom observations are discussed with conclusions drawn at each stage. The timing of the data collection is displayed in Table 6.1.

Table 6.1 The Timing of each Data Collection Point for St Charles’ Year 9

<table>
<thead>
<tr>
<th>Phase</th>
<th>Term</th>
<th>Week</th>
<th>Date</th>
<th>Data Collection Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>8/2</td>
<td>Jenny teacher interview 1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>8/2</td>
<td>Year 9 classroom observation 1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>11/2</td>
<td>Year 9 focus group 1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>9</td>
<td>21/3</td>
<td>Year 9 classroom observation 2</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>9</td>
<td>22/3</td>
<td>Jenny teacher interview 2</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>10</td>
<td>22/3</td>
<td>Year 9 focus group 2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>9</td>
<td>20/6</td>
<td>Jenny teacher interview 3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>9</td>
<td>20/6</td>
<td>Year 9 focus group 3</td>
</tr>
<tr>
<td>4</td>
<td>20/6</td>
<td>Year 9 classroom observation 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----</td>
<td>------</td>
<td>--------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>23/11</td>
<td>Jenny teacher interview 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>23/11</td>
<td>Year 9 focus group 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>No 4th Classroom observation due to examinations</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6.1 St Charles’ Year 9: Setting the Scene.

This Year 9 class followed the 5.2 pathway. All mathematics classes were graded, and this class was ranked 5th out of 7. There were three classes following the 5.2 pathway and this was the third in order of ability. As stated previously based on the marks in the Year 8 end of year examination, this class should have been following the lower 5.1 pathway, but school policy only allowed two 5.1 classes, therefore, it could be argued that these students were completing work at a level beyond their current capacity. Each of the three pathways had their own scope and sequence, but all classes sat the same half yearly and yearly examinations. The examinations had three sections for 5.1, 5.2 and 5.3, and students could attempt all questions even if they had not covered the material in class. A copy of the Year 9, 5.2 scope and sequence is provided in Appendix H. The class comprised twenty-six students with eighteen girls and eight boys. There was no official seating plan, but students usually sat in the same place every lesson.

The first teacher interview took place in Term 1, week 3 on a Tuesday during lesson 1 and the classroom observation followed in the next lesson. The first student focus group discussion took place three days later during lesson 6, the final lesson of the day. The teacher had started using the FQA and Table 6.2 shows the five questions from that lesson. The full detail of the lesson observation is provided in Appendix I.
Table 6.2 *Five Questions from the First Observed 9M5 Lesson*

<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>If $b = 12$ and $h = 6$, find $A$ when $A = 0.5bh$</td>
</tr>
<tr>
<td>2</td>
<td>Simplify $3xy + 2x - 2xy + 8x$</td>
</tr>
<tr>
<td>3</td>
<td>Dallas is paid $12.50$ per hour and works $36$ hours in a week. What is his weekly wage?</td>
</tr>
<tr>
<td>4</td>
<td>Find $10%$ of $624$</td>
</tr>
<tr>
<td>5</td>
<td>The area of a rectangle is given by the formula $A = l \times b$. If the area is $24$ cm$^2$, what values could the base and height be? HINT: base $= 4$ and height $= 6$ Because $A = 4 \times 6 = 24$cm$^2$. What other values could $b$ and $h$ be?</td>
</tr>
</tbody>
</table>

The student focus group selected by the teacher represented students with a range of academic abilities and levels of engagement. As the class consisted of eighteen girls and eight boys the focus group comprised four girls and two boys. Table 6.3 displays the rankings achieved in major examinations by the students in the focus group and their level of engagement as perceived by the teacher based on the description in the FEM. The rankings were taken from the student marks in the final examination from the previous year. The student’s results in the half yearly and yearly examinations in their current year are included for comparison. The teachers were given a definition of engagement from the FEM and asked to rate the level of engagement of each student on a scale of Low (L), Medium (M) or High (H). Interestingly, while none of the six selected students were considered by their teacher to be highly engaged, five of the six did see an improvement in their ranking from Year 8 to Year 9.

Table 6.3 *Ranking and Engagement Level of Year 9 Focus Group Students*
<table>
<thead>
<tr>
<th>Student</th>
<th>Year 8 yearly</th>
<th>Year 9 half</th>
<th>Year 9 yearly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gina</td>
<td>102</td>
<td>M</td>
<td>99</td>
</tr>
<tr>
<td>Jim</td>
<td>121</td>
<td>L</td>
<td>126</td>
</tr>
<tr>
<td>Steven</td>
<td>133</td>
<td>M</td>
<td>112</td>
</tr>
<tr>
<td>Allie</td>
<td>137</td>
<td>M</td>
<td>56</td>
</tr>
<tr>
<td>Shane</td>
<td>148</td>
<td>L</td>
<td>122</td>
</tr>
<tr>
<td>David</td>
<td>164</td>
<td>L</td>
<td>151</td>
</tr>
</tbody>
</table>

The discussion of the first phase of data collection begins by outlining Jenny’s typical approach to teaching.
6.1.1 The typical lesson and the FQA.

The teacher, Jenny, was interviewed before she had used the FQA to collect baseline data on her teaching and thoughts about the class. Her description of the typical lesson was very like that described by the St Patricia’s Year 8 class teachers in Chapter 5. Lessons typically began with notes and examples on the board and students copied these into their books followed by exercises, either textbook or worksheet, which were completed for homework.

Interestingly, Jenny stated that at times the lessons were hands-on discovery type of activities, but that she found this approach had not worked and so she had reverted to the traditional lesson approach. “They didn’t want to think about it themselves and so I’ve tended to go back to the traditional lesson and spoon-feed them pretty much” (Jenny).

Having said this, Jenny expressed the view that the traditional method of teaching was not working either and that something needed to change. She saw that the FQA might enable students to understand and achieve success in the completion of questions resulting in increased cognitive, operative, and affective engagement. Her concern was whether the ‘normal’ content could be sufficiently covered when using the FQA every lesson. She acknowledged that it would be better that the students understood some of the content rather than covering all the content and not understanding any of it.

The focus group students were asked about the types of teaching that they liked. They said they needed good explanations, a teacher who cares, sufficient repetition, and time to develop their understanding. The classroom observation conducted with this class revealed disinterested students who found the FQA questions too difficult.
and waited for individual teacher assistance before answering any questions. In the ‘normal’ part of the lesson the teacher attempted to make the questions relevant to the students by taking percentage discounts off a dress price. The students were mostly attentive during the teacher explanations, but there were disruptions to the lesson flow as some students made irrelevant comments.

The students compared this class using the FQA to their mathematics class from the previous year where they spent a lot of time writing notes or copying from the board and the general feeling was that mathematics class was boring and they did not understand the material. By comparison, with the FQA the students said that they were doing questions and understanding the content. Jim said “…last year we did heaps of writing the whole time pretty much … and I didn’t understand it and now we do the five questions…but over a period of time I get it because we are doing it every day”. Jenny hoped that the FQA would increase the level of engagement of the students.

### 6.1.2 Success and engagement

The Year 9 student focus group view was consistent with the view of the Year 8 students in the previous chapter at St Patricia’s College where they linked successful completion of questions and the development of understanding, with confidence and higher engagement. Again, the students were referring to operative engagement rather than cognitive or affective engagement. The students’ description of their mathematical ability ranged from “horrible”, “appalling”, “bad” to “okay”, and they were very aware of the fact that they were graded and that the students in higher classes must be better at maths than them. David says “Well I think I'm okay, but I
wouldn't say I'm great. Most people are in the higher classes than us, but I know sometimes it's a bit hard, but I still get most of it”. The teacher and focus group students all displayed a fixed mindset on their mathematical ability based on their past performance and allocated graded class, believing that their mathematical ability was set and unchangeable (Murphy & Dweck, 2016). This attitude may have influenced engagement levels as the students believed they could not improve and so did not try.

The second classroom observation showed an improvement in cognitive, operative, and affective engagement and self-efficacy as the students linked the ability to successfully complete questions to their desire to attempt questions. The FEM states that a combination of continuous interaction and substantive conversations increase student engagement. This was evidenced through the actual observation and comments made by students in the second group interview. The FQA questions were attempted with a positive attitude and provided opportunities for all students to achieve a level of success through the correct answering of questions and were challenging for all thus increasing engagement through pedagogical repertoires (Attard 2014). Shane said, “Yeah I feel like because I understand, I know what to do; I just feel I can do it, so I do it”. This confidence in knowing what to do resulted in greater student participation in class. David said “Yeah, this year it’s a lot easier for me because last year I struggled to put my hand up, but this year I put my hand up all the time because the five questions have helped me a lot”. He then commented that he found the FQA mathematics classroom more enjoyable, which appeared to increase his level of engagement. All students made similar comments about understanding leading to greater engagement. This links to the research of Fast and
Hankes (2010), who found increased content knowledge and understanding has a significant influence on student confidence and engagement.

When given a mathematics worksheet to complete some students liked doing questions at their own pace while others felt stressed when they saw the questions. None of them looked forward to completing multiple mathematics questions on the same topic. They expressed the view that there were too many questions to do, and many just wanted to get them done. This was the finding by Rohrer and Taylor (2006) that overlearning, the use of repetition beyond competence, was not effective. One saw it as an opportunity to chat with her friends because everyone else was quiet. However, they all felt that sufficient time and teacher explanation was needed to be successful with the worksheet. “…it’s hard because sometimes I really don’t know what we’re trying to do and at the end when she goes through the answers if I put up my hand and asked for further explanation and she explained it to me that would be good” (Allie). The focus group students generally felt that there was insufficient time to develop an understanding. The importance of allowing time for learning is expanded on in the next section.

6.1.3 Time taken to learn

When interviewed initially, Jenny had a very basic understanding of the FQA and had not implemented it. When the FQA was discussed the repetitive aspect of the review component appealed to her along with the combining of topics and the continual exposure to a concept until the students developed their understanding. Her concern was the amount of lesson time taken up by the FQA reducing the time for ‘normal’ teaching leading to the possibility that the class may fall behind the other
classes. “I am still uncomfortable with getting through less content per lesson” (Jenny). This was not the case as Jenny completed the scope and sequence in the allocated time.

The changes to Jenny’s teaching centred around her organisation and thinking. She intended to focus on students’ understanding of the concept rather than spending the allotted time and moving on to the next topic whether the students understood or not. “…And I am guided by how we’re not all there yet, let’s keep going. Let’s get this, we will get this and then we will move on” (Jenny). By teaching that way she felt that the students successfully completed questions and that they would feel positive about themselves and be more engaged aligning with the research by Rohrer and Taylor (2007) on spacing, the process Jenny is describing. However, as previously noted, while the content may have been covered in the allotted time spent according to the scope and sequence, the students may not have understood the content. “…because I am going over it and not going on to the new concept until they get it. They’re going to get success. My only apprehension I guess is whether I can get through as much content” (Jenny).

Spending time to develop students’ conceptual understanding shows that Jenny is moving towards a mastery learning attitude for class instruction. Students who are in a classroom with a mastery learning approach are more likely to seek conceptual understanding and be engaged in learning (Bonnett et al., 2017).

These Year 9 students, as the Year 8 students had expressed in Chapter 5, felt that under the ‘traditional’ structure there was not enough time to understand each topic. According to Gina “Last year I didn’t understand most of the topics. We have a week or two to finish a topic but really you need more time.” David was in a different class
to Gina and made a similar statement: “Oh I liked my teacher, but I really didn’t understand most of what we did…algebra, percentages, they were just too quick for me”.

The issue of timing raised by Jenny and the Year 9 students is the same as that raised by the Year 8 students in the previous chapter. The next section compares the data collected from the Year 9 class organised into the themes developed in the previous chapter.

6.1.4 Time and understanding.

Some of the same themes from the previous chapter emerged at this initial stage. They were:

- Students who do not understand a topic develop a dislike for that topic.
- There is insufficient time for students to develop understanding or improved self-efficacy.

In contrast to the St Patricia’s Year 8 students’ inherent high level of operative engagement, the St Charles’ Year 9 students had a low level of cognitive, operative, and affective engagement and the classroom observation confirmed that many preferred to chat with their friends rather than complete any mathematics questions. The observation was consistent with the Year 8 teacher’s comment that the more able the student, the more they were engaged. Given that these students were in the fifth out of seven classes it would be reasonable to assume that they had not experienced great success in the developing of mathematical understanding in the mathematics classroom to date.
The classroom observation confirmed the Year 9 students were disengaged with mathematics. During the FQA the students mostly chatted about anything except mathematics until the teacher came to their assistance. Following some explanation, the students generally completed that question but then the same process was repeated. During the observation, it appeared the FQA questions were too difficult for the students to complete independently, as indicated by the apparent lack of operative engagement in the classroom. The teacher had the level of the first four questions above most of the students. According to the suggested structure of the FQA, the only question with real challenge was supposed to be the fifth question.

In the FQA discussion phase the teacher called on several students to answer and was very positive and supportive of their efforts. Unfortunately, there were many comments made by the students during the lesson that indicated that they could not complete the questions. Following further discussion with me, the teacher modified the questions to be more suitable and made the first four questions significantly easier. Over the following weeks she gradually increased the degree of difficulty. The next phase of data collection provided information on these changes.

6.2 Implementation of the FQA

The second teacher interview took place on Tuesday morning six weeks after the initial interview and was followed immediately by the second student focus group discussion. The classroom observation took place the previous Monday in the third lesson of the day but was shortened to thirty minutes due to a school assembly. The questions from that lesson are in Table 6.4. The full detail of the lesson observation is provided in Appendix J.
<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Simon earns $45 for working 5 hours. How much did he earn per hour?</td>
</tr>
<tr>
<td>2</td>
<td>Expand: $3(m+6)$</td>
</tr>
<tr>
<td>3</td>
<td>Jamie earns a salary of $80 000 p.a. and pays $12 000 in tax. What is her net wage?</td>
</tr>
<tr>
<td>4</td>
<td>Simplify $\frac{25xy}{15x}$</td>
</tr>
</tbody>
</table>
| 5      | Carly expanded an expression and got: $10x + 40$ What could the expression have been? There is more than one possibility.  

Hint: $10x + 40 = ___(___ + ___)$ |

### 6.2.1 Success builds confidence

The second classroom observation showed that all students started working on the five questions immediately. The students remained engaged in answering the questions and the classroom was quiet, in contrast to the first classroom observation. The teacher commented that the students all knew to start straight away by writing the first question, then answering and continuing to the next question. While most students had some issues with the questions, they were all attempting them or waiting for assistance from the teacher. There was active discussion of the solutions.
with many students contributing answers and suggestions, showing affective and operative engagement (Attard, 2012).

The students’ attitude towards mathematics had changed positively from the first data collection point. They expressed that they felt better about mathematics because they could understand and had successfully answered questions. Jim summed up the opinions of the group when he said “…last year I just didn’t want to go to math at all, but now this year I understand it a bit more and I kind of like it a bit more now. If I do a question and get it right, I feel good”. Jenny saw that students were achieving success at different points within the lesson and that this caused an increase in their engagement levels. “I think it’s a tricky class … they’re doing pretty well, and by getting those little successes each time they are more engaged, and certainly far more engaged than at the beginning of the year” (Jenny).

The students commented that the class did not move on from a topic until it was understood by most students indicating a change in mindset to mastery learning. They felt it was one of the strengths of the FQA. Jenny said: “…everyone’s involved in making sure that someone understands what we were doing that day. So, they’re engaged in not only their own learning, but in others’ learning, which is good”. The time allocated to topics varied from that allocated in the scope and sequence so that students had sufficient time to understand and this theme is continued in the next section.
6.2.2 Removing unproductive time

Jenny reported that there was less unproductive time at the start of each lesson which provided greater time for learning. The students’ level of retention was still poor with Jenny having to reteach some topics after a break of a few weeks thereby using the spiral curriculum (Bruner, 1966). But she noticed that it took less time for them to understand again and she felt that over time the reteaching would become unnecessary. She felt that the FQA had contributed to an improvement in the students’ memories and recall of procedures. “…it has certainly improved their memory, and they are able to do those procedural questions easier if they are straightforward” (Jenny). She gave the example of the area of plane shapes with the students becoming more “automatic” at completing those questions, and she felt this showed greater understanding, which is a component of the working mathematically NSW K-10 Syllabus outcomes. The students understood the difference between area and perimeter and could answer questions on area given the perimeter of a plane shape and vice versa. This was an example of the progression of understanding from working memory to long-term memory (Baddeley, 2010; Sweller, 1988) and moving from instrumental to relational understanding (Skemp, 1976).

The student focus group agreed that the multiple opportunities and the repetition improved their learning and understanding. They particularly preferred smaller ‘chunks’ as opposed to spending an entire week on the one topic and then moving on to the next topic, never revisiting the previously presented topics. This is the process of interleaving with the content being developed in a spiral manner (Bruner, 1966; Rohrer et al., 2015; Taylor & Rohrer, 2010). When thinking back to last year Allie said, “we do algebra at the start for a week and wouldn’t understand it and then we’d have a test at the end of the term, and we didn’t know it because we only did it at the
The other students in the focus group agreed and felt that the repetition and continual exposure to the questions allowed better understanding and built their confidence.

Jenny applied the ‘yesterday, today and tomorrow’ aspect of the FQA in her class. She used the algebra topic to prepare the class for the future topic on area and volume by providing all the required area and volume formulas as the basis for substitution in algebra. “I’m really focusing on substitution in algebraic formulas; so that we’re doing half of the area and surface area topic already because it is just algebra to them … getting them to make sure that they’re seeing that it is the measurement side of it. So, I think that’s helping as well.” (Jenny).

Jim found that the time spent on questions allowed him to attempt, understand, and answer more questions. “Yeah, last year we just rushed through heaps of stuff and then now this year we actually have time to do the questions and it’s helped me heaps” (Jim). Other students commented that they understood better and that is the content of the next section.

6.2.3 Developing understanding

Jenny had been focusing on teaching algebra since the beginning of the year. She noticed that the students’ procedural skills in the manipulation of algebraic terms had really developed. One aspect she emphasised was the difference between addition and subtraction, and, multiplication and division. She felt that the fifth question had resulted in the students having a more conceptual understanding of the algebraic
concepts. “… they have a better, deeper understanding of algebra for example, the difference that 2x is as opposed to x squared” (Jenny).

The classroom observation of the fifth question confirmed that the students had developed a deeper, more conceptual understanding of algebra. The students were able to delineate between different algebraic representations and explain why some terms could not be combined through arithmetic processes. They recognised algebraic expressions that could not be simplified. Students clearly articulated the difference between 2x and $x^2$ in their discussion of the simplification of algebraic expressions. This then led to the concept of factorising as opposed to simplifying algebraic expressions. Jenny used this particular question 5 to develop the concept of factorising. “So, we did expanding brackets in algebra for a long time, … I just did it as a conceptual question … here’s what they expanded it to, what could possibly have been the question” (Jenny). The student discussion in the classroom observation showed their conceptual understanding of the factorising process by explaining their multiple solutions to question 5. The discussion involved the students clearly explaining why 10x and 40 could not be added together and using factors of 2, 5, and 10 to provide solutions to the question. This according to Cobb (2002) displays the students varied ways of contributing to the overall understanding of a concept or the solution to a problem.

The students felt that the FQA gave them the time to understand the topics and concepts and the development of the questions, slowly increasing in difficulty, assisted their understanding. David felt he was better at mathematics while Gina felt she wasn’t any better, but they both felt they understood more than they did before. Jim said, “I think I’ve got a bit better because I know more stuff.”
The student focus group and teacher Jenny commented that they enjoyed the discussions around the solutions of the five questions. They particularly stated that the discussions of the solutions provided by other students enabled them to understand the concepts being discussed rather than just a solution path. The classroom observation confirmed this, as many students contributed to the solutions, asked questions and made comments. However, students did not present their solutions at the whiteboard, as the St Patricia’s Year 8 class did, they simply gave their method and answer and the teacher wrote and discussed. Unfortunately, that limited the opportunities for the students to satisfy the working mathematically outcomes of reasoning and communication, and to learn from each other.

This concludes the second phase of data collection and the links and themes are discussed in the next section.

6.2.4 Second data collection links and themes.

The focus group students and Jenny commented that the students were attempting and answering more questions because of the success they had with answering questions using the FQA. The students and Jenny confirmed that the extra time spent on topics through the repetition of the FQA had allowed better understanding that resulted in an increased level of engagement, a link to pedagogical repertoires. The classroom observation showed students attempting the five questions, being actively involved in the solution phase, presenting solutions, and asking questions. There was a far greater quantity of quality mathematical discussion taking place.

From the responses by Jenny and the focus group students it appears that the fixed mindset displayed earlier in the year had changed to more of a growth mindset and they believed that through effort they were able to improve (Murphy & Dweck,
The students felt they were learning more, Jenny expressed that the students’ skills were improving, and the level of cognitive, operative, and affective engagement was increasing. All the themes from Chapter 5 listed in Table 5.4 were raised by the Year 9 students except that of examination preparation and stress. The class had not had their half yearly examination and so there was no mention of the examination or any stress leading up to it. That aspect will be discussed in the third phase of data collection which took place after the half yearly examination.

### 6.3 Post Half Yearly Examination Data Collection

The third phase of data collection took place on a Monday in Term 2, week 9, ten school weeks after the previous collection. The teacher interview took place in lesson 1 and was immediately followed by the classroom observation in lesson 2 with the student focus group discussion in lesson 3. The class had been using the FQA for eighteen school weeks. The questions from the classroom observation are in Table 6.5. The full detail of the lesson observation is provided in Appendix K.

<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>In a bag there are 4 blue, 2 yellow and 6 red marbles. What are the chances of picking: a) yellow b) A blue or red</td>
</tr>
<tr>
<td>2</td>
<td>Write $2 \times 3 \times 4 \times 5 \times 6$ with a power.</td>
</tr>
</tbody>
</table>
3. You achieved the following results on maths tests. What is your average mark?

80% 82% 90% 88%

4. Simplify: $y^2 \times y^4$

Hint: Write both in expanded form first.

5. There are 20 marbles in a bag, some red, some blue and some yellow. The chances of choosing a red one is 1 in 4.

What marbles could possibly be in the bag?

Jenny commented that the level of engagement had fluctuated during Term 2 and this is discussed in the next section.

### 6.3.1 Fluctuation in engagement and the half yearly examination

Members of the student focus group expressed that the FQA had resulted in greater engagement and had improved their attitude to mathematics. Shane said, “I never used to listen …now that we have the five questions I probably adapted a bit more … it has changed my understanding…I am not dreading going to maths anymore”.

David and Jim both said that last year they would never have put their hands up to answer questions or contribute, and now they do. Allie reported being more confident to ask for help. The students displayed the growth mindset indicated in the previous data collection with respect to their mathematical learning and ability.

The level of engagement of the Year 9 students had increased due to the success they achieved through the FQA. According to Winheller et al. (2013), students who
achieve success in a topic area will tend to like that topic area. The student focus group agreed with this aspect of success and Allie summarised for the group by saying “It (FQA) was a big influence…getting to learn more about maths and if you’re not good at maths, you don’t like learning it, you don’t engage in class… if you know what you’re doing, you’ll engage”.

Jenny indicated that while the general level of engagement had increased significantly, the engagement of some of the students had fluctuated during Term 2 with decreased engagement of some students due to a lack of success. She said, “towards the middle of the term they started to switch off…they weren’t getting the success, and this was affecting their engagement”. Jim agreed when he said, “I like and don’t like the five questions…maybe something different to the FQA every lesson”. However, most of the focus group said they learnt from the FQA but again felt that it took too much of the lesson time. The classroom observation confirmed that the FQA could continue for too long and that Jenny should have begun the discussion phase earlier.

Following the half yearly examination, the positive performance of many students, and a modification to the degree of difficulty of the first four questions, Jenny noticed that the level of engagement with the FQA increased again. Jenny felt that the students could see the benefits of the FQA after they had completed the half yearly examination. “… after this exam and after that experience I think they got a bigger understanding of why we use the FQA and appreciate that a bit more” (Jenny). One student, not from the focus group, told Jenny that the variety of the FQA had made them much more confident in the examination as they had practice completing questions on multiple topics. Allie said “…when you have an exam it’s like, oh I know how to do this because I do it in five questions, but you don’t
actually realise that in class”. Gina went further when she said “… everyone in the year should (do the FQA), because it really does help when you get to exams, you actually have a better understanding of what you have to do and how to do it”.

Jenny thought back to the first interview where she described the class as more of a 5.1 class than a 5.2 class and considered their development of knowledge using their performance in the half yearly examination as the basis. The structure of the examination is in three sections, each focused on a level. Part A for 5.1, part B for 5.2 and part C for 5.3. All students sat the same examination and due to the structure, the level of a student’s class places a ceiling on the questions that can be attempted. While students are permitted to attempt all questions on the paper there would be questions on mathematical theory that the students would not have encountered in class. In particular Jenny’s class did not complete any of the theory tested in section C of the examination. According to Jenny the class had improved dramatically on the basics and if their part A was compared to the part A of the two 5.1 classes the students would have performed extremely well. Jenny said “… they’ve improved dramatically on that basic understanding and what they’ve achieved in part A I don’t feel is by chance”. Figure 6.1 shows a scatter plot of the rankings of the end of the Year 8 final examination before the FQA, with the half yearly examination rankings for Year 9 after two terms of the FQA. The straight line indicates equal rankings so points above the line show an improvement in ranking. Students that did not have a Year 8 ranking, who were not at the school in Year 8 or did not sit the Year 8 examination, were omitted.
According to Jenny, the improvement in rank compared with the other two 5.2 classes that were graded above them was an even better indication. “I think it was more than half the class have actually improved on their rank over the year, over the cohort” (Jenny). According to Jenny it was the non-linear approach with a variable amount of time on each topic that provided the increased confidence that contributed to the improved examination performance and further detail follows.
6.3.2 Variation and the removal of time restrictions

Jenny found the FQA gave her much greater freedom in the classroom as the time restrictions were removed. “…if it goes somewhere you just let it because that’s okay …you just pick up in the next lesson… I’ve really enjoyed running with whatever is happening and get that understanding” (Jenny). Jim agreed when he said “…and then if we don’t understand it we’ll do it the next day”, demonstrating a growth mindset and mastery learning attitude. The focus group students enjoyed the flexibility and the variety of the FQA. The constant revision and the variety of topics enabled them to develop their understanding. Gina summed up the feeling of the group when she said, “I think I get a better understanding now we seem to do it every day I think I would be able to answer more questions”.

Jenny reflected on the fifth question and believed that the students had developed a deeper understanding of some concepts. The links that were made between topic areas was a benefit of the FQA as these links would not have been made with a traditional teaching approach. “…I think that a huge benefit of the FQA is that it can all just be melded in together at any stage and I think it takes away from the rigidity of one topic at a time” (Jenny). David represented the student focus group when he said, “I think it sort of changed how I learnt, because you’re sort of revising on a whole range of different skills in maths”. This aligned with the view of St Patricia’s Year 8 class and their teachers as described in the previous chapter. The final data collection phase is outlined in the next section.
6.4 Final Data Collection

The final teacher interview took place on a Wednesday in Term 4, week 7, eight weeks after the third data collection. The teacher interview took place in lesson 1 and the student focus group discussion in lesson 2 and the students had been using the FQA for thirty-three weeks. Due to the examinations and post examination school activities a fourth classroom observation was not possible. The next section outlines the results from the yearly examination.

6.4.1 Final examination

The examination results were, in Jenny’s opinion, very good for that class because of their increased cognitive, operative, and affective engagement, a result of the FQA: “…my results from the 5th class a lot of those students are performing up in the 3rd class level. Some performing down but there is certainly quite a chunk that have out-performed their class consistently throughout the year” (Jenny).

Figure 6.2 shows a scatter plot of the rankings of the end of Year 8 final examination, before the FQA with the final examination rankings at the end of Year 9 after a year of the FQA.
Figure 6.2 Year 8 vs Year 9 End of Year Examination Rankings

Clearly there was academic improvement with thirteen students showing significant improvement, three of whom were ranked three classes higher. Three students improved slightly with two staying the same and three performing below expected. It is important to note that as a student moves up then a student performing the same but previously ranked higher will move down one place. With that consideration it is possible that all students except one showed improvement.

The focus group students and the teacher, Jenny, all felt the FQA had helped them to learn and understand. The FQA allowed sufficient time and repetition to understand. The students’ cognitive, operative, and affective engagement increased because of
their success in answering the questions and it prepared them for the examination by building confidence. When asked if they had the choice of a class using the FQA and one that did not, all the focus group students would choose the FQA.

Yeah, I agree with Jim, Dan and Allie because last year when we would learn a topic for however long we learnt it then we’d just move on to another topic and we wouldn’t go back and revise... but with the five questions it’s every day, until we understand it which is good, so, yeah, I’d probably choose the five questions (Gina).

Despite Jenny’s concerns, all the required content for the final examination was completed within the allocated time as set out in the scope and sequence.

6.4.2 Completion of the scope and sequence

The FQA takes time away from what was considered ‘normal’ teaching, and this was the major issue for Jenny at the beginning of the year. However, by the end of Term 2 the class was at the same point as the other classes following the same scope and sequence and this continued through the rest of the year. “At the end of the year I had completed the scope and sequence I had more than caught up…and I was attacking more concepts in the lesson and doing it more often, so they got it” (Jenny).

Addressing more concepts in class and spending more time until the students developed their understanding is an important aspect of the FQA. The students appreciated the extra time available and the repetition. This aspect was a significant influence on the improved engagement of the students. Jim and David said, “I reckon
the FQA helped me because if you don’t really get it, the next day it will be in the five questions again and that’s helped me”. Allie likened the FQA to constant revision and said “…it’s sort of like studying and revising all the time and you don’t even realise you’re doing it because you do it every single day”.

6.5 Conclusions

Jenny commented that there was strong initial resistance to the FQA from the students and she felt that the resistance was due to the students’ previous ‘traditional’ learning experiences. When Jenny pitched the FQA at the correct level there was a high level of cognitive, operative, and affective engagement, but when she made the questions too long or too hard the level of engagement decreased rapidly. This aligns with Vygotsky’s work on the Zone of Proximal Development (ZPD) (Vygotsky, 1978) and the pedagogical relationships where the students’ backgrounds and pre-existing knowledge are not considered. Here the teacher was taking the students too rapidly from their zone of proximal development out of their sphere of comfort and as a result the students became disengaged. The level of operative engagement increased following the half yearly examinations as the students realised how much the FQA had assisted them in their examination preparation.

“A lot of resistance at the beginning of the year…. after exams, it became less resistant because they understood that they were at the same stage as everyone else and that they had done well in their exam because they were constantly revising what was in it. … the results speak for themselves but …they’re just so ingrained with the traditional lesson that
Jenny compared the level of engagement of many of her current students from the previous year as she had taught most them in Year 8. She saw a great difference. “I had most of those students last year - and they were very disengaged, so I was very excited with the FQA. I think it’s had a lot of success for many students.”

The time taken up by the FQA was still seen as an issue even though the scope and sequence was completed and the students learnt and understood more through the questions and repetition. This is related to the fact that the teachers and students have a stereotypical view, rightly or wrongly, of how a mathematics lesson should occur. It is not just the teachers who are ingrained with the traditional teaching approach, but the students as well. As Jenny had started to use the FQA with her other classes in Term 4 she was excited about using the approach with all her classes next year.

“I’m very happy and very excited for the next year now I’ve had this learning year. I feel like next year I’ll be much better placed to implement it to its fullest potential…So excited” (Jenny).

At St Charles, there are two teachers in the mathematics faculty who use what they call ‘five quick questions’ at the start of their lessons. However, there was no structure to the questions as they were simply randomly selected. Whereas the FQA uses the yesterday, today and tomorrow principle for the first four questions as a structure. The major difference is the conceptually challenging fifth question. The FQA uses this to explore and consolidate conceptual understanding and this is absent in the quick question method. “I think it’s that fifth question that makes a big
difference on a traditional quick five at the beginning of the lesson … that fifth question actually has that deeper understanding” (Jenny).

The level of cognitive, operative, and affective engagement of the students increased and in the opinion of the class teacher the results of many of the students improved significantly. The class was the fifth ranked class in Year 9 and many students performed above their expected level. There were 4 students ranked in the second class, including one who ranked 33rd in the year, nine in the third class, and 4 in the fourth class. In total 46% of the class were ranked two or more classes above their current class and 61% of the class were ranked above their current class. Jenny felt this was an outstanding achievement and credited the FQA with the change in ability, attitude and engagement of the students. “I had a talk with the whole class about the FQA when I gave back the results and the majority expressed concern that they wouldn’t be following that approach next year” (Jenny).

The senior management team of St Charles’ College decided to implement the FQA with the entire mathematics faculty and this commenced in the following year. On this adoption of the FQA Jenny said, “I’m really excited that the whole school is going to take on the approach because I feel that it really will increase our own students’ level of understanding”.

The Year 9 St Charles data analysis is complete, and the next chapter will examine the Year 10 St Charles’ data.
7  Analysis of Data St Charles’ Year 10

This chapter focuses on the data collection from St Charles’ Year 10 class taught by Harriette. It follows the collection of the data through the four phases. Each of the three data collection aspects: teacher interviews, student focus group discussions, and classroom observations are discussed. The timing of the data collection is displayed in Table 7.1.

Table 7.1 Timing of each data Collection Point for St Charles’ Year 10

<table>
<thead>
<tr>
<th>Phase</th>
<th>Term</th>
<th>Week</th>
<th>Date</th>
<th>Data Collection Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4</td>
<td>16/2</td>
<td>Harriette teacher interview 1</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>16/2</td>
<td>Year 10 classroom observation 1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>25/2</td>
<td>Year 10 focus group 1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>9</td>
<td>22/3</td>
<td>Harriette teacher interview 2</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>22/3</td>
<td>Year 10 focus group 2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>29/3</td>
<td>Year 10 classroom observation 2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>9</td>
<td>20/6</td>
<td>Harriette teacher interview 3</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>20/6</td>
<td>Year 10 focus group 3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>20/6</td>
<td>Year 10 classroom observation 3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>7</td>
<td>23/11</td>
<td>Harriette teacher interview 4</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td></td>
<td></td>
<td>No 4th observation or focus group due to examinations and end of year activities</td>
</tr>
</tbody>
</table>

7.1 St Charles’ Year 10 class

The Year 10 class was taught by Harriette the head of the mathematics department at St Charles. The class was graded as the fifth of seven classes in Year 10. The class was one of three following the 5.2 pathway and was graded as the third of the three as was the St Charles’ Year 9 class from Chapter 6. Since St Charles’ was a Year 7 to Year 10 school, these students were in their final year at this school. The class comprised twenty-seven students, fifteen girls and twelve boys and there was no official seating plan. The class followed the same scope and sequence as the two
classes ranked above and a copy of the scope and sequence is provided in Appendix L.

The first teacher interview took place in Term 1, week 4 on a Tuesday with the classroom observation in lesson 5 on the same day. The class had not started the FQA and the full details of the classroom observation are provided in Appendix M.

The focus group was chosen by the teacher in the same manner as described in Chapter 6 and comprised equal numbers of girls and boys. Table 7.2 displays the ranking and engagement level of each student. The rankings were taken from the students marks in the final examination from the previous year. The teachers were given a definition of engagement and asked to rate the level of engagement of each student on a scale of Low (L), Medium (M) or High (H).

Table 7.2 Ranking and Engagement Level of St Charles’ Year 10 Focus Group

<table>
<thead>
<tr>
<th>Students</th>
<th>Year 10</th>
<th>Year 9</th>
<th>Engagement</th>
<th>Year 10 half</th>
<th>Year 10 final</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joe</td>
<td>92</td>
<td>L</td>
<td>78</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td>Sara</td>
<td>106</td>
<td>M</td>
<td>93</td>
<td>101</td>
<td></td>
</tr>
<tr>
<td>Allen</td>
<td>107</td>
<td>M</td>
<td>79</td>
<td>72</td>
<td></td>
</tr>
<tr>
<td>Alison</td>
<td>115</td>
<td>L</td>
<td>84</td>
<td>129</td>
<td></td>
</tr>
<tr>
<td>Sonia</td>
<td>136</td>
<td>H</td>
<td>130</td>
<td>77</td>
<td></td>
</tr>
<tr>
<td>Evan</td>
<td>139</td>
<td>L</td>
<td>140</td>
<td>147</td>
<td></td>
</tr>
</tbody>
</table>

The details of the Year 10 class have been given and the discussion of the first phase of data collection begins with teaching and the typical lesson.
7.1.1 Teaching and learning

The teacher, Harriette, described her teaching in the same manner as the teachers in the previous two chapters. Her teaching was mostly traditional, although Harriette stated there were concrete activities completed at times. The typical lesson began with marking and checking homework followed by examples and discussion of the current topic. Students then completed some questions from a worksheet or textbook and were assigned homework. Later in the interview, when summarising her teaching Harriette says, “It is traditional, but I’ve found that, for this school, that works”, the same comment was made by Jenny in Chapter 6. Most of the students in this class were taught by Harriette in the previous year and she made the same comment as Jenny in Chapter 6, that there needed to be a change. Harriette felt there needed to be a change because the students were not engaged in the learning and their examination performance at the end of last year was disappointing. While both Jenny and Harriette commented in their initial interviews that they though the traditional approach was the most appropriate for their school they both indicated that the traditional approach was not working.

In the classroom observation, Harriette was very enthusiastic and talkative with genuine care for her students. The student focus group discussions reinforced this view, and the consensus was that they all liked Harriette’s teaching style. Perhaps because of her natural talkative personality, the lesson observed was entirely teacher centred. During the examples and learning discussions Harriette called on individual students by name to respond to her questions. No voluntary answers were taken. Following the instruction phase a worksheet based on the examples was given for completion. Unfortunately, many students were unable to answer the questions which indicted a low level of understanding. During the instruction phase the
students were compliant but there was no evidence of engagement, only forced participation through targeting the questions at specific students. The engagement aspect is expanded on in the next section.

7.1.2 Engagement of the Year 10 students

The classroom observation showed a low level of student cognitive, operative, and affective engagement and as Harriette described “I’m doing my introduction, and I can see their eyes glazing over… you’ve done this in the past and many can’t remember…so then I spend time on doing that, which frustrates me”. She described her ideal class as one that is prepared and ready to start straight away, but this was not the class that was observed.

As previously mentioned, the classroom teaching was very teacher focused. Engagement was an issue with the class and Harriette felt that she had to force the students to participate by continually talking to the students and calling on the ones who were not paying attention to participate. The classroom observation confirmed that the class behaved well but were not engaged. When discussing the students, Harriette opined that they enjoyed mathematics when they were successful but that she needed to carefully set the level of the questions because if they were too hard then the students gave up immediately. Harriette says, “So they like success and they also know they’re not the best at maths”. Here Harriette displayed the same fixed mindset that Jenny initially expressed in the previous chapter.

One of the aspects that Harriette felt contributed to the students’ poor performance in mathematics was that they did not remember what they had done in the past and did
not have the capacity or desire to go back and find out. Harriette’s summation of the class concluded with this statement: “They like to have success. … if they are getting it and the questions get harder and they’re still having success, they know they’re getting better and that’s what they love. Intrinsically they want to get better”.

Harriette hoped that the FQA may help with success and engagement. The focus group students felt that mathematics was not their favourite subject and while mostly they did not hate it, their attitude was more that they tolerated it. Alison said: “I don’t like maths, but I don’t mind it, it’s not a bad subject for me”. Joe says: “I’m not that confident doing maths, it’s not really my favourite subject”. Harriette wanted improvement in the students’ confidence and learning and felt engagement in the classroom was the key along with sufficient time for learning.

7.1.3 Time for learning

The classroom observation revealed that the students had not learnt the concept that was the focus of the lesson. This was evidenced by the large number of students who were unable to answer the first questions in their homework book, even after they had completed thirty minutes of instruction on that type of question. While the teacher had said that this traditional approach was the most appropriate for these students, the classroom observation did not support this view. The time allocated to the current topic of indices in the scope and sequence was almost exhausted. The students had been exposed to less than half of the allocated content and their understanding of the material that had been presented in class was limited. As a result, based on the current teaching paradigm, the class would be moving on to the next topic prior to completing the current topic. Only during the short revision phase
before the half yearly examination was there opportunity for this topic to be revisited. This aligned with the focus group student concerns that the learning was too rushed to develop understanding and there wasn’t any opportunity to revisit past topics.

The focus group students were concerned that student and teacher absences had impacted on their learning. One student missed school due to illness and another through holidays and both were worried about performing poorly as there was no protocol to deal with the work they missed. Sonia expressed this concern: “…every time you miss a lesson you miss a whole step and then once you get to the next day you’re behind two steps and it’s really hard to get back”. Sara had a similar concern: “I’ve been away for the last week on a holiday and I’m kind of worried that I’m getting behind”.

The focus group students expressed a desire for opportunities to revise past topics to give them time to understand and catch up on work missed through absences. They had been exposed to four lessons involving the FQA and felt that this method may assist with the issues of time, understanding and absences. These aspects are features of the FQA and the next collection of data provided more information.

### 7.2 Implementing the FQA

The second teacher interview took place in Term 2, week 9 on a Tuesday afternoon, six weeks after the initial teacher interview, and was followed immediately by the second student focus group interview. The classroom observation took place the
following Tuesday in the first lesson of the day. The questions from that lesson are
in Table 7.3. The full detail of the lesson observation is provided in Appendix N.

Table 7.3 *Five Questions from the Second Observed 10M5 Lesson*

<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Simplify $2m^3 \times 4m^5$</td>
</tr>
<tr>
<td>2</td>
<td>Solve $4x-5=-13$</td>
</tr>
<tr>
<td>3</td>
<td>A bag contains 6 red and 5 blue disks. Two disks are selected without replacement. What is the probability they are both the same colour?</td>
</tr>
<tr>
<td>4</td>
<td>Find the mean and median of these scores: 10, 12, 12, 13, 15, 33. Which is a better measure of central tendency? Explain.</td>
</tr>
<tr>
<td>5</td>
<td>A square has side length $(x+3)$ cm. Calculate the value of $x$ given that the perimeter is 52cm.</td>
</tr>
</tbody>
</table>

Harriette had just begun to use the FQA and was still developing her ability to write the questions. In particular the question 5 in this example of the five questions is a more complex procedural question rather than a conceptual question. Further advice was given to address this issue and to clearly indicate the type of question required. The classroom observation demonstrated that the students started work on the FQA immediately and this is expanded on in the next section.
7.2.1 Recovering lost time

The teacher, Harriette, described their lessons as being different since introducing the FQA. She described her routine as follows: “I write, they walk in, they sit, they settle, they get their folder out, they write the questions, and off they go” (Harriette). The classroom observation confirmed this as did the student focus group. “Well you go in and get straight to work” (Evan). This aligned with the findings in the previous two chapters.

Unfortunately, there were many lessons missed by Harriette due to other school activities and as head of department and mathematics representative to the Catholic Schools Office (CSO). The replacement teachers were not mathematics trained and ‘traditional’ one topic repetitive worksheets were left for the students. The issue with the repetitive worksheets has been raised in previous chapters.

The yesterday, today and tomorrow aspect of the FQA assisted Harriette in making up for her own and her students’ absences and allowed students to catch up and look ahead. The student focus group commented that the revision of past topics and preparation for future topics through the FQA allowed them to feel more confident in attempting mathematical questions.

7.2.2 Increased confidence and engagement

The focus group students felt they were more confident and able to answer mathematics questions because of their success with the FQA. Sara and Allen agreed with Alison who said, “It’s just a lot easier to understand and I feel more confident answering questions”. The teacher, Harriette, was unable to say if their level of
academic performance had increased but she felt the students were more engaged and confident. She noticed that the students’ level of operative and cognitive engagement with the FQA was much greater than during the traditional part of the lesson. Harriette said “…the students were more engaged doing the FQA than when I was teaching, I had to do a lot of wait for this, you look here, grabbing their attention when I was teaching”. The classroom observation confirmed this aspect as all students were on task during the FQA and discussing the solutions to the questions whereas in the ‘traditional’ teaching phase many students were off task and occupied with non-mathematical conversations. The substantive conversations and variety challenging tasks, elements of pedagogical repertoires evident in the FQA, contributed to increased engagement (Attard, 2012).

Harriette commented that the students were initially opposed to the FQA and questioned the purpose of having questions on past and future topics. However, that negativity waned after about five weeks when the students realised that they better understood the mathematics. When asked how he felt about mathematics after doing the FQA for a while, Allen summed up the opinion of the group when he said: “… I feel more confident going into a lesson knowing what I’m doing…It made me not have to groan or sigh whenever I have maths…I’m a bit more confident doing that work”.

The themes established in Chapter 5 Table 5.4, confirmed for Year 9 in Chapter 6 were all relevant to St Charles’ Year 10. The focus group discussion students reinforced the settling and revision aspects of the FQA, and some commented on the amount of time taken in class away from the ‘normal’ teaching. All the focus group discussion students expressed some anxiety and stress about the half yearly examination especially as their performance in the examination directly affected the
level of mathematics they did and their class allocation. The students all liked the revision aspect of the FQA and felt the revision aspect may help when it came to examinations. The third collection of data that took place after the half yearly examination examined this aspect.

7.3 The Half Yearly Examination

The third teacher interview took place on a Monday morning during lesson 1, eleven school weeks after the second teacher interview. The third classroom observation followed in lesson 2 with the student focus group interview held during lesson 3. The questions from that lesson are provided in Table 7.4. The full detail of the lesson observation is provided in Appendix O.
<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Solve $3x - 11 = 1$</td>
</tr>
<tr>
<td>2</td>
<td>Solve $4 - 5x = 7x - 19$</td>
</tr>
<tr>
<td>3</td>
<td>Calculate the value of $x$</td>
</tr>
</tbody>
</table>

![Diagram](image)

| 4      | Calculate the value of $\theta$ |

![Diagram](image)

| 5      | A boat is at sea and the angle of elevation of the top of a cliff is $65^\circ$. If the height of the cliff is 120m calculate the distance of the boat from the base of the cliff. |

The teacher varied the way in which the FQA was done by only posing questions from the two topics for the half yearly examination and this change is expanded on in the next section.
7.3.1 A change to the FQA

During Term 1, Harriette applied the FQA as directed including the yesterday, today and tomorrow aspects and provision of a variety of questions. However, starting Term 2, before the half yearly examination, Harriette only gave questions on the two topics for the half yearly examination with an emphasis on the procedural questions making the FQA simply a series of graded revision questions deviating significantly from the correct implementation of the FQA. The focus on the two topics, in Harriette’s viewpoint, provided the students with the optimum preparation for the examination. Harriette said “… our two topics for this term were equations and trig and I didn’t really want to confuse them too much by going into next term just yet because we had exams coming up”.

The focus group discussion students all stated that the FQA had really helped with their confidence in answering questions and in their examination preparation. They felt that working on the examination topics and continuous review of the topics through the term assisted in their examination performance. Sara spoke for the group when she said, “I got almost everything right with that trig because we kept focusing on it using the five questions”. The examination performance is expanded on in the next section.

7.3.2 Results in the half yearly examination

The students sat the half yearly examination in week eight of Term 2. Harriette felt that they would perform well and said, “I was really excited to see their results as they were engaged and knew what they were talking about, but bearings was not going to be great because I hadn’t given any questions in the five”. Harriette had
taught most of the students last year when they were in Year 9. Figure 7.1 compared the rankings of the students in the previous yearly examination compared with the half yearly examination.

Figure 7.1 Year 9 Final Examination Rankings vs Year 10 Half Yearly Examination Rankings

The scatter plot indicates that most of the students had improved their rankings from Year 9. The focus group students expressed that the FQA had assisted them in their examination preparation and performance. Joe represented the views of the focus group when he said “… it drilled you with trigonometry I think all the class went very well because we were just doing it every day it was how we got good marks”. 

232
7.3.4 After the half yearly examination

Harriette was very pleased with the performance of her class in the half yearly examination. However, she still had a fixed mindset with respect to the ability of her students and that attitude remained and influenced the way in which she taught the class. She focused on the easier questions and did not try to extend the students. She said “I was concerned, and I just kept going over those basics and making sure they were solid, and I should have spent more time on bearings, but I didn’t. So, it really showed”. The students performed very well in the basic trigonometry questions but poorly in the bearing and more difficult trigonometry section thereby limiting the marks they may have achieved.

However, in Harriette’s defence her class had grown from 27 to 32, as a few students were moved up from the 5.1 class due to behavioural issues and down from the other 5.2 class due to poor examination performance. There was a high level of absenteeism in the class and with Year 10 overall. Harriette said “…it was ridiculous … I’ve got 32 and yet I would say I would maybe have 22 every day because there are so many kids that are out at other things and it was very frustrating”. Harriette followed a similar process, of focusing only on the examination topics with the FQA in preparing for the final examination as she did for the half yearly examination. The yearly examination is the focus of the final phase of data collection.
7.4 Harriette Final Interview

As St Charles is a Year 7 to Year 10 school, the Year 10 students are in their final year at the school. Following the end of year examination, the students are involved in many out of class and out of school activities. As a result, there was no fourth focus student discussion group or classroom observation. Harriette was interviewed in week 7 of Term 4 following the end of year examination.

In the final interview Harriette confirmed the themes previously mentioned and spoke about the FQA process from the beginning to the final stage. Harriette had several students added to her class following the half yearly examination and their attitude when joining the class was interesting. The new students noticed that the current students began work immediately on the FQA and had a far greater recall of topics. “I had some kids come up and for the first two weeks they were floored that my kids were just getting in and doing it and they had no idea as they could not remember” (Harriette).

The completion of the scope and sequence was no issue for Harriette and her class as it was completed in time. The students’ initial resistance to the FQA disappeared with the increased confidence in answering questions and their half yearly results. The yesterday, today and tomorrow aspect of the FQA really appealed to Harriette and her students who appreciated the revision but enjoyed being prepared for upcoming topics. “… they were quite concerned at the very beginning and it seemed to be taking time out of the lesson … but in the end, they loved it especially when they saw that they were actually ahead” (Harriette). This comment is in contrast with school Terms 2 and 4 when the Year 10 FQA focus was on the examination topics only. As the school finishes at Year 10 the is no focus towards Years 11 and 12. As a
result, the tomorrow aspect may not be considered as important as good performance in the yearly examination.

Interestingly, it was the continuation of the fixed mindset by Harriette with respect to her students’ mathematical ability that influenced the way in which the FQA was implemented for the yearly examination, as it did for the half yearly examination. Harriette said, “Leading up to exams I backed off on that fifth question … because I just wanted to hammer into them the basics”. The fixed mindset attitude resulted in a restriction on these students in the examination as they were not prepared for the more difficult questions. The focus on the basics, as Harriette states would have an influence on the students’ ability to problem solve and apply mathematics.

When asked about the yearly examination results Harriette said, “As a whole, phenomenal, I think I even got two Bs but a huge range due to some school avoiders that did not try”. Figure 7.2 compares the rankings of the students in the Year 9 yearly examination from the previous year compared with the Year 10 yearly examination.
The diagram indicates that most of the students performed above their Year 9 ranking. As Harriette mentioned there were two students that did not attend school
for most of Term 4 and performed poorly in the examination as a result. The conclusions from the Year 10 class research are discussed in the next section.

7.5 Conclusions

Similar to the conclusion from Chapter 6, the students were initially resistant to the FQA, but their attitudes changed with the successful answering of questions and development of procedural fluency that they enjoyed both in class and in examinations. The level of cognitive, operative, and affective engagement and confidence in completing mathematical questions increased due to the FQA. The issue of the FQA taking too much time from the ‘normal’ lesson was resolved with the class completing all the scope and sequence with time to revise. Harriette said, "Despite the FQA taking time from the ‘normal’ lesson we completed the entire scope and sequence, and this is the first year we’ve ever actually been on time. Every other year we have struggled to get through it”.

Unfortunately, Harriette did not implement the FQA as intended. In terms 3 and 4 the FQA became a selection of graded revision questions based on the topics for the half yearly examination. While the first 4 questions of the FQA are designed to revise topics, procedures and develop fluency Harriette took this to an extreme by focussing on just two topics. Had the FQA been implemented as designed the questions would have been on more topics, included conceptual investigative questions and focusses on yesterday, today and tomorrow rather than just today. There would have been some preparation for year 11 as year 10 drew to a close. The students performed well in the examination as a result of the repeated opportunity to practice procedures and question answering in a revision style environment. Again,
this is an aspect of the FQA but has taken to an extreme to satisfy a particular teacher’s agenda.

Harriette, as head of the mathematics department, was very excited that the FQA was being adopted by the entire mathematics department. She said, “I’m really excited about using the FQA next year and I think this school needed it so I’m very happy”. Harriette acknowledged that she did not implement the FQA as intended but she did indicate that the FQA would be implemented it as designed in the following year.

This concludes the discussion of the three classes and the next chapter makes further comparisons with the three classes and investigates the case of the students from the Year 8 who were promoted following their strong performance in the half yearly examination.
8 FQA Cross Case Analysis

This study has presented an evaluation of an innovative method for teaching mathematics (the FQA) designed to improve students’ engagement and achievement. The FQA was implemented in slightly different ways in each of the three classes. The similarities and differences of the teaching approaches, the implementation of the FQA, and student academic performance and engagement will be discussed. The unique case of the students from Year 8 who changed classes after the half yearly examination will be discussed in section 8.6.

8.1 Teaching Styles

This study involved four teachers in three classes who implemented the FQA over the course of one year. Prior to using the FQA each of the four teachers taught in a similar manner and all described themselves as ‘traditional’ teachers. The first classroom observations clearly indicated that all the teachers followed the same structure and pathway. The lessons were teacher centred with a focus on procedures only and there was very little discussion in the classrooms other than explanation of the algorithm or process. Examples were presented, the students copied them and then completed similar questions, using the process presented in the examples with unfinished questions set for homework. Any form of discussion was teacher led and there were no conversations between teacher and students or between students, except for closed questions posed by the teacher. According to the Framework for Engagement in Mathematics (FEM) an engaging classroom has continuous interactions and substantive conversations which were lacking in that classroom observation (Attard 2012). The discussion was always in the one direction emanating from the teacher.
The final classroom observations indicated that the teachers had moved, in a significant way, away from the ‘traditional’ teaching approach. There was more discussion about how questions were solved rather than describing a single algorithmic process. There was an emphasis on why a solution path was used rather than simply following a set algorithm to arrive at an answer. The discussion involved the students to a greater extent and a larger number of students in each class were contributing to the discussion. Indicators listed in the FEM show that a combination of continuous interaction and substantive conversations increase student engagement. There was a greater focus on relational understanding and conceptual knowledge through the discussion of different solution pathways.

The changes in teaching from the traditional teaching approach resulted in practices that increased student cognitive, operative, and affective engagement. The FQA had influenced the teaching and learning in all three classrooms but there were differences in the way in which it was implemented. The Year 10 class used the FQA in a different manner to the other classes and the implementation of the FQA is the subject of the next section.

8.2 Implementation of the FQA

All three teachers began implementing the FQA in the same way as intended but all of them had difficulties with the writing and understanding of the conceptual question five. However, while the teachers of Year 8 and 9 continued with the approach as outlined after the half yearly examination, the Year 10 teacher did not. Instead she chose to focus the FQA entirely on the two topics that would constitute the yearly examination. Her reasoning was that as this was the students’ final year at
the school she wanted them to have a good mathematics result on their report. There was no consideration of the preparation of the students for Year 11 mathematics content at their next school. The approach was successful as the Year 10 class had very good results in their final report that was based on the two focus topics. These results were achieved through repeated practice and a focus on procedural fluency which are aspects of the FQA but did not include a variety of topic areas and the aspects of conceptual understanding through multi solution path problem solving and discussion afforded by the correctly written and implemented fifth question.

The FQA is structured so that the first four questions are designed to promote procedural fluency and build student confidence through correctly answering questions. They are meant to be straight forward so that most students complete them successfully and quickly allowing more time on the conceptual fifth question. Students having difficulty with some questions or who were absent from class were targeted by the teacher during this phase. However, the teacher cannot target the required students if too many of the students are having difficulty with the first four questions. The issue of making the first four questions too hard was encountered by all the teachers and this remained a struggle right through the year. The teachers needed to scaffold the questions in terms of the degree of difficulty thus moving students from their ZPD (Vygotsky, 1978) to promote learning. Teachers having to answer multiple questions from students resulted in the FQA taking longer than expected in the classes. The level of difficulty of questions 1 to 4 needs to be appropriate so that the yesterday, today and tomorrow aspect of questions 1 to 4 can be used to revise, consolidate and prepare the students for future topics. The Year 8 and 9 teachers used this principle fully, but the Year 10 teacher focused solely on the two topics for the yearly examination, thus only addressing the today aspect.
While the time taken from the ‘normal’ lesson by the FQA was an issue for all teachers and students, the scope and sequence was still completed by all classes within the required time. At first, all classes were falling behind in the scope and sequence, but by the half yearly examination they were either equal to or ahead of the required time frame. The time taken away by the FQA from the normal lesson was valuable and the review or yesterday aspect gave students more confidence in completing questions during class and in examinations. The teachers said that the student enjoyment of mathematics class, and as a result their affective engagement, was enhanced by the FQA. The positive attitude through success in the FQA increased students’ perceptions that their academic performance was improving and as the results showed, academic performance did improve for many of the students.

The fifth question was designed to be an open-ended task with multiple solution pathways to promote relational understanding and develop conceptual knowledge. At times the fifth question was closer to a multistep procedural question than a multi solution investigation, but the majority of the times in the year 8 and 9 classes it was a conceptual question. As previously discussed the year 10 class did not have a conceptual fifth question throughout terms three and four. The examples in the appendices and through this thesis are a small snapshot of the questions given throughout the year and there were many instances of conceptual question 5 examples in the total of all five questions given.

Class discussion was an integral aspect of the fifth question. The way in which the fifth question was discussed varied between the classes. The Year 8 class had the students talk through their solution from their seat and the teacher wrote on the board and added to the discussion. The students did not write on the board nor did they lead the discussion. The Year 9 and 10 teachers allowed the students to write on the
board and lead the discussion of their solutions. Despite the differences in discussion methods, the teachers all felt the discussion of solutions and student demonstration of their solution methods was valuable in the learning and development of understanding. The accessing of higher order questions through the fifth question and subsequent discussion of multiple solution methods allowed the development of a conceptual rather than procedural understanding of mathematical concepts (Hiebert & Grouws, 2007). The linking between concepts through the FQA increased student understanding. The use of FQA on different topics enhanced student learning by linking topics together and the students said they preferred that to a worksheet with multiple repetitive questions focusing on a single type of question. The students’ performance in the examinations indicated that their academic performance had improved.

Jenny summed up the feeling of the four teachers as she found the FQA gave her much greater freedom in the classroom as the time restrictions were removed. “…if it goes somewhere else you just let it because that’s okay …you just pick up in the next lesson… I’ve really enjoyed the freedom of it and just to run with whatever is happening and get that understanding”. (Jenny Year 9 teacher). Jim agreed when he said “…and then if we don’t understand it we’ll do it the next day” (Year 9 student). For teachers to have this flexibility they need to have strong MKT, MCK, and PCK, creating positive pedagogical relationships (FEM) and syllabus knowledge to be confident in being flexible. All three teachers in this study satisfied these requirements and had the capacity to develop the FQA well, albeit the year 10 teacher choosing not to in terms three and four. A teacher lacking in these skills may have difficulty using the FQA and may be more likely to take the ‘safe’ method of ‘traditional’ teaching.
While there were variations to the implementation of the FQA, all classes achieved some level of increased academic performance using the process. There was improvement academically and an increase in student cognitive, operative, and affective engagement which is expanded on in the next section.

8.3 Engagement

As previously discussed student engagement in the middle years of schooling is a significant issue. There was a difference in the initial engagement level of the three classes. The Year 8 class, 8M2, was described by their teachers as an engaged class and the teachers linked the students’ level of engagement to the fact that they were a high-level class according to the way in which the classes had been graded. This indicated a high level of operative engagement, and while the first classroom observation confirmed the high level of operative engagement it indicated that the level of cognitive engagement was low as the students had no challenge in the work they were completing. Both the Year 9 and Year 10 classes were described as having a low level of engagement and this was attributed by the teacher to the level at which they were graded. In the first classroom observations there was little evidence of cognitive or affective engagement and the operative engagement seemed to be forced on the students. Obviously, the students in higher-level classes have experienced more success in mathematics compared to the students in the lower level classes. The question of whether low engagement preceded poor academic performance or vice versa is of interest but beyond the scope of this study.

Success in the answering of questions and understanding of concepts was clearly important to students and teachers. All teachers in their interviews commented that
the students performed better and were more engaged when they were achieving success. Challenging tasks that are positive and provide opportunities for all students to achieve a level of success result in greater cognitive, operative and affective engagement (FEM). The students made the same comment in their focus group interviews. While teachers and students felt success was important, the initial classroom observations indicated there were very few opportunities for the students to achieve success. In fact, there were many opportunities for the opposite result. The FQA was designed to provide multiple opportunities for success and conversations around mathematical concepts and this was borne out through the study as the level of engagement increased.

Although the students all stated that the FQA took time from the textbook theory work Brenda made the comment:

*I think the 5 questions is better than theory because we’re attempting questions, and you remember sitting there and working out things, and attempting them. And I think that the best lessons are if you remember working it out in your head, and if you get it wrong, sort of talking about it and discussing it is really good (Year 8 student).*

In this student’s words there is the confirmation that the combination of cognitive, operative and affective engagement results in an increase in overall engagement.

The level of student operative engagement remained similar in the Year 8 class but there was an increase in cognitive and affective engagement as evidenced by the discussions centred around the fifth question. Cognitive, operative, and affective engagement increased in the Year 9 and 10 classes and the teachers attributed this to the use of the FQA. Alongside the increase in engagement, a corresponding increase in students’ academic performance was noted.
8.4 Academic Performance

All three classes showed improvement in their half yearly and yearly examination results and the results were discussed in detail in each of the chapters on the individual classes. There was greater improvement in the Year 9 and 10 classes when compared with the Year 8 class, but this could have been because there was significantly greater room for improvement in the level 5 class compared with the level 2 class.

Importantly, all the students felt that they were better at answering questions and would be more likely to attempt mathematics questions because of the FQA. Lewis and Corey said that the repeated practice over time helped them to be better at answering questions. Gemma liked the repetition as well but emphasised that the different techniques and the discussion about all the different ways of answering the question assisted her to gain a greater understanding. The students experienced improvement in their academic performance which altered their initial perception of fixed or predetermined mathematical ability (Dweck, 2015). The academic improvement led to a change in mindset for both the students and teachers, making the realisation that they could improve, that growth was possible with effort.

8.5 A Change in Mindset

The academic improvement noted in the study resulted in a change of mindset in many of the students and all the teachers. When initially interviewed the four teachers all spoke about the grading of their class with respect to expected performance. The students, as well, based their view of their mathematical ability on
the class to which they had been assigned. There was no indication that they were expected to be able to move from the current class or significantly improve. There was a strong fixed mindset with respect to mathematical ability and the students in 9M5 and 10M5 clearly indicated that there were many students better than them at mathematics.

Anne raised the point that, in general, engagement was higher in mathematics classes that contained the more capable mathematics students. The first Year 8 classroom observation showed a well behaved class with high operative engagement but there was little evidence of cognitive engagement. The students appeared to be able to easily answer the questions given but displayed little enthusiasm and the activities did not have any cognitive challenge. The Year 9 and 10 classroom observations showed classes that were not very well behaved nor were they engaged, with significant class time used to manage behaviour and the attention level of the students. The teachers of the Year 9 and Year 10 classes certainly needed to put more effort into ensuring compliance rather than stimulating engagement. Over the course of the year the level of cognitive, operative and affective engagement in those classes increased and the need for classroom management decreased.

By the end of the year there had been a significant shift in mindset, of teachers and students, to a growth mindset, although to a lesser extent for the Year 10 teacher, Harriette. The students had seen improvements made by themselves and their classmates and realised that they were able to learn and improve in mathematics. The teachers saw the improvements made both academically and with engagement indicating a growth mindset. The decision by Harriette to focus only on two topics for the yearly examinations and not prepare the students for Year 11 indicated that she had a short-term mindset to ensure her students had positive final mathematics
results which could indicate she still maintained a partially fixed mindset with respect to her students’ level of mathematical ability. While there was significant growth by the students in her class as evidenced by their results in the yearly examination, it was only in the two areas targeted by the examination and not in the more challenging questions.

Many of the students in the Year 8 class showed significant growth in the half yearly examination and following the examination three Year 8 students, two of which were in the student focus group, were promoted from 8M2 into 8M1 and one student was moved down from 8M1 to 8M2. As only 8M2 used the FQA the change and influence on their learning warrants attention.

8.6 A Change in Class Brings a Change in Attitude.

Sue and Gemma, both members of the student focus discussion group, moved up from the 8M2 class to 8M1 as their half yearly rankings were equal seventh in the year group. Zelda moved down from 8M1 to 8M2 as her half yearly ranking was 63rd in the year group. Zelda was added to the final focus discussion group to provide the view of someone moving into the class using the FQA. Both Sue and Gemma felt that the lack of FQA in the 8M1 class was the main factor in their poorer performance in the yearly examination. As stated earlier the half yearly and yearly examinations were independent with different topics and concepts tested in each. Consequently, a student’s performance in the yearly examination is directly related to the term 3 and 4 class they attended. Their rankings in the half yearly and yearly examinations are displayed in Table 8.1.
Table 8.1 *Half Yearly and Yearly Year 8 Rankings of Students who Entered and Left the FQA Classroom During the Study*

<table>
<thead>
<tr>
<th>Student</th>
<th>Year 7 Ranking</th>
<th>Year 8 Half Yearly Ranking</th>
<th>Year 8 Yearly Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gemma</td>
<td>73</td>
<td>7</td>
<td>67</td>
</tr>
<tr>
<td>Sue</td>
<td>39</td>
<td>7</td>
<td>52</td>
</tr>
<tr>
<td>Zelda</td>
<td>29</td>
<td>63</td>
<td>5</td>
</tr>
</tbody>
</table>

While there are many factors that affect the performance of a student these results indicated that the FQA may have had a positive influence on the performance of Zelda and the lack of FQA a negative influence on the performance of Gemma and Sue. There was a major positive change for Zelda while in the class using the FQA and a major negative change for Gemma and Sue in the class not using the FQA. In the half yearly examination Gemma had attempted all the questions and even if she was not completely sure how to answer a question she was able to make an attempt drawing on her memory from the FQA. She found it difficult to keep up in the new class becoming disengaged because the teacher went through PowerPoint presentations followed by textbook work. A lack of substantive conversation, relevant tasks and variety of tasks could result in a decrease in engagement (FEM). There was no review component or opportunity for class discussion in the lessons and she felt she did not get an opportunity to understand before they moved on to the next aspect. Sue agreed with Gemma.

Zelda commented on the influence of the FQA on her performance in the yearly examination and said “Yeah, I think it absolutely made a huge difference … it’s really really helped having to revise and I think the saying quality not quantity is extremely important there”. Sue did not perform well in the yearly examination. She thought she knew how to do questions as she recognised that she had completed
similar questions in class, but she was unable to answer them in the examination.

She commented:

...while I was in the exam I skipped so many questions and ended up coming back to them and I was freaking out because I thought I knew what I was doing. I just didn’t have any of the confidence I had doing the last exam, it was just a lot of stress; but I never had any of that in 8M2 (Sue, Year 8 student).

Sue and Gemma both missed the FQA and the rhythm of the 8M2 class, knowing what was going to happen each lesson, and Zelda enjoyed that same rhythm and pattern joining the class using the FQA. The link between student attitude and performance in mathematics is coming through strongly in this case. Gemma and Sue have gone from loving to hating mathematics. Sue said “In 8M1 I come in and I’m kind of sitting dreading it because Maths is now my least favourite subject. In 8M1 I just do work I don’t understand”. Zelda is the opposite and said: “It was my least favourite subject, I couldn’t stand it and then I moved down to 8M2 and I’m enjoying it so much more”.

The examples of Gemma, Sue and Zelda indicated that the 8M2 classroom environment improved their learning outcomes, increased their confidence and enjoyment of mathematics and resulted in a positive examination performance. Each of those students felt that it was the FQA that allowed this to occur. They all felt that the, as they described it, ‘traditional’ learning environment of the 8M1 class detracted from these aspects. Using the FQA and the aspect of repetition with variation and spacing allowed students to engage with mathematics and develop confidence in answering questions and developing their understanding. The multiple
opportunities to attempt questions and develop understanding through class discussions came through as a very strong positive aspect of the FQA.

8.7 Conclusion

There were many changes as a result of the teachers using the FQA. The most noticeable was the change in teaching style from a ‘traditional’ approach as previously described with a focus on procedural understanding to an approach that provided opportunities for the development of conceptual understanding through the fifth question. This was particularly evidenced by the increase in time spent on the solutions and discussion of the fifth question a practice that did not exist in any of the classrooms prior to the FQA. The student voice that was initially absent in all three classrooms became more prevalent with a combination of continuous interaction and substantive conversations. While at times the fifth question was a more complex procedural question that a conceptual question there was still discussion of the solution.

While there were variations in the implementation of the FQA the approach was implemented as designed in the Year 8 and 9 classes. While the Year 10 teacher focused on the current topics and procedural questions only this was rationalised, as her intent was to have strong student performance in the yearly examination that was completely based on the two topic areas. The students were in the final year at that school and the tomorrow aspect was not considered in this case.

The overall level of engagement increased in all classes and there were a number of factors that contributed. While student behaviour and application increased,
particularly in the Year 9 and 10 classes, it was the increase in cognitive, operative, and affective engagement that was so compelling. This was due to the change from repetitive worksheets and the success of the students in answering questions, along with the increase in student voice. The teachers and students enjoyed and perceived the FQA as a positive influence on the learning and understanding in the classroom.

This chapter has outlined the differences in classes and implementation of the FQA. The change in mindset to a growth mindset came through clearly, the specific case of the three students that changed Year 8 classes showed an interesting insight into the influence of the FQA. The final chapter will draw conclusions based on the research.
9 Conclusion

It is crucial that we engage more students in mathematics to improve Australia’s results in international testing and to halt the decline in students selecting higher levels of mathematics. We know that traditional approaches to teaching mathematics with students following teacher-led examples, completing repetitive exercises, and marking questions without discussion are disengaging. A new approach, the Five Question Approach (FQA) to teaching mathematics was developed and tested in this research over the course of a school year. Data revealed that the FQA significantly increased student cognitive, operative, and affective engagement and resulted in improved student academic achievement.

The major sections of this chapter summarise the results of this research and discuss the implications of the study and future directions for the FQA. The purpose of this study was to investigate the influence of the use of the FQA at the commencement of each mathematics lesson on the academic performance of students, perceived and actual, and the engagement of students. The central research questions were:

1. What influence does the FQA have on perceived and actual academic performance of students in mathematics?

2. What influence does the FQA have on student engagement in mathematics?

The questions were informed by the following sub-questions:

- Are there specific aspects of FQA that contribute to student engagement?

- What influence do teachers perceive the FQA has on learning in the classroom?
To address the research questions three mathematics classes from two schools participated in a longitudinal study over the course of a school year. All participating teachers took part in four interviews and their classes were involved in four classroom observations during that time, excepting one Year 10 class which did not have a final observation due to end of school activities and one Year 9 class due to examinations. Each class teacher selected a group of six students to take part in four focus group discussions throughout the year. These students represented a range of academic abilities and levels of cognitive, operative, and affective engagement. This qualitative data was gathered, transcribed and analysed as described in Chapters 5, 6, 7, and 8. The quantitative data was gathered from the half yearly and yearly examinations with the previous year’s final examination rankings used as a baseline to determine relative changes in academic performance. This data was analysed in Chapters 5, 6, 7, and 8 and a summary of the results follows.

9.1 Summary of Results

The study was motivated by the issues of declining engagement of students and the resulting decrease in participation rates in higher levels of mathematics in Australia. The decline in engagement levels and academic performance was related to the issue of ‘traditional’ teaching and the way in which mathematics was taught in the classroom. The need to move away from ‘traditional’ teaching was described in Chapter 1 as urgent in 2003 and remains urgent today. As described in Chapter 2 many studies had been undertaken and ‘better’ ways of teaching students were described in section 2.1.
However, while there is a wealth of research into the teaching of mathematics, a shift from traditional mathematics pedagogy continues to remain a challenge. In this study, the FQA has been shown as a means to accomplish this change. The FQA did not require the teachers in this study to completely change their teaching praxis. It was an addition to their classroom teaching that evolved over time. Despite initial concerns from teachers and students that the FQA would take time away from what they considered to be the ‘real’ teaching, the textbook, this was not the case. Not only did they complete all the assigned content from the scope and sequence within the allocated time frame, but the students performed better in the examinations, had more confidence and genuinely engaged in the teaching and learning activities within the classroom. Not only was the ‘traditional’ textbook centred teaching paradigm ingrained in the teachers, it appeared to be with the students as well.

The FQA changed the teachers’ pedagogy methods by aligning their teaching praxis with best practice as described in Chapter 2. The FQA eased the teachers into a relational and conceptual approach to teaching through the investigative, open-ended fifth question. The time spent investigating and the discussion ensuing from the fifth question allowed students to develop a deeper, conceptual understanding of the power of using mathematical principles to solve problems, thereby linking areas of mathematics. This was enhanced through the development of procedural fluency through the first four questions, providing the students with a greater assortment of skills that could be applied to strategies that solve more complex problems.

The FQA positively influenced engagement, as the repetitive nature of worksheets, blindly following a set procedural approach, was replaced with genuine investigative, multi-solutioned, mathematical challenges. The addition of the discussion phase particularly from the fifth question, and greater involvement of
students in their learning significantly increased behavioural, cognitive and affective engagement.

The changes in cognitive, operative, and affective engagement were the result of the teaching and learning activities used in the FQA in the classroom. The teachers and students had a change in mindset from one that was fixed and strongly aligned to the graded mathematics class that they were assigned, to a growth mindset where teachers and students felt they could learn and improve in mathematics. Importantly, they felt they had improved, with the results indicating that many improved significantly, relative to the other students in their year level and students’ enjoyment of mathematics increased.

Linking back to figure 2.1 the change in the teaching and learning approaches by incorporating the FQA, whether fully or partially, resulted in more engaged higher achieving learners. The students responded positively to receiving feedback and partaking in discussions resulting in greater understanding. The students had an increase in confidence developed a growth mindset and had reduced examination anxiety due to the development of procedural fluency through the first four questions and conceptual understanding through the fifth question.

The study has added to current research on effective teaching and student engagement by providing a deeper understanding of influences on student engagement and academic performance within the secondary mathematics classroom through use of the FQA. Engagement and academic performance were the subject of the research questions and the FQA positively influenced the level of engagement and academic performance in all three classes thus favourably addressing the two central research questions.
9.2 Research Question 1

What influence does the FQA have on perceived and actual academic performance of students in mathematics?

The first research question was answered positively as academic performance improved in all three classes. The focus group discussions clearly indicated that the students felt they were better at mathematics. They were able to answer more questions and were more inclined to attempt more complex questions where the solution path was not obvious. The examination results clearly indicated significant improvement for all classes with some individual students demonstrating exceptional improvement.

The FQA provided multiple spaced opportunities to support the students understanding of concepts rather than in the ‘traditional’ classroom following the scope and sequence and stopping work on the topic when the time had elapsed. The teacher could use the FQA to focus on the areas of the curriculum causing issues for the students until they had the opportunity to develop their understanding. By addressing multiple content areas every lesson, the students were able to make links between areas of mathematics that they were not able to do when completing a single topic at a time. The result was the ability to recall procedures from multiple content areas, linking them together to solve multiple step mathematical questions. The students commented that during the examinations they were able to think back to the FQA, when they were working on similar questions, which allowed them to attempt more questions in the examination. The students felt a reduction in examination anxiety due to the continual revision aspect of the FQA combined with more time
and opportunities to develop understanding. Major examinations contain questions from all content areas and as such the FQA appeared to benefit students through the posing of questions from all content areas every day.

The examination results indicated improvement for most students as discussed in Chapters 5, 6, 7, and 8. The Year 8 half yearly examination results had nearly half the class ranked in the top thirty even though they were initially graded from position thirty to sixty. With two students promoted to the class above, based on the half yearly results, the yearly examination results revealed that eight students improved significantly, nineteen performed as expected, and five performed below expectations. Unfortunately, as detailed in Chapter 8, the two students promoted following the half yearly examination performed very poorly in the yearly examination having experienced a return to a ‘traditional’ teaching approach without the FQA for the second half of the year. As the concepts studied for the yearly were not related to those for the half yearly then the difference was the teaching praxis without the FQA.

Most of the students in the Year 9 class showed significant improvement in their examination rankings. As the 5th ranked class out of seven classes, their initial rankings were all below 100, and the yearly rankings had ten students with rankings above 90, including two above 60 and one in 33rd position. Clearly there was significant growth for most of the students in that class as evident in the detailed results in Chapter 6. The Year 10 class showed similar growth. Initially there were two students ranked above 100 with rankings down as far as 180th position. The yearly examination results had twelve students ranked above 100 including three above 80 and two above 60. The detailed results were presented in Chapter 7.
The improvement was not limited to academic performance in examinations. The students spoke of increased confidence and a greater willingness to attempt questions and be involved in class discussions. They linked that confidence with the success in answering questions correctly in class and the willingness to extend their knowledge through more challenging questions. Initially the students had a fixed mindset about their ability, and that changed as the year progressed with many students expressing enjoyment associated with their greater ability to correctly answer questions and a belief that they could improve with effort. The reduction in examination anxiety had a positive influence on examination performance with students able to attempt more questions by drawing on the class experiences in which they were engaged during the FQA.

9.3 Research Question 2

What influence does the FQA have on student engagement in mathematics?

The FQA had a significant positive influence on cognitive, operative, and affective engagement in all three classes. Students enjoyed the variety of questions that allowed them to improve procedural fluency, have multiple opportunities to develop understanding, and carefully consider mathematical concepts in depth. Students received positive and diagnostic feedback and the learning needs of each student were determined by the individual interaction between the teacher and students every lesson during the FQA. Combined with the teacher’s enthusiasm for the learning of mathematics the classroom environment displayed positive pedagogical relationships resulting in engaged students.

Initially, the Year 8 class was well behaved but not engaged cognitively but the cognitive challenge increased significantly during the year through the FQA.
Students enjoyed the challenge of the open-ended or investigative fifth question. Rather than following an algorithm and completing far more questions than needed to learn a procedure rather than understand a concept, they were challenged to develop their own method of solution and share that method with the other members of the class thus growing the understanding of the entire class through a mathematical conversation. This indicates an improvement in affective engagement. Some of this can be attributed to the increase in substantive conversation that was not just teacher to student, but also student to student. The tasks within the FQA allowed all students to achieve success and the fifth question provided multiple exit points so that all students were able to engage with the same challenging question and achieve to their maximum potential. These engaging pedagogical repertoires combined with a diverse range of activities contributed to an engaging classroom.

The Year 9 and 10 classes were not engaged at the start of the study. The level of appropriate behaviour, engagement and interest increased significantly through the FQA reflecting an increase in cognitive, operative, and affective engagement. The early classroom observations showed disinterested and disengaged students, but this changed and later classroom observations revealed engaged students that were taking an interest in, and responsibility for, their learning. The FQA facilitated this change by providing more confidence to the students through the repeated exposure to questions until the students understood the concepts and success was obtained through the first four questions. The students commented that they enjoyed the challenge and discussion of the fifth question.

Overall, the FQA had a significant influence on the level of engagement of most students in the study. Initially the teachers linked engagement with the level of the class they had been allocated. The Year 8 teachers stated that the higher the
academic level of the class the greater the level of engagement. However, when more closely examined it was a combination of academic success and behavioural compliance indicating operative engagement only that was evident in the early classroom observations. As described in Chapter 5 the Year 8 class was very well behaved which the teachers interpreted as highly engaged but there was little evidence of cognitive engagement with students easily completing all tasks. The classroom observation indicated a lack of substantive conversation, challenging tasks or even a variety of tasks, or constructive feedback. As the year progressed aspects of engagement that were absent at the start of the year were found to be present because of the FQA. The class remained well behaved, but through the FQA the level of cognitive engagement increased during the year. The classroom evolved from a teacher-centred environment to one in which the student voice was valued and many students contributed to the overall learning.

As previously stated, initially, the Year 9 and 10 classes did not have a high level of engagement and really did not display any signs of engagement. During the first classroom observations none of the descriptors of an engaging classroom from the Framework for Engagement in Mathematics (FEM) were apparent.

Most of the students did not value mathematics, were not actively participating in classroom activities and had no voice in the classroom. The students expressed that their mathematical capacity was limited and that students in higher mathematics classes were ‘smarter’ than them. The teachers expressed the same belief. This fixed mindset on ability in mathematics changed during the year as students developed more competence and confidence through the FQA. The level of student participation and involvement in the lesson increased over the year. By the second half of the year many aspects of the FEM were apparent in the classroom.
observations. Specifically, students were keen to provide responses and discuss solution methods taking a greater interest in their mathematical learning. The later classroom observations indicated classrooms with substantive conversations taking place, tasks that were varied and challenging, linking to the students’ pre-existing knowledge and building on that knowledge. There was continuous interaction and constructive feedback. Students were more engaged, and this had an influence on their academic performance, and in turn, their academic improvement increased their cognitive, operative, and affective engagement. The downward spiral that they had experienced until the introduction of the FQA appeared to have reversed and mathematics lessons became a positive and uplifting and more enjoyable classroom experience. Students, through the change to a growth mindset developed a propensity to take risks and make an effort to attempt questions that they may not have attempted before, knowing that if there was an error in their response the error provided an opportunity to learn, thus reversing the disengagement trend.

In the final interview the students were asked if given the choice, whether or not they would want to attend a class using the FQA. The teachers were also asked if they would continue to use the FQA or return to their previous teaching style. All students and teachers indicated that they would like to continue with the FQA in the following year with one school intending to adopt the practice across the entire faculty. During the study the lack of involvement of an entire faculty was a limitation as was the scale. These and other limitations need to be considered as a precursor to further research.
9.4 Limitations

The study had some limitations which will be outlined in this section. Only secondary schools were included in the study and both schools were secondary Catholic schools. Both schools graded their classes, and as a result, the students had a strong focus on their examination performance as that determined their class allocation. It would be difficult to predict the results of the FQA in schools where students are grouped heterogeneously, particularly in Years 7 and 8. Such a context might provide a greater challenge for the teacher in determining the optimal set of five questions for each class. Also, the study only included students in Years 8, 9, and 10 therefore classes in other secondary years or even primary classes were not included.

The design of the five questions requires teachers to have strong MCK and deep PCK and as a result, teachers not specifically trained in mathematics may find the question writing difficult. Teachers within a mathematics faculty would necessarily have different levels of experience, training, MKT, and their own personal level of engagement with the teaching of mathematics in the classroom. All teachers in the study had a high level of personal and professional engagement at the commencement of the study and this remained throughout. However, they all committed to increasing their level of PCK, MCK and ultimately MKT to improve the classroom experience particularly with respect to the fifth question. The teachers’ depth of engagement increased with a stronger emphasis on pedagogical relationships and pedagogical repertoires. A teacher with a low level of engagement and/or PCK may not be able to implement the FQA successfully and teacher engagement would be a limiting factor in the success of the FQA. Professional
development may be required to assist teachers in the development of their teaching knowledge.

Professional development would be required to assist teachers within the faculty to effectively write and use the FQA. The teachers in this study received professional development training in the FQA at the commencement of the study and at multiple times during the study over the entire school year, at no cost. As described in previous chapters, the FQA as implemented by the teachers needed fine tuning throughout the year, particularly around the difficulty level of the questions and the concepts that needed exploration through the fifth question. Therefore, teachers receiving an initial professional development day at the commencement of the school year to implement the FQA would need further support throughout the year. This would necessitate a cost to either the school or system which would require a financial commitment, perhaps not a limitation but an investment in better outcomes and student cognitive, operative, and affective engagement, to ensure the FQA was implemented for maximum positive effect.

A consistent approach to the FQA by the entire mathematics faculty is an important consideration and may also be a limitation. As clearly demonstrated by the case of the two students promoted from 8M2 to 8M1 and their subsequent drastic decline in a class without the FQA, there is a need for whole faculty involvement in the use of the FQA. In addition, the entire mathematics faculty must be consistent in the implementation of the FQA and their teaching approach using the FQA, otherwise gains achieved in one class will be lost in another class. All three classes in the study implemented the FQA as intended for the first half of the year, but the Year 10 class varied the approach in the second half of the year. The Year 10 FQA focus was solely on the two topics that were in the yearly examination in order to maximise
students’ performance and provide the best possible mathematics results on their final school report. No consideration was given for the students’ preparation for Year 11, particularly as this would be in another school. The application of the FQA to ensure the approach is implemented correctly requires monitoring for consistency across the faculty.

A final limitation concerns the length and depth of the study. The study could be extended beyond one school year to investigate ongoing influence of the FQA on academic results and engagement.

9.5 Additional Findings

The study revealed some unanticipated outcomes of the implementation of the FQA. These included the increased use of and familiarity with the syllabus particularly the content and processes, increased teacher discussions within the faculty and greater teacher engagement. In a search to find suitable fifth questions, and determine which concepts needed further development all the teachers referred to the NSW K-10 Mathematics syllabus to more clearly determine the aim of the syllabus in that content area. This was not the usual practice as the teachers in the past followed the scope and sequence and teaching program developed by the school without necessarily directly using the syllabus. The quest for appropriate fifth questions resulted in an increase in mathematical discussion in the mathematics faculties with teachers exchanging ideas and resources thereby promoting a community of practice. The teachers realised a need to move beyond the syllabus and find other sources of inspiration for the questions.
The FQA brought about a change in teacher engagement. At the commencement of the study it was observed that all the teachers were dedicated and hard-working professionals. However, there was a noticeable change in their level of engagement with the content and the classroom interactions with the students over the year. There was an increase in pedagogical relationships with improved awareness of the students’ progress and pre-existing knowledge. The questions in the FQA were appropriate for the students, there was continuous interaction in the classroom, and positive, constructive feedback. The teachers’ pedagogical repertoires increased with a variety of challenging and relevant tasks that promoted substantive conversations. The FQA was designed to achieve these outcomes with the students and it was unanticipated that this would also occur with the teachers. Combining the results, limitations and unanticipated outcomes, suggestions for further research follow.

9.6 Suggestions for Future Research

The focus of this study was the effect of the FQA on engagement and perceived and actual academic performance. The study showed overwhelmingly that the FQA contributed positively in answering these questions. This research has the potential to be expanded by involving a broader range of schools and school systems, both secondary and primary, teachers, and students and including non-denominational schools, other school systems, and schools using a mixed ability approach to mathematics classes. The collection of data from two schools and three different year levels provided some insight into the value of the FQA. A longitudinal study that tracks the students into the senior years of schooling and extension of the study into more schools may provide a deeper understanding of the potential of the FQA to
alleviate some of the issues involved with a 'traditional' teaching environment with a fixed time linear approach to teaching mathematics.

A continued study of the same students would reveal whether the improvements continue into future school years. Some of the students would continue in classes using the FQA and others would not. The students’ level of engagement and academic performance would be of interest after a year in a class without the FQA. Further study of students whose next class also uses the FQA and any fluctuation in their engagement and academic performance would provide further insight into the value of the FQA. Additionally, a study of the teachers who used the FQA and any additional changes to their teaching because of the continued use of the FQA would prove valuable.

As stated earlier, a duplication of the study across different school contexts would provide further insights into the influence of the FQA. Further, an investigation of gender and engagement and academic performance across school years may be of interest particularly to school systems with single sex and coeducational schools. Additionally, an adaptation and investigation of the FQA in other subject areas may be of assistance to teachers in other subject areas and provide further data on engagement in mathematics in comparison with other subjects. Finally, further study to determine if the FQA is a vehicle for teacher and student engagement and academic performance may show that the FQA generates a need for professional learning which is of benefit to teachers, schools, systems and the education of pre-service teachers.
9.7 Conclusion

This study contributed to a deeper understanding of student engagement in mathematics and factors affecting academic performance and was highly successful for the classes involved with significant increases in student engagement and perceived and actual academic performance. Prior to the commencement of the study, consideration was given to whether the success in the classrooms using the FQA was due to myself, the researcher as a teacher or the FQA itself. The study has clearly shown that the FQA stands alone and regardless of the teacher can positively influence cognitive, operative, and affective engagement and academic performance. In fact, the use of the FQA and the search for appropriate conceptual problem-solving questions evolved into professional development that positively influenced teaching and learning. The FQA is not limited to the secondary mathematics classroom and the approach is potentially transferable and may possibly be replicated in other subject areas. It is possible that the FQA could influence educational policy, through dissemination of the results and further research, at the school level, system level and also change aspects of pre-service teachers’ education.

As outlined in Chapter 1 there are volumes of research on best teaching practice but the uptake of different teaching praxis to ‘traditional’ teaching in the mathematics classroom is slow. The great strength of the FQA is that it is an operational way for teachers to improve teaching and learning outcomes for their students. Teachers can use the FQA alongside their current techniques and the success of the FQA in increasing engagement and academic performance can facilitate a change in the way teaching takes place in the classroom. This study showed the change in three classrooms from ‘traditional’ to classrooms with an increase in substantive discussion. There was a focus on problem solving with multiple solution pathways.
and an increase in student engagement. All content was successfully completed within the required time and a high level of procedural fluency achieved without the use of repetitive, disengaging worksheets.

The study provided evidence that the FQA influenced students to become more engaged and confident in mathematics, which then resulted in an increase in their enjoyment of mathematics with improved academic performance as the outcome. The FQA has been shown to facilitate the achievement of the two goals of the Melbourne Declaration (2008) as described in Chapter 2. Goal 1 was achieved through the promotion of high-quality teaching by using the FQA and the focus on conceptual knowledge and relational understanding particularly through the fifth question with its emphasis on problem solving and the development of understanding. The teachers’ MCK, PCK, and MKT increased because of the FQA which improved their effectiveness as teachers. Goal 2 was achieved through the success of students through the FQA and the increase in confidence that empowered these young Australians to become successful and confident learners in mathematics. As the study progressed all three classes displayed an increase in cognitive, operative, and affective engagement and increased student confidence, all promoted by the FQA.

Through their experience of the FQA, students may approach mathematics with more confidence and less anxiety and see that not only is success possible, but that mathematics is an interesting and valuable academic pursuit. The FQA may empower students to study mathematics at higher levels and change negative attitudes to mathematics with fewer people celebrating the fact that they can’t do mathematics. Perhaps the FQA can promote more ‘mathematics’ people who have fond positive memories of mathematics at school and carrying that attitude into their
everyday lives. Society, as a whole may then benefit from more mathematically literate citizens.
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insights


Ministerial Council on Education, Employment, Training and Youth Affairs


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Appendix A

School of Education
University of Western Sydney
Locked Bag 1797
Penrith NSW 2751

Teacher Participant Information Sheet Teacher

**Project Title:** A Five Question Approach to the Teaching of Mathematics

**Project Summary:** This project will investigate the effectiveness of the use of five questions at the commencement of mathematics lessons on the engagement and academic performance of students.

You are invited to participate in a research study being conducted by John Ley, doctoral candidate, University of Western Sydney under the Supervision of Dr Catherine Attard.

**How is the study being paid for?**

No funding is applicable.

**What will I be asked to do?**

You will be asked to undertake some training in the focus question approach to the teaching of mathematics that will help you implement this approach. You will be asked to participate in four 40 minute interviews, start of the year, end term 1, end term 2 and end of the school year, to help us understand whether the program was effective in increasing students’ engagement and academic performance.

There will be four classroom observations, at the start of the year, at the end of term 1 and term 2 and the end of term 4. The purpose of the observations is to collect
data on the engagement of the students in the 5 questions and the rest of the mathematics lesson.

While your involvement will not be confidential during the collection of the data, you will be provided with a pseudonym to ensure anonymity and confidentiality in the publication of the results.

You have been chosen to participate in the research because you are an experienced mathematics teacher who is established in your school and are currently using some form of questions at the commencement of your mathematics lessons.

**How much of my time will I need to give?**

The initial training will take approximately one hour. The four interviews will be approximately 40 minutes each in duration.

**What specific benefits will I receive for participating?**

The project will be an opportunity for you to participate in professional learning in the area of question writing, engagement and mathematics. The professional learning on question writing and mathematics will include aspects of procedural and conceptual knowledge, instrumental and relational understanding.

**Will the study involve any discomfort for me? If so, what will you do to rectify it?**

No discomfort.

**How do you intend to publish the results?**

Please be assured that only the researchers will have access to the raw data you provide.
The findings of the research will be published in my doctoral thesis. Mathematics education journals and educational conference proceedings.

*Please note that the minimum retention period for data collection is five years.

**Can I withdraw from the study?**

Participation is entirely voluntary, and you are not obliged to be involved. If you do participate, you can withdraw at any time without giving a reason. If you do choose to withdraw, any information that you have supplied will be destroyed.

**Can I tell other people about the study?**

Yes, you can tell other people about the study by providing them with the chief investigator’s contact details. They can contact the chief investigator to discuss their participation in the research project and obtain an information sheet.

**Data storage**

There are a number of government initiatives in place to centrally store research data and to make it available for further research. For more information, see [http://www.ands.org.au/](http://www.ands.org.au/) and [http://www.rdsi.uq.edu.au/about](http://www.rdsi.uq.edu.au/about). Regardless of whether the information you supply or about you is stored centrally or not, it will be stored securely, and it will be de-identified before it is made available to any other researcher.

**What if I require further information?**

Please contact John Ley should you wish to discuss the research further before deciding whether or not to participate. John Ley, doctoral candidate, University of Western Sydney, email 18224098@student.uws.edu.au

**What if I have a complaint?**
This study has been approved by the University of Western Sydney Human Research Ethics Committee. The Approval number is H11276.

If you have any complaints or reservations about the ethical conduct of this research, you may contact the Ethics Committee through the Office of Research Services on Tel +61 2 4736 0229 Fax +61 2 4736 0013 or email humanethics@uws.edu.au.

Any issues you raise will be treated in confidence and investigated fully, and you will be informed of the outcome.

If you agree to participate in this study, you may be asked to sign the Participant Consent Form.
This is a project specific consent form. It restricts the use of the data collected to the named project by the named investigators.

Project Title: A Five Question Approach to Teaching Mathematics

I, _________________________________ [name of participant] consent to participate in the research project titled: A Focus Question Approach to the Teaching of Mathematics.

I acknowledge that:

I have read the participant information sheet [or where appropriate, ‘have had read to me’] and have been given the opportunity to discuss the information and my involvement in the project with the researcher/s.

The procedures required for the project and the time involved have been explained to me, and any questions I have about the project have been answered to my satisfaction.

I consent to the interviews, audio taping and classroom observations.

I understand that my involvement is confidential, and that the information gained during the study may be published but no information about me will be used in any way that reveals my identity.

I understand that I can withdraw from the study at any time, without affecting my relationship with the researcher/s now or in the future.

Signed:
This study has been approved by the University of Western Sydney Human Research Ethics Committee. The Approval number is: H11276

If you have any complaints or reservations about the ethical conduct of this research, you may contact the Ethics Committee through the Office of Research Services on Tel +61 2 4736 0229

Fax +61 2 4736 0013 or email humanethics@uws.edu.au. Any issues you raise will be treated in confidence and investigated fully, and you will be informed of the outcome.
Project Title: A Five Question Approach to the Teaching of Mathematics

Project Summary: This project will investigate the effectiveness of the use of five questions at the commencement of mathematics lessons on the engagement and academic performance of students.

You are invited to participate in a research study being conducted by John Ley, doctoral candidate, University of Western Sydney under the Supervision of Dr Catherine Attard.

How is the study being paid for?
No funding is applicable.

What will I be asked to do?
You will be asked to participate in four 40 minute interviews, start of the year, end term 1 and term 2 and the end of term 4, to help us understand whether the program was effective in increasing students’ engagement and academic performance.
There will be four classroom observations, at the start of the year, at the end of term 1 and term 2 and at the end of term 4. While your involvement will not be confidential during the collection of the data, you will be provided with a pseudonym to ensure anonymity and confidentiality in the publication of the results.

How much of my time will I need to give?
The four interviews will be approximately 40 minutes each in duration.

What specific benefits will I receive for participating?
The project will be an opportunity for your child to participate in professional learning in the area of teaching and learning mathematics.

Will the study involve any discomfort for me? If so, what will you do to rectify it?
No discomfort.

How do you intend to publish the results?
Please be assured that only the researchers will have access to the raw data you provide.

The findings of the research will be published in my doctoral thesis. Mathematics education journals and educational conference proceedings.
*Please note that the minimum retention period for data collection is five years.

**Can I withdraw from the study?**
Participation is entirely voluntary, and you are not obliged to be involved. If you do participate, you can withdraw at any time without giving a reason.

If you do choose to withdraw, any information that you have supplied will be destroyed.

**Can I tell other people about the study?** [remove if not relevant]
Yes, you can tell other people about the study by providing them with the chief investigator’s contact details. They can contact the chief investigator to discuss their participation in the research project and obtain an information sheet.

**Data storage**
There are a number of government initiatives in place to centrally store research data and to make it available for further research. For more information, see [http://www.ands.org.au/](http://www.ands.org.au/) and [http://www.rdsi.uq.edu.au/about](http://www.rdsi.uq.edu.au/about). Regardless of whether the information you supply or about you is stored centrally or not, it will be stored securely, and it will be de-identified before it is made available to any other researcher.

**What if I require further information?**
Please contact John Ley should you wish to discuss the research further before deciding whether or not to participate.
John Ley, doctoral candidate, University of Western Sydney,
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If you agree to participate in this study, you may be asked to sign the Participant Consent Form.
Human Research Ethics Committee
Office of Research Services

Participant Consent Form

This is a project specific consent form. It restricts the use of the data collected to the named project by the named investigators.

Project Title: A Five Question Approach to Teaching Mathematics

I, ___________________________________________ [name of participant] consent to participate in the research project titled [insert title].

I acknowledge that:

I have read the participant information sheet [or where appropriate, 'have had read to me'] and have been given the opportunity to discuss the information and my involvement in the project with the researcher/s.

The procedures required for the project and the time involved have been explained to me, and any questions I have about the project have been answered to my satisfaction.

I consent to the [insert specific activities] [if applicable] [list all components of involvement, e.g. audio/video taping to ensure participants can indicate their willingness to participate in all or some of the research]

I understand that my involvement is confidential, and that the information gained during the study may be published but no information about me will be used in any way that reveals my identity.

I understand that I can withdraw from the study at any time, without affecting my relationship with the researcher/s now or in the future.

Signed:

Name:

Date:

Return Address: Return to class teacher

This study has been approved by the University of Western Sydney Human Research Ethics Committee. The Approval number is: [enter approval number]

If you have any complaints or reservations about the ethical conduct of this research, you may contact the Ethics Committee through the Office of Research Services on Tel +61 2 4736 0229 Fax +61 2 4736 0013 or email humanethics@uws.edu.au. Any issues you raise will be treated in confidence and investigated fully, and you will be informed of the outcome.
Appendix B

Classroom Observation Guide

LESSON 1: TOPIC__________________ DATE:__________________

_**The Learning Experiences**_

<table>
<thead>
<tr>
<th>High cognitive</th>
<th>Evidence that students were:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• Reflectively involved in deep understanding of mathematical concepts and applications, and expertise</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High affective</th>
<th>Evidence that students were:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• Genuinely valuing – this learning will be useful to me in my life outside the classroom</td>
</tr>
<tr>
<td></td>
<td>• ‘enjoying’ mathematics activities and learning</td>
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</table>

<table>
<thead>
<tr>
<th>High operative</th>
<th>Evidence that students were:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• Actively participating – five questions, group discussions, practical, relevant activities and homework tasks</td>
</tr>
<tr>
<td>Messages</td>
<td>What it means to students</td>
</tr>
<tr>
<td>----------</td>
<td>--------------------------</td>
</tr>
<tr>
<td><strong>Knowledge</strong>&lt;br&gt;reflectively constructed access to contextualised and powerful knowledge</td>
<td>“we can see the connection and the meaning”</td>
</tr>
<tr>
<td><strong>Ability</strong>&lt;br&gt;feelings of being able to achieve and a spiral of high expectations and aspirations</td>
<td>“I am capable”</td>
</tr>
<tr>
<td><strong>Place</strong>&lt;br&gt;valued as individual and learner and feelings of belonging and ownership over learning</td>
<td>“it’s great to be a kid from”</td>
</tr>
<tr>
<td><strong>Voice</strong>&lt;br&gt;environment of discussion and reflection about learning with students and teachers playing reciprocal, meaningful roles</td>
<td>“we share”</td>
</tr>
</tbody>
</table>

*Engaging Messages*

Additional Notes:
Appendix C

Year 8 2016 Scope and Sequence –

<table>
<thead>
<tr>
<th>Term 1</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<th>8</th>
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<tr>
<td>Review of Year 7</td>
<td>Number and Algebra</td>
<td>Pythagoras</td>
<td>Percentages</td>
<td>Algebra Techniques</td>
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<tr>
<td>Probability 2</td>
<td>Measurement</td>
<td>Equations 2</td>
<td>Review Catch Up</td>
<td>Exams</td>
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<td></td>
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<tr>
<td></td>
<td><em>Perimeter, Area, Volume and Time</em></td>
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<th>8</th>
<th>9</th>
<th>10</th>
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</thead>
<tbody>
<tr>
<td>Equations 2</td>
<td>Ratios and rates</td>
<td>Coordinate Geometry</td>
<td>Circles and Cylinders</td>
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<td>Statistics</td>
<td>Revision Catch Up</td>
<td>Exams</td>
<td>Reasoning in Geometry</td>
<td>EOYA</td>
<td>Pupil Free</td>
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</table>

The Teaching Program follows Signpost Mathematics Year 8 as a basic resource.
Appendix D

Year 8 First classroom observation

The observation took place in the third lesson of the day immediately following the morning tea break. The lesson commenced with the teacher writing the five questions on the whiteboard. The students started working on the questions immediately and most completed the first four without teacher intervention. The teacher provided positive reinforcement to the students as she went around the room during the solution phase of the five questions. Some of the comments she made were “well done”, “you have good working out”, “that is a good way of doing that question”. “you are nearly there just think about it a bit more”. The teacher then drew the attention of the class to the front and they discussed the solutions to the questions. Students were asked to present their solutions to the class. The five questions from that lesson are shown below.

<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10 ÷ 2 = $</td>
</tr>
<tr>
<td></td>
<td>$-10 ÷ 2 = $</td>
</tr>
<tr>
<td></td>
<td>$10 ÷ -2 = $</td>
</tr>
<tr>
<td></td>
<td>$-10 ÷ -2 = $</td>
</tr>
<tr>
<td>2</td>
<td>Simplify</td>
</tr>
<tr>
<td></td>
<td>$12ab - 7ba$</td>
</tr>
<tr>
<td>3</td>
<td>Calculate 10% of 75</td>
</tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>4</td>
<td>Between which 2 whole numbers does ( \sqrt{90} ) lie?</td>
</tr>
<tr>
<td>5</td>
<td>Michael said the answer to ( 9 - 3x ) m is 6m. What did he do wrong? What is the correct answer?</td>
</tr>
</tbody>
</table>

The students were engaged in the solution phase of the lesson as the majority of students were raising their hands to offer ideas and solution methods. Their level of engagement remained high for the entire lesson with students constantly adding their solution ideas to the ideas of other students. The discussion of question 5 led to further discussions on square numbers and surds. The teacher stated that these would form the basis of a question in tomorrow’s five questions. A student provided a novel solution to the question and explained the method to the class and as a result another student presented a further modification of that method for discussion.

Karen then asked the students to raise their hands to indicate the number of questions correctly answered. Nearly all students had either 4 or 5 correct which demonstrated that most had a good understanding of the questions being attempted and were able to develop that understanding further through the discussions particularly on question 5. After the lesson the I asked the teacher about the discussion generated by the fifth question. The teacher stated that the discussion was great but that it would not have normally occurred in her “traditional” class. It was the style of the fifth question that allowed deep and meaningful discussion to happen. Karen was very pleased with the students’ development of their understanding. She stated that it moved away from the procedural, that was the norm in her class, to a more conceptual understanding.
The students in the class showed their active participation and engagement by raising their hands and contributing ideas. There was a strong mathematical discussion taking place during the solution phase of the five questions. In question 3 some of the methods of finding 10% were finding one tenth and moving the decimal point one place to the left. This led to a discussion on place value and the equivalence of one tenth and 0.1, thus linking fractions and decimals. Discussion on square numbers resulted from question 4 with many students commenting that 90 was not a square number and that the square numbers either side of 90 were 81 and 100. Some stated that the answer was between 81 and 100 instead of 9 and 10. The teacher then asked if $\sqrt{90}$ was closer to 9 or 10 and further discussion ensued with an answer of 9, although many said it was in the middle and that was very close as the difference is very small. In question 5 many students knew the correct answer but were unable to articulate the reason. After the teacher mentioned order of operations there was discussion about order of operations and the rules of algebraic operations with several students contributing ideas.

The ‘new’ content for the lesson had centred on Pythagoras’ theorem and right angled triangles. Some students had difficulties with the squaring aspects and were directed back to the previous lesson’s five questions which had specific questions on square numbers. The students were then able to make the link between the square numbers and the question they were attempting and were able to progress without further teacher assistance. The students were then given a worksheet with about 20 questions that required the calculation of the hypotenuse of a triangle.

The students were engaged in their learning as they were attentive to the teacher during her explanations but there was very little discussion during this ‘repeated practice’ phase of the lesson in direct contrast to the five questions phase.
Appendix E

Year 8 second classroom observation

The classroom observation took place the lesson before the student interviews. Karen was teaching the class and they entered the room at the direction of the teacher and took their seats. The teacher began writing the five questions on the board and the students started doing the questions immediately. There were 4 students who arrived late to class and they began the five questions immediately. After writing the questions on the board the teacher moved about the room looking at student work and answering questions as required.

After a period of time the teacher called the class to stop working and pay attention to the front of the room. She then began to discuss the answers to the questions. There was some discussion on the first four questions but that was mainly clarification of the method of solution. The fifth question generated a significant amount of discussion. Three students, one at a time, presented different solutions to the fifth question on the board and these were discussed by the class. The questions are displayed below.
<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Find the value of m.</td>
</tr>
<tr>
<td>2</td>
<td>Simplify $10m \times 2 + 8m \div 4$</td>
</tr>
<tr>
<td>3</td>
<td>Decrease 1600g by 14%</td>
</tr>
<tr>
<td>4</td>
<td>How many mm in 2.6 metres?</td>
</tr>
<tr>
<td>5</td>
<td>What was the base price, before GST was added, to give a retail price of $21.45?</td>
</tr>
</tbody>
</table>

This discussion lasted for about 15 minutes with students asking questions that were answered by the students presenting the solutions with the teacher adding to the discussion. The discussion led to further work involving the unitary method and precisely deriving the rule of dividing by 11 to find the GST. Many of the students had learnt that rule but did not know where it came from or why it worked. The teacher then asked what would happen if the GST was 20% and not 10%, would you still divide by 11. The students worked out that the new number would be 12 not 11. This showed deep understanding of percentages and the unitary method. From my experience, the unitary method is often found to be quite difficult for students.
Two of the first four questions were a little too difficult and this prompted a number of students to seek assistance and the teacher had difficulty getting to them in a reasonable time. This could be resolved with an adjustment to the level of difficulty. The students were very engaged in the five question phase of the lesson as evidenced by the amount of discussion between students concerning the questions. Students were not asking ‘how do I do this’ type of questions but were discussing the solution to the question.

Following the five questions the teacher began the ‘normal’ part of the class. There was very little discussion with the teacher presenting a number of examples and asking closed questions that prompted a single answer response from selected students. A significant amount of time was spent copying the examples and then the students were given an exercise from the textbook to complete. That exercise was directly related to the examples and the questions were very repetitive. The students were compliant in the completion of the exercise but there was no discussion on the questions or their solutions. There was a low level of engagement demonstrated by a number of student conversations unrelated to mathematics or the current lesson. There were three pairs of students who did not complete any of the exercises and talked to each other for the remainder of the lesson. Yet these same students were actively answering questions and engaged during the five questions. It appeared that the teacher’s focus on textbook use was so ingrained that she did not consider changing the remainder of the lesson.
Year 8 third classroom observation

As with the second data collection the classroom observation took place in the lesson before the student focus group interviews. Anne was observed, and the class entered the room, collected their books, as they had been taken earlier in the week by Anne to be marked. Anne began writing the five questions on the whiteboard and the students began immediately. The students began working in silence but after about 10 minutes conversations began. However, the conversations were all related to the questions and methods of solutions and comparison of answers. This demonstrated that the students were engaged with the five questions. The actual questions are shown below.

<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$4x + 2 = -10$</td>
</tr>
</tbody>
</table>
| 2      | Harry rides his bike at 24km/h  
   a) How far will he ride in 10 minutes?  
   b) How far will he ride in 50 minutes? |
| 3      | $4\text{h 10min} – 2\text{hr 40 min}$ |
| 4      | Calculate mentally $30 \times 99$ |
| 5      | Calculate the volume |
The students became unsettled after 25 minutes indicating they were ready for the solutions, but the teacher was helping some individual students and continued to do that for another 5 minutes. Then the solutions were started. All students were listening and contributing during this time with a variety of solution methods discussed. The discussion was not limited to teacher to student with lots of student to student discussion through the teacher regulating turn taking. This showed strong engagement with the five questions. There was positive mathematical discussion on the question 30 x 99 with students providing good explanations of their thought process showing working mathematically occurring. Students appeared to be enjoying the discussions.

One student proposed incorrectly that the answer to 30 x 99 was five million four hundred. The teacher wrote this as 5,400,000 and about half the class immediately raised their hands and stated that it was incorrect and should be written as 5 400 000 but the correct answer was 2970. The teacher considered the way the large number was written and stated that the students were correct with their formatting of the
number. She then checked with the observer who confirmed that the students were correct. This showed a cooperative classroom where all members are respected for their contributions, the learning is shared with the teacher and the students playing reciprocal and meaningful roles.

Question 3 had two different solution methods given, one of which was the traditional bridging technique. However, a student presented an alternate solution method.

\[ 4h\ 10\text{min} - 2h\ 40\text{min} = 2h - 30\text{min} = 1h\ 30\text{min} \]

Two other students had the same method. The student, with support from the other two, explained the solution method that used a strong understanding of negative numbers. The usual Year 8 lesson on this type of question would use the bridging technique followed by a series of repetitive questions solved in that manner. The teacher commented that she would not have used that solution method but that it was a very efficient and correct method. In this case the students were exposed to a method of solution that otherwise they would not have seen.

The students were then asked about the number of questions they answered correctly. One student scored three with all the rest scoring 4 or 5. The students seemed quite pleased with their performance and were quick to raise their hands when asked. This aligns with success resulting in engagement discussed previously.

Following the completion of the five questions the class were then instructed to take out the worksheet from last lesson and six minutes were spent with the teacher.
calling out the answers and the students marking a very repetitive worksheet. There was no discussion on the worksheet. The teacher then handed out another worksheet and the students were instructed to start working. There was no discussion, examples or instruction. The level of engagement decreased from that observed during the five questions with at least 6 students chatting and not working.

The observation showed that the level of engagement of students was higher during the five question phase. This links with the student comments about five questions on a variety of topics versus a repetitive worksheet. The issue of the five questions taking too much time was confirmed by the observation. The teacher allowed approximately 10 minutes too long for the students to complete their answers. As a result, some students did not start straight away, and others went off task as they had completed all the questions.
Appendix G

Year 8 fourth classroom observation

The fourth classroom observation was conducted in Term 4, week 7 immediately following the students final examination period. As a result, the teacher, Anne, had not seen the class for over a week and this was their first mathematics lesson since the examinations ended. The students were very keen to have their papers returned and find out their marks.

Anne wrote the five questions on the board and about 80% started immediately with a few pairs still chatting. They started more slowly than previous observations and some students were only copying down the questions without completing the answers. The questions were at a higher level than previous observations and question 4 was introducing new material. The questions appear below.

<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Factorise fully $24m^3n^2 + 60m^2n^4$</td>
</tr>
<tr>
<td>2</td>
<td>The price of a shirt is increased by 20%. If the new price is $48, what was the original price?</td>
</tr>
<tr>
<td>3</td>
<td>Find the value of $\Theta$. Give a reason.</td>
</tr>
</tbody>
</table>

![Diagram of a triangle with angle $\Theta$ and $40^\circ$]
Find the most common score of 4, 4, 5, 6, 6, 6, 7

The “mode” is_____?

The median is _____?

The range is ______?

Two circles are exactly the same size. What other parts of the circles are exactly the same?

Some students were observed looking back at previous questions to help solve current questions displaying an independent learning attitude. The teacher assisted students with the questions. As in the third observation the time allocated to the five questions was too long by about 10 minutes.

During the solution phase students were all engaged and the solutions to the percentage question showed a very good understanding of percentages and the unitary method with some interesting solutions. Normally this would have been solved using the usual unitary method shown:

120% is $48

1% is $48 ÷ 120

100% is $48 ÷ 120 x 100 = $40.

While many students used this method, there were two other solutions given by students. The first was:
48 ÷ 120 = 0.4 therefore 20% is 0.4 x 20 = 8 so price is $48 - $8 = $40. The calculation of 20% and subtracting was different to the usual solution of directly finding 100%.

The second was:

48 ÷ 6*5 because 20% is 1/5 and if it has been added then there are 6 fifths, so divide by 6 to get the number of fifths and multiply by 5 to get the whole amount.

There was significant discussion on these solutions. Both showed that students really understood the concept of percentages through a link with fractions and decimals.

The teacher then returned the examination papers and the students looked through these and compared marks with each other.
Appendix H

Year 9 2016 Scope and Sequence –

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<tr>
<th>Term 1</th>
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<td></td>
<td>Financial Maths</td>
<td></td>
<td>Algebra</td>
<td>Tools Down</td>
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<table>
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<tr>
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<td>Area</td>
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<td>Surface Area</td>
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<td></td>
<td>Single Data Analysis</td>
<td>Trigonometry</td>
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<td>Tools Down</td>
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<th>Term 4</th>
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<td>10</td>
</tr>
<tr>
<td></td>
<td>Indices and numbers of any magnitude</td>
<td></td>
<td>Exam</td>
<td>Geometry</td>
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</tbody>
</table>

The Teaching Program follows Insight Mathematics Year 9 as a basic resource.
Appendix I

Year 9 classroom observation 1

The classroom observation began with me, the researcher in the room as the students entered. It took about 10 minutes before the majority of the class were settled and the lesson began. The students were mostly attentive during the teacher explanations but there were disruptions to the lesson flow as some students asked irrelevant questions.

The teacher attempted to make the questions relevant to the students by taking percentage discounts off a dress. With respect to the five questions the majority of the students copied the first question and then chatted amongst themselves. They were not engaged with the questions at all. The actual questions are in the table below.

<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>If ( b = 12 ) and ( h = 6 ), find ( A ) when ( A = 0.5bh )</td>
</tr>
<tr>
<td>2</td>
<td>Simplify ( 3xy + 2x - 2xy + 8x )</td>
</tr>
<tr>
<td>3</td>
<td>Dallas is paid $12.50 per hour and works 36 hours in a week. What is his weekly wage?</td>
</tr>
<tr>
<td>4</td>
<td>Find 10% of %624</td>
</tr>
<tr>
<td>5</td>
<td>The area of a rectangle is given by the formula ( A = l \times b ). If the area is ( 24 \text{ cm}^2 ), what values could the base and height be? HINT: base = 4 and height = 6</td>
</tr>
</tbody>
</table>
Because $A = 4 \times 6$

$= 24\text{cm}^2$.

What other values could $b$ and $h$ be?

Despite a number of students being off task and chatting, the teacher remained very positive with the whole class and provided the students with encouragement. In the discussion phase the teacher called on a number of students to answer questions and was very positive and supportive of their efforts. Unfortunately, there was a very strong atmosphere amongst the students that reflected that they could not do the questions.

During the five questions the students mostly chatted until the teacher came to their assistance. Following some explanation, the student completed that question, but the same process was continually repeated. It appeared that the level of the five questions was pitched too high. Following the lesson in discussion with the teacher she stated that the first question was too difficult and that had put off many of the students. She decided that she would give a similar but easier version of the same question in the next days’ five questions.

The class was not focused on the material in the lesson and the five questions chosen were too difficult for the students at that time.
Appendix J

Year 9 classroom observation 2

The class was reduced from the normal time due to a full school assembly held in the morning. As the class immediately followed the assembly some students were slow to enter. As the students entered they unpacked and immediately started on the questions. The students were all working on the questions and the classroom was much quieter than the first observation. Some students were having difficulty with the algebra questions but were looking back in their book to find the method of solution. This was a contrast to the first observation where the students chatted about non-mathematical things while waiting for the teacher’s assistance. A copy of the five questions are displayed below.

<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Expand and simplify $4(x+3) + 5(3x+10)$</td>
</tr>
<tr>
<td>2</td>
<td>Sally earns 5% commission on her sales of $58,000. How much did she earn?</td>
</tr>
<tr>
<td>3</td>
<td>Simplify $\frac{x}{6} + \frac{x}{3}$</td>
</tr>
<tr>
<td>4</td>
<td>James earns a gross salary of $120,000pa. He pays $30,000 in tax and $2000 in union fees. What is his net salary?</td>
</tr>
<tr>
<td>5</td>
<td>Sally expanded an expression and got: $24x + 18$</td>
</tr>
</tbody>
</table>
What could the expression have been? There is more than one possibility.

Hint: $24x + 18 = \text{(___ + ___)}$

After about twenty minutes the class moved on to the discussion phase of the FQA. All students participated in this phase, even the students that were very disengaged in the first classroom observation. Many students provided solutions and explanations of solutions.

Question 1 was not totally completed by some students as they expanded but did not simplify and a further example was given to consolidate this aspect. For question 2 the teacher gave another example using 5% commission and a number of students responded with suggestions for the solution. Question 4 had four different responses from the students and the teacher went back to earlier questions and the pizza analogy to consolidate their understanding. The definitions of gross and net income were presented. There was participation by all members of the class and the questions asked indicated cognitive engagement.

The FQA took the entire shortened lesson but the students’ learning experiences were operating at a highly affective level as they appeared to be enjoying the activities and the learning. The teacher was happy with the learning gains of the students through the questions and discussion of the solutions indicating high operative engagement through the active participation.
Appendix K

Year 9 classroom observation 2

The classroom observation took place in lesson 6 the final lesson of the day which follows a short afternoon tea break. As in the previous classroom observation some students were slow to enter. But as they entered they unpacked and immediately started on the questions. However, all five questions were ‘new’ to the students as they were all on topics for the new semester. This is not how the FQA should operate. As a result, there were a significant number of questions and a number of students off task. During the discussion phase, the FQA was used as a teaching tool to introduce the new topics for Term 3. A copy of the five questions are displayed below.

<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
</table>
| 1      | In a bag there are 4 blue, 2 yellow and 6 red marbles. What are the chances of picking:  
        | a) yellow  
        | b) A blue or red |
| 2      | Write $2 \times y \times y \times y \times y \times y$ with a power. |
| 3      | You achieved the following results on maths tests. What is your average mark?  
<pre><code>    | 80% 82% 90% 88% |
</code></pre>
<p>| 4      | Simplify: $y^2 \times y^4$ |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>There are 20 marbles in a bag, some red, some blue and some yellow. The chances of choosing a red one is 1 in 4. What marbles could possibly be in the bag?</td>
</tr>
</tbody>
</table>

Even though some students were disengaged there was strong engagement in the solution and discussion phase showing high operative engagement. This was evidenced by students asking questions, offering solutions and variations to solutions. In particular the probability question 5 involved a great deal of discussion of probability with students seeing the connections with ratio and probability.

Question 4 was answered by adding the powers and this was the expected correct response. The teacher then asked students to explain the reasoning behind the rule and a number provided an appropriate explanation.

The FQA should have had at least three familiar questions with two introducing the new topics. There were too many ‘new’ topics for the students to engage effectively. This was discussed with the teacher and modifications made for future lessons. The lesson still had far greater engagement than the first observed lessons with the high cognitive engagement evidenced in the discussion particularly around the probability question. The students were enjoying the questions and learning indicating high affective engagement. The FQA took all but the last five minutes of the lesson but given the amount of ‘new’ learning this would have been considered to be time well spent.
## Appendix L

**Year 10 2016 Scope and Sequence** –

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<th>Term 1</th>
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<tr>
<td></td>
<td>Indices</td>
<td>Probability</td>
<td>Data Analysis</td>
<td>Tools Down</td>
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<tr>
<th>Term 2</th>
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<tbody>
<tr>
<td></td>
<td>Equations</td>
<td>Trigonometry</td>
<td>Exams</td>
<td>Retreat</td>
<td>Tools Down</td>
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<th>Term 3</th>
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<td></td>
<td>Volume</td>
<td>Linear Relationships</td>
<td>Non-Linear relationships</td>
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<tbody>
<tr>
<td></td>
<td>Functions and Graphs</td>
<td>Exams</td>
<td>Awareness Work Activities</td>
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</tbody>
</table>

The Teaching Program follows Insight Mathematics Year 10 as a basic resource.
Appendix M

Year 10 classroom observation 1

The class observation occurred on the same day as the teacher interviews. The lesson did not use the five question approach and was a typical lesson before implementing the five question approach.

The lesson was very teacher focused with the teacher posing closed questions to the students and then calling on specific students to provide answers. The teacher tried to have the students pose their own question for others to answer to consolidate their knowledge. Examples were then developed from the discussion and students were asked to copy these down. From the observer’s perspective the teacher-centred focus was to maintain student involvement in the discussion. While the teacher saw this as engagement it was really just compliance. During this teacher-led discussion there was significant praise and positive reinforcement given to all the students.

Following the discussion, the students were posed a number of questions to complete from the homework book. About half of the students were able to commence working on the questions while the others copied the question and were not sure what to do. A number of students completed the questions, but their answers were incorrect. It certainly appeared that many of the students still did not understand despite this being the third lesson on the particular aspect of algebra. The level of student compliance was high, but the level of engagement was low.

During the completion of the work phase the teacher moved about the room and had one to one discussion with the students providing further advice where required. The
students were set homework for the following lesson which consisted of more questions from the homework book.

While the teacher attempted to involve the students in the lesson and discussion there was little time for them to reflect on the concept or to develop their own understanding. There were simply a number of rules and they had to learn them. The lesson was very procedural in nature without conceptual development. The students did appear to enjoy the lesson, but this seemed to be more based on the entertainment of an enthusiastic teacher.
Appendix N

St Charles’ Year 10 classroom observation 2

The class was observed during the first lesson of the day. The teacher had not seen the class for two weeks due to teacher meetings as head of department and illness. The lesson began with the five questions being written on the whiteboard. The questions are in the following table:

<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Simplify $2m^3 \times 4m^5$</td>
</tr>
<tr>
<td>2</td>
<td>Solve $4x - 5 = -13$</td>
</tr>
<tr>
<td>3</td>
<td>A bag contains 6 red and 5 blue disks. Two disks are selected without replacement. What is the probability they are both the same colour?</td>
</tr>
<tr>
<td>4</td>
<td>Find the mean and median of these scores: 10, 12, 12, 13, 15, 33. Which is a better measure of central tendency? Explain.</td>
</tr>
<tr>
<td>5</td>
<td>A square has side length $(x+3)$ cm. Calculate the value of $x$ given that the perimeter is 52cm.</td>
</tr>
</tbody>
</table>

The students arrived at class over a period of five minutes due to an extended tutor group. All except four commenced the five questions immediately on arrival at the class. The four had commenced within two minutes. The teacher did not move about the class communicating with the students as recommended in the approach but answered questions from the front of the room. The students were all on task and discussion between students centred on the questions.
The students connected the questions with previous work and could see the links.
The questions were well graded so that every student had an opportunity for success.
The discussion phase was teacher centred and all students were included in the
discussion as the teacher chose which student answered the questions. There were a
number of solutions given for question 5 showing the link between algebra and
measurement. Some students still struggled with the indices question.

Harriette deviated from the FQA as she gave the students five question homework.
This was done as the school had a policy that all mathematics classes were given
homework every lesson. The students’ level of engagement decreased during the
discussion of the five question homework and it appears that two sessions of five
questions were too much for the one lesson.

The ‘normal’ lesson then began with the teacher completing examples on the board
which the students copied into their books. Again, the entire process was teacher
centred and the students were forced to pay attention as they could be randomly
asked a question. The students were compliant but not engaged during this phase of
the lesson.

The lesson concluded with homework set from the homework book for the next
lesson.
Appendix O

St Charles’ Year 10 classroom observation 3

The class was observed in Term 2, week 9 during the second lesson of the day. The lesson began with the five questions being written on the whiteboard. The questions are in the following table.

<table>
<thead>
<tr>
<th>Number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Solve $3x-11=1$</td>
</tr>
<tr>
<td>2</td>
<td>Solve $4-5x = 7x -19$</td>
</tr>
<tr>
<td>3</td>
<td>Calculate the value of $x$</td>
</tr>
<tr>
<td>4</td>
<td>Calculate the value of $\Theta$</td>
</tr>
</tbody>
</table>

![Diagram](image)
A boat is at sea and the angle of elevation of the top of a cliff is $65^\circ$. If the height of the cliff is 120m calculate the distance of the boat from the base of the cliff.

From the beginning of Term 2 the teacher varied the way in which the FQA was done by only posing questions from the two topics for the half yearly examination. The focus was on the today aspect of the FQA with yesterday and tomorrow being omitted. Harriette focused on the two topics in the examination which enabled her students to perform to the best of their ability in the examination.

The students entered the room and immediately began the five questions. The teacher moved about the room assisting the students as required. A few students had issues with the equation from question 2 as a result of not keeping the balance of the equation. Questions 3 and 4 posed no difficulty but there were a number of students confused in question 5 with the positioning of the angle of elevation.

During the discussion phase the teacher gave a number of questions involving angles of elevation and depression to consolidate the students’ understanding. There was high operative engagement with all the students actively participating in the discussion resulting from question 5.

This observation showed a lesson quite different from the teacher-directed first observation with the student voice taking a more prominent role in the learning. The lesson then continued with a discussion of bearings as the ‘normal’ part of the lesson. The examples were presented in a less teacher-centred manner with greater voluntary input from students indicating they valued the learning thus displaying
high affective engagement. The lesson concluded with homework questions on trigonometry.