Chapter 1 – Introduction

1.1 Astronomy requirements of the new antenna design concepts

The main driving force for building a new large radio telescope, is for astronomers to look at the distant universe to observe the processes which first imposed structure and order, allowing galaxies and stars to form, more commonly known as the “Big Bang Theory”. It is believed that the quasi-CW microwave radiation from hydrogen atoms – the HI emission needs to be observed, which is in the frequency range between 0.2GHz – 2 GHz [71]. Sensitivity can only be improved by increasing the collecting area, and this in turn is only practically feasible if it is in the form of an array [71]. Multibeaming is another requirement for reasons of science, effective mitigation and observing efficiency as it increases the number of simultaneous observations at any one time [71, 34].

1.2 The Square Kilometer Array (SKA) Antenna Specifications

The antennas for the SKA could comprise of a large number of small steerable satellite dishes, spherical radio lenses or flat sheets which are electronically steered. The SKA could be constructed by being dense in the middle of a station of antennas and sparse at the outer limits of the array of antennas [68].

1.2.1 Necessary Specifications

The Necessary specifications for the SKA are: [209, 81]

- Effective Area of 1 Km²
- Large dynamic Range.
- Frequency Range of 0.2 to 2 GHz and later extending to 10GHz.
- Dual Polarisation.
- Full Sky Coverage.
- Multi-Beaming possibilities. [13, 95]
- Instantaneous bandwidth 100-200 MHz.
1.2.2 Desirable criteria

Other criteria which is desirable for the SKA are: [92]

- Multi-Frequency Operation.
- Interference Excision.

1.2.3 Understanding the Requirements

The main factors of interest are listed below:

- The antenna would have to be very inexpensive, due to the limited funding. Hence, it is very important to obtain an optimal antenna design within the budget (research funding), and due to the non-commercial aspects of the project.
- It would have to be repeatable, for ease of building and future replacements of components for the large required effective area of 1 Km².
- The frequency range would have to be 0.2 to 2 GHz with an instantaneous bandwidth of 100-200 MHz.
- Dual-Polarisation and full sky coverage desirable.
- Multi-Beaming desirable to maximise usage for a device of such calibre.
- The constant increase in interference due to modern technology would require interference excision.

1.3 Examples of some large Radio-telescopes

A notable large radio telescope which was developed is the 300m diameter fixed spherical reflector telescope at Arecibo, Puerto Rico, where the beam was originally steered by moving the linear feed [42], and in more recent times by a Gregorian Focal Cabin.

A versatile flat and large radio telescope is the Soviet RATAN-600 Radio Telescope, which has four different modes of operation, and these can be used separately or together to provide simultaneous observations [99]. This antenna forms a strip of a paraboloid, made of individually adjustable plates [99][42].
The Kraus-type of antenna has been quite popular as well and was built originally in Ohio, USA with a larger one built later in Nancay, France. This design combines a fixed parabolic reflector combined with a tiltable plane reflector [42].

Perhaps the most popular way of constructing antennas with large collecting areas is to use several smaller reflective dishes together in some particular configuration as an array, to provide an overall large collecting area. An example is the VLA (Very Large Array) in New Mexico.

1.4 Comparing various antenna types

1.4.1 Non-reflector directional antenna

These include log periodic, helical, yagi or horn antennas, of which the log periodic antenna is perhaps the lightest and easiest to mechanically steer whereas the remainder provide a higher gain [42, 32]. Examples of some other efficient but odd shaped antennas are trihedral twist-grid polarimetric reflectors [113], and modified folded monopoles using the genetic algorithm method [5].

1.4.2 Phased arrays [29, 14, 45, 190]

1.4.2.1 Advantages of Phased Arrays

- Flat.
- Multi-Beam capability
- Repeatable, so hence easy for mass production.
- No cumbersome mechanical structures for manoeuvring the antenna.
- Phased arrays in the form of Fractal antennas can provide the frequency range required.

1.4.2.2 Disadvantages of Phased Arrays

- Expensive.
- Generally narrow band in operation unless used as a fractal antenna configuration.
- Coupling effects.
• Grating Lobes.
• Loss of Gain, beam shape and polarisation discrimination under scanning conditions.

1.4.3 Reflector Antennas

1.4.3.1 Advantages of Reflector Antennas

• Relatively cheap.
• Can provide full sky coverage as required, provided the necessary mechanical steering is available.
• Inherently broadband and limited only by the feed arrangements.

1.4.3.2 Disadvantages of Reflector Antennas

• Large and cumbersome mechanical structures for manoeuvring the antenna.
• Generally only a single beam capability.

Lightweight Reflectors made by wires [65] and mesh [43] make it feasible to use these antennas in space and it also makes them cheaper [48-51], [128], hence a similar method of construction could be utilised for the purposes of the SKA as well.

1.4.4 Analysis of Antennas using Optics

The two methods of interest are the Physical Optics Method and the Geometric Optics Method, which will be briefly described in the following sub-sections.

1.4.4.1 Physical Optics

Physical Optics calculates the reflected fields from the induced currents on the reflector surface and then re-radiating them [57]. The most commonly used concept is the Physical Optics approximation, which states that if an object is large the total field can be assumed to be negligible in the shadow region and hence the surface current is simply given by $J_s = 2n \times H^i$ where $n$ is the local normal to the surface and $H^i$ is the incident field [76].
1.4.4.2 Geometrical Optics

Geometrical Optics is used to calculate the aperture fields from the feed patterns reflected or refracted by surfaces/structures on the aperture antennas [180]. The basis of Geometrical Optics [121-123] is that energy flows along straight ray paths in TEM modes. The corresponding phase is determined by the path lengths and the amplitude is determined by the convergence/divergence of the ray tubes [180].

1.4.5 Lens Antennas

A historical variety of types of lens antennas is discussed below, outlining advantages and disadvantages for the problem at hand.

1.4.5.1 Reflectarray technology

This new method is of particular interest, as it combines microstrip array technology and traditional paraboloid reflector antenna, and is designed to have very high gain and efficiency and can also be scanned electronically by phase shifters or mechanically by large angles up to 50° [82]. Reflectarrays have dual-linear and dual-circular polarisation capabilities [82]. High costs which are usually incurred in beam scanning antennas can be eliminated by the use of micro-controllers in each element [82].

1.4.5.2 Luneburg Lens

The Luneburg Lens is a spherically symmetric, delay type lens formed of a dielectric, where the incident plane wave is brought to focus on the opposite side [26]. Signals can be received simultaneously with a Luneburg lens from anywhere in space and the number of signals would equal the number of receivers placed on the lens [101]. Hence lenses such as the Luneburg lens are flexible with complete angular coverage, but their construction is more complicated than that of a reflector, and it gets more expensive with increased performance requirement [26].

1.4.5.3 Metal-Lens Antennas

A metal lens antenna uses the optical properties of a radio wave, it consists of a number of conducting plates of proper shape and spacing and is a lens, that focus waves [98]. The lens is similar to a parabolic reflector as it transforms the spherical
wave into a uniphase wavefront at the aperture [98]. It can be used successfully for a wide range of frequencies. One dimensional sectoral horns flared out with a lens at the aperture form an important horn antenna, where wires can be used instead of metal lenses to reduce material costs and weight at lower frequencies. It has been observed that many optical instruments could be duplicated at radio frequencies by the use of metal plate refractive mediums. Metal plate optics have also proven to have exceptional wide angle scanning properties [165].

1.4.5.4 Bootlace Lens

The bootlace lens uses a planar, homogenous wave-transmission region, which has a set of feed ports on one side and the lens ports on the other [173]. It is a variation on the metal plate waveguide [96]. Bootlace Lens were initially popular in the 1950's for rapid mechanical scan by means of feed motion, and later for their capabilities of a multiple beam system [173]. The R-2R, bootlace lens, focuses perfectly at an infinite number of points in practice [96]. An example of an R-2R lens is a pair of parallel circular sheets, spaced less than one half wavelength apart, of radius R that are placed against a pair bounded by a circular arc having a radius 2R [56].

1.4.5.5 Rotman Lens

A Rotman lens is a trifocal bootlace lens with a parallel plate waveguide which is symmetrical about the lens axis and has one focal point on the axis and a circular feed arc [158]. Rotman lenses are wide angle electronically scanned lens antennas [160, 118]. Fine scanning can be achieved in two ways, either by mechanically moving a horn along the input arc, or by electronically commutating a weighting function over the input probes [46]. The excitation of a single input produces nearly uniform illumination across the aperture, and the far field pattern is a sinc function [46]. In designing Rotman lenses it is important to be very wary of mutual coupling between the feed and collector [118] and also internal reflections, as these will affect the amplitude and phase distribution of the radiating arrays.
1.4.5.6 Pillbox Antenna

The first model of the microwave analogue of the Schmidt was quite different in appearance to its wide-angled optical counterpart. This was known as the pillbox type antenna, as it had reflectors within a parallel plate region. The initial studies were carried out using a two-layer pillbox, fed with a TEM wave [38-39]. The optical Schmidt itself consists of a spherical mirror, with a thin refractive lens located at the mirror’s centre of curvature.

Rapid scanning by physical movement is impractical for a paraboloid antenna, but this can be better achieved by placing the feed between parallel plates, which produces a line feed, when scanning is easily possible [132]. When rapid scanning is required in radar applications, it is convenient to use microwave radiation focussed in one plane only [198]. This is done using a pair of parallel conducting sheets, which gives a fan shaped beam, but is also advantageous to use at another location with an added lens in front, giving a pencil shaped beam. Metal lenses generally offer a larger freedom of design when compared to optical lenses [56].

The double layer pillbox is a microwave parallel plate system in which the image and object space, relative to a two-dimensional internal reflector are electrically separated by a metal septum. The double layer prevents shadowing effects and the impedance mismatch which is often found in a single layer system and also corrects optical aberrations. Circular reflectors as mentioned before are advantageous as the lens can be spun around for rapid scanning at lower frequencies [161] and circular aberrations can be corrected by lenses.

A wide angle double layer pillbox is ideal for multiple beam generation or reception, as there is no aperture blockage usually caused by multiple feeds. This system thereby allows the use of many independent beams either sequentially or simultaneously. Lens fed multiple beam arrays have been used extensively and come in linear, planar, arc or cylindrical shapes which can be flat or curved, thereby giving adequate flexibility for the purpose of the design [6].
For a smoother bend, at a lower cost, different shaped slots and dielectric loaded slots have been used, thus showing that the insertion of a properly designed dielectric insert proved quite beneficial, as it reduced the reflection levels of the fold [143-148].

1.4.6 Methods for analysing Lens antennas

There are several ways of solving Maxwell’s Equations, which describe electromagnetic fields in any medium. The methods discussed briefly in this section include ‘The Mode Matching Method’, ‘The Moment Method’, ‘The Reciprocity Method’ and ‘The Finite Difference Method’. Finite parallel plate waveguide arrays have also been successfully analysed using the integral equation method [203].

The parallel plate waveguide was used for the “Interscan” project at CSIRO [143-148], where the methods suggested for solving the parallel plate system are the “Mode Matching” method, “Moment” method and the “Variational” approach [143-148]. The “Moment” method as well as the “Reciprocity” method make use of the exact Green functions, although the “Reciprocity” method has been shown to be computationally more accurate [78-79].

1.4.6.1 Mode Matching

Initial stages of mode matching were referred to as the point matching method [9], which is the conventional modal matching method and uses the sequence of least squares for the solution of perfectly conducting waveguides [137]. Mode matching has been used extensively to calculate fields in arbitrarily shaped waveguides and junctions [36, 155]. It is possible to expand the two port mode matching analysis of a parallel plate 180° bend to a three port mode matching analysis [17]. A set of scattering parameters to characterise the entire device can be evaluated, such as is used in characterisation and circuit design of transistors [17]. When considering arrays of lenses the radiation pattern with the sharpest cut-off for the sidelobe level is a critical issue, and needs to be considered as well [112].
The “Mode Matching” method, was shown capable of solving waveguide coupling problems in the “Interscan” project and is of particular interest [11]. It was in this case used similar to a waveguide, assuming waves travelling at angles i.e. higher modes, rather than the simple TEM wave [143-148].

1.4.6.2 Moment Method and Reciprocity Method

Methods such as the Moment method [75] and the reciprocity method are also popular methods for solving parallel plate problems [78, 79]. The Method of Moments is a general procedure for solving linear equations and owes its name to the process of taking moments by multiplying with appropriate weighting functions and integrating [130]. Two numerical methods that are used for solving these equations are the conjugate gradient and pseudoinverse method [183]. These methods will not be used in this thesis, hence they will not be discussed.

1.4.6.3 Finite Element Method

The finite element method can be applied to a wide variety of analytical problems, which include the areas of engineering and physics [183]. The Finite Element Method is implemented on an integral formulation of the boundary value problem. It has the advantage over the Finite Difference Method of using triangular elements, which can fit curved boundaries more easily. The Finite Element method is essentially applied to the cross section of a device rather than the propagating wave [183]. However the Finite element method is not the best method to be used for waveguide structures [183].

1.4.6.4 Finite Difference Time Domain Method

The Finite Difference Time Domain (FDTD) Method was first formulated by K.S. Yee in 1966. The method involves solving the time dependant Maxwell’s equations, where the partial difference equations are replaced by a set of central finite difference equations [207]. By using this method the waveforms at a particular instance in time within a structure can be derived, hence making this method quite desirable for the solution of fields within a parallel plate network. The advantages of the FDTD method are numerous, it works for a wide range of frequencies, stimuli, objects, environments,
response locations, and can be made as accurate as required by increasing the cell number, although it is accurate for even a small number of cells [104].

1.5 Other Antenna Interests based on the SKA project

Within the Australian SKA program most antenna design effort is presently centred on the investigation of spherical three-dimensional radio lenses, known as Luneburg Lenses [70] and a cylindrical reflector doublet antenna [31].

1.5.1 Luneburg Lenses

Luneburg lenses are expensive and heavy and the number of lenses required for a large scale antenna project such as SKA would be several (100’s of) very large lenses [93]. Hence, effort is targeted towards using cheaper and lighter dielectric material which more or less possess the same performance as traditionally used material [69, 70]. Two types of dielectric materials include, an artificial dielectric comprising dielectric spheres embedded in a low-permittivity foam and foamed plastic [100]. Some examples of artificial dielectric materials that have been investigated are “Polyurethane”, “Polystyrene”, “Fused Quartz” and “Alumina Ceramic” [100].

The virtual source Luneburg lens is an alternative solution, to reducing weight and hence costs, in this configuration a hemispherical Luneburg lens would be situated over a conducting ground plane [93]. The energy from the source entering the lens is reflected from the ground plane to exit the lens as though from a virtual source from the ‘missing’ hemisphere below the ground [93].

1.5.2 Cylindrical Reflector Doublet Antenna

This consists of two offset fed cylindrical surfaces supported at an angle of 45° and mounted on an azimuth drive [31]. Phasing of the line feed provides elevation steering and with the azimuth drive full sky coverage is achieved and at any one time the field of view extends from the horizon to zenith [31]. This antenna is supposed to satisfy practically all the specifications of SKA [33], but the mechanical aspects of the design may be cumbersome.
1.6 Choice of Antenna Design to Satisfy SKA Specification

A parallel plate pillbox antenna actually uses the advantages of both a reflector and a phased array type antenna and provides a possible solution to the main requirement of the 1kT antenna.

Sections 1.4 and 1.5 described several antenna proposals for the SKA project. The parallel plate lenses have been chosen to be analysed and proven with measurements for this thesis as a prospective SKA Antenna. The reason for the choice of a parallel plate lens is because it promises to satisfy several of the SKA requirements discussed in Section 1.2.3, such as multi-beaming capabilities, low cost, dual polarisation, easy to manufacture and replicate and can cover the desired frequency range.

The Parallel plate lenses that have been chosen would be of a size closer to a phased array element and can make up an array sub-element rather than using traditional reflector antennas, which can be large and cumbersome. These lenses placed in both a horizontal and vertical alignment to each other can also solve the dual polarisation requirement.

Only one feed point is required to cover the entire bandwidth. The beam can be simply steered by moving the feed point, rather than using mechanical steering or individual phase shifters [91].

The other possibilities with this method are also multi-beaming, multi-frequency operation and interference excision [91]. There is no blockage due to the feed or the sub-reflector, and hence as many reflectors as required can be used for maximum steering (sky coverage) and minimum alteration of reflector profile or feed location.

1.7 Simplification of the Problem

The analysis of the parallel plate pillbox as an overall 3-dimensional structure would be quite a complicated task. By simplifying the device into two separate 2-dimensional problems, the final individual results for each cross section can be obtained. A Finite Difference Time Domain method or a Mode Matching method can be used to analyse the
vertical cross section of the parallel plate. A physical optics or geometrical optics method can be used for the reflector analysis in the horizontal cross-section. Hence, if the S-parameter result of each cross-section of the device is combined by cascading, then an overall 3-dimensional result can be obtained.

As a result of the work which has been carried out, there is little doubt that the full 3-dimensional analysis of this problem could be achieved using the Finite Difference Time Domain Method.

1.8 Project Definition

1. The two most suitable analysis methods chosen in this thesis to solve the fields in a two dimensional parallel plate pillbox are Mode Matching and Finite Difference Time Domain (FDTD) Method.

Mode Matching determines the fields at the output of a device by cascading the S-Parameter results of each section of the device, and obtaining an overall set of S-parameters [197]. One of the objectives in this thesis is to develop a set of equations by using the same approach as a T-Junction and also to study the possibility of extending the 2-Dimensional analysis to achieving a 3-port S-parameter solution for the entire structure.

FDTD uses the method of representing Maxwell’s equation (usually represented by partial differential equations), by a set of finite difference equations. The other objective of this thesis is to investigate the efficiency of evaluating fields for a parallel plate pillbox with a discontinuity using the FDTD method. The two main types of discontinuity that this thesis will concentrate on are a 90° bend and a 180° bend. The analysis will be carried out for a 2D solution however the possibility of extending the analysis to achieving a 3 dimensional solution for the entire structure will be investigated and results will be presented.
2. Design a physical parallel plate pillbox, in order to make comparisons between measurements and the analysis in objective 1, using either Mode Matching, FDTD or a combination of both as the analysis method.

The pillbox structure cannot be designed or constructed for the experimental set-up using the frequency specifications required for the SKA due to the large wavelength. A scaled model at a higher frequency would prove less cumbersome and still satisfy the aim of this part of the thesis by providing a physical insight into an experimental model of a pillbox.

3. Analyse the fields within a parallel plate device by the most suitable method chosen in objective 1. Carry out 2-dimensional analyses for a 90° bend and a 180° bend parallel plate pillbox.

4. Measure and record practical measurements and compare them to the analysis and theoretical results obtained in the previous section. Establish the success of the method used for the parallel plate pillbox.

5. Once the 2-dimensional analysis has been established successfully in objectives 1 to 4, it is essential to resolve the 3rd dimension of the parallel plate pillbox, the reflector profile. Study various reflector profiles such as circular or parabolic, and using the method of optics analyse the structure.

The optics methods that are selected to be reviewed are physical optics and geometric optics and these are studied in detail.

6. Using both methods in objective 5 study ways of reconfiguring the reflector profile to scan the beam, rather than steering the entire reflector. Also study multi reflector set-ups to reduce the focal length and to minimise the level of re-configuration required for the reflector profiles, to increase flexibility within the parallel plate design.
Establish the preferred method to complete the analysis of the 3rd dimension of the parallel plate pillbox.

7. Analyse and record measurements from practical set-ups of monopoles, dipoles and reflector surfaces. Consider new innovative methods of achieving multiple frequencies using direct and indirect switching in the proposed monopole structure.
Chapter 2 – Finite Difference Time Domain Method and Mode Matching Method

2.1 Summary of FDTD Equations

It is known that Maxwell’s equations, usually represented by partial differential equations, can be replaced by a set of finite difference equations [207]. The speed and memory capacity of present day computers, permit the implementation of these difference equations to produce numerical solutions quite easily. A conducting surface is approximated as surfaces of cubes, each parallel to the co-ordinate axis. Applying boundary conditions sets the tangential components of electric field and the perpendicular components of magnetic field to zero at the conducting surface.

Maxwell’s Equations in an isotropic medium are: [179]

\[
\frac{\partial B}{\partial t} + \nabla \times E = 0 \tag*{E2. 1}
\]

\[
\frac{\partial D}{\partial t} - \nabla \times H = J \tag*{E2. 2}
\]

\[B = \mu H \tag*{E2. 3}\]

\[D = \varepsilon E \tag*{E2. 4}\]

The 2-dimensional Finite Difference equations for Maxwell’s equations above are given as follows [207],

\[
H^{n+0.5}_z(i + 0.5, j + 0.5) =
\]

\[
H^{n-0.5}_z(i + 0.5, j + 0.5) + \frac{\Delta t}{\mu} \left[ \frac{E^n_y(i + 1, j + 0.5) - E^n_y(i, j + 0.5)}{\Delta y} \right] - \frac{\Delta t}{\mu} \left[ \frac{E^n_y(i + 0.5, j + 1) - E^n_y(i + 0.5, j)}{\Delta z} \right]
\]

\tag*{E2. 5}
\[ E_{y}^{n+1}(i + 0.5, j) = E_{y}^{n}(i + 0.5, j) - \frac{\Delta t}{\varepsilon \Delta x} \left[ H_{z}^{n+0.5}(i + 0.5, j + 0.5) - H_{z}^{n+0.5}(i - 0.5, j + 0.5) \right] \]

\[ E_{x}^{n+1}(i + 0.5, j) = E_{x}^{n}(i + 0.5, j) - \frac{\Delta t}{\varepsilon \Delta y} \left[ H_{z}^{n+0.5}(i + 0.5, j + 0.5) - H_{z}^{n+0.5}(i + 0.5, j - 0.5) \right] \]

The derivation of these equations, from the basic Maxwell's equation is given in Appendix A2 to this chapter and that derivation follows Yee's development [207].

2.2 Background

Finite differences have been used for years as an approximation of the integral solution, which otherwise become lengthy and unwieldy to handle especially in the case of irregular shapes [179].

In 1966, K.S. Yee developed the first finite difference equations in the time domain [207], and this in conjunction with later developments by others has been used in this chapter and for the analysis of the various bends in a parallel plate device.

Comparison of the FDTD method with the Method of Moments for modelling reflector systems, suggests less computational effort and more precision for the FDTD method [97,187,194]. The FDTD method has been used for the analysis of a parallel plate discontinuity in this chapter.

90° bends/discontinuities can also be analysed by truncating the bend and representing the problem in terms of a series of first order differential equations, which can then be solved [37,129]. This is another way to study parallel plate bends, but the FDTD method is considered to be a neater approach [134, 135, 83].

When the fields are required in an unbounded antenna system, the use of absorbing conditions is useful, as this prevents numerical reflections from the source. [131,
186,188]. Hence it is important not to extend beyond the source plane unless using absorbing conditions at the source plane.

For a parallel plate pillbox, which is generally several wavelengths in length, it would be very inefficient to model the entire structure, in terms of computation time and memory requirements. The reason behind this is the exponential increase in the size of the evaluating matrix when a large structure is evaluated using the same grid for the whole structure.

A method of sub-gridding can be used for structures where the grid can be fine around the vicinity of the discontinuity and can be quite large when there is no change in profile of the device. This obviously, is a logical and simple process to minimise unnecessary computing time [208].

2.3 Input Signal

The choice of the input signal is very critical to the overall results and the analysis of the device under test. The reason for choosing two different input pulses is mainly to show consistency of the results from the FDTD equations used for the parallel plate lenses, whilst exciting it with pulses that are either symmetrical (gaussian) or cyclic (sinusoidal) waveforms in the time domain.

The FDTD method can be used to evaluate fields for a limited portion of the parallel plate pillbox until such time as when the forward and reflected wave settles down, which depends on the nature of the discontinuity and also the input signal.

In all the computer analysis carried out within this chapter, two types of input waveforms were used as the excitation. The first is a half wave sine pulse at the required frequency of 10 GHz, with a normalised amplitude of 1. The sinusoidal pulse that was used in the preliminary field calculations is given in Figure 2.1, and the FFT of the pulse is given in Figure 2.2.
Figure 2.1 Half wave Sinusoidal pulse used for the FDTD calculations.

Figure 2.2 FFT of the sinusoidal pulse used for the FDTD calculations.

The second input pulse is a Gaussian pulse which is usually used as an excitation in time-domain calculations, due to its smoothness in time, and the ease with which it can be adjusted for a specific pulse width [110, 104, 23, 25]. The Fourier Transform of a
Gaussian time pulse results in a Gaussian pulse in the frequency domain. The relationship of the time pulse and its FFT are shown in Figure 2.3 and 2.4 respectively.

The Gaussian excitation pulse is given as

\[ E_p(t) = e^{-\left(t-t_0\right)^2} \]  \hspace{1cm} \text{E2.8} 

where 

\[ f = \text{frequency} \] 
\[ t = \text{time} \] 
\[ t_0 \text{ ensures the causality of the system \cite{110}.} \]

\[ \text{Figure 2.3 The Gaussian pulse used for the FDTD calculations.} \]
2.4 Stability Criterion

A Stability Criterion needs to be used whenever a discontinuity occurs in a structure. Figure 2.5 illustrates the positions of the electric and magnetic field components about a unit cell of the FDTD lattice in Cartesian coordinates [Yee]. Velocity is chosen as $c$, which is the speed of light in free space and satisfies the analysis undertaken.
The choice of $\delta$ and $\Delta t$ is selected for accuracy and stability respectively. $\delta$ must be small compared to the wavelength to ensure the accuracy of the spatial derivatives of the EM fields.

To ensure the stability of the time stepping algorithm expressed in $E_x$ and $E_z$, $\Delta t$ is chosen to satisfy the inequality

$$\Delta t \leq \frac{1}{c_{\text{max}} \sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}}$$

E2. 9

where

$\delta x = \delta y = \delta z$, is the space increment shown in Figure 2.6.

$\delta t$ is the time increment

The general 3 dimensional stability criterion is given [185] as

$$\left(\frac{1}{\delta x^2} + \frac{1}{\delta y^2} + \frac{1}{\delta z^2}\right)^{\frac{1}{2}} \leq \frac{1}{\delta t}$$

$$\Delta t \leq \frac{\delta}{c_{\text{max}} \sqrt{3}}$$

for a 3D (dimensional) lattice and, .................................................... E2. 10

$$\Delta t \leq \frac{\delta}{c_{\text{max}} \sqrt{2}}$$

for a 2D lattice, when $\delta z = 0$ .................................................... E2. 11

![2 D and 3D lattice showing space increments](image)

Figure 2. 6 2D and 3D lattice showing space increments
Because the H field is calculated at every half a time step $\Delta \tau = c\Delta t$, it also establishes the criteria that the ratio $\frac{\Delta \tau}{\Delta x} = \frac{\Delta \tau}{\Delta y}$ cannot be larger than $\frac{1}{\sqrt{2}}$ for a 2-dimensional grid, to provide a real solution [155].

Figure 2.7 Distribution of fields in a cell of a FDTD model.

Figure 2.7 graphically displays the distribution of the fields in a FDTD model cell, which is used as the basis for the analysis of any structure/device, which needs to be sub-divided into cells. A pictorial representation of the fields along and between the plates are given in Figures 2.8 and 2.9, and the explanation of the development is provided alongside the results.

2.5 Simple manual verification of the FDTD Method for a transmitting and reflected wave.

The arrow along the cells in Figure 2.8 show the location of the front edge of the input pulse (which is a 5 point sinusoidal pulse where the point values for $E_y = [0.5, 0.866, 1, 0.866, 0.5]$), as it travels through the structure. At time step $n=2$, when the pulse reaches $i=1$, the FDTD method approximates the value to be 0.404, when it should have actually been 0.5, but the size of the cell which is $\frac{\lambda}{12}$, causes a 8.3% error at $n=5$, which can be drastically improved with a smaller grid size. The same error trend continues as the FDTD method approximates the subsequent predictions. As can be seen at $n=5$ where at
i=1 the value for Ey is 0.917 instead of 1.0 and at i=2 the value for Ey=0.67 instead of 0.5. The values of Hz at i=0.5 and i=1.5 are 0.002 instead of 0.0023.

Using a 5 point sinusoidal pulse where the point values for Ey = [0.5, 0.866, 1, 0.866, 0.5]

Figure 2.8 Distribution of fields for various time steps for a $\frac{\lambda}{12}$ grid size.
Figure 2.9 Distribution of fields for various time steps for a $\lambda/24$ grid size.

It can be seen in Figure 2.9 at n=5 and i=0 that the value for $E_y$ is 0.9659 instead of 1. Hence, using a finer mesh gives better results immediately, as the values making up the sine wave in the structure are closer to the input sinewave, than with the courser grid size of Figure 2.8. The smaller grid size results in a smaller error factor of 3.4% at n=5 as shown in Figure 2.9.

The grid size that gave optimum results with minimum computer effort was $\lambda/64$, which has been used for all calculations in this Chapter and the next (Chapter 3). The input Gaussian pulse signal is shown in Figure 2.3, and the plot of the 500 point FFT of the Gaussian is shown in Figure 2.4.
2.6 Computer verification of the FDTD Method for a transmitting and reflected wave

An electric field pulse with a sinusoidal half wave shape, is fed into a structure with a reflector located at a certain distance. As the wave hits the reflector, it is expected that the wave would be reflected back with a 180° phase shift, to ensure zero electric field tangential to the reflector, giving negative results to the initial values of the sinusoidal pulse.

The equations were interpreted and written into a computer program, and the program was run using various grid sizes. A more accurate and smoother sinusoidal pulse is achieved by using a denser grid.

At the time when half the pulse has hit the reflector, the forward and backward waves cancel and give a null resultant field.

The results for the various grid sizes are similar, but more accurate with a smaller grid size, as in all the cases of grid size, the pulse is reflected after it hits the reflector, where the electric field is set to 0. At the minimum x value the reflected wave bounces again, as the value of the field is zero before the minimum value, the wave returns to the reflector, like the initial forward wave. This simple exercise satisfies the general electromagnetic wave theory, and also verifies the interpretation of the Finite Difference Time Domain equations.

2.7 Verification of the FDTD Method for a parallel plates with a step up discontinuity

![Figure 2. 10 Parallel plates showing a step up discontinuity.](image)

Figure 2. 10 Parallel plates showing a step up discontinuity.
A parallel plate with a step up discontinuity is shown in Figure 2.10 and Figure 2.11, which was used as the simplest discontinuity to verify the written program and the FDTD equations. The number of divisions in a wavelength (\( \lambda \)) is 60 points, each \( \lambda/12 \) cell is further divided into 5 to give more accurate results. The time step is equivalent to \( \lambda/120 \) which is half the size of a distance step, which is \( D_x = D_y = \lambda/60 \) as shown in Figure 2.11. The stability criteria was discussed in Section 2.4, and expressed by Equation E 2.10 as

\[
\delta t \leq \frac{\delta}{c\sqrt{2}} = \frac{0.707\delta}{c}.
\]

From this equation it is appropriate to choose the ratio of 'time increments' to 'distance on velocity increments' to be \( \frac{1}{2} \), which allows a more convenient visual conceptualisation of the fields in a device.
When the stability criteria is not followed, the results appear to be correct when there is no discontinuity in the device, but the instant a discontinuity is present in the device, the results become quite erratic and incorrect.

The results for various cross-sections for a parallel plate of $1\lambda$ in length on either side of the step (Figure 2.11) are shown in Figure 2.12, Figure 2.13 and Figure 2.14. There is no loss before the input signal encounters the step discontinuity, as shown in Figure 2.12. A slight loss in magnitude of the input Gaussian field which is normalised to 1, can be seen in these figures, after it crosses the step discontinuity. The reduced magnitude is caused by the multiple reflections that occur due to the step, and these can be seen in Figures 2.13 and 2.14. Although there are differences in the fields initially at the presence of a discontinuity, they settle down very quickly almost within half a wavelength of the operating frequency. The reflected fields remain slightly different, but the ongoing/transmitted part of the pulse is a smooth sine pulse as can be seen in Figure 2.14. The FDTD method has been used quite successfully to analyse a step discontinuity in a parallel plate.

![Graph of Normalised Magnitude vs Distance from Input - Wavelength (Lambda)](image)

**Figure 2.12** The forward pulse for all the cross-sections of the step discontinuity, at a time snapshot of $n=t^*c=120$ (t=0.4μsec), before the input pulse reaches the discontinuity.
$A t \ y = \left( \frac{\lambda}{3} \right) + \left( \frac{\lambda}{24} \right)$

$A t \ y = \frac{\lambda}{4}$

$A t \ y = \frac{\lambda}{6}$

$A t \ y = \frac{\lambda}{12}$

Figure 2.13 The forward and reflected pulse for the cross-sections of the step discontinuity, at a time snapshot of $n = t^*c - 180$ ($t = 0.6 \mu$sec).
Figure 2.14 The forward and reflected pulse for the cross-sections of the step discontinuity, at a time snapshot of n=x*T=240 (t=0.8μsec).
2.8 The Point Matching Method and the origins of Mode Matching

Determining the electromagnetic fields in the vicinity of a waveguide discontinuity has many different methods of solution. An exact solution using the integral-transform technique is not always possible, and hence approximations become essential [117]. These will be discussed in this chapter.

The lack of any systematic method for estimating 'a priori' the validity of such hypothesis, was the main difficulty in applying the point matching method [106-109, 167]. But this can be overcome by the method of overlapping regions used to solve waveguide discontinuity problems and can be used in turn for a parallel plate problem as well. An initial assumption can be iterated in a few steps to the final solution by Schwarz's method in combination with the point matching method [85].

Waveguide bends and the analysis of waveguides using mode matching methods has been dealt with elaborately, by Wexler [197] and others in more recent times, however very little work has actually been carried out on parallel plate networks [136]. Mode matching can also be used efficiently to model small antenna systems, which are generally difficult to analyse due to the presence of multiple interactions between feed, subreflector and main reflector [191].

A right angle bend can be subdivided into 3 areas as shown in Figure 2.12 below [107].

The field in region I is given as \( H_y = e^{-j\beta z} + \sum_{n} R_n e^{i \xi z} \cos \left( \frac{n \pi z}{b} \right) \) \( \ldots \ldots \ldots \ldots \quad \text{E2. 12} \)

where \( k = \left( \frac{2\pi}{\lambda} \right) \).

The field in region II is given as \( H_y = T_o e^{-j\beta z} + \sum_{n} T_n e^{-i \xi z} \cos \left( \frac{n \pi z}{b} \right) \) \( \ldots \ldots \ldots \ldots \quad \text{E2. 13} \)

Where \( R \) and \( T \) represent the reflected and transmitted wave amplitudes respectively.
In region III the H field is given as, $H_y = \int_0^\infty f(n) \cos \left( \frac{n\pi x}{b} \right) \cosh[\Gamma n(z - b)] dn \quad \text{E2. 14}$

Where the trigonometric forms give zero tangential electric fields on the metal walls. The derivatives of the fields in region I and III and region II and III can be matched at their respective discontinuities to give a complete and rigorous solution.

Various modifications [166, 171] to the original modal matching method have been used to solve for the characteristics of various types of discontinuities [20-22]. Another hybrid method is the full-wave mixed potential mode matching, which is used successfully to analyse planar and quasi-planar transmission lines [192]. Orthogonal-series expansions of the fields can be used to analyse microstrip discontinuities and T junctions [202]. The mode matching method has been used successfully to analyse the Magic-T-junction [177], and corrugated waveguide mode converters [90].

Arbitrarily shaped waveguide discontinuities have been solved using a full-wave boundary contour mode-matching method [156]. E-plane right angle waveguide bends (both mitered and sharp cornered bends) have been successfully analysed [4, 199] using the mode matching method [64]. A combined mode matching [54] and finite difference method can be used which takes advantage of the high numerical efficiency and versatility of the two methods respectively [126].
Comparisons have been made between the various techniques such as differential equation [84], mode matching, circuit theory [171], resonator method, standing wave formulation, transverse resonance method [20] and others [205,206].

These hybrid methods are not necessary for the parallel plate pillbox arrangement, as the structure is quite basic and straightforward.

Cascading several sections of discontinuity is however an important step in the overall analysis of the waveguide or parallel plate structure [114, 115].

2.9 Mode Matching Analysis [145, 186]
The method of mode matching has been used [197] to analyse waveguide “Tee” structures and also [205] the parallel plate structure. The latter work has been confirmed by a GTD approach [133] but there was no report of application to the 90 degree case. The present work concentrates on the mode matching approach. While mode matching may be reasonably straightforward for a uniform parallel plate bend the present structure is a circular reflector in the plane perpendicular to the 90° bend. Reflectors are very successfully analysed using Physical Optics so that the aim of the present work is to combine the two quite separate approaches. There is also the possibility of achieving a 3-port S-Parameter solution for the entire structure, similar to that used for a transistor [172].

The method of mode matching determines the fields at the output of a device, by cascading the S-parameter results of each section of the device, and obtaining an overall set of S-parameters [197]. A set of equations has been developed by using the same approach as a T junction [193].
There are forward and backward travelling waves in regions I and II, and these can be written in terms of S-parameters, which can be derived from basic field theory, and involve the dimensions of the device. At each discontinuity higher modes are generated.

The parallel plate pillbox is analysed as two right angle bends such as Figure 2.16, placed back to back with a very short region II, this is shown in Figure 2.17. Ideally the width needs to be infinite in size to minimize any edge effects, but this is not possible in practice and hence an arbitrary finite size is assumed.
The analysis of a waveguide 180° bend has been coded in Matlab (as shown in Appendix B2), to perform the various calculations which are necessary to obtain the overall S-Parameters. In this analysis the number of modes is optimized to achieve a satisfactory result with reasonable computer effort.

The analysis of a parallel plate pillbox is slightly different, as only TEM waves are excited between the two sheets [178]. However higher order TEM modes will be generated at each discontinuity, and these appear as “waves” effectively bouncing at different angles between the two parallel plates, which travel in forward and backward directions [80]. An example of two such plane waves are shown in Figure 2.18.

![Figure 2.18 Direction of travel of higher modes in a parallel plate guide.](image)

Just as $TE_{m0}$ modes in rectangular waveguides can be developed by adding plane waves and introducing waveguide walls, this parallel plate case can be envisaged as $TE_{m0}$ modes which are unrestricted in the ‘x’ direction in Figure 2.18. The electric vector of such a mode is perpendicular to the plates of the lens.

The modes in a parallel plate region are given as angles $\theta_n$ in region I and $\theta_k$ in region II, where the sum of all the modes (fields at various angles) give the total field in the region. The waves can be represented by the following equations:

The total forward waves in region I is given by $\sum_{\theta_n=0}^{N} A_{n} e^{j(kx - \beta \sin \theta_n z - \beta \cos \theta_n z)}$, and

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the total backward waves in region I is given by \( \sum B'_n e^{i(kz + \beta \sin \theta_x + \beta \cos \theta_z)} \), where \( F'_n \) and \( B'_n \) are the forward and reflected (backward) fields respectively.

It is expected that the higher order modes die away some wavelengths from the discontinuity and will be accounted for in the field analysis as an attenuation factor \( \alpha \). The theoretical development is presented in the following sections.

2.10 Derivation of Mode matching equations for a 90° waveguide bend.

![Diagram of 90° waveguide bend](image)

Figure 2.19 90° waveguide bend.

![Diagram of field distribution in a single section bend analysis](image)

Figure 2.20 Distribution of fields in a single section bend analysis.

Figure 2.19 gives a 2 dimensional representation of a 90° waveguide bend showing the forward and reflected fields. Figure 2.20 expresses the vector fields in a block diagram format for ease of representation of the fields and to aid in presentation of the equations for a 90° waveguide bend.
The analysis of a 90° waveguide bend follows the method [193] developed for a waveguide T junction. The detailed evaluation for the equations of a 90° waveguide bend is presented in this section.

\[ H = \frac{j}{\omega \mu} \Delta X \Delta X A_n \]  \[ H_z = \frac{j}{\omega \mu} \left( -\frac{\partial \psi}{\partial y} A_n(z, y) - \frac{\partial \psi}{\partial z} A_n(z, y) \right) \]

The field in region I is given as

\[ A_{\text{I}} = \sum_{n=0}^{\infty} \sin\left(\frac{n\pi y}{a}\right) \frac{\cos\left(\frac{n\pi y}{a}\right)}{\sqrt{1 + \delta_{\text{I}}}} \left( F_n^{\text{I}} e^{ik_\text{I}z} - B_n^{\text{I}} e^{-ik_\text{I}z} \right) \]

and similarly the field in region II is given as

\[ A_{\text{II}} = \sum_{n=0}^{\infty} \sin\left(\frac{n\pi y}{a}\right) \frac{\cos\left(\frac{n\pi y}{a}\right)}{\sqrt{1 + \delta_{\text{II}}}} \left( F_n^{\text{II}} e^{ik_{\text{II}}z} - B_n^{\text{II}} e^{-ik_{\text{II}}z} \right) \]

Choose the field in region III \((A_{\text{III}})\), as a combination of the fields in region I and II, but choose arbitrary amplitudes \(C_i\) and \(C_i^\prime\).

\[ A_{\text{III}} = \sum_{n=0}^{\infty} \sin\left(\frac{n\pi y}{a}\right) \frac{\cos\left(\frac{n\pi y}{a}\right)}{\sqrt{1 + \delta_{\text{I}}}} \cos(k_{\text{III}} z) + \sum_{n=0}^{\infty} C_i \sin\left(\frac{k\pi y}{a}\right) \frac{\cos\left(\frac{k\pi y}{a}\right)}{\sqrt{1 + \delta_{\text{II}}}} \cos(k_{\text{II}} z) \]

At the discontinuity \(z = 0\)

\[ E'_i = F_{\text{III}} \frac{\partial A_{\text{I}}}{\partial z} + \frac{\partial A_{\text{III}}}{\partial z} \]

Solving for the unknown \(C_i\) with \(z = 0\), gives
\[ C_n^l = j \frac{(F_n^l + B_n^l)}{-\sin(k_n c)} \]  \hfill (E2.21)

At the discontinuity \( y = b \)

\[ E_z^q = E_z^u \quad \frac{\partial A_{kn}^u}{\partial y} = \frac{\partial A_{kn}^u}{\partial y} \]  \hfill (E2.22)

therefore,

\[ C_k^u = j \frac{(F_k^u + B_k^u)}{\sin(k_n c b)} \]  \hfill (E2.23)

Now use the \( H \) fields

\[ H_z^l = H_z^u \quad \frac{\partial A_{kn}^l}{\partial y} = \frac{\partial A_{kn}^u}{\partial y} \]  \hfill (E2.24)

Evaluate equation (E2.24), then multiply both sides of the equation by \( \frac{\cos \left( \frac{n \pi y}{b} \right)}{\sqrt{1 + \delta_n}} \), then integrate \( \int_0^b dy \). Use the orthogonality relations, rearrange the equation and then substitute the equations for \( C \) to give

\[ F_n^l - B_n^l = \frac{j}{\tan(\alpha_0 c)}(F_n^l + B_n^l) + D \quad \frac{2j}{\sin(\alpha_0 c h)} \sqrt{k_{n,\alpha}^u} \left( \frac{\cos \left( \frac{n \pi y}{b} \right) \cos(k_{n,\alpha}^u)}{\sqrt{1 + \delta_{\alpha}}} \right) dF_k^u + B_k^u \]  \hfill (E2.25)

where

\[ D = \left[ \begin{array}{c} k_{n,\alpha}^u + \left( \frac{n \pi}{h} \right) \\ k_{n,\alpha}^u + \left( \frac{n \pi}{h} \right) \\ \end{array} \right] \]  \hfill (E2.26)

Similarly \( H_z^u = H_z^u \) gives
\[
F^\prime - B^\prime = \frac{j}{\tan(k_{\text{in}} b)} F^\prime + B^\prime + \frac{1}{D \sin(k_{\text{in}} c)} \frac{2j(-1)^n}{c} \cos\left(\frac{k_{\text{in}} z}{c}\cos\left(k_{\text{in}} (z-c)\right)\int_{\gamma} F^\prime + B^\prime dy\right)
\]

Writing equations E2.26 and E2.27 in simpler form gives

\[
F^\prime - B^\prime = -D_c (F^\prime + B^\prime) + L_c^2 (F^\prime + B^\prime)
\]

E2.28

\[
F^\prime - B^\prime = L_c^2 (F^\prime + B^\prime) + D_c (F^\prime + B^\prime)
\]

E2.29

Rearranging equations E2.28 and E2.29 to

\[
B^\prime = S^{11} F^\prime + S^{12} B^\prime
\]

E2.30

\[
F^\prime = S^{21} F^\prime + S^{22} B^\prime
\]

E2.31

which can be written in matrix form as

\[
\begin{pmatrix}
F^\prime \\
B^\prime
\end{pmatrix} =
\begin{pmatrix}
S^{11} & S^{12} \\
S^{21} & S^{22}
\end{pmatrix}
\begin{pmatrix}
F^\prime \\
B^\prime
\end{pmatrix}
\]

E2.31

where

\[
S^{11} = \frac{(D_c - 1)(D_c + 1) + (L_i^2 L_i)}{(D_c - 1)(1 - D_c) - L_i^2 L_i}
\]

E2.32

\[
S^{12} = \frac{2L_i}{(D_i - 1)(1 - D_c) - L_i^2 L_i}
\]

E2.33

\[
S^{21} = \frac{2L_i}{(D_i - 1)(1 - D_c) - L_i^2 L_i}
\]

E2.34

\[
S^{22} = \frac{-D_c + 1 - L_i^2 L_i - D_c^2 + D_c^2 L_i + D_c L_i^2 L_i}{(D_i - 1)(1 - D_c) - L_i^2 L_i}
\]

E2.35
By substituting the parameters for the parallel plate pillbox into the above equations, the S-parameter values can be obtained. The S-parameters from each section of the pillbox need to be cascaded to obtain the overall S-parameter set.

Given two sets of S-parameters:
\[
\begin{bmatrix}
S^1_{11} & S^1_{12} \\
S^1_{21} & S^1_{22}
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
S^2_{11} & S^2_{12} \\
S^2_{21} & S^2_{22}
\end{bmatrix}
\]

the overall S-parameter is given as
\[
\begin{bmatrix}
S^t_{11} & S^t_{12} \\
S^t_{21} & S^t_{22}
\end{bmatrix}
\]
where
\[
S^t_{11} = S^1_{11} + S^1_{12} S^1_{21} W \quad \text{...........................................} \quad \text{F2. 36}
\]
\[
S^t_{12} = S^1_{12} S^1_{21} W + S^1_{11} S^1_{22} \quad \text{...........................................} \quad \text{F2. 37}
\]
\[
S^t_{21} = S^1_{21} W \quad \text{...........................................} \quad \text{F2. 38}
\]
\[
S^t_{22} = S^1_{21} S^1_{22} W + S^1_{22} \quad \text{...........................................} \quad \text{F2. 39}
\]
\[
W = (I - S^1_{22} S^1_{11})^{-1} \quad \text{...........................................} \quad \text{F2. 40}
\]

2.11 Derivation of equations for a 90° discontinuity in a parallel plate region

No references in the literature have been found which treat the parallel plate transmission line with TEM modes, which is derived in this section. An attenuation factor \( \alpha \) is incorporated into the equations from the previous part and the combined fields in each section of parallel plate is given below.

The field in region 1 is given as
\[
A' = \sum_n F_n e^{j \beta n} - \sum_n H_n \left| e^{j \omega z} \right| = e^{j \omega z} \quad \text{...........................................} \quad \text{F2. 41}
\]
and similarly the field in region II is given as
\[ A' = \sum_{\theta_1} F_{n'} e^{i(\alpha - jk)k(y-b)} + e^{\left(-\frac{\alpha - jk}{2}\left(\frac{z}{\sin \theta_1} + \frac{y-b}{\cos \theta_1}\right)\right)} \] .......................... E2. 42

Choose the field in region III (A''_{nr}), as a combination of the fields in region I and II, but choose arbitrary amplitudes C'_{n} and C''_{k}.

\[ A'' = \sum_{\theta_1} C'_{n} \cos(\beta(z-c)) + \sum_{\theta_2} C''_{k} \cos(\beta y) \] .......................... E2. 43

At the discontinuity z=0
\[ E_{y}' = E_{y}'' \xrightarrow{z=0} \frac{\partial A'}{\partial z} = \frac{\partial A''}{\partial z} \] .......................... E2. 44

\[-jk \sum_{\theta_1} F'_{n} e^{-jkz} - \sum_{\theta_1} B' \left[ (\alpha + jk) e^{i(\alpha + jk)z} + \left(-\frac{\alpha + jk}{2\cos \theta_1}\right) e^{\left(-\frac{\alpha + jk}{2}\left(\frac{z}{\sin \theta_1} + \frac{y-b}{\cos \theta_1}\right)\right)} \right] = -\beta \sum_{\theta_1} C' \sin(\beta(z-c)) \] .......................... E2. 45

Solving for the unknown C'_{n} with z=0, gives
\[ \sum_{\theta_1} C' = -jk \sum_{\theta_1} F'_{n} e^{-jkz} - \sum_{\theta_1} B' \left[ (\alpha + jk) + \left(-\frac{\alpha + jk}{2\cos \theta_1}\right) e^{\left(-\frac{\alpha + jk}{2}\left(\frac{z}{\sin \theta_1} + \frac{y-b}{\cos \theta_1}\right)\right)} \right] \] .......................... E2. 46

At the discontinuity y=b
\[ E_{y}'' \xrightarrow{y=b} \frac{\partial A''}{\partial y} = \frac{\partial A''}{\partial y} \] .......................... E2. 47
\[
\sum_{\theta_i} F^* \left[ (\alpha - jk)e^{(i\theta - jk)r - \nu} - \frac{\alpha - jk}{2 \cos \theta_i} \left( \frac{\nu}{2 \sin \theta_i} \right)^{\frac{\nu}{\sin \theta_i} \cos \theta_i} \right] = -\beta \sum_{\theta_i} C^* \sin(\beta r) \quad \text{......}
\]

Solving for the unknown \( C^* \) with \( y = b \), gives

\[
\sum_{\theta_i} C^* = \frac{\sum_{\theta_i} F^* \left[ (\alpha - jk) + \frac{\alpha - jk}{2 \cos \theta_i} \left( \frac{\nu}{2 \sin \theta_i} \right)^{\frac{\nu}{\sin \theta_i} \cos \theta_i} \right]}{-\beta \sin(\beta r)} \quad \text{......}
\]

Therefore,

\[
A^* = \frac{\cos(\beta(z - c))}{-\beta \sin(-jk)} - jk \sum_{\theta_i} F^* \sum_{\theta_i} B^* \left[ (\nu \pm i jk) + \frac{\alpha + jk}{2 \cos \theta_i} \left( \frac{\nu}{2 \sin \theta_i} \right)^{\frac{\nu}{\sin \theta_i} \cos \theta_i} \right] \quad \text{......}
\]

Now use the H fields

\[
H_{\nu}^* = H_{\nu}^* \rightarrow k \frac{\partial A^*}{\partial y} - k \frac{\partial A^*}{\partial y} \quad \text{......}
\]

First evaluate the equations for the H fields and then equate them to exactly determine the forward and backward waves.

\[
H_{\nu}^* = k \left[ \sum_{\theta_i} F^* e^{\nu j} - \sum_{\theta_i} B^* e^{\nu j} e^{\frac{\nu}{2 \sin \theta_i} \cos \theta_i} \right] \quad \text{......}
\]

\[
\left[ \sum_{\theta_i} B^* e^{\nu j} + \frac{\alpha + jk}{2 \sin \theta_i} \left( e^{\nu j} \left( \frac{\nu}{2 \sin \theta_i} \right)^{\frac{\nu}{\sin \theta_i} \cos \theta_i} \right) \right] - \sum_{\theta_i} F^* e^{\nu j} \quad \text{......}
\]
\[
H_y = k^2 \left[ \sum_{n_y} F''' \left[ e^{(\alpha - jk)z} + e^{\left(-\frac{\alpha - jk}{2} \bigg| \begin{array}{l} \frac{z}{\sin \theta}, \\ \frac{v}{\cos \theta} \end{array} \right)} \right] \right] 
\]

\[
\sum_{n_y} F' \left[ (\alpha - jk)^2 \left( \begin{array}{l} \frac{z}{\sin \theta}, \\ \frac{v}{\cos \theta} \end{array} \right) \right] + \left( \frac{-\alpha - jk}{2\cos \theta} \right)^2 \left( \begin{array}{l} \frac{z}{\sin \theta}, \\ \frac{v}{\cos \theta} \end{array} \right) \right] 
\]

\[
H'' = k^2 \left[ \frac{\cos(\beta(z - c))}{-\beta \sin(-\beta k)} \left[ -jk \sum_{n} F'' - \sum_{n} B' \left( (\alpha + jk) + \left( \frac{-\alpha + jk}{2\cos \theta} \right) \right) \right] \right] + \left[ \frac{\cos(\beta(z - c))}{-\beta \sin(\beta k)} \left[ \sum_{n} F' \left( (\alpha - jk) + \left( \frac{-\alpha - jk}{2\cos \theta} \right) \right) \right] \right] 
\]

\[
\frac{\beta \cos(\beta h_1)}{-\beta \sin(\beta k)} \left[ \sum_{n} F'' \left( (\alpha + jk) + \left( \frac{-\alpha + jk}{2\cos \theta} \right) \right) \right] + \left[ \frac{\beta \cos(\beta h_1)}{-\beta \sin(\beta k)} \left[ \sum_{n} F' \left( (\alpha - jk) + \left( \frac{-\alpha - jk}{2\cos \theta} \right) \right) \right] \right] 
\]

\]

Now equate the H equations

\[
H' = H'' - \frac{\partial^2 F''}{\partial y^2} = k^2 A'' - k^2 A''' 
\]

Which can be written in simpler form for ease of programming as

\[
Z \sum_{n} F'' = Z \sum_{n} B' + Z \sum_{n} F' 
\]

\[
Y \sum_{n} F'' = Y \sum_{n} B' + Y \sum_{n} F' 
\]
This gives 2 simultaneous equations with two unknowns which are \( F' \) and \( B' \), the value for \( F' \) is fixed to a normalised value of 1.

Therefore \( B' = \frac{(Y_1Z_1 - Y_2Z_1)F'}{Y_1Z_2 - Y_2Z_1} \) \hspace{1cm} \text{E2. 59}

and \( F'' = \frac{Z_1B' + Z_1F'}{Z_1} \) \hspace{1cm} \text{E2. 60}

2.12 Results for a 90° discontinuity in a parallel plate device

The Modes were represented as various angles in a parallel plate and some of these are given below with a list of the forward and reflected power results from the programs that were written. Theta I and Theta II were made equal, hence referred to a theta and given in degrees.

Values of Theta between 2° and 89° with increments of 1° and values of Theta between -2° and -89° with increments of -1° result in the total absolute reflected power \( B = 0.99 \) and the total absolute forward power \( F = 0.01 \).

Values of Theta which include 11.25°, 22.5°, 33.75°, 45°, 56.25°, 67.5°, -11.25°, -22.5°, -33.75°, -45°, -56.25° and -67.5° result in the total absolute reflected power \( B = 0.82 \) and the total absolute forward power \( F = 0.18 \).

Values of Theta which include 22.5°, 45°, 67.5°, -22.5°, -45° and -67.5° result in the total absolute reflected power \( B = 0.69 \) and the total absolute forward power \( F = 0.31 \).

Values of Theta which include 11.25°, 33.75°, 56.25°, -11.25°, -33.75° and -56.25° result in the total absolute reflected power \( B = 0.75 \) and the total absolute forward power \( F = 0.25 \).

Values of Theta which include 60° and -60° or -45° and 45° result in the total absolute reflected power \( B = 0.5 \) and the total absolute forward power \( F = 0.5 \).
Figure 2.18 represents the results in Section 2.12, demonstrating that the method is accurate only when a large number of modes are used, as it is expected that most of the transmitted wave will be reflected for a 90 degree discontinuity.

![Graph showing magnitude of reflected and transmitted wave with various number of modes using the method of Mode Matching.]

**Figure 2.21** Magnitude of reflected and transmitted wave with various number of modes using the method of Mode Matching.

### 2.13 Conclusion

From the above results it can be seen that if many modes are used, a larger reflected power is obtained, compared to a weak or zero transmitted power. Although mode matching is a simple method there are very many assumptions that have to be made, such as the attenuation of modes, the number of modes and perfectly absorbing receivers, which are not always the case in practice. Hence the Finite Difference Time Domain method is likely to be the more accurate and superior method to use when the field distribution is required within a device, and not just the $S$ parameter values. The first part of this chapter presented the Finite Difference Time Domain Method, where the results for a simple step discontinuity in a parallel plate was evaluated and the results presented in plots. The results were very convincing and can be very easily adapted to solve for bends within the parallel plate pillbox. The detailed analysis of the Finite Difference Time Domain Method will be presented in Chapter 3.
Appendix A2

Appendix A2-1

Maxwell’s Equations in an isotropic medium were given in equations E2.1 to E2.4

Hence rearranging E2.1 and E2.2

\[- \frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E} \]

\[
\frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} + (\nabla \times \mathbf{H})
\]

Substituting E2.3 and E2.4 into A2-1 and A2-2

\[- \mu \frac{\partial \mathbf{H}}{\partial t} = \nabla \times \mathbf{E} = \begin{vmatrix}
i & j & k \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
E_x & E_y & E_z
\end{vmatrix} \]

\[
\varepsilon \frac{\partial \mathbf{E}}{\partial t} = \mathbf{J} + (\nabla \times \mathbf{H}) = \begin{vmatrix}
i & j & k \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
E_x & E_y & E_z
\end{vmatrix}
\]

The 2-dimensional fields for the x-y plane can be visually shown as follows,
Figure A2-1 Distribution of fields in a cell of a FDTD model.

Evaluating A2-3

\[-\mu \frac{\partial H_z}{\partial t} = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \] ................................................................. A2-5

\[-\mu \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial y} - \frac{\partial E_x}{\partial z} \] ................................................................. A2-6

\[-\mu \frac{\partial H_x}{\partial t} = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \] ................................................................. A2-7

From Figure A2-3 it can be seen that \( H_x = 0, H_y = 0 \) and Hz exists so

\[\mu \frac{\partial H_z}{\partial t} = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \] ................................................................. A2-8

Evaluating A2-5

\[\varepsilon \frac{\partial E_z}{\partial t} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \] ................................................................. A2-9

\[\varepsilon \frac{\partial E_y}{\partial t} = \frac{\partial H_z}{\partial y} - \frac{\partial H_z}{\partial x} \] ................................................................. A2-10
\[
\varepsilon \frac{\partial E_x}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_z}{\partial y} \tag{A2-11}
\]

This time from Figure A2-3 it can be seen that \( E_x \) and \( E_y \) exist and \( E_z = 0 \) so

\[
\varepsilon \frac{\partial E_x}{\partial t} = -\frac{\partial H_z}{\partial x} \tag{A2-12}
\]

\[
\varepsilon \frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} \tag{A2-13}
\]

Expressing A2-9 in finite difference form:

\[
\mu \left[ \frac{H_z^{n+0.5}(i+0.5, j+0.5) - H_z^{n-0.5}(i+0.5, j+0.5)}{\Delta t} \right] = \mu \left[ \frac{E_y^n(i+1, j+0.5) - E_y^n(i, j+0.5)}{\Delta y} \right] - \mu \left[ \frac{E_y^n(i+0.5, j+1) - E_y^n(i+0.5, j)}{\Delta z} \right] \tag{A2-14}
\]

Rearranging A2-14 gives the predictive shape:

\[
H_z^{n+0.5}(i+0.5, j+0.5) = H_z^{n-0.5}(i+0.5, j+0.5) + \frac{\Delta t}{\mu} \left[ \frac{E_y^n(i+1, j+0.5) - E_y^n(i, j+0.5)}{\Delta y} \right] - \frac{\Delta t}{\mu} \left[ \frac{E_y^n(i+0.5, j+1) - E_y^n(i+0.5, j)}{\Delta z} \right] \tag{A2-15}
\]

Expressing A2-15 in finite difference form:

\[
\varepsilon \left[ \frac{E_y^{n+1}(i+0.5, j) - E_y^n(i+0.5, j)}{\Delta t} \right] = -\frac{H_z^{n+0.5}(i+0.5, j+0.5) - H_z^{n+0.5}(i-0.5, j+0.5)}{\Delta x} \tag{A2-16}
\]

Rearranging A2-16 gives the predictive shape:
\[ E_x^{n+1}(i+0.5,j) = E_x^n(i+0.5,j) - \frac{\Delta t}{\varepsilon \Delta x} \left[ H_z^{n+0.5}(i+0.5,j+0.5) - H_z^{n+0.5}(i-0.5,j+0.5) \right] \]

Expressing A2-17 in finite difference form:

\[ \varepsilon \left[ \frac{E_x^{n+1}(i+0.5,j) - E_x^n(i+0.5,j)}{\Delta t} \right] = \frac{H_z^{n+0.5}(i+0.5,j+0.5) - H_z^{n+0.5}(i+0.5,j-0.5)}{\Delta y} \]

Rearranging A2-18 gives the predictive shape:

\[ E_x^{n+1}(i+0.5,j) = E_x^n(i+0.5,j) - \frac{\Delta t}{\varepsilon \Delta y} \left[ H_z^{n+0.5}(i+0.5,j+0.5) - H_z^{n+0.5}(i+0.5,j-0.5) \right] \]
Chapter 3 – Design and Analysis of Parallel Plate Device

3.1 Design of the device

An experimental model of the parallel plate structure was developed at X-band. The gap size at the bend was made adjustable, in order to study the variations in pattern with various gap sizes. Two circular reflector cross-sections were used, in order to study the behavior due to the various configurations, one with a flat cross-section and the other a 90° wedge shape as shown in Figures 3.1 and 3.2.

![Diagram](image)

**Figure 3.1** Parallel plate profile (side view) - Flat Reflector.

![Diagram](image)

**Figure 3.2** Parallel plate profile (side view) - Wedge Reflector.
Figure 3.3 shows the perpendicular view of the circular reflector. The field pattern (magnitude and phase) at the output (receiving) aperture or anywhere in the device, can be measured by probing [24]. The pillbox was fed by a H-plane sectoral horn, which was incorporated between the bottom two sheets within the pillbox arrangement. No external waveguide connections were required except an input probe for the horn, at the bottom sheet at a fixed distance of $\lambda/4$ from the edge, refer to Figure 3.3.

Figure 3.3 Parallel plate profile (top view) – Circular Reflector.
Figure 3.4 90° Parallel plate bend showing all the dimensions

It is of interest to see how the fields behave within a parallel plate bend of 90° instead of 180°. Hence the following parallel plate 90° bend as shown in Figure 3.4 was designed.

The simple prototypes mentioned above gives a quick understanding of the fields in parallel plate media. The design can be used to develop a practical experimental setup at X-band (10 GHz) and the same structure can be analysed theoretically using the Finite Difference Time Domain Method as well, thus providing comparison data. If studies show the feasibility of analysing this structure and obtaining reasonable practical results, it can most definitely be a contender for the SKA project. Although it is anticipated that the final scanning arrangement will consist of at least a double reflector system, where the parallel plates will have two 180° bends with the output being some form of directly radiating aperture.
3.2 Analysis of a parallel plate 90 degree bend using the FDTD Method with a Gaussian pulse as input.

![Graph showing the reflected pulse in a parallel plate with a 90° bend of length $2\lambda$.](image1)

Figure 3.5 Plot showing the reflected pulse in a parallel plate with a 90° bend of length $2\lambda$.

![Graph of the FFT for the reflected pulse in a parallel plate with a 90° bend of length $2\lambda$.](image2)

Figure 3.6 Plot of the FFT for the reflected pulse in a parallel plate with a 90° bend of length $2\lambda$. 

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Figure 3. 7 Plot showing the transmitted pulse in a parallel plate with a 90° bend of length $2\lambda$.

Figure 3. 8 Plot of the FFT for a transmitted pulse in a parallel plate with a 90° bend of length $2\lambda$. 
Figure 3. 9 Plot showing the reflected pulse in a parallel plate with a 90° bend of length $5\lambda$.

Figure 3. 10 Plot of the FFT for the reflected pulse in a parallel plate with a 90° bend of length $5\lambda$. 
Figure 3.11 Plot showing the transmitted pulse in a parallel plate with a 90° bend of length $5\lambda$.

Figure 3.12 Plot of the FFT for a transmitted pulse in a parallel plate with a 90° bend of length $5\lambda$. 
Figures 3.5 and 3.9 show the reflected pulse for a parallel plate with a 90° bend, the result for both the 2$\lambda$ and 5$\lambda$ size parallel plate device is a smooth and distinctive Gaussian pulse with a reduced amplitude. Figures 3.7 and 3.11 show the transmitted pulse which is very small and almost insignificant. Figures 3.6 and 3.10 show the FFT of the reflected pulse for the 2$\lambda$ and 5$\lambda$ length devices, which are similar. Figures 3.8 and 3.11 show the FFT of the transmitted pulse for the 2$\lambda$ and 5$\lambda$ length devices, which are similar again. The FFT plots are used to obtain the phase for the S parameter values.

From the results it can be concluded that a 90° bend transmits very little energy, and is a lossy device, as the pulse that is also reflected back is reduced in amplitude.
3.3 Analysis of a parallel plate 180 degree bend using the FDTD Method with a Gaussian pulse as input.

![Graph 1](image1)

**Figure 3. 13** Plot of the transmitted wave for a $2\lambda$ parallel plate device, with a 180° bend and a Gaussian input.

![Graph 2](image2)

**Figure 3. 14** Plot of the FFT of the transmitted wave for a $2\lambda$, parallel plate device with a 180° bend and a Gaussian input.
Figure 3. 15 Plot of the reflected wave for a $2\lambda$ parallel plate device with a 180° bend and a Gaussian input.

Figure 3. 16 Plot of the FFT of the reflected wave for a $2\lambda$ parallel plate device with a 180° bend and a Gaussian input.
Figure 3.17 Plot of the transmitted wave for a $10\lambda$, parallel plate device, with a $180^\circ$ bend and a Gaussian input.

Figure 3.18 Plot of the FFT of the transmitted wave for a $10\lambda$, parallel plate device with a $180^\circ$ bend and a Gaussian input.
Figure 3.19 Plot of the reflected wave for a $10\lambda$ parallel plate device with a $180^\circ$ bend and a Gaussian input.

Figure 3.20 Plot of the FFT of the reflected wave for a $10\lambda$ parallel plate device with a $180^\circ$ bend and a Gaussian input.
Figure 3. 21 Plot of the transmitted wave for a $18\lambda$ parallel plate device, with a $180^\circ$ bend and a Gaussian input.

Figure 3. 22 Plot of the FFT of the transmitted wave for a $18\lambda$, parallel plate device with a $180^\circ$ bend and a Gaussian input.
Figure 3. 23 Plot of the reflected wave for a 18\(\lambda\) parallel plate device with a 180° bend and a Gaussian input.

Figure 3. 24 Plot of the FFT of the reflected wave for a 18\(\lambda\) parallel plate device with a 180° bend and a Gaussian input.
Figures 3.13, 3.17 and 3.21 plot the transmitted field magnitude for parallel plate lengths of \(2\lambda\), \(10\lambda\) and \(20\lambda\), they are all very similar, and produce similar frequency plots given in Figures 3.14, 3.18 and 3.22.

Figures 3.15, 3.19 and 3.23 plot the reflected field magnitude for parallel plate lengths of \(2\lambda\), \(10\lambda\) and \(20\lambda\), they are all very similar, and produce similar frequency plots given in Figures 3.16, 3.20 and 3.24.

The similarity which is noted in both cases indicates a steady state response.

The plots show that the input pulse is split in two with approximately half going through as the transmitted wave and the remainder reflected back. There are multiple reflections caused by the bend at the corner, and this is apparent from all the results that have been presented so far.

### 3.4 Evaluating the S-parameter from the FDTD solution

The transient results are easily obtained using the FDTD method. It is then possible to obtain the S-parameter of a device (both magnitude and phase) by the Fourier transform of these transient waveforms [144, 24, 35].

If

\[
[V] = [S][V]' 
\]

E3.61

where \([V]',[V]'\) and \([V]'\) are the incident, transmitted and reflected field values respectively.

Then

\[
S_{11}(\omega) = \frac{3[V]'}{3[\nu]'} 
\]

E3.62

and

\[
S_{21}(\omega) = \frac{3[V]'}{3[\nu]'} 
\]

E3.63
The magnitude and phase S-parameter results for a 180° parallel plate bend are plotted in Figures 3.25 to 3.28 for various lengths of parallel plate transmission line. The values on either side of the frequency of interest have also been included in Table A3.1 in Appendix A3. The values of S21 are very consistent, both in magnitude and in phase and do not vary much with change in position along the length of the parallel plate and this is shown in Figures 3.27 and 3.28. It can also be seen from these same plots that the transmitted signal stabilises within 3 wavelengths. The values of S11 are consistent for the magnitude shown in Figure 3.25, but are more sensitive for the phase in Figure 3.26. The magnitude settles for S11 within 3 wavelengths as well, but the phase only settles down after about 7 wavelengths.

The values obtained for S11 and S21 from the practical measurements are presented in Chapter 4 and compared with the theoretical results.
Figure 3. 25 Magnitude of S11 for various lengths of the 180° parallel plate for 3 frequencies.

Figure 3. 26 Phase of S11 for various lengths of the 180° parallel plate for 3 frequencies.
Figure 3. 27 Magnitude of S21 for various lengths of the 180° parallel plate for 3 frequencies.

Figure 3. 28 Phase of S21 for various lengths of the 180° parallel plate for 3 frequencies.
<table>
<thead>
<tr>
<th>L</th>
<th>Initial Input</th>
<th>Reverse</th>
<th>Forward</th>
<th>S11</th>
<th>S21</th>
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<td>10.7°95°</td>
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<td>-8.5°210°</td>
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<tr>
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<td>26°-153°</td>
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<td>9.9°65°</td>
<td>-20°153°</td>
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<td>10.4°125°</td>
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<td>-9.3°201°</td>
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<td>3.1°20°</td>
<td>10.2°96°</td>
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</tr>
<tr>
<td></td>
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</tr>
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</tr>
<tr>
<td></td>
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<td>9.9°68°</td>
<td>-18.2°138°</td>
<td>-8.4°221°</td>
</tr>
</tbody>
</table>

Table A3. 1 Table of S11 and S21 results for various lengths (L in lambda) of parallel plate region in the 180° bent device.
Chapter 4 – Practical Results for a 90° and 180° bend in a Parallel Plate Pillbox

4.1 Introduction

A parallel plate pillbox has several advantages as mentioned in Chapter 1. This chapter compares the results of the experiments carried out for a parallel plate bend with the theoretical results. Section 4.2 presents and describes the results from the experiments carried out for a 90° parallel plate bend. Section 4.3 presents and describes the results from the experiments carried out for a 180° parallel plate bend.

4.2 90° bend in a parallel plate region

Essentially a 180° parallel plate bend is made up of two 90° bends with a very short distance separating them. Since this concept was used in the previous chapter on mode matching, it was considered essential to also verify the results experimentally.
Figure 4.1 90° parallel plate bend showing actual dimensions and structural layout.

4.2.1 Analysis of the incoming wave

If the incoming wave is approaching the bend at an angle $\phi$, there are two possibilities in which the incoming wave could travel after the bend:

- The wave could continue at an angle of $\phi$ even after crossing the bend, or
- The wave could travel at an angle perpendicular to the bend, at an angle normal to the angle $\phi$.

This concept is shown in Figure 4.2.
Figure 4.2 Two possibilities for ray travel paths after the bend.

Figure 4.2 shows two possibilities for the rays to travel after the bend. These are given by an arrow that splits in two after the bend. By measuring the phase along both directions the direction in which the ray actually travels after the bend can be determined. Figure 4.3 shows a diagram of the 90 degree bend flattened out.

Figure 4.3 Possibilities of Ray travel paths for a 90° bend showing it flattened for simplicity.
4.2.2 Measurements along 10° rays from input

These measurements were carried out at intervals of $\lambda/2$ which corresponds to a 180° phase difference between the probed holes. Measurements were made in

- a continuing 10° angle after the bend.
- a straight path centrally perpendicular to the bend after the bend.
- a straight path perpendicular (10° shift from centre) to the bend after the bend.

Because of the large number of holes that were drilled $\lambda/2$ apart along each of the paths described above, there were two sets of measurements carried out. One was with a piece of metal sheet over the parallel plate blocking the holes and the other was with the holes uncovered. Comparing the results in Figures 4.4 and 4.5, it can be seen that there was no difference in the measurements with and without cover, hence it was not a necessary requirement for the measurements of the other paths. It can be concluded that the probing holes have minimal effect on the parallel plate propagation. The results of Figure 4.4 and 4.5 for a 10° angle after the bend compare well with the results for the centre axis after the bend.

The phase difference for a 10° straight path after the bend is not 180° as can be seen in Figure 4.7. Therefore it can be concluded that the bend does not affect the path of travel of the incoming wave and hence continues on in the same angle it approaches the bend of the parallel plate.
Figure 4.4 Phase and phase difference for a 10° angle after the bend (refer to Figures 4.2 and 4.3 for explanation).
Figure 4.5 Phase and phase difference for a 10° angle after the bend (with metal sheet covering unused holes on the surface).
Figure 4.6 Phase and phase difference along the centre axis (refer to Figures 4.2 and 4.3 for explanation).

The phase difference along the centre axis is 180° as expected and is shown in Figure 4.14.
Figure 4. 7 Phase and phase difference for a 10° straight path after the bend (refer to Figures 4.2 and 4.3 for explanation).
4.2.3 Measurements for probing along the aperture

S-parameters represent the reflection and the transmission of a particular network as a set of parameters, and the subscripts denote the port numbers. The ratio of the output of port 2 to the incident wave on port 1 is designated $S_{21}$. Likewise, for reflected waves, the signal comes in and out of the same port, hence the S-parameter for the input reflection is designated $S_{11}$.

For a two port network, (using matched sources and loads) $S_{11}$ is the reflection coefficient of the input $S_{22}$ is the reflection coefficient of the output $S_{21}$ is the forward transmission gain $S_{12}$ is the reverse transmission gain (from output to input).

Phase Shift is the shift of a periodic signal, measured referenced to its zero crossing, compared to a reference calibration signal of the same frequency.

The equipment used to carry out the S parameter measurements in Chapters 2, 3, and 4 are as follows:

- a Hewlett Packard 8510C Network Analyser,
- a 8517B, 45MHz - 50GHz S-Parameter Test Set, and
- a 836651B, 10MHz - 50GHz Synthesized Sweeper.

These were calibrated prior to recording any measurements according to the manufacturers specifications.

Phase and magnitude values of $S_{11}$ and $S_{21}$ were measured and recorded, by probing across the aperture, and these were carried out via holes drilled at $\lambda/2$ apart, at the following cross-section locations along the parallel plate after the bend:

- On the edge of the parallel plate region (a distance of $18*\lambda$ after the bend).
- Half way up the sheet after the bend (a distance of $10*\lambda$ after the bend).
- Right after the bend (a distance of $\lambda/2$ after the bend).
Figures 4.8 to 4.15 present the S21 and S11 magnitude and phase plots for the 90° parallel plate discontinuity. The S11 values for magnitude and phase remain consistent (of a constant value) across the aperture for all three locations, at the bend, centre and at the edge, as expected. The Figures show that the phase and amplitude for the transmitting wave S21, is symmetrical and is a maximum at the centre of the aperture and drops off to the edges, as expected. Comparison plots of the phase and magnitude at the 3 locations are presented in Figures 4.14 and 4.15.

Figure 4.8 Phase plot for 90° bend at the edge of the parallel plate device.
Figure 4. 9 Magnitude plot for 90° bend at the edge of the parallel plate device.

Figure 4. 10 Phase plot for 90° bend at the centre of the parallel plate device.
Figure 4.11 Magnitude plot for 90° bend at the centre of the parallel plate device.

Figure 4.12 Phase plot for 90° bend at the bend of the parallel plate device.
Figure 4. 13 Magnitude plot for 90° bend at the bend of the parallel plate device
Figure 4. S11 and S21 phase comparison between the 3 aperture measurements.
4.3 90° bent parallel plate region – Horn transmitter and horn receiver

Figure 4. 16 S11 and S21 for a 90° bend discontinuity in a parallel plate device using the original horns for transmitting and receiving the input and output signals.

Figure 4. 17 90° parallel plate bend showing layout with the very large horn (which extends for the whole distance of the parallel plate setup).
Figure 4. 18 S11 and S21 for a 90° bend discontinuity in a parallel plate device using very large horns for transmitting and receiving the entire input and output signals.

Figure 4. 19 S11 and S21 for a 90° bend discontinuity in a parallel plate device using very large horns for transmitting and receiving the entire input and output signals – with the probing holes covered from previous experimentation.
Figure 4.17 shows a drawing of the very large horn, which extends for the entire length of the parallel plate and hence was used to capture maximum power between input and output. Figure 4.16 and 4.18 show the S-parameter values between using the very long horn as shown in Figure 4.17 and the normal size horn whose dimensions are shown in Figure 3.20. The large horn gives the same results as the short horn, so it was not necessary to repeat all the variations in the set-up with both horn sizes.

It can be seen from the above results in Figure 4.18 and 4.19, that the large number of probing holes actually affect the measurements when using the very large horn. Therefore, it is important to cover up the holes with some metal tape, which works better than when a metal sheet placed on top of the parallel plate covering the holes.

4.4 Parallel plate with 180° bend – Horn transmitter and Horn receiver

Two reflector profiles were used, a flat circular reflector and a circular reflector with a wedge shaped profile, perpendicular to the bend. The gaps between the centre sheet and reflector were varied and the S11 and S21 values were recorded and plotted.

On comparing the results for the 90° parallel plate bend which has a constant 1 cm gap, to a 180° parallel plat bend with a wedge reflector at a gap of 1 cm, it can be seen that the return loss is similar. But the transmitted signal S21 (shown in Figures 4.20 and 4.22), is a lot better for the 180° bend compared to the 90° bend, as it is about –11 dB for the 180° bend as compared to a –23 dB for a 90° bend.
Figure 4. 20 S11 and S21 plots for various gaps between centre sheet and reflector (Using a horn transmitter, horn receiver and a wedge reflector) – All Measurements carried out at 10 GHz.

Figure 4. 21 Flat reflector S-parameter plot for various gap sizes between centre sheet and reflector.
Figure 4. 22 Output power plot for a wedge shaped circular reflector with input power in all cases being –1 dBm.

Figure 4. 23 Output power plot for a flat shaped circular reflector with input power in all cases being –1 dBm.
Figure 4. 24 Comparison of the input and output power between the two reflector profiles with input power in all cases being \(-1\) dBm.
Figure 4.25 The laboratory set-up of the 180° parallel plate bend – This shows the transmitting horn and the circular wedge reflector of the lower parallel plate section only without the upper and middle plates.
Figure 4.26 The laboratory set-up of the 180° parallel plate bend – This shows the parallel plates, input and output probes and the absorbers surrounding the device under test.
4.5 180° bent parallel plate region – Horn transmitter and Probe receiver.

Again a simple probe was used to measure the fields at various points across the aperture, and again this was carried out for both reflector profiles, and can be compared for various gap sizes between the reflector and the centre sheet.

Figure 4.27 and Figure 4.29 plot S11 and S21 magnitudes for both reflector cross sections, where magnitude is expressed as an amplitude in decibels relative to the reference level, which is normalised to 0dB. Figure 4.28 and Figure 4.30 plot the S11 and S21 phase comparing both reflector cross sections.
Figure 4. 27 S11 Magnitude for a wedge and flat shaped circular reflector.
Figure 4.28 S11 Phase plot for a wedge and flat shaped circular reflector.
Figure 4. 29 S21 Magnitude plot for a wedge and flat shaped circular reflector.
Figure 4. 30 S21 Phase plot for a wedge and flat shaped circular reflector.
The above probing results for a 180° parallel plate bend looks similar to the results for a 90° bend where S11 and S21 are symmetrical about the centre axis as expected. The results are not as good a comparison as the results using the horns as there are significant losses in both systems, and maximum energy could be captured using horns to measure the fields.

4.6 Phase Measurement in the parallel plate with the 180° discontinuity

S-parameter measurements, both magnitude and phase were recorded using the same horn arrangement used earlier in Section 4.5. The phase measurements were compared between the initial arrangement of the receiving horn to the second position of the receiving horn.

The receiving horn was first moved by 2 wavelengths at the operating frequency of 10 GHz, and then it was moved by a further 0.5 of a wavelength, which was 2.5 wavelengths in total from the initial position of the feed horn. The Magnitude and Phase results from these measurements are presented in Figures 4.31 and 4.32 respectively.

It can be seen from these figures that the S-parameter magnitude values remain the same, irrespective of the location of the receiving feed horn, both for S11 and S21. However, the phase measurements of the S21 values depend on the location of the receiving horn, whereas the S11 phase values do not. At a position of 2.5 wavelengths from the original location of the horn, the S21 phase is expected to be 180° out of phase compared to the S21 phase measurement at 2 wavelengths from the original location. Figure 4.32 shows that at 2 wavelengths from the initial horn location, the phase remains the same and at a 0.5 wavelength change the phase varies by it’s equivalent in wavelength, that is 180°. These results therefore are as expected, and compare well with the expected and the theoretical measurements from Chapter 3.
Figure 4.31 Horn Feed with fixed probe feed.

Figure 4.32 Horn Feed with fixed probe feed.
4.7 Testing of a Loop Antenna

A waveguide feed with probe for the horn antenna is not very flexible as the probe is fixed and the distance inside the horn/waveguide cannot be varied, hence it is difficult to optimise with different geometries and frequencies. One simple way of overcoming this is to use a loop antenna to feed the horn. This can be mounted at the end of the horn allowing its position to be varied to obtain the best return loss for the horn.

![Diagram of horn feed with fixed probe feed](image1)

**Figure 4.33** Horn Feed with fixed probe feed.

![Diagram of horn feed with variable loop feed](image2)

**Figure 4.34** Horn Feed showing variable loop feed.
Before introducing the new feed to the horn, a simple experiment was conducted to study the performance of a loop antenna. A double sided microstrip board was cut to fit across a waveguide, and a connector was fitted to the centre. Wire of various widths were soldered to the centre conductor, bent around in a loop and soldered on to the board to earth it, refer to Figure 4.34 and Figure 4.35. The orientation of the loop was perpendicular to the magnetic field (i.e. width across the narrow width of the waveguide), for the fields to surround it. Care was taken not to allow the loop to touch the sides of the waveguide and get shorted. The waveguide was terminated with a matched load. Measurements were made for various lengths and widths of the loop and the frequency where the best return loss was observed was noted, refer to Table 4.1 and Table 4.2.

![Diagram of loop antenna](image)

**Figure 4.35 Mounting board and loop antenna.**

<table>
<thead>
<tr>
<th>Freq. (GHz)</th>
<th>$\lambda$ (mm)</th>
<th>Length (L mm)</th>
<th>Width (W mm)</th>
<th>S11 (dB)</th>
<th>$L/\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.000</td>
<td>27.0</td>
<td>19.0</td>
<td>7.5</td>
<td>-26</td>
<td>0.70</td>
</tr>
<tr>
<td>10.200</td>
<td>29.4</td>
<td>21.5</td>
<td>3.5</td>
<td>-20</td>
<td>0.73</td>
</tr>
<tr>
<td>8.975</td>
<td>33.4</td>
<td>24.0</td>
<td>3.0</td>
<td>-20</td>
<td>0.72</td>
</tr>
<tr>
<td>6.675</td>
<td>45.0</td>
<td>38.0</td>
<td>2.5</td>
<td>-20</td>
<td>0.84</td>
</tr>
</tbody>
</table>

**Table 4.1 Relationship between frequency, length and width of the antenna**

(wire width = 0.71 mm).
The ratio between the length and wavelength of the 0.71mm wide antenna was about 0.75. For a frequency of 10GHz the length should be 22.5mm as the wavelength ($\lambda$) is 30mm.

<table>
<thead>
<tr>
<th>Freq. (GHz)</th>
<th>$\lambda$ (mm)</th>
<th>Length (mm)</th>
<th>Width (mm)</th>
<th>S11 (dB)</th>
<th>L/$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.65</td>
<td>25.75</td>
<td>22.5</td>
<td>4.5</td>
<td>-26</td>
<td>0.87</td>
</tr>
<tr>
<td>10.67</td>
<td>28.11</td>
<td>23.4</td>
<td>5.5</td>
<td>-42</td>
<td>0.83</td>
</tr>
<tr>
<td>10.38</td>
<td>28.90</td>
<td>24.5</td>
<td>6.5</td>
<td>-25</td>
<td>0.85</td>
</tr>
<tr>
<td>9.55</td>
<td>31.40</td>
<td>26.0</td>
<td>6.0</td>
<td>-34</td>
<td>0.82</td>
</tr>
</tbody>
</table>

**Table 4.2 Relationship between frequency, length and width of the antenna (wire width = 1.1mm).**

The ratio between the length and wavelength of the 1.1 mm wide antenna was about 0.85. For a frequency of 10GHz the length should be 25.5mm as the wavelength ($\lambda$) is 30mm. Measurements were also made for wires of thickness 0.193mm and 0.4 mm, and the following conclusions can be made.

- thinner the wire, larger the wavelength
- Frequency of operation is quite critically dependant on wire thickness, length of loop and width of loop.

For the purposes of this experiment, a 0.5mm thick wire provides the best compromise, as it is easier to bend, but stiff enough to retain the initial shape. A width of about 2.5mm is chosen for the loop, as it has to fit inside a waveguide width of about 10mm and cannot touch the sides of the waveguide. The length would be about 30mm based on the experimental results for the wire thickness and width. It should be noted that this waveguide feed can be implemented as a printed circuit, which will be repeatable and precise.
4.7.1 Conclusion

The aim of this experiment was to determine an optimal wire thickness for the loop antenna, which operates at a specific frequency for a given length and width of loop.

The main conclusion that can be drawn from the waveguide measurements, is that the thinner the width of the wire the lower the frequency of operation and hence the larger the wavelength for the same length of loop antenna. As expected the frequency of operation is dependant on wire thickness, length of loop and width of loop.

In view of the above conclusion, a wire of thickness 0.5 mm provides the best compromise, as it is easier to bend but stiff enough to retain the initial shape.
Chapter 5 – Geometrical Optics for a dual Cassegrain reflector with a variable sub-reflector for beam steering

5.1 Summary

Basically reflector antennas have scanning limitations [16], this is generally because the reflector is fixed and the feed is displaced [3, 103]. But it is a known fact that scanning can be improved [27], and hence can result in high gain by using two or even three reflectors, as this can reduce the amount of reflector shaping [10, 87] or feed motion [189, 163, 168, 197]. The sub-reflector in a Cassegrain antenna system can be shaped to provide uniform illumination and hence a more efficient antenna reflector [89, 201, 40, 127].

Two slightly different approaches using “Geometrical Optics” are investigated, and these are described briefly.

Method 1: A program was written using a synthesis method based on Geometrical Optics, using Snell’s law, the Conservation of energy and constant phase front (Theorem of Malus), to derive reflector profiles for a dual reflector system [62]. The program had limitations as the equations were derived using certain fixed conditions and hence did not have as much flexibility as was required for the purposes of this project.

Method 2: Due to the limited conditions of Method 1, a program was written using the Deschamps method [55] based again on the theories of Geometrical Optics (ray tracing) to derive a sub-reflector profile, with the main reflector and focal point chosen, in a dual reflector system [88]. Variable surfaces for various scanning angles were determined for a two dimensional dual reflector system.

Results from both methods will be presented and discussed in this chapter. The main drawback of using Geometrical Optics is the requirement that the reflectors should be large and have a large radius of curvature with respect to the wavelength.
5.2 Reflector antennas [47], [164], [52], [58], [154], [152], [153]

There are two main configurations of dual reflector antennas Cassegrain and Gregorian, of which the main reflector is a paraboloid and the sub-reflector can either be a hyperboloid or an ellipsoid [73], [41], [163], [200], [124]. To avoid blockage by the feed the sub-reflector is usually offset from the main reflector [138][195][94] [15][28][176] [175]. The beam can be steered in two ways, either by moving the reflectors mechanically or by reflector shaping [3], [180]. In this chapter a dual, on axis Cassegrain antenna system will be studied with a variable sub-reflector for beam steering [138][195][94].

5.3 Synthesis Method (Method 1) [62]

Here an analytical expression of the principles of Geometrical Optics is used with the geometry of the reflectors, to derive first order non-linear ordinary differential equations. These equations when subject to boundary conditions give the reflector cross-sections. The reflectors are given on the x-y plane, where x1-y1 is the plane for reflector 1 and x2-y2 is the plane for reflector 2.

![Diagram of reflector profiles and variables](image)

Figure 5.1 Reflector profiles and variables defining the positions with respect to two co-ordinate systems [62].
\[
\frac{d\theta_i}{dx_2^i} = f_i(\theta_i; x_2^i) = \frac{I_{3i}(x_2^i)x_2^i}{I_i(\theta_i) \sin \theta_i}
\]

\[
\frac{dy_i}{dx_2^i} = f_i f_i \left[ \beta_i \tan \theta_i + (\beta - y_i) \sec^2 \theta_i \right]
\]

\[
\frac{d}{d(1 + f_i \tan \theta_i)}
\]

Using \( \frac{d\theta_i}{dx_2^i} \) and \( \frac{dy_i}{dx_2^i} \), the values for \( \theta_i(x_2^i) \) and \( y_i(x_2^i) \) can be respectively evaluated, which give the profile of the sub-reflector. Equations E5.1 and E5.2 are derived in Appendix A5-1.

Figure 5.2 shows both reflector half axis profiles, this compares identically to the example in the reference paper by Galindo [62, 63], which was used in this section. The reflectors are plotted with the separation between the individual axis and Figure 5.3 plots the same reflector profiles without the gap between them.

![Reflector Profiles to Produce a Plane Wave](image)

**Figure 5.2** Reflecter Profiles with the actual distance shown between them.
Figure 5.3 Reflector Profiles plotted on the same axis.

Figure 5.4 Complete reflector profiles with a 0° phase drift across the aperture.
Figure 5.5 Reflectors profiles with a 30° phase drift across the aperture.

The program equations were varied slightly to calculate values for a complete reflector surface and this is given in Figure 5.4. Reflector 2 was shifted up by the required amount of phase shift and reflector one was fixed as before. Then by varying the phase at the aperture from 0° to a phase delay, the reflector profiles could be varied to create the required phase delay. Figure 5.5 shows the plot for a phase shift of 30°. The Matlab code for evaluating the entire reflector surface for any phase drift is given in Appendix B5-1.

The reflector profiles \((x_1,y_1,x_2,y_2)\) were stored as data files and using Microsoft Excel the path lengths were calculated and verified as being correct for the various scan angles.

From the results it can be seen that the reflector is not really shaped but instead tilted. It was concluded that the above results would not be suitable for the purposes of this project, as the variations of the reflector position due to scanning with respect to the zero scan case would be too large to implement for active shaping.
5.4 Geometrical Optics (Method 2)[88]

5.4.1 Summary of programming steps

- The rays for a parabolic main reflector were derived, and it was verified that they converge at the chosen focal point, when there is no phase shift.
- However when the incoming plane wave was varied to go off axis, with small angle increments, the focus spreads out as expected and no longer converges to a point.
- A 50% sub-reflector size with respect to the main reflector size was introduced, to intercept the rays reflected off the main reflector. The sub-reflector profile for an on axis incoming plane wave was derived, using constant path length (Fermat’s principle).
- The sub-reflector was verified as a hyperbolic function, and the rays reflected off the sub-reflector converged to the chosen focus.
- The incoming plane wave was varied to go off axis, with small angle increments. The rays intersect with the secondary reflector at a different location, causing the focus to spread out and not converge to a point anymore.
- Used the optimization method of least mean squares, to derive the optimal focal point for a particular scan angle.
- The starting point of the secondary reflector was chosen as the intersection point of the sub reflector and the extreme incoming ray from the main reflector.
- Used the new focal point and the original main reflector, and derived the new sub-reflector shape for various scan angles.
- The size of the sub-reflector was varied (made larger and smaller), and changes to the sub reflector were observed.

Some measures taken for the program to work effectively

- Increased the number of points in the matrix for more accurate results.
- Bypassed the points with very large first and second derivatives of the profile, which occurs around the stationary point of the sub-reflector.
The above steps were carried out with an aim to determine the reflector surfaces for a dual reflector parallel plate pillbox. This was carried out successfully for a 2 dimensional analysis for an incident plane wave with a curvature of 0.

5.4.2 Derivation of equations (Method 2)

Initially equations were programmed for a single reflector without scanning (incoming wave angle set to 0), and later with scanning (change incoming wave angle). Plots were created for the reflector and also for the ray path to see the convergence and to verify that this occurred at the chosen focal point.

The profile of a parabolic reflector is given as $\tau(\delta)$

$$\tau(\delta) = \frac{F}{\cos^2\left(\frac{\delta}{2}\right)} \quad \text{E5. 3}$$

$$\tau'(\delta) = \tau \tan\left(\frac{\delta}{2}\right) \quad \text{E5. 4}$$

$$\tau''(\delta) = \frac{\tau^2(2 - \cos(\delta))}{2F} \quad \text{E5. 5}$$

The $\rho$-$z$ components are

$$\rho = \tau(\delta) \sin \delta \quad \text{E5. 6}$$

$$z = \tau(0) - \tau(\delta) \cos(\delta) \quad \text{E5. 7}$$

The normal $n$ in figure 3.6 is given as

$$\hat{n} = \hat{z} \sin \beta - \hat{\rho} \cos \beta \quad \text{E5. 8}$$
Figure 5.6 Reflector Co-ordinates with source, field point and vectors shown.

Figure 5.7 Reflector Profile.
\[
\cos \beta = \frac{dz}{d\Gamma} = \frac{dz}{d\delta} \frac{d\delta}{d\Gamma} = \frac{\tau \sin \delta - \tau' \cos \delta}{\sqrt{\tau^2 + \tau'^2}} \quad \text{E5. 9}
\]

\[
\sin \beta = \frac{d\rho}{d\Gamma} = \frac{d\rho}{d\delta} \frac{d\delta}{d\Gamma} = \frac{\tau \cos \delta + \tau' \sin \delta}{\sqrt{\tau^2 + \tau'^2}} \quad \text{E5. 10}
\]

\[
\beta = \tan^{-1} \left[ \frac{\tau \cos \delta + \tau' \sin \delta}{\tau \sin \delta - \tau' \cos \delta} \right] \quad \text{E5. 11}
\]

\[
\sin \nu_i = \cos \theta_i (\tan \theta_i \sin \beta + \cos \beta) = \sin \theta_i \sin \beta + \cos \theta_i \cos \beta \quad \text{E5. 12}
\]

\(\nu_i\) is the angle between the incoming ray and the normal to the surface the ray intersects with.

The curvature matrix for reflection in the \(\rho-z\) plane is written as

\[
C_0 = \begin{bmatrix}
-\frac{1}{\rho_i^c} & 0 \\
0 & -\frac{1}{\rho_j^c}
\end{bmatrix} = \begin{bmatrix}
\frac{-(\tau \sin \delta - \tau' \cos \delta)}{\tau \sin \delta (\tau^2 + \tau'^2)^{1/2}} & 0 \\
0 & \frac{-(\tau^2 - \tau \tau'' + 2\tau'^2)^{1/2}}{\tau^2 + \tau'^2}
\end{bmatrix} \quad \text{E5. 13}
\]

where \(\rho_{i,j}^c\) are the principle radii of curvature for \(\Gamma\).

The reflected field curvature matrix is given by

\[
Q'(0) = \begin{bmatrix}
2C_{11} \cos \nu_i & 2C_{12} \\
2C_{12} & 2C_{22} \sec \nu_i
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\frac{-2 \cos \nu_i (\tau \sin \delta - \tau' \cos \delta)}{\tau \sin \delta (\tau^2 + \tau'^2)^{1/2}} & 0 \\
0 & \frac{-2 \sec \nu_i (\tau^2 - \tau \tau'' + 2\tau'^2)^{1/2}}{\tau^2 + \tau'^2}
\end{bmatrix} \quad \text{E5. 14}
\]
\[
\frac{1}{\rho_{12}'} = \frac{1}{2} \left[ Q_{11}'(0) + Q_{22}'(0) \pm \sqrt{(Q_{11}'(0) - Q_{22}'(0))^2} \right] \]

E5. 15

\[
\Rightarrow \frac{1}{\rho_1'} = Q_{11}'(0) \text{ and } \frac{1}{\rho_2'} = Q_{22}'(0) \]

E5. 16

And finally the geometrical optics reflected field can be written as

\[
\begin{bmatrix}
E_x'(s') \\
E_y'(s')
\end{bmatrix} = \begin{bmatrix}
-1 & 0 \\
0 & 1
\end{bmatrix} \begin{bmatrix}
E_x'(P_0) \\
E_y'(P_0)
\end{bmatrix} \frac{\rho_1' \rho_2'}{(\rho_1' + s')(\rho_2' + s')} \frac{1}{2} e^{-jks'}
\]

E5. 17

if the input electric fields are normalised to 1.

\[
E_y'(s') = -\frac{\rho_1'}{(\rho_1' + s')} e^{-jks'} \]

E5. 18

\[
s' = \sqrt{(\tau_2^2 + r_2^2 - 2 \tau_2 \chi_2)} \]

E5. 19

\[
\chi_2 = \sin \theta_2 \sin \delta \cos(\phi_2 - \Phi) - \cos \theta_2 \]

E5. 20

When using numerical differentiation \( \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{(x + h) - x} \) the result is in the x+h/2 interval. Therefore the points have to be interpolated so that it is in the same interval as y and x. As a result of this, the first and last terms of the array are eliminated.
5.4.3 Variation in Equations for Programming

\[ \tau'(n) = \frac{\tau(n+1) - \tau(n-1)}{2(\Delta \delta)} \quad \text{E5. 21} \]

\[ \tau''(n) = \frac{\tau(n+1) - 2\tau(n) + \tau(n-1)}{(\Delta \delta)^2} \quad \text{E5. 22} \]

When the sub reflector was obtained for the on-axis case the shape was verified as a hyperbola, which is as expected when the main reflector is a parabola.

A polynomial of degree three was derived to fit the reflector curve and hence the equation of the sub reflector was obtained.

The incoming wave angle was altered.

The equation for each ray was derived and the point of intersection of the sub reflector and the incoming rays were evaluated.
The new angle made by the incoming ray at the sub reflector with respect to the horizontal was evaluated.

The focus does not converge when the incoming wave is off axis and the trace of the new focus is evaluated by determining where the rays end up after they reflect from the sub reflector.

A new optimal focus is chosen using the least squares method as described in the next section.

Using the first point of intersection at the sub reflector of the incoming wave, the new angles, the new focal point and all the curvature and reflector equations used previously for the on axis case, the new reflector profile is derived. A cubic spline is fitted through the points to give a reflector profile [189, 60, 111, 105].

The incoming rays and the reflector profiles were plotted and the point of convergence of the rays was verified as the optimal focal point that was derived using the least squares method.

5.4.4 Least Squares Method to evaluate an optimal focal point [102]

Figure 5. 9 Focal trace for the reflected rays from the sub-reflector.
\[ d_i = \left| r_o - q_i \right| = \text{distance} \]

\[ I = \sum d_i^2 \]

\[ \frac{\partial I}{\partial x_0} = \frac{\partial I}{\partial y_0} = 0 \]

Solve the linear equations for \( x_0 \) and \( y_0 \) where \( r_o = (x_0, y_0) \)

The variables used in the program and the derivation of the equations to give the new focal point when the incoming wave is off axis is given below.

The aim is to find the equation of the line through \( r_o \) and perpendicular to each of the rays.

An arbitrary point \( p_1(x_1, y_1) \) is picked on the line (ray) and the equation of the line between \( P_1 \) and \( r_o \) is \( (x_0 - x_1)i + (y_0 - y_1)j \)

The equation for the normal of a line \( P_1P_2 \) (the reflected ray from the sub reflector) is given as

\[ N = \frac{y_1 - y_2}{x_1 - x_2}i - j \]

\[ |N| = \sqrt{\left(\frac{y_1 - y_2}{x_1 - x_2}\right)^2 + 1} \]

The shortest distance (normal) from \( r_o \) to each line (ray) is \([102]\)

\[ d_i = \left[ (x_0 - x_1)i + (y_0 - y_1)j \right] \cdot \left[ \frac{y_1 - y_2}{x_1 - x_2}i - j \right] = \frac{(x_0 - x_1) \left( \frac{y_1 - y_2}{x_1 - x_2} - (y_0 - y_1) \right)}{\sqrt{\left(\frac{y_1 - y_2}{x_1 - x_2}\right)^2 + 1}} \]

where \( m = \frac{y_1 - y_2}{x_1 - x_2} \)
\[ I = d_i^2 = m^2 x_0^2 - 2mx_0 y_0 - 2my_0 x_0 + 2my_1 x_0 + m^2 x_i^2 + 2y_0^2 + 2mx_0 y_0 - 2mx_1 y_1 - 2y_1 y_4 + y_1^2 \]
\[ \frac{\partial I}{\partial x_0} = 2m^2 x_0 - 2my_0 - 2m^2 x_i + 2my_1 \]
\[ \frac{\partial I}{\partial y_0} = -2mx_0 + 2y_0 + 2mx_1 - 2y_1 \]

\[
\begin{bmatrix}
2m^2 & -2m \\
-2m & 2
\end{bmatrix}
\begin{bmatrix}
x_0 \\
y_0
\end{bmatrix}
= 
\begin{bmatrix}
2m^2 x_i - 2my_1 \\
-2mx_1 + 2y_1
\end{bmatrix}
\]

\(x_0\) and \(y_0\) are equal to the x and y co-ordinates of the new focal point when the incoming wave is scanned off axis.

### 5.4.5 Results

Some plots obtained from the program are shown below, in which the parameters that can be compared are phase shift and sub-reflector size.

![Dual Reflector Profiles with 0 degree scan, and incoming plane wave](image)

**Figure 5.10** 50% sub reflector to main reflector ratio for 0° scan.
Figure 5.11 50% sub reflector to main reflector ratio for 5° scan (unfocussed).

Figure 5.12 50% sub reflector to main reflector ratio for 10° scan (unfocussed).
Figure 5.13 10\% sub reflector to main reflector ratio for 0° scan.

Figure 5.14 10\% sub reflector to main reflector ratio for 1° scan (unfocussed).
Figure 5. 15 10% sub reflector to main reflector ratio for 1° scan (focussed).

5.4.6 Conclusion to Geometrical Optics

Equations were derived and programmed for a dual reflector case, for the case when the main reflector profile and focal point are given and the profile of the sub-reflector is derived. This worked well for an on axis case. As expected for various scan angles other than 0° a focal point is not achieved but rather a focal locus. This focal locus is optimised using a least mean square method and an optimal focal point is achieved. Using the new focal point and the original main reflector the sub-reflector profile was obtained, as shown in Figure 5.15. As expected there is a limitation using the method of Geometrical Optics, as the program works efficiently only when the sub-reflector to main reflector ratio is more than 10%. In a parallel plate setup where the sub-reflector to main reflector ratio is larger than 10%, this constraint will not pose a problem. Overall the program works well when the parameters are varied, and can be adapted well for the parallel plate pillbox model, with variable reflectors for scanning.
Appendix A5

Appendix A5 - 1 Derivation of Equations (Method 1) [62]

Snell’s Law at Reflector 1 gives

\[
\frac{dy_1}{dx_1} = \tan \left[ \frac{\theta_1 - \theta_2(x_1, y_1, x_2, y_2)}{2} \right]
\].................................A5- 1

where

\[
\theta_2 = \tan^{-1} \left[ \frac{x_2 - x_1}{\alpha + \beta + y_2 - y_1} \right]
\].................................A5- 2

The distance between the origins of the 2 co-ordinate systems is determined by the location of the focus and is given as

\[
\alpha(\theta_1) + \beta(\theta_1) = \kappa
\].................................A5- 3

Snells Law at Reflector 2 gives

\[
\frac{dy_2}{dx_2} = -\tan \left[ \frac{\theta_2 - \theta_3}{2} \right]
\].................................A5- 4

where

\[
\theta_3 = \tan^{-1} \left[ \frac{x_2' - x_2}{\gamma_2} \right]
\].................................A5- 5

Using conservation of energy

\[
I_1(\theta_1) \sin(\theta_1) \left( \frac{d\theta_1}{dx_1} \right) = I_{3a}(x_2')x_2'
\].................................A5- 6

where \( I_1(\theta_1) \) is the power density of the primary illumination and \( I_{3a}(x_2') \) is the power density flow normal to the aperture of the second reflector.

Integrating both sides

\[
I_{\text{in}} \int_{\theta_{2\text{in}}}^{\theta_{2\text{ap}}} I_1(\theta_1) \sin(\theta_1) d\theta_1 = \int_{x_2'_{\text{in}}}^{x_2'_{\text{ap}}} I_{3a}(x_2')x_2' dx_2'
\].................................A5- 7
with the phase at the aperture given as $C_p(x'_j)$, and a constraint set such that

$$C_p(x'_j) = r_1(\theta_1, y_1) + \rho_2(\theta_1, y_1, x_2, y_2) + \rho_3(x_2, y_2, x'_2) + C_p(\theta_1) \hspace{1cm} A5-8$$

where

$$r_1 = [\beta(\theta) - y_1] \sec \theta_1 \hspace{1cm} A5-9$$

$$\rho_2 = \sqrt{(x_2 - x_1)^2 + (\alpha + \beta + y_2 - y_1)^2} \hspace{1cm} A5-10$$

$$\rho_3 = \sqrt{(x'_2 - x_2)^2 + y_2^2} = \frac{y_2}{\sqrt{1 - \left(\frac{dC_p}{dx'_2}\right)^2}} \hspace{1cm} A5-11$$

where

$$\frac{dC_p}{dx'_2} = \sin \theta_1 \hspace{1cm} A5-12$$

Solving the two differential equations A5.1 and A5.4

$$y_2(\theta_1, y_1; x'_2) = -\left[\frac{(x_1 - x'_2)^2 + (\alpha + \beta + y_1)^2 - B^2}{2(\alpha + \beta - y_1) + \frac{dC_p}{dx'_2}(x_1 - x'_2) + B} \right] \sqrt{1 - \left(\frac{dC_p}{dx'_2}\right)^2} \hspace{1cm} A5-13$$

$$x_2(\theta_1, y_1; x'_2) = x'_2 + \left[\frac{(x_1 - x'_2)^2 + (\alpha + \beta - y_1)^2 - B^2}{2(x_1 - x'_2) + \left(\frac{dC_p}{dx'_2}\right)^2(1 - \left(\frac{dC_p}{dx'_2}\right)^2) + B} \right] \hspace{1cm} A5-14$$

where

$$B = C_p - (\beta - y_1) \sec(\theta_1) - C_p(\theta_1) \hspace{1cm} A5-15$$

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\[
\frac{dy_1}{dx_2'} = f_1(\theta_1, y_1; x_2') \frac{dx_1}{dx_2'} = \frac{\left(\beta_0 \tan \theta_1 + (\beta - y_1) \sec^2(\theta_1)\right) f_1 f_i}{(1 + \tan \theta_1 f_i)}
\]

\[
f_1(\theta_1, y_1; x_2') = \tan \left(\frac{\theta_1 - \tan^{-1}\left(\frac{x_2 - x_1}{\alpha + \beta + y_2 - y_1}\right)}{2}\right)
\]

\[
\frac{dx_1}{dx_2'} = \frac{dx_1}{d\theta_1} \frac{d\theta_1}{dx_2'} + \frac{dx_1}{dy_1} \frac{dy_1}{dx_2'} \quad \text{A5-18}
\]

\[
\frac{dx_1}{d\theta_1} = (\beta_0 \tan \theta_1 + (\beta - y_1) \sec^2 \theta_1)
\]

\[
\frac{d\theta_1}{dx_2'} = \frac{\frac{f_{a_0}(x_2')_x_2'}{f_i(x_2')_x_2'}}{\sin \theta_1}
\]

\[
\frac{dx_1}{dy_1} = -\tan \theta_1 \quad \text{A5.21}
\]

\[
\frac{dy_1}{dx_2'} = f_2(\theta_1, y_1; x_2') = f_i \frac{dx_1}{dx_2'} = f_i \frac{dx_1}{d\theta_1} \frac{d\theta_1}{dx_2'} + f_i \frac{dx_1}{dy_1} \frac{dy_1}{dx_2'} \quad \text{A5-22}
\]

\[
\frac{dy_1}{dx_2'} = \frac{f_i f_a \left(\beta_0 \tan \theta_1 + (\beta - y_1) \sec^2 \theta_1\right)}{(1 + f_i \tan \theta_1)} \quad \text{A5-23}
\]
This program worked well after correcting equation (32) in [62].

If

$$y_2(\theta_i, y_i; x'_2) = -\frac{\left[ (x_i - x'_2)^2 + (\alpha + \beta - y_i)^2 - B^2 \right]}{2 \left( \alpha + \beta - y_i \right) + \frac{dC_p}{dx_2} (x_i - x'_2) + B \sqrt{1 - \frac{(dC_p)^2}{dx'_2}}}$$

and $B = C_p - (\beta - y_i) \sec(\theta_i) - C_{p_0}(\theta_i) = C_p - (\beta - y_i) \sec(\theta_i)$

as $C_{p_0}(\theta_i) = 0$

and $B = C_p - r$ as $r = (\beta - y_i) \sec(\theta_i)$ so this should give

$$y_2 = y_2(\theta_i, y_i; x'_2) = \frac{(C_p - r_i)^2 - (x_i - x'_2)^2 - (\alpha + \beta - y_i)^2}{2(\alpha + \beta - y_i + C_p - r_i)}$$

which gives correct results and not

$$y_2 = y_2(\theta_i, y_i; x'_2) = \frac{(C_p - r_i)^2 - (x'_2 - x_i)^2 - (\alpha + \beta - y_i)^2}{2(\alpha + \beta - y_i - C_p - r_i)}$$

which has two errors, and gives incorrect results.
Chapter 7 – Re-configurable Monopole

7.1 Background

Optical reconfiguring of monopole antennas has been studied in the past, where the antenna segments are switched on or off with light transmitted through optical fibres [61]. The optical fibres are electromagnetically transparent, and hence eliminate the problems of switching with wires, which would cause interference in the antenna pattern.

Alternative methods of switching that are currently under development include magnetic tuning and photo detection. Rainville and Harackiewicz, state that “Magnetic tuning of the radiation frequency of microstrip antennas on ferrite film can control a switch” [151]. Photo detectors can offer a variety of light sensitive switches, which eliminates the requirement for DC biasing [86] and the undesirable copper feed lines.

Another means of extending the bandwidth of a dipole/monopole is by capacitively loading it via varactor diodes to produce a travelling wave dipole [159]. For a high antenna Q and electronic tuning over more than a decade, a normal mode resonant helix is very efficient [139]. An array of reconfigurable monopoles can be created on a groundplane and can efficiently have its length altered to operate at various frequencies [182, 61].

This Chapter looks at ways of increasing the bandwidth by varying an antenna element physically, by simple means.

7.2 Simulation and Testing of a re-configurable monopole using MAFIA

7.2.1 MAFIA and The Finite Integration Technique

MAFIA stands for MAXwell's equations by the Finite Integration Algorithm and it is a multipurpose ECAD system, which is used in industry and research laboratories. The applications of this computer program include solving problems in the simulation of
electromagnetic fields from statics to the high frequencies. MAFIA is based on the Finite Integration Technique [210].

The Finite Integration Technique (FIT) is a theoretical basis for solving Maxwell's equations in integral form. The analytical equations are discretized onto two grids which are orthogonal to each other. This yields a set of matrix equations, each of which is the discrete analogue to one of the original Maxwell equations. MAFIA codes consist of a pre-processor, a postprocessor, and several solver modules for the different applications. The interactive command input is easy to use, and many graphics options are available [211].

7.2.2 Aim

The aim of conceptualising this design was to acquire an understanding of the antenna behaviour and also to study the simulation package MAFIA, and establish its feasibility for use in such problems. A re-configurable microstrip monopole with diodes as switches was simulated, fabricated and tested using ideal cases (a short circuit and an open circuit) in place of actual diodes.

7.2.3 MAFIA design

Filaments were modelled between the antenna patches, the location where the feed excites the antenna was modelled as a current source and the gap between the patches was modelled as a capacitor of 0.4 pF or a conductance of 1 mho/m. These values were obtained from simulation and experiments with diodes and were therefore chosen to represent a reverse biased and forward biased diode respectively. Figure 2.1 gives an overview of the design.
Figure 7.1 Set-up of design used in MAFIA and practically.

7.2.4 Varying Parameters

The parameters which were varied in the simulation include substrate size, ground plane size, gap size between elements, gap size between feed point and element, distance above ground, width of element, width of component joining elements and number of mesh points for the finite element solution. Generally a coarse mesh was chosen as this saved computational time and a rough and quick solution could be obtained. An optimum set of parameters were chosen for the fabrication and testing process.

7.2.5 Optimum Parameters

The parameters chosen were as follows:
- Substrate size (0.2 m square)
- Substrate width size (0.0023 m)
- Ground Plane (0.4 m square)
- Both gap sizes (0.01 m)
- Height above ground (0.35 m)

The dielectric constant for the material used ε_r = 4.5
A frequency between 500 MHz and 1 GHz was used, so the antenna segments as shown in the figure were \( \lambda/4 \) in size at 1 GHz and were each 75 mm in length. The width used was 70 mm, 50 mm and 30 mm.

First, 5, then 3 and then 1 microstrip filaments were used instead of forward biased diodes between the two antenna segments, as these were only preliminary experiments. The idea was, that if the number of filaments to connect two antenna segments could be established, then the number of diodes and biasing that may be required for more detailed experimentation can be established in a less cumbersome fashion. The results obtained are discussed in Section 7.2.6.

### 7.2.6 Results

Figure 7.2 and Figure 7.3 show how the antenna works when there is an open circuit between the antenna segments and the antenna should be matched at 1 GHz. The S11 magnitude is well matched at a frequency of 1 GHz and not so well at 0.5 GHz, especially when considering a gap size of 0.015 cm or 0.02 cm.

It should be noted that the gaps in the following Figures 7.2, 7.3, 7.4 and 7.5 are not the same, only Figure 7.3 and 7.4 provide are a direct comparison, the other figures give various combinations of gaps and frequencies showing the variations in the S11 (magnitude).
Figure 7. 2 Magnitude of S11 for a 1.0 GHz Monopole versus varying widths of antenna for a 0.015 cm gap.

Figure 7. 3 Magnitude of S11 for a 1.0 GHz Monopole versus varying widths of antenna for a 0.02 cm gap.
Figure 7.4 Magnitude of S11 for a 0.5 GHz Monopole versus varying widths of antenna for a 0.02 cm gap.

Figure 7.5 Magnitude of S11 for a 0.5 GHz Monopole versus varying widths of antenna for a 0.01 cm gap.
Figure 7.4 and Figure 7.5 show that the filament(s) connecting the two segments is perhaps not sufficient, as a lower frequency of operation is not obtained and again the antenna is best matched at the higher frequency of 1GHz. Several other measurements were made and some of the results and comparisons when parameters are varied are given in Appendix A7. The theoretical results from MAFIA compared reasonably well with the measured results.

7.2.7 Conclusion

It can be seen from Figures 7.2 and 7.3 that a smaller gap gives a better matched antenna at the higher frequency of 1.0 GHz as expected. Figures 7.4 and 7.5 again show that the smaller gap gives a better matched antenna at the higher frequency although a match at the lower frequency was the expected outcome.

But this method of testing was inconclusive as the connecting filaments between the two antenna segments were not sufficient to join the two segments and to make them look electrically like a continuous antenna segment to operate at the lower frequency of 0.5 GHz. Recommendations now follow.

It was concluded that the second ground plane in the x-y plane, is not required. By using a monopole instead of a dipole, the second ground plane can be eliminated, as the monopole already has a ground plane. The idea for future experimentation would be to keep it simple, and as the construction of such devices is so very basic, the theoretical simulation is unnecessary. The next sections discuss various experiments, results and conclusions. Innovative ideas for the construction and operation of re-configurable dipoles/monopoles are introduced.

7.3 Fabrication and Testing of Re-configurable Monopoles

The objective of eliminating the simulation procedure was to achieve a quick result using a simple antenna model using inexpensive and readily available material. This was achieved by using a copper tube generally used as residential water pipes. The
practicability of re-configurability rather than the frequency of operation was investigated initially and this will be demonstrated in the sections that follow.

7.3.1 Design of a simple monopole using plumbing material

The length was chosen as 300 mm as this was λ/4 at 500 MHz, the inner diameter of the copper tube was about 17mm and the outer diameter was 19mm. The tube was soldered into a plumbing fitting which was readily available from a hardware shop. A connector was fabricated, which was secured to a 1m by 1m ground plane to which the tube could be easily screwed on or off. Details of the connector are given in the next section.

7.3.2 Design of a connector (for use with monopole made of plumbing material)

The connector active part was made of brass, to match the material of the plumbing fitting and the ground section was made of aluminium. Special non-conducting glue was used between the two sections to make the overall connector firm and to prevent it from breaking, especially the inner conductor where it joins the p-type connector, as it is fragile. The dimensions of the connector and a cross section diagram are given in Figure 7.6. The dimensions (which are all in mm) were obtained using basic impedance transformation formulae.

![Cross Section of the connector for a monopole.](image)

Figure 7.6 Cross Section of the connector for a monopole.
The formula used is \( Z_0 = 138 \log \frac{D}{d} \) where \( D \) is the inner dimension of the outer conductor, \( d \) is the outer dimension of the inner conductor and \( Z_0 \) is the characteristic impedance \([94]\). The smaller diameters are given the subscript 2 and the larger diameters the subscript 1. The coaxial connector has a characteristic impedance of \( 50\Omega \) and the monopole has a characteristic impedance of close to \( 36.5\Omega \). This was obtained from the self impedance of a dipole which is known to be \( 73+j42.5\Omega \) \([94]\), and since the monopole is half the size of the dipole, the impedance will be half the size as well \([67]\). Using these values of impedance and assuming dimensions for the inner conductor as \( d_1=10 \) and \( d_2=4 \), the calculations are

\[
D_1 = \log^{-1}\left[\frac{37.5}{138}\right] d_1 \quad \text{and} \quad D_2 = \log^{-1}\left[\frac{50}{138}\right] d_2
\]

which gives the values for the outer conductor as \( D_1 = 18.7 \) and \( D_2 = 9.4 \) (again all dimensions are in mm).

**Figure 7.7** Variation of frequency of operation for various gaps in the copper monopole using a Styrofoam separator.
Figure 7.8 Variation of frequency of operation for various gaps in the copper monopole between cardboard and plastic separators.

Figures 7.7 and 7.8 show coupling between the two copper segments which form the monopole. The separation was achieved by using simple separators of three different types, which include cardboard, plastic and Styrofoam. Cardboard and plastic have their individual characteristics and as expected are not as good as the Styrofoam separator which has the closest characteristic to air. Each set of measurements are repeatable when carried out on different days, providing consistent and conclusive answers.

Figure 7.9 below shows coupling again between the copper tubes, but this time with a thin wire of 0.5mm diameter. The success of this stage enables the inclusion of a diode as a switch in the middle of the centre wire and in the centre gap between the monopole segments. A gap of 1cm or more should give a frequency almost double that of the initial continuous monopole if diodes were used.

The next step calls for the construction of a DC bias connector, which generally is the same as what has been used up to this stage. The only difference is the return path from the centre wire exits through a small hole beneath the ground plane. The drawing and
picture of the DC bias connector are given in Figures 7.10 and 7.11 respectively. Appendix A7 provides detailed drawings of the connector.

![Graph](image)

**Figure 7. 9 Variation of frequency of operation for various gaps in the copper tubes with a 0.5mm wire through the centre.**

![Diagram](image)

**Figure 7. 10 Drawing of cross section of DC-bias connector.**
Figure 7.11 Picture of DC-bias connector.

Figure 7.12 Return loss (S11) for a Simple Re-configurable Monopole.
7.3.2 Conclusion

Figure 7.12 shows the final results of a working re-configurable monopole, which is very promising, as the monopole is switched successfully between 250 and 390 MHz. It was quite obvious from the coupling measurements that a doubling of the frequency (1:2 ratio) will not be possible but a ratio of 1:1.6 is quite acceptable.

7.4 Indirect Switching of a Re-configurable Monopole

Although the basic design of a re-configurable monopole works as seen in section 7.2, the question of noise caused by the switching of the diode arises and that is where the design suggestion of an indirectly switched monopole comes in the picture.

![Diagram of a re-configurable monopole with labeled parts: Diode, Monopole Segments, Insulated thin wire for Diode bias return to ground, Ground plane, Small Hole in structure to allow diode biasing, n-type connector.]

Figure 7.13 Cross Section for a Proposal for an indirectly switched monopole.
The basic principle used here is the repetition of a wavelength of a wave, if a short circuit or an open circuit occurs at a particular position in a transmitting media, then a wavelength away, a short or open circuit will occur again, and this happens in increments of wavelength. This way although diodes will still be used they will not however be in the direct path of the wave and thus will not interfere with the transmission or reception of the antenna.

7.4.1 Effects of Coupling

As can be seen in Figure 7.13, any indirectly switched monopole has a complicated structure and this is analysed in simpler steps. The main area of study is the coupling between the metallic tubes within the outer monopole structure and also the coupling between each of the monopole segments for various gaps between the segments. The interaction between the metal segments were made for the following set-ups:

- Vary gap between the two copper tubes
  - Without centre conductor
  - With continuous centre conductor
  - With varying gap in middle of the centre conductor
- With additional continuous metal tube between copper tubes and inner conductor
- With additional metal tube inside with gap in the middle
- Repeat varying width of inner conductor

7.4.2 Results of the various matched frequency effects from experiments

The measurements for the matched frequency between the various set-ups were carried out and the same conductor sizes are used throughout this chapter. The cross section arrangement with the conductor dimensions is shown in Figure 7.14, where ‘D’ is the outer diameter of the metal tube and ‘d’ is the inner diameter of the metal tube.
Figure 7.14 Cross Section showing dimensions of the conductors used in the measurements.

The description for each of the plots in Figure 7.15 is:

- Al+Cu1 stands for the first copper segment and the inner aluminium conductor and the plot shows the matched frequency for varying gaps within the aluminium tube.
- Al+Cu1+Cu2 stands for both copper segments and the inner aluminium segments. The outer copper conductors have a fixed gap size of 2 cm, while the varying gap between the aluminium conductors is plotted.
- Al+Cu1+Cu2+Br is the same as that above but also includes the innermost conductor which is a brass conductor made of two segments and varied exactly like the aluminium conductor.

The comparison here is not between the various metals, but rather a comparison of effects between the metal tubes used, which happened to be of different metals due to the cross section diameter of each conductor.
Figure 7. 15 Variation of frequency of operation for various gaps between the three conductors.

Figure 7. 16 Variation of frequency of operation for various gaps in the inner conductor with no variation in the outer conductor.
The various plots in Figure 7.16 show various gap sizes between the two copper conductors and the gap size against which the plots are plotted are the gap sizes between the inner brass conductor. A gap of 1 cm between the inner brass conductor is sufficient, but the gap between the outer conductor needs to be a lot larger and an appropriate size can be chosen depending on frequency range required for the final switchable monopole.

Figure 7.17 shows the plot of matched frequency, this time with the aluminium conductor between the thin brass rod and the outer copper conductor. The aluminium being larger than the brass increases the coupling effects and as a result the frequency of operation decreases. A 0.5 cm gap size for the two inner conductors together is sufficient and an appropriate gap size can be chosen between the outer copper conductors.

![Graph showing frequency of operation vs. gap size](image)

**Figure 7.17** Variation of frequency of operation for various gaps in both the inner conductors with no variation in the outer conductor.
7.4.3 Conclusion

After studying the various effects on the matched frequency due to several conductors one inside the other, with varying gap sizes between the segments, it can be concluded that continuous inner conductors do not give the frequency shifts as much as those with gaps between the segments. The indirectly switched monopole of Figure 7.13 proposes a biasing problem, as the diodes between the various segments need to be switched. This however is not possible without the interference of DC biasing wires within the radiation fields of the monopole. Still basing the theory on an indirectly switched re-configurable monopole, a new physical antenna design is suggested which is shown in the Figures 7.18 and 7.19.

Figure 7.18 Cross section of a proposal for a new re-configurable monopole.
When the diodes are reverse biased the monopole will operate at about 450 MHz and when forward biased the monopole will operate at about 250 MHz. The two innermost diodes form the short across the inner conductor and the 1 wavelength meandering line through which the wave travels on either side of the inner conductor will go up, down, up and finally down to form 1 wavelength at the lower frequency. Thus a short is formed at the output conductor, and the monopole will operate at the lower frequency. Therefore the electrical noise from the diodes should not affect the properties of the monopole, however the diodes when conducting will add noise to the antenna output.
Appendix A7

Appendix A7.1 Measured Results

Figure A7.1. 1 Figure A7.1 1: $S_{11}$ at 0.5 GHz when number of shorts are increased.

Figure A7.1. 2 $S_{11}$ at 0.5 GHz with varying distance above ground, width and substrate size.
**Figure A7.1.3** S11 with varying number of filaments between monopole segments.

**Figure A7.1.4** S11 with varying distance above ground plane, gap and feed size.
Figure A7.1. S11 with varying gap size with respect to fixed feed size.
Appendix A7.3

Figure A7.3. 1 CAD Drawing (1) of a DC Bias Connector.

Figure A7.3. 2 CAD Drawing (2) of a DC Bias Connector.
Chapter 6 – Physical Optics

6.1 Background

Physical Optics uses the method of calculating the reflected fields from the currents on the reflector surface and then re-radiating them. The equivalent currents produced by a feed antenna on a surface coincident with the reflector surface are obtained. The currents from the source antenna are multiplied by the dyadic Green’s function to give the fields at the respective locations in space corresponding to the source current. A considerable part of this chapter will be dedicated to the dyadic Green’s function, as it plays a crucial role in the solution process to the Physical Optics problem.

Because real space has objects, the use of the free-space Green’s function can only be an approximation [57]. Physical Optics improves on the free-space Green’s function approach by adding the radiation from currents excited on nearby objects [57]. If a problem has a particular geometry, a Green’s function solution will satisfy the boundary conditions and produce the fields for given currents and charges [57]. It is important to keep in mind that Physical Optics does have limitations, because it assumes the currents on the shadowed side are zero when in fact they are finite [57].

There are two approaches to the currents on the reflector. The first method finds the electric currents induced on the reflector by applying boundary conditions with incident magnetic field [57]. A second method first reflects the wave from the surface and then replaces it by a combination of electric and magnetic currents of the Huygens source an infinitesimal distance away from the reflector [57].

Although in most cases there is blockage created by the feed, struts and the sub-reflector, these are not a problem in a parallel plate set-up, as each reflector is offset from the other slightly through the plates. The far-field calculation uses the combination of currents from the main reflector, the struts, the feed, and any central blockage [57]. A subreflector increases the effective f/D of the reflector, which allows the use of feeds with narrow
beamwidths such as corrugated horns [57]. The dual reflector gives an extra degree of freedom to the design [57]. Shaping the subreflector can increase the gain when small compensatory changes are made to the main reflector [57]. As long as the shapes are known, PO can calculate the new reflector with the same effort as an unshaped reflector [57]. The finite size of the subreflector distorts the geometrically reflected pattern by diffraction and is easily calculated by PO [57].

### 6.1.1 Dyadic Greens Function [157]

A differential equation,

\[
\frac{d^2 u}{dx^2} + k^2 u = -f(x)
\]

with boundary conditions,

\[0 < x < 1\]

and forcing function \( f(x) \), whose solution can be written in the following form,

\[u(x) = \int_0^1 f(y) G(x, y) dy\]

gives rise to the Green’s function, which is generally represented as \( G(x, y) \). From this Green’s function the required solution to a differential equation can be obtained with specified boundary conditions, and forcing function.

#### 6.1.1.1 Near and Far Field Patterns [57]

\[
G_{\text{E}_{	ext{j}}} (R) = j \omega \mu \frac{e^{-jkr}}{4\pi R} \left[ \left( 1 - \hat{R} \hat{R} \right) \left( 1 - \frac{1}{k^2 R^2} - j \frac{1}{k R} \right) + \hat{R} \hat{R} \left( \frac{2}{k^2 R^2} + j \frac{2}{k R} \right) \right] \quad \text{E6.1}
\]

\[
G_{\text{H}_{	ext{j}}} (R) = j \omega \epsilon \frac{e^{-jkr}}{4\pi R} \left[ \left( 1 - \hat{R} \hat{R} \right) \left( 1 - \frac{1}{k^2 R^2} - j \frac{1}{k R} \right) + \hat{R} \hat{R} \left( \frac{2}{k^2 R^2} + j \frac{2}{k R} \right) \right] \quad \text{E6.2}
\]

\[
G_{\text{H}_{z}} (R) = \left( \frac{1}{R} + jk \right) \frac{e^{-jkr}}{4\pi R} \hat{R} \times \hat{z} \quad \text{E6.3}
\]

\[
G_{\text{E}_{z}} (R) = \left( \frac{1}{R} + jk \right) \frac{e^{-jkr}}{4\pi R} \hat{R} \times \hat{z} \quad \text{E6.4}
\]
\[ E_x(r, J) = - \int \frac{G_{E}(R)}{4\pi k^2 R^3} (1 - \hat{R} \hat{R}) \cdot J(r') dS' \] \[ H_m(r, M) = - \int \frac{G_{H}(R)}{4\pi k^2 R^3} (1 - \vec{R} \vec{R}) \cdot M(r') dS' \] \[ H_x(r, J) = - \int \frac{G_{H}(R)}{4\pi k^2 R^3} \vec{R} \times J(r') dS' \] \[ E_m(r, M) = \int \frac{G_{E}(R)}{4\pi k^2 R^3} \vec{R} \times M(r') dS' \]

6.1.1.2 Reactive Near Field Equations [57]

\[ E_x(r, J) = - j \alpha \mu \int \frac{e^{-j k R}}{4\pi R^2} \left(1 - \hat{R} \hat{R}\right) \cdot J(r') dS' \] \[ H_m(r, M) = - j \alpha \varepsilon \int \frac{e^{-j k R}}{4\pi R^2} \left(1 - \vec{R} \vec{R}\right) \cdot M(r') dS' \] \[ H_x(r, J) = - \int \frac{e^{-j k R}}{4\pi R^2} \hat{R} \times J(r') dS' \] \[ E_m(r, M) = \int \frac{e^{-j k R}}{4\pi R^2} \hat{R} \times M(r') dS' \]

6.1.1.3 Radiative Near Field Equations [57]

\[ E_x(r, J) = - j \alpha \mu \int \frac{e^{-j k R}}{4\pi R} \left(1 - \hat{R} \hat{R}\right) \cdot J(r') dS' \] \[ H_m(r, M) = - j \alpha \varepsilon \int \frac{e^{-j k R}}{4\pi R} \left(1 - \vec{R} \vec{R}\right) \cdot M(r') dS' \] \[ H_x(r, J) = - j k \int \frac{e^{-j k R}}{4\pi R} \hat{R} \times J(r') dS' \] \[ E_m(r, M) = j k \int \frac{e^{-j k R}}{4\pi R} \hat{R} \times M(r') dS' \]

6.1.1.4 Radiative Far Field Equations [57]

\[ E_x(r, J) = - j \alpha \mu \int \frac{e^{-j k R}}{4\pi R} J(r') dS' \] \[ H_m(r, M) = - j \alpha \varepsilon \int \frac{e^{-j k R}}{4\pi R} M(r') dS' \]
\[ H_e(r, J) = -jk \hat{R} \times \int \frac{e^{-jkR}}{4\pi R} J(r')dS' \] ..........................E6.19

\[ E_m(r, M) = jk \hat{R} \times \int \frac{e^{-jkR}}{4\pi R} M(r')dS' \] ..........................E6.20

\[
(\hat{l} - \hat{R} \hat{k}) \cdot M = \begin{bmatrix}
(1 - R_{xx}^2)M_x - R_{xx}R_{yy}M_y - R_{xx}R_{zz}M_z \\
- R_{yy}R_{xx}M_x + (1 - R_{yy}^2)M_y - R_{yy}R_{zz}M_z \\
- R_{zz}R_{xx}M_x - R_{zz}R_{yy}M_y + (1 - R_{zz}^2)M_z
\end{bmatrix} \] ..........................E6.21

\[ \hat{R} \times M = \begin{bmatrix}
R_{yy}M_z - R_{zz}M_y \\
R_{zz}M_x - R_{xx}M_z \\
R_{xx}M_y - R_{yy}M_x
\end{bmatrix} \] ..........................E6.22

6.1.2 Equivalence Principle [76, 77, 57]

Two sources producing the same field within a region of space are said to be equivalent within that region. Huygen’s equivalence principle is used to obtain radiation patterns of an antenna in the presence of physical objects that may absorb, reflect or alter part of the radiated fields.

The volume distribution at the source is given as

\[ H'(r, J) = -\int r' \left( \frac{1}{R} + jk \right) \frac{e^{-jkR}}{4\pi R} \hat{R} \times J(r')dV' \] ..........................E6.23

\[ E'(r, M) = \int r' \left( \frac{1}{R} + jk \right) \frac{e^{-jkR}}{4\pi R} \hat{R} \times M(r')dV' \] ..........................E6.24

\[ R = r - r' \]

\( r \) is the equivalent surface points and \( r' \) is the source points.

\[ H_n' = \frac{1}{4\pi} \sum_m \left( \frac{1}{R_{mn}} + jk \right) \frac{e^{-jkR_{mn}}}{R_{mn}} \hat{R}_{mn} \times J_m \Delta S_m \] ..........................E6.25

\[ E_n' = \frac{1}{4\pi} \sum_m \left( \frac{1}{R_{mn}} + jk \right) \frac{e^{-jkR_{mn}}}{R_{mn}} \hat{R}_{mn} \times M_m \Delta S_m \] ..........................E6.26

\( m \) is the source point increment, \( n \) is the field point increment and \( R_{mn} \) is the vector between the mth source point and the nth field point.
\[ E'(r, J) = Z_o \int_{V'} \left( \frac{1}{R} + jk \right) \frac{e^{-jkR}}{4\pi R} \hat{R} \times \hat{R} \times J(r') dv' \]

\[ H^s(r, M) = Y_o \int_{V'} \left( \frac{1}{R} + jk \right) \frac{e^{-jkR}}{4\pi R} \hat{R} \times \hat{R} \times M(r') dv' \]

\[ E_n' = \frac{Z_o}{4\pi} \sum_{m}^{M} \left( \frac{1}{R_{nm}} + jk \right) \frac{e^{-jkR_{nm}}}{R_{nm}} \hat{R}_{nm} \times \hat{R}_{nm} \times J_m \Delta S_n \]

\[ H_n^s = \frac{Y_o}{4\pi} \sum_{m}^{M} \left( \frac{1}{R_{nm}} + jk \right) \frac{e^{-jkR_{nm}}}{R_{nm}} \hat{R}_{nm} \times \hat{R}_{nm} \times M_m \Delta S_n \]

The total fields are given as
\[ E_n = E_n' + E_n^s \]
and
\[ H_n = H_n' + H_n^s \]

6.1.3 Equivalent Currents [77]

Equivalent currents are those that occur on the equivalent surface and replace the surface fields so that they reradiate in accordance with Huygen’s theorem. \( \hat{n} \times E_i = -M_i \) and \( \hat{n} \times H_i = J_s \)

6.2 Analysis

A modularised program was written in Matlab using physical optics to solve for both far and near field patterns. The flexibility of using modules for each aspect of the problem allows otherwise complicated problems with multiple reflectors of various profiles and shapes to be solved with ease. If more than one sub-reflector is used, then the same calculations as the previous sub-reflector are used and a separate file written with the dimensions of the new sub-reflector. This new file (module) is called in the main program and the far field pattern added to give the final output.

The steps for the physical optics solution are given below. Section 6.2.1 deals with the overall analysis, with emphasis on parabolic or hyperbolic reflectors with circular or elliptical apertures. Section 6.2.2 describes the analysis of a parabolic or hyperbolic
reflector with rectangular aperture. Section 6.2.3 converts the reflectors to flat sheets and determines their output. Section 6.2.4 describes the analysis of a single flat reflector fed with a dipole, instead of a horn. This last part is used mainly to verify the program performance, using a relatively simple problem, which has a known solution—a dipole with a conducting sheet a certain distance away from it.

6.2.1 Programming steps for the physical optics solution for a system of circular or elliptical apertures and parabolic or hyperbolic structures fed with a horn (Cassegrain system).

6.2.1.1 Step 1 - Define Variables

![Figure 6.1 Paraboloid showing sample area.](image)
Figure 6.2 Parabola showing a very small sampling segment (which equals a flat surface).

**Main reflector dimensions:**

$D_x$ - $x$-axis diameter.

$D_y$ - $y$-axis diameter.

F-focal point.

Vertex.

Aperture center.

Sampling spacing.

The aperture boundary which is parallel to the $x$-$y$ plane and elliptical is given by

$$\frac{(x-x_o)^2}{a_x^2} + \frac{(y-y_o)^2}{a_y^2} = 1 \quad \text{...................................................................... E6.33}$$

The paraboloidal surface is given by

$$z = \frac{x^2 + y^2}{4F} - F \quad \text{...................................................................... E6.34}$$
Let $x=0$ in E6.34
\[ z = \frac{y^2}{4F} - F \] .................................E6.35

The sampling interval and area differential can be obtained by evaluating equation E6.35 at $(z1, y1)$ and $(z2, y2)$, which is a very small sampling area. This gives
\[ z1 = \frac{y1^2}{4F} - F \quad \text{and} \quad z2 = \frac{y2^2}{4F} - F \] respectively.

The difference between the two $z$ values is given as
\[ z1 - z2 = \frac{y1^2}{4F} - F - \frac{y2^2}{4F} + F = \frac{y1^2 - y2^2}{4F} \] .................................E6.36

From E6.36 the sampling segment distance $a$ in Figure 6.2 can be obtained by using the pythagoras theorem
\[ a = \sqrt{(z1 - z2)^2 + (y1 - y2)^2} = \sqrt{\left(\frac{y1^2 - y2^2}{4F}\right) + (y1 - y2)^2} \] .................................E6.37

If $y1 = y2 = y$ as the sample is assumed to be infinitesimally small then
\[ a = \sqrt{1 + \left[\frac{y}{2F}\right]^2} \] .................................E6.38

Similarly if $y=0$ in equation E6.34 then
\[ z = \frac{x^2}{4F} - F \] .................................E6.39

and $a$ with respect to $x$ is given as
\[ a = \sqrt{1 + \left[\frac{x}{2F}\right]^2} \] .................................E6.40
The area of the samples is given by
\[
\Delta S = \int_{x-\Delta x/2}^{x+\Delta x/2} \int_{y-\Delta y/2}^{y+\Delta y/2} dS 
\]
………………………………………………………………………………………………………………..E6.41

where \( \Delta x \) and \( \Delta y \) are the x and y sampling intervals and the area differential \( dS \) can be obtained from equations E6.38 and E6.40, and is given as
\[
dS = \Delta x \Delta y \sqrt{1 + \left( \frac{x}{2F} \right)^2} \sqrt{1 + \left( \frac{y}{2F} \right)^2} 
\]
………………………………………………………………………………………………………………..E6.42

\[
dS = dS_x dS_y 
\]
………………………………………………………………………………………………………………..E6.43

Substituting equation E6.42 into equation E6.41 and expressing it as equation E6.43 gives
\[
dS_x = F \left[ \ln \left( \frac{x}{2F} + \sqrt{\left( \frac{x}{2F} \right)^2 + 1} \right) + \frac{x}{2F} + \sqrt{\left( \frac{x}{2F} \right)^2 + 1} \right]_{x-\Delta x/2}^{x+\Delta x/2} 
\]
………………………………………………………………………………………………………………..E6.44

\[
dS_y = F \left[ \ln \left( \frac{y}{2F} + \sqrt{\left( \frac{y}{2F} \right)^2 + 1} \right) + \frac{y}{2F} + \sqrt{\left( \frac{y}{2F} \right)^2 + 1} \right]_{y-\Delta y/2}^{y+\Delta y/2} 
\]
………………………………………………………………………………………………………………..E6.45

Sub-reflector dimensions:
Generally the parameters required are similar to the ones used above, but if the surface is elliptical [180, 102] then the parameters required need to be evaluated based on the surface.

Positions of the feed, sub-reflector and main reflector:
The location with respect to each other is represented in the cartesian coordinate system, like the dimensions of the main reflector and the sub-reflector.

Horn parameters:
Unit vectors.
Horn dimensions.
Sampling spacing.
Modes (TE or TM or a combination).

**Pattern angles:**
These are expressed as spherical coordinates and are -
\( \theta \text{start} \).
\( \theta \text{stop} \).
\( \Delta \theta \).
\( \phi \) angle for \( \theta \) cut.
Number of \( \theta \) simultaneous angles.

6.2.1.2 Step 2 – Feed Currents and Patterns

**Feed aperture surface definition:**
Number of samples in the x-axis.
Number of samples in the y-axis.
Total number of samples.

**Feed aperture currents** \( J_z \) and \( M_z \):
Aperture lengths.
Phase curvature.
x and y wavenumbers (cutoff).

**Feed forward patterns:**
Use the feed aperture currents \( J_z \) and \( M_z \) in the far field routine to find E and H.
Sum the E and H fields.
And express the final result as power in dB.

6.2.1.3 Step 3 – Sub-reflector Surface Definition

\( z \) offset.
Number of radial samples.
Radial increment.
Number of circular samples.
Total samples.
Array of unit normals.
Sample areas.
z positions.
$\alpha$ angles.
$\phi$ angles.

### 6.2.1.4 Step 4 – Sub-reflector Currents

Calculate H and E from $J_s$ and $M_s$ by using near field function.

Then evaluate the cross products $J_{re} = n \times H$ and $M_{re} = -n \times E$

$M_{ne} = \text{SampleArea} \times M_{re}$

$J_{ne} = \text{SampleArea} \times J_{re}$

$M_{rne} = M_{ne}$ and $J_{rne} = J_{ne}$ (re-radiation currents).

$M_{de} = M_{ne}$ and $J_{de} = J_{ne}$ (blockage currents).

At this stage if there is another sub-reflector then calculate near field patterns using $M_{rne}$ and $J_{rne}$, otherwise go straight to step 7 to calculate the far field pattern.

### 6.2.1.5 Step 5 – Main Reflector Surface Definition

x and y half axis.
x and y limits.
x and y increments.
x and y samples.
x, y and z co-ordinates.
x and y integral limits.
Values of $\phi$ using atan(x/y).
Surface areas of samples.
6.2.1.6 Step 6 – Main Reflector Currents

Repeat step 4 with main reflector parameters, however there are no blockage currents only re-radiation currents.

6.2.1.7 Step 7 – Reflector Patterns

Use re-radiation currents from main reflector to calculate the far field pattern of the reflector. Normalize and calculate power in dB.

Use sub-reflector blockage currents and calculate far field pattern.

Add far field patterns of the main reflector, sub reflector and feed.

Express E in terms of power in dB.

6.2.2 A system of rectangular apertures and parabolic or hyperbolic structures fed with a horn.

The sub-reflector and main reflector surface definitions in step 3 and step 5 are altered and the routines for current and field calculations remain the same. When a circular aperture was used the sample areas were varied radially, with more samples at the edge and fewer samples in the centre of the reflector. However, with a rectangular aperture the same values of x and y increments are used over the entire aperture forming a grid and creating a matrix of x and y values on the aperture.

![Diagram of a rectangular aperture showing sample areas over the aperture.](image)

Figure 6. 3 Rectangular aperture showing sample areas over the aperture.
It can be seen from Figures 6.3 and 6.4 that for a rectangular aperture it is better to use rectangular sampling areas such as that shown in Figure 6.1. This simplifies the routine as all the areas will be of the same size.

![Circular and rectangular apertures showing circularly sampled areas over the aperture.](image)

**Figure 6.4** Circular and rectangular apertures showing circularly sampled areas over the aperture.

6.2.3 A system of flat reflectors with rectangular profiles fed with a horn.

![Reflector Profiles for a dual flat reflector system, y-z plane.](image)

**Figure 6.5** Reflector Profiles for a dual flat reflector system, y-z plane.
Again for a change in profiles only the sub-reflector and main reflector surface definitions in Step 3 and Step 5 need to be altered, the other routines remain the same. The reason for creating routines with flat rectangular apertures will be discussed in the next section.

Figure 6. 6 Reflector Profiles for a dual flat reflector system, x-y plane.

6.3 Results

The results from each of the alterations to the initial program are given below, as radiation plots. The plots are given as normalised power in the phi and theta planes in the far field.
6.3.1 Single reflector with Rectangular Aperture using a horn feed

Figure 6.7 Radiation pattern in the phi plane for a rectangular aperture with a parabolic single reflector.

Figure 6.8 Radiation pattern in the theta plane for a rectangular aperture single reflector.
6.3.2 Dual reflector with Rectangular Aperture using a horn feed

Figure 6. 9 Radiation pattern in the theta plane for a rectangular aperture dual reflector.

Figure 6. 10 Radiation pattern in the phi plane for a rectangular aperture dual reflector.
6.3.3 Flat Dual reflectors with Rectangular Aperture using a horn feed

Figure 6.11 Plot of radiation power in the phi plane.

Figure 6.12 Plot of radiation power in the theta plane.
It is quite obvious from Figures 6.11 and 6.12 that dual flat reflectors do not allow the rays of field to converge but instead they diverge and the fields add up on either side on the central axis in a slightly erratic fashion.

6.4 Conclusion

The reason the analysis of a rectangular aperture parabolic profile reflector is important to this thesis, is in order to be able to analyse the 3rd dimension of the parallel plate lens setup. The 2 dimensional analysis was discussed in detail in Chapters 2, 3 and 4, using the Finite Difference Time Domain Method.

It can be concluded that the rectangular aperture parabolic profile reflector works very satisfactorily, as the radiation patterns presented in section 6.3.1 are smooth and perfectly centred for the on-axis illumination.

Results were also obtained for a dual parabolic reflector system with rectangular apertures shown in Figures 6.9 and 6.10, as this would be essential for a dual reflector parallel plate pillbox system.

The results in Fig. 6.11 and Fig. 6.12 for the dual flat reflector profile, was to show a comparison between using a flat and a curved reflector.

The smooth results for curved reflectors as opposed to the erratic results for the flat reflectors re-emphasises the improved performance which is using a curved reflector for the parallel plate pillbox structure that was proposed in the earlier chapters.
Chapter 8 – Re-configurable Reflector

8.1 Introduction

The concept behind stepped reflector antennas, is the combination of reflector antennas and phased arrays. Reflector antennas predominantly use mechanical methods for scanning. Phased arrays on the other hand, are an array of antennas, that have a phase drift introduced across the array, which simulates the same effect as if the beam is steered. Phased arrays can be expensive as a result of the phase delays that are required for each element, i.e. expensive electronics. For phased arrays, the effective aperture decreases, with increase in scan angle, resulting in loss of efficiency. The idea behind step reflectors, is to use increments in path differences, across the surface of a parabolic reflector, in the same way as phase drifts, in phased arrays. This proposition will be discussed in detail, in this chapter.

Segmented reflector antennas have been shown to reduce cost and weight amongst other advantages. A large reflector can be represented as smaller subapertures, and the radiation pattern can be worked out for each subaperture separately and this also reduces the computer calculation time [44].

It is known that in reflector gratings (as shown in Figure 8.1) the grating resonates and becomes completely transparent when the length of the slots is approximately $\lambda/2$, thus acting like a simple band pass filter [18, 19]. When the system is operated at a frequency below its resonant frequency it behaves like a reflector.

The complement of the system described above, is an array of dipoles, which can be used similarly as a reflector or a non-reflector (Babinet’s principle) [18].
Some of the methods for enlarging the bandwidth of microstrip antennas are, using multilayer techniques, tuning elements, short circuit pins, or placing into a radiating element of resonant frequency F another element of resonant frequency F+Δf, and using electromagnetic coupling to feed the latter [35].

A stepped reflector antenna consists of a number of confocal paraboloid segments arranged around a spherical surface with the centre as the focus [142]. Utilising a stepped reflector in a parallel plate lensing setup gives more flexibility for scanning and eliminates problems such as spherical aberration. Stepped reflectors have a major disadvantage that is small bandwidth and astigmatism during scanning [140, 141].

8.2 Aim.

The aim is to understand the amount of reflector surface alteration, in order to obtain the required phase drift, which can be applied to produce the desired scanning. Aperture field theory is used to study the radiation patterns of a two-dimensional stepped reflector, especially when the surface is varied.
8.3 Analytical approach.

8.3.1 Array of point sources \([74, 101]\).

The equation for the electric field at a distant point for an array of point sources is shown in Figure 8.2 and is given by.

\[
E = 1 + e^{i\phi} + e^{i2\phi} + e^{i3\phi} + e^{i4\phi} + \cdots + e^{i(n-1)\phi} \quad \text{E8. 1}
\]

where

\[
c = 3 \times 10^8 \text{ ms}^{-1}
\]

\(d = \text{distance between the points in the array.}\)

\(f = \text{frequency of operation.}\)

\(\phi = \text{angle of rotation.}\)

\(\delta = \text{phase drift applied across the point sources, to create beam steering.}\)

\[
\beta = \frac{2\pi}{\lambda}
\]

\[
\lambda = \frac{c}{f}
\]
\[ \varphi = \frac{2\pi d \cos \phi}{\lambda} + \delta \]
\[ d_r = \frac{2\pi d}{\lambda} \]

The Simplified equation can be written as

\[ E_{\text{normalized}} = \frac{\sin \left( \frac{n\varphi}{2} \right)}{n \sin \left( \frac{\varphi}{2} \right)} \] \hspace{1cm} \text{E8. 2} \]

where \( E_{\text{max}} = n \), and the working is shown in Appendix A8-1.

8.3.2 Continuous arrays [101]

Consider the array of point sources as shown in Section 8.3.1, but let the number of point sources tend to infinity and let the distance separating the point sources become infinitesimal. Without dealing with complex mathematics, an analogy can be drawn, between the array of point sources and a “continuous” array. If \( a = \) aperture width, then from before \( n*d = a \), then \( E_{\text{normalized}} \) and \( \varphi \) are varied to satisfy continuous arrays.

\[ E_{\text{normalized}} = \frac{\sin \left( \frac{\varphi'}{2} \right)}{\varphi'} \] \hspace{1cm} \text{E8. 3} \]

where,

\[ \varphi' = \beta a \cos \phi \] \hspace{1cm} \text{E8. 4} \]

8.3.3 Step Reflector

The equation for a step reflector can be obtained by using an array of point sources, which gives
\[ E_{\text{normalised}} = \frac{\sin\left(\frac{b \cos \phi}{2}\right) \sin\left(\frac{a(\cos \phi - \delta)}{2}\right)}{\left(\frac{b \cos \phi}{2}\right) \left(\frac{a(\cos \phi - \delta)}{2}\right)} \] \text{E8. 5}

where \( b \) = length of segment with the same phase drift \( \equiv n1*d \), and \( a = n2*b \). The derivation of this equation is given in Appendix A8-2.

### 8.3.4 Relating \( \delta \) to the geometry of a parabola

For varying the phase across the parabola, the path length from the focal point to a distant point needs to be varied. This can be achieved by using parabolas with the same focal point but different \( f/D \) ratios, in order to give the same effect as increasing \( \delta \). Let \( \delta \) be equal to the normalised distance, which is given by the sum of the distances from the focal point \( F(0,p) \) of the parabola to a point on the parabola \( P1(x1,y1) \), and from the point \( P1 \) to a distant point \( P2(x,y) \), this length should be measured, perpendicular to the array of point sources. This is illustrated in Figure 8. 3.

![Figure 8. 3 2-dimensional parabola with an example of 1 ray.](image)

Actual distance = \( \delta + y + p \) \text{E8. 6}
In order to evaluate the equation for the total distance, let a point on the parabola equal \((x_1, y_1)\).

Total distance = distance_1 + distance_2 \(\text{E8. 7}\)

\[
distance_1 = \sqrt{(p - y_1)^2 + (x_1)^2} \quad \text{E8. 8}
\]

\[
distance_2 = y - y_1 \quad \text{E8. 9}
\]

Equating actual distance and total distance and simplifying gives,

\[
(\delta + p + y_1)^2 = (p - y_1)^2 + x_1^2 \quad \text{E8. 10}
\]

but it is known that,

\[
y_1 = \frac{x_1^2}{4(p + s)} - s \quad \text{E8. 11}
\]

where, \(s\) equals the amount the parabola needs to be altered, in order to create the correct amount of phase drift across the parabola, in order for the parabola to scan.

Substituting \textbf{E8. 12} in \textbf{E8. 13} and expanding the equation gives,

\[
2(p + s)(\delta^2 + 2\delta p - 2\delta s - 4ps - x_1^2) + \delta x_1^2 + 2px_1^2 = 0 \quad \text{E8. 12}
\]

Writing equation \textbf{E8. 14} as a quadratic in terms of \(s\) gives,

\[
(-4\delta - 8p)s^2 + (-8p^2 + 2\delta^2 - 2x_1^2)s + (2p\delta^2 + 4\delta p^2 + 5x_1^2) \quad \text{E8. 13}
\]

Solve for \(s\) using the quadratic equation.
\[ s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

where \[ a = -4\delta - 8p \], \[ b = -8p^2 + 2\delta^2 - 2x1^2 \] and \[ c = 2p\delta^2 + 4\delta p^2 + \delta x1^2 \]

### 8.4 Programming

Matlab was used as a tool, to carry out all the programming. This was carried out in steps, starting from basics, and building up, such as the equations given in the previous part.

The steps are:

- Point sources without any phase drift,
  - using sum of exponentials.
  - using ‘sin’ functions.
- Continuous apertures.
- Interferometer.
- Point sources with a phase drift,
  - using sum of exponentials.
  - using ‘sin’ functions.
- Continuous step reflector array.
- Relation to geometry of parabola, what is the value of s (the amount a parabola needs to be altered) for a particular phase shift.
- Compare if s and \( \delta \) are consistent.

The Matlab code for each of the topics mentioned above is included in Appendix B8.

### 8.5 Results

#### 8.5.1 Point sources

When the program was run initially, using the sin approximation for the value of E, a decrease in amplitude when the beam was scanned, was obtained. This was only expected, when the effective area, would be taken into account, which suggests that when a beam is scanned off axis, the effective area drops by the cosine of the angle of scan.
This however, did not occur, when the sum of exponentials was used. This was due to a quantisation error, as the beam became very narrow, when scanned off axis. This was overcome, by increasing the number of points of $\phi$. The plots of Electric field versus phase in rectangular co-ordinates is given in Figures 8.4 and 8.5. These graphs show a comparison of an array of point sources without and with a phase shift.

![Field pattern of a phased array exp, 40 elements, lambda/2 apart](image)

**Figure 8.4 Field pattern using sum of exponentials with 40 elements, $\lambda/2$ apart**
Figure 8. 5 40° shifted field pattern with 40 elements, \( \lambda/2 \) apart (sum of exponentials).

8.5.2 Interferometer

Variations in the program for an array of point sources were made in order to study the results of an interferometer. From Figures 8.6 to 8.9 it can be seen that when the distance between points is \( \geq \lambda \), then grating lobes occur. If there are only two point sources, then the side lobe is of the same magnitude as the main lobe and hence cannot be distinguished, unless the position of the lobe is expected before hand. If grating lobes occur when the beam is scanned, then the beam could be smaller in magnitude, compared to the sidelobes.
Figure 8.6 Interferometer pattern for 2 elements, $0^\circ$ shift, $\lambda$ apart.

Figure 8.7 Interferometer pattern for 2 elements, $10^\circ$ shift, $\lambda$ apart.
Figure 8. 8 Interferometer pattern for 2 elements, $0^\circ$ shift, 10 $\lambda$ apart.

Figure 8. 9 Interferometer pattern for 2 elements, $10^\circ$ shift, 10 $\lambda$ apart.
8.5.3 Step Reflector of point sources

In this case the phase is shifted in steps, and if the number of point sources with the same phase drift across them are a length $\geq \lambda$, then grating lobes occur, as shown in Figures 8.10 and 8.11. In Figure 8.12 the length of the number of points with the same phase drift is less than $\lambda/2$, hence the pattern for the E field has no grating lobes. When a $20^\circ$ phase shift is applied to the result in Figure 8.12, Figure 8.13 is obtained, where the main lobe is offset by $20^\circ$. If however grating lobes were present then the main lobe cannot be identified as shown in Figure 8.14.

Figure 8. 10 Step array (sum of exponentials), 40*40 elements, $\lambda/10$ apart, $0^\circ$ shift.
Figure 8. 11 Step array (sum of exponentials), $10 \times 40$ elements, $\lambda/10$ apart, $0^\circ$ shift.

Figure 8. 12 Step array (sum of exponentials), $5 \times 40$ elements, $\lambda/10$ apart, $0^\circ$ shift.
Figure 8. 13 Step array (sum of exponentials), 5*40 elements, λ/10 apart, 20° shift.

Figure 8. 14 Step array (sum of exponentials), 10*40 elements, λ/10 apart, 20° shift.
8.5.4 Continuous Step Reflector

Figures 8.15 and 8.16 show the effect of phase shift for a continuous step reflector, larger phase shifts result in a larger drop in the magnitude of the E field.

Figure 8.15 Step array (integral) \(15\lambda\) aperture, \(\lambda/10\) segment, \(50^\circ\) shift.
Figure 8. 16 Step array (integral) $15\lambda$ aperture, $\lambda/10$ segment, $10^\circ$ shift.

8.5.5 Effects of Segment and Aperture size on the Radiation Pattern

Figures 8.17 to 8.19 show the drop in magnitude of the E-field with respect to the phase shift. It is obvious that the larger the phase shift, the larger the drop in the main-lobe of the E-field. And as expected, the smaller the segments of the parabola, the smaller is the drop in the main-lobe of the E-field. These results are quite obvious from the graphs, as a whole range of varying segment and aperture sizes have been evaluated and results for a $50\lambda$ aperture have been plotted in Figures 8.17 to 8.19.
Figure 8. 17 Effect on field for various phase shifts in a step array (integral), $50\lambda$ aperture, $\lambda/2$ segment.

Figure 8. 18 Effect on field for various phase shifts in a step array (integral), $50\lambda$ aperture, $\lambda/5$ segment.
Figure 8. 19 Effect on field for various phase shifts in a step array (integral), $50\lambda$ aperture, $\lambda/20$ segment.

8.5.6 Variations in the Parabolic Reflector Surface

For a particular phase shift, particular values for $s$ (the factor which varies the parabola, to have the same focal point and varying focal length to diameter ratio) were obtained which are shown in Figure 8.20. It can be seen that for a larger phase shift, a larger vertical step is required for the parabola. But, because a larger step size in the step reflector increases losses, smaller steps can be used such as that shown in Figure 8.21 instead of that in Figure 8.22.

On comparing the path lengths resulting from the stepping of the parabolic surface and $\delta$ (the phase drift applied across the aperture), they were exactly the same as expected.
Figure 8. 20 Vertical shift in a parabola to get the required phase drift.

Figure 8. 21 Step reflector surfaces for various phase drifts.
8.6 Conclusion of the theoretical analysis of step reflectors

A two-dimensional step reflector has been studied and the results obtained are as expected. The formulae were derived from basic antenna array theory and the geometry of a parabola, and these have been supported by graphs.

8.7 Experimental analysis

It is known that at a particular frequency, a conductor more than \( \lambda/2 \) in length is a reflector and less than \( \lambda/2 \) in length is transparent. This knowledge can be used to create an array of wires, that can be used as a reflective or transparent surface. Layers of these re-configurable surfaces can be used to form a frequency dependant re-configurable reflector. If a sheet of wire segments was created to resonate at a particular frequency (500MHz). It is expected that the wire sheet will be transparent at the resonant frequency and lower, and reflective at frequencies higher that 500 MHz.

There are two methods that were studied practically,
• Method 1 used a horn to illuminate wire segments that formed a sheet.
• Method 2 used a monopole to illuminate wire segments of various sizes.

The results from each of these methods will be discussed below in further detail.

8.7.1 Method 1: Horn illumination to experimentally study a re-configurable surface

A closed field set-up was used, where the wire sheet was fed with a rectangular horn. A large low frequency horn was used with a 1m by 1.25m foam, which was fitted across the opening of the horn. The horn was tuned to operate at the frequency that the wire sheet was designed for. Reflector coefficient measurements were carried out using absorbers across the horn opening for an open circuit (transmitting case), which was well matched over the frequency of interest. The reflection coefficient for the reflecting case was measured with a metal sheet across the horn opening, used as a short circuit, which resulted in all the power being reflected back. Hence, the results for both the open circuit and the short circuit measurements were as expected.

Figure 8.23 Picture of Laboratory setup of large horn and absorber.
A picture of the laboratory set-up is shown in Figure 8.23 and Figure 8.24. The wire sheet reflector acted as a good reflector at the tuned frequency. But when the wire size was decreased, the wire reflector was expected to appear transparent at the same frequency which it did not. This was attributed to the circular fields emerging from the horn, as in order for the surface to reflect or be transparent to the maximum, plane waves that are horizontally and vertically polarised would be more practical. This was not the case at the opening of the horn, as the fields do not settle down into linearly polarised waves, hence not giving the expected results. In order for the set-up to perform satisfactorily the wire sheet needs to be placed a couple of wavelengths from the horn, which at the low operating frequency of the horn, would be too large a distance to accommodate in an anechoic chamber, surrounded by absorbers.
8.7.2 Method 2: Monopole illumination to experimentally study a re-configurable surface

A simplified version of a re-configurable surface was investigated, this time using a 'Yagi-Uda' configuration and similar to a re-configurable monopole, refer to Figure 8.25, for the general layout. A monopole at 500Mhz of length $\lambda/4$ (15cm) was used as the transmitting antenna. The monopole was mounted centrally on a large circular ground plane of $4\lambda$ in diameter.

Another element was used at varying distances between 0.15$\lambda$ and 0.33$\lambda$ from the transmitting monopole on the same ground plane. The length of the element was varied between $\lambda/6$ and $2\lambda/3$.

A half wave dipole at 500 MHz was used to measure the transmitted/reflected far fields about 5$\lambda$ away from the transmitting monopole. The experiment was carried out in the middle of paddocks, which minimised reflection due to any external structures. The set-up is shown in Figures 8.26 and 8.27.

![Diagram of re-configurable surface](image)

Figure 8.25 Layout for measuring a practical re-configurable surface.
At a particular gap size, there were four different set-ups and measurements that were carried out and recorded. These are given abbreviations which are used in the graphs and are explained below, with the help of Figure 8.28, Figure 8.30, Figure 8.32 and Figure 8.34. The experiments were carried out for various gap sizes between the two elements and all the results were as expected, as the plots followed similar trends. Figure 8.36 shows the comparison between the 4 set-ups, to give an overall perspective. Refer to Appendix A8-3 for a table of the data that was recorded in this experiment.

![Figure 8.26 Picture of re-configurable surface experimental setup in the paddocks.](image)
Figure 8. Picture of re-configurable surface experimental setup (Showing receiving dipole and transmitting monopole).
Figure 8.28 Setup for measuring far field transmission and reflected pattern, using a grounded variable element (Measured forward of monopole).

The set-up in Figure 8.28, gives the results in Figure 8.29 which shows that when the variable element is longer than the transmitting monopole the received signal is stronger, showing that it is a combination of both the transmitted and the reflected signal from the variable element. When the variable element is shorter than the transmitting monopole, the received signal is less, which clarifies that no signal gets reflected by the variable reflector, and only the transmitted signal is recorded.

Figure 8.29 Through connected reflector/director – Not near the measuring dipole.
Figure 8.30 Setup for measuring far field transmitted and reflected pattern, using an ungrounded variable element (Measured forward of monopole).

The set-up in Figure 8.30 gives the results in Figure 8.31 which shows that when the variable element is longer than the transmitting monopole, the received signal is not as strong as in the case of the set-up in Figure 8.28. The reason for this is because the variable element is not grounded and does not reflect as much of the signal as in the grounded case. The length of the variable element has no major consequence, for reflection, when it is not grounded.

Figure 8.31 Unconnected reflector/director (Not near the measuring dipole).
Figure 8. 32 Setup for measuring far field transmitted pattern, using a grounded variable element (Measured behind reflector).

Measuring the transmitted signal in the set-up of Figure 8.32, gives the results in Figure 8.33 which shows that not much signal is received when the variable reflector is longer than the transmitting monopole. When the variable reflector is shorter than the transmitting monopole then a large amount of signal is received.

Figure 8. 33 Through connected reflector/director – Near the measuring dipole.
Figure 8.34 Setup for measuring far field transmitted pattern, using an ungrounded variable element (Measured behind reflector).

Measuring the transmitted signal in the set-up of Figure 8.34, gives the results in Figure 8.35 which shows that the signal is received even when the variable reflector is longer than the transmitting monopole, as it is not grounded this time. When the variable reflector is shorter than the transmitting monopole the signal is still received.

Figure 8.35 Unconnected reflector/director – Near the measuring dipole.
Figure 8. 36 Comparison of Field measurements between the various reflector/director heights, at a gap of 0.25 lambda from the source.

8.8 Conclusion

From the antenna measurements it can be concluded that a re-configurable antenna, promises to be a novel method for varying the shape of a reflector at a particular frequency, simply by adding or removing segments of conductor by means of simple diode switching. It is also interesting to note the effect of grounding the variable element, and it can be concluded that the element needs to be grounded when it is used as a reflector, and not grounded when it is required to be transparent. This can again be accomplished using diodes to connect the element to the ground or not, by forward or reverse biasing them. Varying the operating frequency by switching the antenna segments on/off using diodes was proven practically in Chapter 7 on re-configurable monopoles.
Appendix A8

Appendix A8-1: Simplifying the equation for an array of point sources.

\[ E = 1 + e^{j\varphi} + e^{j2\varphi} + e^{j3\varphi} + e^{j4\varphi} + \ldots + e^{j(n-1)\varphi} \] \[ \text{A8-1} \]

where
\[ c = 3 \times 10^8 \text{ ms}^{-1} \]
\[ d = \text{distance between the points in the array}. \]
\[ f = \text{frequency of operation}. \]
\[ \phi = \text{angle of rotation}. \]
\[ \delta = \text{phase drift applied across the point sources, to create beam steering}. \]

\[ \beta = \frac{2\pi}{\lambda} \]
\[ \lambda = \frac{c}{f} \]
\[ \varphi = \frac{2\pi d \cos \phi}{\lambda} + \delta \]
\[ d_r = \frac{2\pi d}{\lambda} \]

Simplify equation (A8.1), by multiplying throughout by \( e^{j\varphi} \) to give,

\[ E e^{j\varphi} = e^{j\varphi} + e^{j2\varphi} + e^{j3\varphi} + e^{j4\varphi} + \ldots + e^{j(n-1)\varphi} \] \[ \text{A8-2} \]

subtracting (A8.1) from (A8.2) and cross multiplying gives,

\[ E = \frac{1 - e^{jn\varphi}}{1 - e^{j\varphi}} = e^{j\varphi} \left( \begin{array}{cc} jn\varphi & -jn\varphi \\ e^2 & -e^2 \end{array} \right) \left( \begin{array}{cc} jn\varphi & -jn\varphi \\ e^2 & -e^2 \end{array} \right) \] \[ \text{A8-3} \]
This can be written as,
\[ E = e^{j\delta} \frac{\sin \left( \frac{n\phi}{2} \right)}{\sin \left( \frac{\phi}{2} \right)} \] .......................... A8- 4

When the phase is referred to the centrepoint of the array, and E is normalised to \( E_{\text{max}} = n \), this gives,
\[ E_{\text{normalized}} = \frac{\sin \left( \frac{n\phi}{2} \right)}{n \sin \left( \frac{\phi}{2} \right)} \] ............................................ A8- 5

Appendix A8-2 : Derivation of Step Reflector equations

Using an array of point sources the following is obtained :

\[ E = 1 + e^{j(n_1\phi + \delta_0)} + e^{j(2\phi + \delta_0)} + e^{j(3\phi + \delta_0)} + \cdots + e^{j((n-1)\phi + \delta_0)} + e^{j((n+1)\phi + \delta_1)} + e^{j((n+2)\phi + \delta_1)} + \cdots + e^{j(2(n-1)\phi + \delta_1)} + e^{j(2n\phi + \delta_2)} + \cdots + e^{j(3(n-1)\phi + \delta_2)} + \cdots + e^{j((n-2)\phi + \delta_{n-2})} + e^{j((n-2)\phi + \delta_{n-2} + \delta_0)} + \cdots + e^{j((n-1)\phi + \delta_{n-2})} \] ............................................ A8- 6

where,
\( \phi = d \cos \phi \)

Equation (A8.6) can now be simplified by collecting like terms,
\[ E = e^{j(0n_1\phi + \delta_0)} [1 + e^{j(\phi + \delta_0)} + e^{j(2\phi + \delta_0)} + e^{j(3\phi + \delta_0)} + \cdots + e^{j((n-1)\phi + \delta_0)}] \]
\[ e^{j(n_1\phi_1 + \delta_1)}[1 + e^{j(\phi_1 + 50)} + e^{j(2\phi_1 + 50)} + e^{j(3\phi_1 + 50)} + \cdots + e^{j((n_1-1)\phi_1 + 50)}] \]
\[ e^{j(2n_1\phi_1 + \delta_2)}[1 + e^{j(\phi_1 + 50)} + e^{j(2\phi_1 + 50)} + e^{j(3\phi_1 + 50)} + \cdots + e^{j((n_1-1)\phi_1 + 50)}] \]
\[ e^{j((2n_2-n_1)\phi_1 + \delta_n2)}[1 + e^{j(\phi_1 + 50)} + e^{j(2\phi_1 + 50)} + e^{j(3\phi_1 + 50)} + \cdots + e^{j((n_1-1)\phi_1 + 50)}] \]

Which can be further simplified,
\[ E = \cdots \cdots [1 + e^{j(\phi_1 + 50)} + e^{j(2\phi_1 + 50)} + e^{j(3\phi_1 + 50)} + \cdots + e^{j((n_1-1)\phi_1 + 50)}] \]
\[ [1 + e^{j(n_1\phi_1 + \delta_1)} + e^{j(2n_1\phi_1 + \delta_2)} + \cdots + e^{j((2n_2-n_1)\phi_1 + \delta_n2)}] \]

Let \( \phi_s = n_1\cos\phi + n_1\delta \), then,

\[ E_{\text{normalized}} = \frac{\sin\left(\frac{n_1\phi_s}{2}\right) \sin\left(\frac{n_2\phi_s}{2}\right)}{n_1 \sin\left(\frac{\phi_s}{2}\right) \sin\left(\frac{\phi_s}{2}\right)} \]

Converting the derivation in A8.9 to continuous arrays gives,

\[ E_{\text{normalized}} = -\frac{\sin\left(\frac{b\cos\phi}{2}\right) \sin\left(\frac{a\cos\phi - \delta}{2}\right)}{b \cos\phi \sin\left(\frac{a\cos\phi - \delta}{2}\right)} \]

where \( b = \) length of segment with the same phase drift \( = n_1*d \), and \( a = n_2*b \).
### Appendix A8-3: Results

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<td>C-NI</td>
<td>-13 -13 -13</td>
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<table>
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<th>Connected - no Intrusion Transmitted and Reflected</th>
</tr>
</thead>
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</tr>
<tr>
<td>0.31Lambda</td>
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<tr>
<td>0.33Lambda</td>
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</tbody>
</table>

<table>
<thead>
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<th>Not Connected - no Intrusion Transmitted and Reflected</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.2Lambda</td>
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<table>
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<th>Connected With intrusion-transmitted</th>
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<tbody>
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</tr>
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</tr>
<tr>
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<th>Not Connected With intrusion – Transmitted</th>
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Table A8.1 Results from Experiment for re-configurable Yagi-like set-up.
Chapter 9 - Conclusion

9.1 Project Overview

The extreme antenna design criteria stipulated by the requirements of SKA (Square Kilometer Array) a future large Radio Telescope, generated an immense interest both internationally and within Australia, amongst both astronomers and antenna design engineers alike. Although the initial requirement was an effective area of 1 square kilometer, there is the possibility of requiring an effective aperture of 10 square kilometer over time. The main consensus was the urgent need of innovative and new research towards developing an antenna that did not exist, but which would satisfy the extremely demanding specifications outlined by the astronomers for the requirements of the antenna.

The present thesis originated while keeping the specifications mentioned in Chapter 1 in perspective. The demanding design and economic specifications of the required antenna resulted in investigating numerous antenna design possibilities.

Various new and old antenna technologies were reviewed, and the initial study was dedicated to broadbanding simple and inexpensive antennas. The area of antenna re-configurability was studied in detail, both by computer simulations as well as experimentally. This work provided very promising results for broadbanding an antenna and also for variable reflector surfaces where scanning can be achieved by varying the surface, by frequency sensitive elements or panels.

A detailed theoretical study of re-configurable reflectors was undertaken and a program was written using Geometrical Optics which determined the new reflector profiles and the feed location when the beam was scanned off-axis by a certain degree. However, the results were very limited and was not a good direction to proceed in.

The search for an antenna that could satisfy the required specifications led to the investigation of various lens antennas, which proved to be a very promising solution.
The research progressively focussed on lenses especially the parallel plate pillbox type, due to their versatility and their ability to satisfy many of the required specifications, which include:

- Their broadband capabilities.
- A wide angle, multi-beaming system due to no blockage by feeds in the offset system of parallel plate lenses.
- Rapid scanning, spun around very easily, or some form of electronic scanning.
- Fine scanning which can be achieved by moving a horn along the input arc.
- Low costs as expensive electronics for phase shifters would not be required.
- Arranging the lenses in a vertical and horizontal grid takes care of the dual polarisation requirement.
- Flaring the output end of the parallel plate can increase the effective collecting/receiving area, of course doing that would however reduce the beamwidth.

Methods for analysing the fields within the structure were considered and there were two promising approaches. The first was the Mode Matching method and the second was the Finite Difference Time Domain (FDTD) method. The FDTD method proved very efficient for the purposes of the parallel plate device.

Working programs were written using the theory of FDTD, which analysed both a 90° and a 180° bend discontinuity very well. The results were verified by practical measurements carried out on the respective lens arrangements, which agreed very well with the theory.

The practical measurements also gave the opportunity to study the direction of propagation of the fields after the bend and the drastic differences between the 90° and 180° bend discontinuities in a parallel plate. The theory confirmed the propagation of the fields after the bend, the large energy transmission around the bend for a 180° bend as opposed to the 90° bend, which hardly transmitted any of the input signal after the bend.
The distribution of the fields along the aperture and at various locations along the bend were also measured and compared satisfactorily with the theoretical analysis. The success of the FDTD method leaves the possibility of analysing the entire three-dimensional parallel plate device.

9.2 Project Outcome

The seven main objectives of this thesis outlined in Section 1.8 ‘Project Definition’ will be revisited here and each objective will be assessed to establish achievements.


A survey of mode matching techniques was presented using S-parameters to analyse a parallel plate pillbox. A solution was derived for a 90° waveguide bend, which was then modified to treat a 90° discontinuity in a parallel plate guide for the first time.

A background description of the FDTD method was presented using different input signals (Sinusoidal wave and Gaussian pulse) and illustrating the relationship between grid size and accuracy. The FDTD method was established as consistent by successfully analysing a parallel plate with a step discontinuity.

2. Chapter 3 covers the design and fabrication of a practical pillbox fed by a H-Plane sectoral horn. The FDTD method was used to study a 90° parallel plate bend, which showed that most of the energy from the input signal was reflected back due to the bend and only a very small percentage of the signal was actually transmitted.

The FDTD method was again used to study a 180° bend in a parallel plate set-up, which showed that multiple reflections occur and about half of the input pulse is reflected and the other half is transmitted.

From the S-parameter results for the 180° bend parallel plate it was shown that $S_{21}$ stabilizes within 3 wavelengths and $S_{11}$ requires 7 wavelengths to stabilize.
3. In Chapter 4 practical measurements were carried out using the parallel plate set-ups that were fabricated in Objective 2.

S Parameters $S_{11}$ and $S_{21}$ were measured at distances of $18\lambda$, $10\lambda$ and $0.5\lambda$ after the bend. A comparison of the performance between the $90^\circ$ parallel plate with a constant 1 cm gap and $180^\circ$ parallel plate bend with a wedge reflector at a 1cm gap were made. It was shown that the $180^\circ$ bend had a superior transmitted signal $S_{21}$ compared to the $90^\circ$ bend.

Experiments were carried out with a loop antenna to feed a horn antenna to provide increased flexibility and an optimal size for the loop antenna was determined.

Geometrical and Physical Optics were studied in detail in Chapter 5 and Chapter 6.

4. Geometrical Optics was initially used to predict the reflector profiles which are required to produce a plane wave using Matlab. The phase at the aperture was varied from $0^\circ$ to $30^\circ$, which showed the reflector tilt in order to account for the phase shift. The variation in the reflector position proved too large to be implemented by shaping. Hence, another approach was used to solve for a shaped sub reflector instead of shaping the main reflector. A Least Means Square method of optimisation was used to evaluate the feed point for a specific scan angle.

The second method worked effectively for sub reflector to main reflector ratios greater than 10 degrees. Chapter 5 concluded that Geometrical Optics is a suitable method for studying parallel plate pillbox structures with variable reflector for scanning. It was clearly shown that increasing the number of reflectors would reduce the variation required in reflector profile for scanning large angles of the central axis.

Physical Optics was used in Chapter 6, this method used the Dyadic Greens function to evaluate E and H fields on the reflector using boundary conditions. A modular program was developed using Matlab to solve for both far and near fields. The benefit of using a
modular program was to analyse multiple reflectors with ease. The complex program was verified successfully by analysing a flat sheet reflector fed by a dipole as well as several types of reflectors, both with circular and rectangular apertures similar to the parallel plate reflector.

5. Chapter 7 and Chapter 8 demonstrated practical methods of making monopoles and reflectors re-configurable.

Chapter 7 concentrated on re-configurable monopoles, which aimed to increase the bandwidth by varying an antenna element. A multi-purpose ECAD program – MAFIA was used to examine a re-configurable microstrip dipole with switches. A practical antenna was fabricated and the measurements were compared with the simulation results, which compared within acceptable limits.

As the structure was relatively simple and cheap to fabricate, practical experiments using copper tubes as monopole segments and diodes were used to assess the re-configurability of a monopole. Switching was successfully carried out using diodes between 380 MHz and 500 MHz using the copper tube re-configurable monopole.

A new antenna design concept was introduced to achieve indirect switching of the monopole structure so that the noise generated from the diodes do not interact with the radiation pattern of the monopole antenna.

6. Chapter 8 studied re-configurable reflectors using simple Matlab programs, initially examining stepped surface parabolic reflectors and then using aperture field theory to study the radiation patterns of a 2 dimensional stepped reflector with a varying surface.

A very large horn antenna was used to study the reflecting/transmitting properties of a rectangular wire mesh. The results were inconclusive due to the distortion of fields at the horn aperture. Hence the expected behaviour was not obtained. To successfully carry out
this line of investigation using a wire mesh would require using a suitable anechoic chamber and placing the rectangular mesh a certain distance away from the horns opening. An alternate method to measure re-configurability was studied and the description follows.

A re-configurable surface was studied outdoors to minimize reflections using a monopole at 500 MHz as a transmitter on a circular reflective ground plane. Reflection using variable length elements was successfully represented by measurement of reflected and transmitted signals at a point beyond the monopole transmitter. This method proved that a reflector could be successfully re-configured.

9.3 Project Original Work

It was discovered that no theoretical analyses of fields within the parallel plate structure of lens antennas had been presented anywhere in the literature, inspite of the large usage of these types of antennas. The other areas of work in this thesis and the analytic development of the lens antenna, unquestionably provide a strong potential candidate for the SKA project as it satisfies a large portion of the required specifications.

This thesis makes a notable contribution to the study of the microwave lens family in presenting a theoretical analysis which agrees well with practical measurements for the first time.
References


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APPENDIX B

This Appendix consists of all the written programs which are referred to by the section in the thesis they are used in, preceded by a "B".

Appendix B2-1

Program written in Matlab to work out the step discontinuity in a parallel plate region

% Define the variables
Z=120*pi; % Impedance of free space.
m=5; % magnification factor for the cell
lam=0.03; % value of Lambda in meters.
jmin=2; % y start value.
imin=2; % x start value.
xc=lam/3; % Height of the parallel plate.
yc=lam/3; % Width of the parallel plate.
c=0.5;

% These are variables and can be varied for changing cell size
% and the settling of backward waves.

delx=lam/(12*m); % cell width.
dely=lam/(12*m); % cell height.
dx=lam; % x length upto the bend.
dy=1*lam/3; % y height after the bend.

% calculations for the limits of x, y and n.
jlim=jmin+round(yc/dely); % The limit that sets the height of the parallel plate.
jmax=jlim+round(dy/dely); % The maximum value of j which is the y variable.
ilim=imin+round(dx/delx); % The limit that sets the distance from the input point to the bend.
imax=ilim+round(xc/delx); % The maximum value of i which is the x variable.
jstepmax=round(1.25*(jlim-jmin))-(jlim-jmin);
for n=1:2
  for i=1:imax*4+1
    for j=1:jmax+1
      Ey(n,i,j)=0;
      Hz(n,i,j)=0;
      Ex(n,i,j)=0;
    end
  end
end

count=(180/((lam/2)/delx));
inew=1;
for n=1:m*12*2*2
  for i=1:inew
    for j=1:jlim-2
      if (n*count)<180
        Ey(1,2,j+2)=sin((n*count)*pi/180);
      else
        Ey(1,2,j+2)=0;
      end
    end
  end
  if i<iлим-2
    for j=1:jlim-2
      Hz(2,i+2,j+2)=Hz(1,i+2,j+2)+(c/Z)*((Ex(1,i+2,j+1))-(Ex(1,i+2,j)))-(c/Z)*(Ey(1,i+2,j+2)-Ey(1,i+1,j+2));
      Hz(2,i+2,jmin)=Hz(2,i+2,jmin+1);
      Hz(2,i+2,jlim+1)=Hz(2,i+2,jlim);
    end
  elseif i>iлим-2
    for j=1:jlim-2
      Hz(2,i+2,j+2)=Hz(1,i+2,j+2)-(c/Z)*((Ex(1,i+2,j+1))-(Ex(1,i+2,j)))+(c/Z)*(Ey(1,i+2,j+2)-Ey(1,i+1,j+2));
      Hz(2,i+2,jmin)=Hz(2,i+2,jmin+1);
      Hz(2,i+2,jlim+1)=Hz(2,i+2,jlim);
    end
  end
for j=1:jlim-2+jstepmax
    Hz(2,i+2,j+2)=Hz(1,i+2,j+2)+(c/Z)*((Ex(1,i+2,j+1))-(Ex(1,i+2,j)))-(c/Z)*(Ey(1,i+2,j+2)-Ey(1,i+1,j+2));
    Hz(2,i+2,jmin)=Hz(2,i+2,jmin+1);
    Hz(2,i+2,jlim+1+jstepmax)=Hz(2,i+2,jlim+jstepmax);
end
elseif i==ilim-2
    for j=1:jlim-2+jstepmax
        if j>=jlim-2
            Hz(2,i+2,j+2)=Hz(2,i+3,j+2);
        else
            Hz(2,i+2,j+2)=Hz(1,i+2,j+2)+(c/Z)*((Ex(1,i+2,j+1))-(Ex(1,i+2,j)))-(c/Z)*(Ey(1,i+2,j+2)-Ey(1,i+1,j+2));
            Hz(2,i+2,jmin)=Hz(2,i+2,jmin+1);
            Hz(2,i+2,jlim+1)=Hz(2,i+2,jlim);
        end
    end
end
Hz(2,ilim+1,jlim+1)=Hz(2,ilim,jlim);

if i<=ilim-2
    for j=1:jlim-2
        Ex(2,i+2,j)=Ex(1,i+2,j)+Z*c*(Hz(2,i+2,j+2)-Hz(2,i+2,j+1));
    end
else
    for j=1:jlim-2+jstepmax
        Ex(2,i+2,j)=Ex(1,i+2,j)+Z*c*(Hz(2,i+2,j+2)-Hz(2,i+2,j+1));
    end
end
end
inew=1+inew;
for i=1:inew

    if i<=ilim-2
        for j=1:jlim-2
            Ey(2,i+1,j+2)=Ey(1,i+1,j+2)-Z*c*(Hz(2,i+2,j+2)-Hz(2,i+1,j+2));
        end
    else
        for j=1:jlim-2+jstepmax
            Ey(2,i+1,j+2)=Ey(1,i+1,j+2)-Z*c*(Hz(2,i+2,j+2)-Hz(2,i+1,j+2));
        end
    end
end
end
end

Appendix B2-2

% Routine for evaluating the parallel plate 90 degree bend problem

%%%Part I

%Declaring variables

freq=10*10^9;   % Frequency

\[ c=3*10^8; \]  
% Distance between the parallel plates after the bend,
\% which is the same before the bend

\texttt{lam=c/freq; \% Wavelength in m}
\texttt{k=2*pi/lam; \% Propagation constant}
\texttt{mult=10000; \% Multiplication or division factor for integration,}
\texttt{\% the number of steps taken.}
\texttt{b=0.01; \% height of the parallel plate (distance between the plates)}
\texttt{c=0.01; \% Distance between the parallel plates after the bend,}
\texttt{\% which is the same before the bend}
\texttt{l=lam/4; \% Distance from bend discontinuity in the z plane,}
\texttt{\% where the field can be assumed as a TEM wave.}
\texttt{thetaI=[[2:89]*pi/180,[2:89]*pi/180];}
\texttt{\% Modes represented as various angles in a parallel plate.}
\texttt{thetaII=thetaI; \% Modes represented as various angles in a parallel plate.}
\texttt{\% Arbitrary values are chosen here.}
\texttt{alph=4; \% attenuation factor for the modes}
\texttt{beta=k; \% Propagation constant}
\texttt{n=length(thetaI);}
\texttt{\%}
\texttt{for a=1:n, F1(a)=(1/n)^(1/2); end}

\% Initialising
\texttt{intz_1=0;}
\texttt{intz_2=0;}
\texttt{inty_1=0;}
\texttt{inty_2=0;}
\texttt{\% Integrating across the aperture y=0 to y=b}
\texttt{for y=1:b*mult}
\texttt{inty1=exp(((alph+j*k)/2)*((y-1)/mult)./(sin(thetaI))};
\text{inty}_2 = \text{beta} \times \frac{\cos(k \times (y-1) / \text{mult})}{\sin(\text{beta} \times b)};
\text{inty}_1 = \text{inty}_1 + \text{inty}_1;
\text{inty}_2 = \text{inty}_2 + \text{inty}_2;
\text{end}
\text{inty}_1 = \text{inty}_1 / \text{mult};
\text{inty}_2 = \text{inty}_2 / \text{mult};

\% Integrating across the aperture \( z=0 \) to \( z=c \)
\text{for} \ z = 1 \cdot c \times \text{mult}
\text{intz}_1 = \frac{\cos(k \times (z-1) / \text{mult})}{\sin(k \times c)};
\text{intz}_2 = \exp(-\frac{(\text{alph}+j \times k) \times (z-1) / \text{mult}}{\sin(\text{thetaII})})�\frac{\text{intz}_1 + \text{intz}_1}{\text{intz}_2 + \text{intz}_2};
\text{end}
\text{intz}_1 = \text{intz}_1 / \text{mult};
\text{intz}_2 = \text{intz}_2 / \text{mult};

Y_1 = k^2;
Y_2 = k^2;
Y_3 = ((k^2) - ((-\text{alph}+j \times k) / (2 \times \sin(\text{thetaI}))) \times (2 \times \text{inty}_1);\text{inty}_1;
Y_4 = (((-\text{alph}+j \times k) / (2 \times \cos(\text{thetaI}))) \times (2 \times k^2) \times \text{inty}_1 \times ((-\text{alph}+j \times k) / (2 \times \cos(\text{thetaI}))) \times (\cot(-\text{beta} \times c) / (-\beta));
Y_5 = j \times (k^2 - 1) \times (\cot(-\text{beta} \times c));
Y_6 = (k - (1 / k)) \times \text{alph}+j \times k) \times (\cot(-\text{beta} \times c));
Y_7 = -2 \times \text{alph}+j \times k) \times (\text{inty}_2);
Y_8 = -2 \times ((-\text{alph}+j \times k) / (2 \times \cos(\text{thetaII}))) \times (\text{inty}_2);

Z_1 = (k^2 - (\text{alph}+j \times k)^2);
Z_2 = ((k^2) - ((-\text{alph}+j \times k) / (2 \times \cos(\text{thetaII}))) \times (2 \times \text{intz}_2);
\[ Z_4 = (k^2 - (-a + j \cdot k) \cdot (2 \cdot \sin(\theta_1))^{\cdot 2}) \cdot (-(-a + j \cdot k) \cdot (2 \cdot \cos(\theta_1))) \cdot \exp(b \cdot (-a + j \cdot k) \cdot (2 \cdot \sin(\theta_1))) \cdot \text{intz}_1 \cdot (-k) \]
\[ Z_5 = j \cdot \text{intz}_1 \cdot k^2 \]
\[ Z_6 = -k \cdot (-a + j \cdot k) \cdot \text{intz}_1 \]
\[ Z_7 = (-2 \cdot k) \cdot (-a + j \cdot k) \cdot (2 \cdot \cos(\theta_1)) \cdot (\cot(\beta \cdot b)) \]
\[ Z_8 = (-2 \cdot k) \cdot (-a + j \cdot k) \cdot (2 \cdot \cos(\theta_1)) \cdot \text{intz}_2 \cdot (\cot(\beta \cdot b)) \]

\[ F_1 = 1 \]
\[ a_1 = Y_7 - Y_8 \]
\[ b_1 = Y_2 + Y_3 + Y_4 + Y_6 \]
\[ c_1 = Y_1 \]
\[ a = Z_2 + Z_1 - Z_7 - Z_8 \]
\[ b = Z_4 + Z_6 \]
\[ c = Z_5 \]

\[ B = \text{abs}(F_1 \cdot (c_1 \cdot a - a_1 \cdot c) / (a_1 \cdot b - b_1 \cdot a)) \]
\[ F = \text{abs}((b_1 \cdot B + c \cdot F_1) / a) \]

\[ B_{\text{norm}} = (B^2) / \max(B^2) \]
\[ F_{\text{norm}} = (F^2) / \max(F^2) \]
\[ B_{\text{avg}} = (\sum(B_{\text{norm}})) / n \]
\[ F_{\text{avg}} = (\sum(F_{\text{norm}})) / n \]
\[ B_{\text{back}} = (B_{\text{avg}}) / (B_{\text{avg}} + F_{\text{avg}}) \]
\[ B_{\text{forw}} = (F_{\text{avg}}) / (B_{\text{avg}} + F_{\text{avg}}) \]

**Appendix B2-3**

% Routine to cascade the two 90° bends back to back to provide the 180° bend
function St=cascade(Sl,Sr)
xx=length(Sl)/2;
for x=1:xx
    for y=1:xx
        SL11(x,y)=[Sl(x,y)];
        SL12(x,y)=[Sl(x+xx,y)];
        SL21(x,y)=[Sl(x,y+xx)];
        SL22(x,y)=[Sl(xx+x,y+xx)];
        Sr11(x,y)=[Sr(x,y)];
        Sr12(x,y)=[Sr(x+xx,y)];
        Sr21(x,y)=[Sr(x,y+xx)];
        Sr22(x,y)=[Sr(xx+x,y+xx)];
    end
end

W=inv(eye(length(Sl)/2)-Sl22*Sr11);
St11=Sl11+Sl12*Sr11*W*Sl21;
St12=Sl12*(eye(length(Sl)/2)+Sr11*W*Sl22)*Sr12;
St21=Sr21*W*Sl21;
St22=Sr21*W*Sl22*Sr21;
St=[St11 St12;St21 St22];

\[\text{Appendix B3-1}\]

Program written in Matlab to work out the 90° bend in a parallel plate region

function [Ey] = step90b

% Define the variables
Z=120*pi; % Impedance of free space.
m=5; % magnification factor for the cell
lam=0.03; % value of Lambda in meters.
jmin=2; % y start value.
imin=2; % x start value.
xc=lam/3; % Height of the parallel plate.
yc=lam/3; % Width of the parallel plate.
c=0.5;

% These are variables and can be varied for changing cell size
% and the settling of backward waves.

delx=lam/(12*m); % cell width.
dely=lam/(12*m); % cell height.
dx=2*lam; % x length upto the bend.
dy=2*lam; % y height after the bend.

% calculations for the limits of x, y and n.
jlim=jmin+round(yc/dely); % The limit that sets the height of the parallel plate.
jmax=jlim+round(dy/dely); % The maximum value of j which is the y variable.
ilim=imin+round(dx/delx); % The limit that sets the distance from the input point to the
% bend.
imax=ilim+round(xc/delx); % The maximum value of i which is the x variable.
jstep=1;

for n=1:2
    for i=1:imax+1
        for j=1:jmax*20+1
            Ey(n,i,j)=0;
            Hz(n,i,j)=0;
            Ex(n,i,j)=0;
        end
    end
end
count=(180/((lam/2)/delx));
new=1;
Dt=delx/((2)*(3*10^8));% 0.833 pico seconds
T=15*10^(-12);
to=3*T;

for n=1:480
    for i=1:new

        for j=1:jlim-2
            Ey(1,2,j+2)=exp(-(((n*Dt)-to)/T)^2);
            E(n)=Ey(1,2,j+2);
        end

        if i<=ilim-2
            for j=1:jlim-2
                Hz(2,i+2,j+2)=Hz(1,i+2,j+2)+(c/Z)*((Ex(1,i+2,j+1))-(Ex(1,i+2,j)))-(c/Z)*((Ey(1,i+2,j+2)-Ey(1,i+1,j+2)));
                Hz(2,i+2,jmin)=Hz(2,i+2,jmin+1);
                Hz(2,i+2,jlim+1)=Hz(2,i+2,jlim);
            end
        end
        elseif ((i>ilim-2) & (i<=imax-2))
            for j=1:jlim-2+jstep
                Hz(2,i+2,j+2)=Hz(1,i+2,j+2)+(c/Z)*((Ex(1,i+2,j+1))-(Ex(1,i+2,j)))-(c/Z)*((Ey(1,i+2,j+2)-Ey(1,i+1,j+2)));
                Hz(2,i+2,jmin)=Hz(2,i+2,jmin+1);
            end
        end
    end
end
for j=1:jlim-2+jstep
Hz(2,imax+1,j+2)=Hz(2,imax,j+2);  
end
for j=jlim-1:jlim-2+jstep  
  Hz(2,ilim,j+2)=Hz(2,ilim+1,j+2);  
end

Hz(2,ilim+1,jlim+1)=Hz(2,ilim,jlim);

if i<=ilim-2  
  for j=1:jlim-2  
    Ex(2,i+2,j)=Ex(1,i+2,j)+Z*c*(Hz(2,i+2,j+2)-Hz(2,i+2,j+1));  
  end
else  
  for j=1:jlim-2+jstep  
    Ex(2,i+2,j)=Ex(1,i+2,j)+Z*c*(Hz(2,i+2,j+2)-Hz(2,i+2,j+1));  
  end
end

if inew<imax-2  
  inew=1+inew;
else
  inew=inew;
end

for i=2:inew

  if i<=ilim-1  
    for j=1:jlim-2  
      Ey(2,i+1,j+2)=Ey(1,i+1,j+2)-Z*c*(Hz(2,i+2,j+2)-Hz(2,i+1,j+2));  
    end
  elseif i>ilim-1
for j=1:jlim-2+jstep
    Ey(2,i+1,j+2)=Ey(1,i+1,j+2)-Z*c*(Hz(2,i+2,j+2)-Hz(2,i+1,j+2));
end
end

for j=1:jlim-2+jstep
    Hz(1,i+1,j+2)=Hz(2,i+1,j+2);
    Ex(1,i+1,j)=Ex(2,i+1,j);
    Ey(1,i+1,j+2)=Ey(2,i+1,j+2);
end
end

if n>ilim-2
    jstep=jstep+1;
end
end

**Appendix B3-2**

Program written in Matlab to work out the 180° bend in a parallel plate region

% Define the variables

\[
Z=120*\pi; \quad \text{% Impedance of free space.}
\]
\[
m=5; \quad \text{% magnification factor for the cell}
\]
\[
lam=0.03; \quad \text{% value of Lambda in meters.}
\]
\[
jmin=2; \quad \text{% y start value.}
\]
\[
imin=2; \quad \text{% x start value.}
\]
\[
xc=lam/3; \quad \text{% Height of the parallel plate.}
\]
\[
yc=lam/3; \quad \text{% Width of the parallel plate.}
\]
\[
c=0.5;
\]
\[
k=0;
\]

% These are variables and can be varied for changing cell size
% and the settling of backward waves.
delx=lam/(12*m);  % cell width.
dely=lam/(12*m);  % cell height.
dx=10*lam;  % x length upto the bend.

% calculations for the limits of x, y and n.
jlim=jmin+round(yc/dely);  % The limit that sets the height of the parallel plate.
jmax=jmin+round(2*yc/dely);  % The maximum value of j which is the y variable.
ilim=imin+round(dx/delx);  % The limit that sets the distance from the input point to the bend.
imax=ilim+round(xc/delx);  % The maximum value of i which is the x variable.
jstep=1;

for n=1:2
    for i=1:imax+1
        for j=1:jmax+1
            Ey(n,i,j)=0;
            Hz(n,i,j)=0;
            Ex(n,i,j)=0;
        end
    end
end

count=(180/((lam/2)/delx));
inew=1;
Dt=delx/((2)*(3*10^8));  % 0.833 pico seconds
T=15*10^(-12);
to=3*T;

for n=1:300
for i=1:i:leinew
    for j=1:j:lim-2
        Ey(1,2,j+2)=exp(-(((n*D)-to)/T)^2);
        E(n)= Ey(1,2,j+2);
    end

    if i<=i:lim-2
        for j=1:j:lim-2
            Hz(2,i+2,j+2)=Hz(1,i+2,j+2)+(c/Z)*((Ex(1,i+2,j+1))-(Ex(1,i+2,j)))-(c/Z)*(Ey(1,i+2,j+2)-Ey(1,i+1,j+2));
            Hz(2,i+2,jmin)=Hz(2,i+2,jmin+1);
        end
    elseif ((i>i:lim-2) & (i<=i:max-2))
        for j=1:j:lim-2+j:step
            Hz(2,i+2,j+2)=Hz(1,i+2,j+2)+(c/Z)*((Ex(1,i+2,j+1))-(Ex(1,i+2,j)))-(c/Z)*(Ey(1,i+2,j+2)-Ey(1,i+1,j+2));
            Hz(2,i+2,jmin)=Hz(2,i+2,jmin+1);
        end
    end

    for j=1:j:lim-2+j:step
        Hz(2,i:max+1,j+2)=Hz(2,i:max+j+2);
    end

    if i<=i:lim-2
        for j=1:j:lim-2
            Ex(2,i+2,j)=Ex(1,i+2,j)+Z*c*(Hz(2,i+2,j+2)-Hz(2,i+2,j+1));
        end
        Ex(2,i+2,j:lim)=0;
    else
        for j=1:j:lim-2+j:step
\[
Ex(2,i+2,j)=Ex(1,i+2,j)+Z^*c^*(Hz(2,i+2,j+2)-Hz(2,i+2,j+1));
\]
end
end

if i<=ilim-2
for j=1:jlim-2
\[
Ey(2,i+2,j+2)=Ey(1,i+2,j+2)-Z^*c^*(Hz(2,i+3,j+2)-Hz(2,i+2,j+2));
\]
end
elseif i>ilim-2
for j=1:jlim-2+jstep
\[
Ey(2,i+2,j+2)=Ey(1,i+2,j+2)-Z^*c^*(Hz(2,i+3,j+2)-Hz(2,i+2,j+2));
\]
end
end
for j=1:jlim-2+jstep
\[
Hz(1,i+2,j+2)=Hz(2,i+2,j+2);
\]
Ex(1,i+2,j)=Ex(2,i+2,j);
Ey(1,i+2,j+2)=Ey(2,i+2,j+2);
end
if inew<imax-2
\[
inew=1+inew;
\]
else
\[
inew=inew;
\]
end
end

% PART TWO FOR 180 DEGREE BEND
if n>(ilim-1)
if k==ilim-3
k=k;
else
k=k+1;
end

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end
for i=(ilim-2-k):(ilim-2)
  for j=jlim-1:jlim-1+jstep
    Hz(2,i+2,j+2)=Hz(1,i+2,j+2)+(c/Z)*((Ex(1,i+2,j+1))-(Ex(1,i+2,j)))-(c/Z)*(Ey(1,i+2,j+2)-Ey(1,i+1,j+2));
  end
  Hz(2,i+2,jmax+1)=Hz(2,i+2,jmax);
  for j=jlim-1:jlim-1+jstep
    Ex(2,i+2,j)=Ex(1,i+2,j)+Z*c*(Hz(2,i+2,j+2)-Hz(2,i+2,j+1));
  end
  Ex(2,i+2,jlim)=0;
  for j=jlim-1:jlim-1+jstep
    Ey(2,i+2,j+2)=Ey(1,i+2,j+2)-Z*c*(Hz(2,i+3,j+2)-Hz(2,i+2,j+2));
  end
  for j=jlim-1:jlim-1+jstep
    Hz(1,i+2,j+2)=Hz(2,i+2,j+2);
    Ex(1,i+2,j)=Ex(2,i+2,j);
    Ey(1,i+2,j+2)=Ey(2,i+2,j+2);
  end
end
end

if n>ilim-2 & n<imax-2
  jstep=jstep+1;
elseif (jstep==yc/dely)
  jstep=jstep;
end

t(n)=(n-1)*Dt;
end

location=10*(10^9)*500*Dt; %500 is number of fft points
Appendix B5 - 2

% This program calculates the reflector profiles for dual reflector antennas,
% with arbitrary phase and amplitude distributions.

% User defined independent variables.

a1=0;
a2=0;
b1=0;
b2=0;
theta1max=30*pi/180;% maximum angle for reflector 1 from feedpoint expressed in radians.
x2max=1;% normalized maximum value for height of reflector 2.
x1max=0.1;% normalized maximum value for height of reflector 1.
alpha=0.1;% distance between co-ordinate 2 and feed point.
theta3=-30*pi/180;% phase at aperture.
p=16;% power of the feed cosine term.
x2(1)=x2max;% initial value of x2.
h=0.01;% incremental step size for runge-kutta method, used to solve the differential equation.
N=200;% maximum limit of for loop.
theta1(1)=theta1max;

% Dependent variables.

beta=(x1max)/(tan(theta1max));% distance between co-ordinate 1 and feedpoint.
y1(1)=0;% initial value of y1.
M=1-(cos(theta1max))**(p+1);% constant for solution of theta 1.
theta2max=atan((x2max-x1max)/(alpha+beta));% maximum angle for reflector 2 from feedpoint.
Cpmax=(beta/cos(theta1max))+((alpha+beta)/cos(theta2max));% phase constant due to path length.
Cp(1)=Cpmax;
x1new(1)=x1max;

%Main Algorithm.
for n=1:N

I1(n)=cos(theta1(n))^(p);

r1(n)=(beta-y1(n))*sec(theta1(n));

x1(n)=(beta-y1(n))*tan(theta1(n));

y2(n)=(((Cp(n)-r1(n))^2-(x1(n)-x2(n))^2-(alpha+beta-y1(n))^2)/(2*((alpha+beta-
y1(n))+(sin(theta3)*(x1(n)-x2(n)))+(Cp(n)-r1(n)))/(1-sin(theta3))^(1/2)));

x(n)=x2(n)+((-Cp(n)-r1(n))^2+(x1(n)-x2(n))^2+(alpha+beta-y1(n))^2)/(2*((x1(n)-
x2(n))+(alpha+beta-y1(n))*(1-sin(theta3))^2)/(1/2)+(Cp(n)-r1(n))/(sin(theta3))));

f1(n)=tan((theta1(n)-atan((x2(n)-x1(n))/(alpha+beta+y2(n)-y1(n)))))/2);

I3a(n)=1-a1*exp(-b1*(1-x2(n)))-a2*exp(-b2*x2(n));%current magnitude at aperture.

f2(n)=((13a(n)*x2(n))/(I1(n)*sin(theta1(n))))/(quad('Pout',0,x2max)/
quad('Pin',0,theta1max));

f(n)=f1(n)*f2(n)*((beta-y1(n))*(sec(theta1(n))^2))/(1+f1(n)*tan(theta1(n)));

f3(n)=sin(theta3);

%f3(n)=(x2(n)-x(n))/((x2(n)-x(n))^2+y2(n)^2)^(1/2);

k1(n)=h*f(n);

k12(n)=h*f2(n);

k13(n)=h*f3(n);

x2n(n)=x2(n)-0.5*h;

y1n(n)=y1(n)+0.5*k1(n);

theta1n(n)=theta1(n)+0.5*k12(n);

Cpn(n)=Cp(n)+0.5*k13(n);

I1n(n)=cos(theta1n(n))^(p);

r1n(n)=(beta-y1n(n))*sec(theta1n(n));

x1n(n)=(beta-y1n(n))*tan(theta1n(n));
y2n(n)=((Cpn(n)-r1n(n))^2-(x1n(n)-x2n(n))^2-(alpha+beta-y1n(n))^2)/(2*((alpha+beta-y1n(n))+(sin(theta3)*(x1n(n)-x2n(n))/(Cpn(n)-r1n(n)))/(1-sin(theta3))^(1/2));
xn(n)=x2n(n)+((Cpn(n)-r1n(n))^2+(x1n(n)-x2n(n))^2+(alpha+beta-y1n(n))^2)/(2*((x1n(n)-x2n(n))+(alpha+beta-y1n(n))*(1-sin(theta3))^2)^(1/2)+(Cpn(n)-r1n(n)))/(sin(theta3)));
f1n(n)=tan((theta1n(n)-atan((x2n(n)-x1n(n))/(alpha+beta+y2n(n)-y1n(n)))))/2);
l3an(n)=1-a1*exp(-b1*(1-x2n(n)))-a2*exp(-b2*x2n(n));%current magnitude at aperture.
f2n(n)=((l3an(n)*x2n(n))/(l1n(n)*sin(theta1n(n))))/(quad('Pout',0,x2max)/quad('Pin',0,theta1max));
fn(n)=f1n(n)*f2n(n)*((beta-y1n(n))*(sec(theta1n(n)))^2)/(1+f1n(n)*tan(theta1n(n)));
f3n(n)=sin(theta3);

k2(n)=h*fn(n);
k22(n)=h*f2n(n);
k23(n)=h*f3n(n);
x2nn(n)=x2(n)-0.5*h;
y1nn(n)=y1(n)+0.5*k2(n);
theta1nn(n)=theta1(n)+0.5*k22(n);
Cpnn(n)=Cp(n)+0.5*k23(n);
I1nn(n)=cos(theta1nn(n))^(p);
rl1nn(n)=(beta-y1nn(n))*sec(theta1nn(n));
x1nn(n)=(beta-y1nn(n))*tan(theta1nn(n));
y2nn(n)=((Cpnn(n)-rl1nn(n))^2-(x1nn(n)-x2nn(n))^2-(alpha+beta-y1nn(n))^2)/(2*((alpha+beta-y1nn(n))+(sin(theta3)*(x1nn(n)-x2nn(n))+(Cpnn(n)-rl1nn(n)))/(1-sin(theta3))^(1/2));
xnn(n)=x2nn(n)+((Cpnn(n)-rl1nn(n))^2+(x1nn(n)-x2nn(n))^2+(alpha+beta-y1nn(n))^2)/(2*((x1nn(n)-x2nn(n))+(alpha+beta-y1nn(n))*(1-sin(theta3))^2)^(1/2)+(Cpnn(n)-rl1nn(n))/(sin(theta3)));)
f1nn(n)=tan((theta1nn(n)-atan((x2nn(n)-x1nn(n))/(alpha+beta+y2nn(n)-y1nn(n)))))/2);
l3ann(n)=1-a1*exp(-b1*(1-x2nn(n)))-a2*exp(-b2*x2nn(n));
%current magnitude at aperture.
f2nn(n)=((l3ann(n)*x2nn(n))/(l1nn(n)*sin(theta1nn(n))))/(quad('Pout',0,x2max)/quad('Pin',0,theta1max));
fnn(n)=f1nn(n)*f2nn(n)*((beta-y1nn(n))*(sec(theta1nn(n))*2))/(1+f1nn(n)*tan(theta1nn(n)));
f3nn(n)=sin(theta3);
k3(n)=h*fnn(n);
k32(n)=h*f2nn(n);
k33(n)=h*f3nn(n);
x2nnn(n)=x2(n)-h;
y1nnn(n)=y1(n)+k3(n);
theta1nnn(n)=theta1(n)+k32(n);
Cpnnn(n)=Cp(n)+k33(n);

l1nnn(n)=cos(theta1nnn(n))^p(n);
r1nnn(n)=(beta-y1nnn(n))*sec(theta1nnn(n));
x1nnn(n)=(beta-y1nnn(n))*tan(theta1nnn(n));
y2nnn(n)=((Cpnnn(n)-r1nnn(n))^2-(x1nnn(n)-x2nnn(n))^2-(alpha+beta-y1nnn(n))^2)/(2*((alpha+beta-y1nnn(n))+sin(theta3)*(x1nnn(n)-x2nnn(n))+Cpnnn(n)-r1nnn(n)))/(1-sin(theta3))^4/(1/2));
xnnn(n)=x2nnn(n)+((Cpnnn(n)-r1nnn(n))^2+(x1nnn(n)-x2nnn(n))^2+(alpha+beta-y1nnn(n))^2)/(2*((alpha+beta-y1nnn(n)+((alpha+beta-y1nnn(n))*(1-sin(theta3))^2)/(1/2)+(Cpnnn(n)-r1nnn(n))/sin(theta3)));)
flnnn(n)=tan(theta1nnn(n)-atan((x2nnn(n)-x1nnn(n))/(alpha+beta+y2nnn(n)-y1nnn(n))))/2);
l3ann(n)=1-a1*exp(-b1*(1-x2nnn(n)))-a2*exp(-b2*x2nnn(n));
%cumulative magnitude at aperture.
f2nnn(n)=((l3ann(n)*x2nn(n))/(l1nn(n)*sin(theta1nn(n))))/(quad('Pout',0,x2max)/quad('Pin',0,theta1max));
fnnn(n)=f1nn(n)*f2nn(n)*((beta-y1nn(n))*(sec(theta1nn(n))*2))/(1+f1nn(n)*tan(theta1nn(n))));
f3nn(n)=sin(theta3);
k4(n)=h*f3nn(n);
k42(n)=h*f2nnn(n);
k43(n)=h*f3nn(n);
x2(n+1)=x2(n)-h;
x1new(n+1)=x1new(n)-(2*x1new(1)/N);
y1(n+1)=(y1(n)-(1/6)*(k1(n)+2*k2(n)+2*k3(n)+k4(n))); theta1(n+1)=(theta1(n)-(1/6)*(k12(n)+2*k22(n)+2*k32(n)+k42(n))); Cp(n+1)=(Cp(n)-(1/6)*(k13(n)+2*k23(n)+2*k33(n)+k43(n))); Focus(n)=beta;
end

x1(201)=(beta-y1(201))*tan(theta1(201));
r1(201)=(beta-y1(201))*sec(theta1(201));
y2(201)=((Cp(201)-r1(201))^2-(x1(201)-x2(201))^2-(alpha+beta-
y1(201))^2)/(2*((alpha+beta-y1(201))+(sin(theta3)*(x1(201)-x2(201))+(Cp(201)-
r1(201))/(1-sin(theta3))^2(1/2))); z2=y2+alpha+beta;
plot(z2,x2,y1,x1,Cp,x2);
xlabel('y values, reflectors shown with separation'),ylabel('xvalues, alpha=0, beta=0.87'); title('Reflector Profiles to Produce a 30 degree phase drift across Reflector 2'); axis([-0.1,2,-1,1]); gtext('Reflector 1')
gtext('Reflector 2')
gtext('Phase Drift across Reflector 2')

function y=Pin(x)
y=sin(x).*(cos(x)).^16;
function y=Pout(x)
y=x;
Appendix B5 - 3

% Setting initial variables.

F=4;
dd=0.05;
thetaScan=1*pi/180; % determines angle of incoming wave w.r.t. the z axis
input=10; % Incident wave from plotting distance
input1=1000000; % Incident wave from infinity distance
h=0.004;
N=250;
F=5*10^9;
lamda=3*10^8/F;
k=2*pi/lamda;

% Subroutine for calculating main reflector field.

for n=1:N+1
    delta(n)=((n-(1+(N/2)))/(2*N/3));
end

tau=F/((cos(delta/2).*cos(delta/2)));
dtau=tau.*tan(delta/2);
ddt=tau.^2.*(2-cos(delta))./(2*F);

rho=tau.*sin(delta); % rectangular co-ordinates
z=tau((N/2)+1)-tau.*cos(delta); % rectangular co-ordinates
rhoScan1=(input1-z).*tan(thetaScan); % rays from infinity
rhoScan=(input-z).*tan(thetaScan); % rays for plotting start point
theta1=atan((rho+rhoScan1)./(input1-F));

beta=atan((tau.*cos(delta)+dtau.*sin(delta))/(tau.*sin(delta)-dtau.*cos(delta)));

v=asin((sin(theta1).*sin(beta))+cos(theta1).*cos(beta));

% angle of incidence and reflection
p12c=((tau.^2+dtau.^2).^(3/2))/(tau.^2-tau.*dtau+2.*dtau.^2);
%principle radii of curvature for the reflected wavefront
C22=-1./p12c;%surface curvature matrix
Qr22=2.*C22.*sec(vi);%reflected field curvature matrix
p12r=-1./Qr22;%principle radii of curvature for the reflected wavefront
s=F+z;%ray focusses at a parabolic focus
E=(-p12r.*exp(-j*k*sr))./((p12r+sr)); %Geometrical Optics reflected field
plot(abs(E),rho); %plotting E against rho

%Calculating where rays from reflector 1 end up.

for n=1:N/2
    rayv2(n)=rho(n)+p12r(n)*sin(2*vi(n)+thetascan);
    rayh2(n)=+z(n)+p12r(n)*cos(2*vi(n)+thetascan);
end
for n=1:(N/2)+1
    rayv2((N/2)+n)=rho((N/2)+n)-p12r((N/2)+n)*sin(2*vi((N/2)+n)-thetascan);
    rayh2((N/2)+n)=+z((N/2)+n)+p12r((N/2)+n)*cos(2*vi((N/2)+n)-thetascan);
end

%subroutine for calculating reflector 2 profile, set starting point
fx=0.5; %location of focus in the x axis
fy=0; %location of focus in the y axis
rho2(N+1)=1*rho(N+1)/10;% co-ordinates of sub reflector
z2(N+1)=F-((rho2(N+1))/(tan(delta(N+1))));
C=(input-z(N+1))+((rho(N+1)-rho2(N+1))^2+(z(N+1)-z2(N+1))^2)^(1/2)+((fy-
  rho2(N+1))^2+(fx-z2(N+1))^2)^(1/2);
%path length from input to focus
for q=1:N
    nn=N+1-q;
    if delta(nn)==0

\[
\text{rho2(nn) = 0; else for } x = 1:2 \\
mrho2(nn, x) = rho2(nn + 1) + ((1 - x) \times h); \\
\text{end for } m = 1:20 \\
\text{for } n = 1:2 \\
z2(nn, n) = F - ((mrho2(nn, n)) / (\tan(delta(nn)))); \\
\text{Gunc(nn, n) = -C + (input - z(nn)) + ((rho(nn) - mrho2(nn, n))^2 + (z(nn) - z2(nn, n))^2)^{1/2} + ((fy - mrho2(nn, n))^2 + (fx - z2(nn, n))^2)^{1/2}; end for } m = 1:20 \\
\text{for } n = 1:2 \\
z2(nn, 3) = F - ((mrho2(nn, 3)) / (\tan(delta(nn)))); \\
\text{Gunc(nn, 3) = -C + (input - z(nn)) + ((rho(nn) - mrho2(nn, 3))^2 + (z(nn) - z2(nn, 3))^2)^{1/2} + ((fy - mrho2(nn, 3))^2 + (fx - z2(nn, 3))^2)^{1/2}; if Gunc(nn, 1) * Gunc(nn, 3) < 0 \\
\text{mmrhot2(nn, 2) = mrho2(nn, 3); } \\
\text{mmrhot2(nn, 1) = mrho2(nn, 1); else } \\
\text{mmrhot2(nn, 1) = mrho2(nn, 3); } \\
\text{mmrhot2(nn, 2) = mrho2(nn, 2); end end end } \\
rho2(nn) = mrho2(nn, 1); \\
\text{end end z2 = F - ((rho2)./(tan(delta))); } \\
z2((N/2) + 1) = z2(N/2) - 0.0001; \]
tau2=rho2./sin(delta);
Cpp=(input-z)+((rho-rho2).^2+(z-z2).^2).^((1/2))+((fy-rho2).^2+(fx-z2).^2).^((1/2));

% Calculating equations for rays and curve fit for reflector 2 and
% finding the point of intersection of the two.

C1=(rayv2-rho)/(rayh2-z);
C2=rho+((rayv2-rho)/(rayh2-z)).*-z;
pol2=polyfit(rho2,z2,3); % fit a third degree polynomial curve through the points
for n=1:N+1
    d(n)=C1(n)*pol2(1);
    a(n)=C1(n)*pol2(2);
    b(n)=C1(n)*pol2(3)-1;
    c(n)=C1(n)*pol2(4)+C2(n);
end
ppp=[d(n),a(n),b(n),c(n)]; % equation of rays intersecting the polynomial
r=roots(ppp);
% roots of the polynomial determine the intersecting points of the rays and the reflector
end
yint(n)=r(3);
end
xint=pol2(1).*yint.^3+pol2(2).*yint.^2+pol2(3).*yint+pol2(4);

% Calculating the curvature of reflector 2.

newangle=atan(C1);
z1=xint;
rho1=yint;
for n=1:N-1
    gradang(n+1)=atan((-yint(n)+yint(n+2))/(xint(n)+xint(n+2)));
end
%Calculation of end sub reflector points.
if thetascan==0
delta0=((-1+(N/2))/(2*N/3));
delta32=((N+2-(1+(N/2)))/(2*N/3));
tau0=F/((cos(delta0/2)*cos(delta0/2)));
rho0=tau0*sin(delta0);
z0=tau((N/2)+1)-tau0*cos(delta0);
C10=(rayv2(1)-rho0)/(rayh2(1)-z0);
C20=rho0+((0-rho0)/(4-z0))*z0;
d0=C10*pol2(1);
a0=C10*pol2(2);
b0=C10*pol2(3)-1;
c0=C10*pol2(4)+C20;
r0=roots([d0,a0,b0,c0]);
yint0=r0(3);
tau32=F/((cos(delta32/2)*cos(delta32/2)));
rho32=tau32*sin(delta32);
z32=tau((N/2)+1)-tau32*cos(delta32);
C132=(rayv2(N+1)-rho32)/(rayh2(N+1)-z32);
C232=rho32+((0-rho32)/(4-z32))*z32;
d32=C132*pol2(1);
a32=C132*pol2(2);
b32=C132*pol2(3)-1;
c32=C132*pol2(4)+C232;
r32=roots([d32,a32,b32,c32]);
yint32=r32(3);
xint0=pol2(1)*yint0^3+pol2(2)*yint0^2+pol2(3)*yint0+pol2(4);
xint32=pol2(1)*yint32^3+pol2(2)*yint32^2+pol2(3)*yint32+pol2(4);
gradang(1)= atan((yint0-yint2)/(xint0-xint2));
gradang(N+1)=atan((yint32-yint(N))/(xint32-xint(N)));
else
  % assume something close to previous point
  gradang(1)=gradang(2);
  gradang(N+1)=gradang(N);
  xint0=xint(1);
  xint32=xint(N+1);
  yint0=yint(1);
  yint32=yint(N+1);
  delta0=atan(yint0/(F-xint0));
  delta32=atan(yint32/(F-xint32));
end

%%% the incident angle is positive for both top and bottom be aware of it.
  normang=atan(-1./tan(gradang));
  incang=newangle+normang;
  tau3=((F-xint).^2+yint.^2).^1/2;
  delta3=atan(yint./(F-xint));
for n=1:N-1
  nn=N+1-n;
  dtau3(nn)=(tau3(nn+1)-tau3(nn-1))/(delta3(nn+1)-delta3(nn-1));
  ddttau3(nn)=(tau3(nn+1)-2*tau3(nn)+tau3(nn-1))/((delta3(nn+1)-
  delta3(nn))*(delta3(nn)-delta3(nn-1)));
end
  tau332=((F-xint32).^2+yint32.^2).^1/2;
  tau30=((F-xint0).^2+yint0.^2).^1/2;
  ddttau3(N+1)=(tau332-2*tau3(N+1)+tau3(N))/((delta32-delta3(N+1))*(delta3(N+1)-
  delta3(N)));
  dtau3(N+1)=(tau332-tau3(N))/(delta32-delta3(N));
  ddttau3(1)=(tau30-2*tau3(1)+tau3(2))/((delta0-delta3(1))*(delta3(1)-delta3(2)));
  dtau3(1)=(tau30-tau3(2))/(delta0-delta3(2));
p22c=((tau3.^2+ddtau3.^2).^3/2)./(tau3.^2-tau3.*ddtau3+2.*ddtau3.^2);
p22c=1./p22c;
p22i=p12r;
snewi=((rho-yint).^2+(xint-z).^2).^(1/2);
winew=(atan((rho-yint)./(xint-z))+thetascan)./2;
Q22i=-1../(p22i-snewi);
Q22r=(2*Q22c./cos(incang))+Q22i;
p22r=-1../(Q22r);

%Calculating where rays from reflector 2 end up.

for n=1:N+1
    rayh(n)=xint(n)-p22r(n)*cos(incang(n)+normang(n));
    rayv(n)=yint(n)-p22r(n)*sin(incang(n)+normang(n));
end

%Calculating the optimal focal point, using least squares method.
rayv(1)=rayv(2);
rayv(N+1)=rayv(N);
rayh(1)=rayh(2);
rayh(N+1)=rayh(N);

for s=2:N/2
    m(s)=(rayv(s)-yint(s))/(rayh(s)-xint(s));
    aa(s)=2*m(s).^2;
    ab(s)=-2*m(s);
    ba(s)=-2*m(s);
    bb(s)=2;
    ca(s)=2*(m(s).^2)*xint(s)-2*m(s)*yint(s);
    cb(s)=-2*m(s)*xint(s)+2*yint(s);
end
for s=(N/2)+2:N
\begin{align*}
m(s) &= (\text{rayv}(s)-\text{yint}(s))/(\text{rayh}(s)-\text{xint}(s)) ; \\
\text{aa}(s) &= 2 \times m(s)^2 ; \\
\text{ab}(s) &= -2 \times m(s) ; \\
\text{ba}(s) &= -2 \times m(s) ; \\
\text{bb}(s) &= 2 ; \\
\text{ca}(s) &= 2 \times (m(s)^2 - \text{xint}(s) - 2 \times m(s) \times \text{yint}(s)) ; \\
\text{cb}(s) &= -2 \times m(s) \times \text{xint}(s) + 2 \times \text{yint}(s) ; \\
\text{end}
\end{align*}

\begin{align*}
\text{A} &= [\text{sum}(\text{aa}) \ \text{sum}(\text{ab}) \ \text{sum}(\text{ba}) \ \text{sum}(\text{bb})] ; \\
\text{B} &= [\text{sum}(\text{ca}) \ \text{sum}(\text{cb})] ; \\
\text{D} &= \text{A} \backslash \text{B} \\
\hline
\end{align*}

\%plotting the ray path

\begin{align*}
\text{yy} &= [\text{input} \ z(1) \ zl(1) \ \text{rayh}(1)] ; \\
\text{xx} &= [\text{rho}(1)+\text{rhoscan}(1) \ \text{rho}(1)+\text{rho1}(1) \ \text{rayv}(1)] ; \\
\text{yy1} &= [\text{input} \ z(1+(N/10)) \ zl(1+(N/10)) \ \text{rayh}(1+(N/10))] ; \\
\text{xx1} &= [\text{rho}(1+(N/10))+\text{rhoscan}(1+(N/10)) \ \text{rho}(1+(N/10)) \ \text{rho1}(1+(N/10)) \ \text{rayv}(1+(N/10))] ; \\
\text{yy2} &= [\text{input} \ z(1+(2*N/10)) \ zl(1+(2*N/10)) \ \text{rayh}(1+(2*N/10))] ; \\
\text{xx2} &= [\text{rho}(1+(2*N/10)) \ +\text{rhoscan}(1+(2*N/10)) \ \text{rho}(1+(2*N/10)) \ \text{rho1}(1+(2*N/10)) \ \text{rayv}(1+(2*N/10))] ; \\
\text{yy3} &= [\text{input} \ z(1+(3*N/10)) \ zl(1+(3*N/10)) \ \text{rayh}(1+(3*N/10))] ; \\
\text{xx3} &= [\text{rho}(1+(3*N/10)) \ +\text{rhoscan}(1+(3*N/10)) \ \text{rho}(1+(3*N/10)) \ \text{rho1}(1+(3*N/10)) \ \text{rayv}(1+(3*N/10))] ; \\
\text{yy4} &= [\text{input} \ z(1+(4*N/10)) \ zl(1+(4*N/10)) \ \text{rayh}(1+(4*N/10))] ; \\
\text{xx4} &= [\text{rho}(1+(4*N/10)) \ +\text{rhoscan}(1+(4*N/10)) \ \text{rho}(1+(4*N/10)) \ \text{rho1}(1+(4*N/10)) \ \text{rayv}(1+(4*N/10))] ; \\
\text{yy5} &= [\text{input} \ z(1+(5*N/10)) \ zl(1+(5*N/10)) \ \text{rayh}(1+(5*N/10))] ; \\
\text{xx5} &= [\text{rho}(1+(5*N/10)) \ +\text{rhoscan}(1+(5*N/10)) \ \text{rho}(1+(5*N/10)) \ \text{rho1}(1+(5*N/10)) \ \text{rayv}(1+(5*N/10))] ; \\
\end{align*}
yy6 = [input z(1+(6*N/10)) z1(1+(6*N/10)) rayh(1+(6*N/10)) ];
xx6 = [rho(1+(6*N/10)) + rhoscan(1+(6*N/10)) rho(1+(6*N/10)) rho1(1+(6*N/10))
rayv(1+(6*N/10)) ];
yy7 = [input z(1+(7*N/10)) z1(1+(7*N/10)) rayh(1+(7*N/10)) ];
xx7 = [rho(1+(7*N/10)) + rhoscan(1+(7*N/10)) rho(1+(7*N/10)) rho1(1+(7*N/10))
rayv(1+(7*N/10)) ];
yy8 = [input z(1+(8*N/10)) z1(1+(8*N/10)) rayh(1+(8*N/10)) ];
xx8 = [rho(1+(8*N/10)) + rhoscan(1+(8*N/10)) rho(1+(8*N/10)) rho1(1+(8*N/10))
rayv(1+(8*N/10)) ];
yy9 = [input z(1+(9*N/10)) z1(1+(9*N/10)) rayh(1+(9*N/10)) ];
xx9 = [rho(1+(9*N/10)) + rhoscan(1+(9*N/10)) rho(1+(9*N/10)) rho1(1+(9*N/10))
rayv(1+(9*N/10)) ];

yy10 = [input z(1+N) z1(N+1) rayh(N+1) ];
xx10 = [rho(N+1) + rhoscan(N+1) rho(N+1) rho1(N+1) rayv(N+1) ];

yy11 = [input z(N) z1(N) rayh(N) ];
xx11 = [rho(N) + rhoscan(N) rho(N) rho1(N) rayv(N) ];

yy12 = [input z(N) z1(N) rayh(N) ];
xx12 = [rho(N) + rhoscan(N) rho(N) rho1(N) rayv(N) ];

plot(z,rho,z2,rho2,yy,xx,xint,yint,yy1,xx1,yy2,xx2,yy3,xx3,yy4,xx4,yy5,xx5,yy6,xx6,yy7,
xx7,yy8,xx8,yy9,xx9,yy10,xx10);

xlabel('z axis, height of reflector, f=0.5,F=4');
ylabel('rho axis, diameter of reflector');
title('Dual Reflector Profiles with 10 degree scan, and incoming plane wave');
axis([0,4,-4,4]);

%Comparing plots for the curve fitted elliptical polynomial
%with the numerical points for reflector 2.

for zz=1:N+2
y(zz) = (zz-((N/2)+1))/10;
x(zz) = pol2(2)*((y(zz))^2+pol2(4));

end
end

%plot of one of the rays to determine if in correct direction.
for xxx=1:N+10+1
    x1(xxx)=xxx/10;
    y1(xxx)=C1(N+1)+x1(xxx)+C2(N+1);
end
%plot(z2,rho2,x,y,x1,y1);
fx=D1(1);
fy=D2(1);
xint(xxx)=xint1;
yint(xxx)=yint1+C2;
for b=1
    for n=N:N+1
        Nz3(n)=xint1(n);
        Nrho3(n)=yint1(n);
        Ntau3(n)=((F-Nz3(n))^2+(Nrho3(n))^2)^(1/2);
    end
    Nsnewi(N+1)=(rho(N+1)-Nrho3(N+1))^2+(Nz3(N+1)-z(N+1))^2)^(1/2);
    NQ22i(N+1)=1/(p22i(N+1)-Nsnewi(N+1));
    NQ22r(N+1)=1/(((Nz3(N+1)-fx)^2+((Nrho3(N+1)-fy)^2)^(1/2);
    incx(N+1)=(rho(N+1)-Nrho3(N+1))/(Nz3(N+1)-z(N+1));
    incy(N+1)=(Nrho3(N+1)-fy)/(Nz3(N+1)-fx);
    for q=1:N-1
        nn=N+1-q;
        Nsnewi(nn)=(rho(nn)-Nrho3(nn))^2+(Nz3(nn)-z(nn))^2)^(1/2);
        NQ22i(nn)=1/(p22i(nn)-Nsnewi(nn));
        NQ22r(nn)=1/(((Nz3(nn)-fx)^2+((Nrho3(nn)-fy)^2)^(1/2);
        incx(nn)=(rho(nn)-Nrho3(nn))/(Nz3(nn)-z(nn));
        incy(nn)=(Nrho3(nn)-fy)/(Nz3(nn)-fx);
        if ((delta3(nn-1))<=0.005) & ((delta3(nn-1))>=-0.005)
\( N_{\rho 3}(n-1) = 0; \)
\( N_z3(n-1) = N_z3(nn); \)
\( N_{\tau 3}(n-1) = (F - N_z3(nn-1)); \)
\( N_{\rho 3}(nn-2) = N_{\rho 3}(nn); \)
\( N_z3(nn-2) = N_z3(nn); \)
\( N_{\tau 3}(nn-2) = N_{\tau 3}(nn); \)
\( N_{\rho 3}(nn-3) = N_{\rho 3}(nn+1); \)
\( N_z3(nn-3) = N_z3(nn+1); \)
\( N_{\tau 3}(nn-3) = N_{\tau 3}(nn+1); \)
\( N_{\rho 3}(nn-4) = N_{\rho 3}(nn+2); \)
\( N_z3(nn-4) = N_z3(nn+2); \)
\( N_{\tau 3}(nn-4) = N_{\tau 3}(nn+2); \)
\( N_{\rho 3}(nn-5) = N_{\rho 3}(nn+3); \)
\( N_z3(nn-5) = N_z3(nn+3); \)
\( N_{\tau 3}(nn-5) = N_{\tau 3}(nn+3); \)
\( N_{\rho 3}(nn-6) = N_{\rho 3}(nn+4); \)
\( N_z3(nn-6) = N_z3(nn+4); \)
\( N_{\tau 3}(nn-6) = N_{\tau 3}(nn+4); \)

for \( n=1:6 \)
\[
\text{nun}=\text{nn}-\text{n}+1;
\]
\[
N_{\Delta \tau 3}(\text{nun},1) = (N_{\tau 3}(\text{nun}+1) - N_{\tau 3}(\text{nun}-1))/(\delta_{\tau 3}(\text{nun}+1) - \delta_{\tau 3}(\text{nun}-1));
\]
\[
N_{\Delta \Delta \tau 3}(\text{nun},1) = (N_{\tau 3}(\text{nun}+1) - 2*N_{\tau 3}(\text{nn}) + N_{\tau 3}(\text{nun}-1))/((\delta_{\tau 3}(\text{nun}+1) - \delta_{\tau 3}(\text{nun})))\times((\delta_{\tau 3}(\text{nn}) - \delta_{\tau 3}(\text{nun}-1)));
\]
\[
N_{\Delta \tau 3}(\text{nun},2) = (N_{\tau 3}(\text{nun}+1) - N_{\tau 3}(\text{nun}-1))/(\delta_{\tau 3}(\text{nun}+1) - \delta_{\tau 3}(\text{nun}-1));
\]
\[
N_{\Delta \Delta \tau 3}(\text{nun},2) = (N_{\tau 3}(\text{nun}+1) - 2*N_{\tau 3}(\text{nn}) + N_{\tau 3}(\text{nun}-1))/((\delta_{\tau 3}(\text{nun}+1) - \delta_{\tau 3}(\text{nun})))\times((\delta_{\tau 3}(\text{nn}) - \delta_{\tau 3}(\text{nun}-1)));
\]
\[
N_{\Delta \tau 3}(\text{nun},3) = (N_{\tau 3}(\text{nun}+1) - N_{\tau 3}(\text{nun}-1))/(\delta_{\tau 3}(\text{nun}+1) - \delta_{\tau 3}(\text{nun}-1));
\]
\[
N_{\Delta \Delta \tau 3}(\text{nun},3) = (N_{\tau 3}(\text{nun}+1) - 2*N_{\tau 3}(\text{nn}) + N_{\tau 3}(\text{nun}-1))/((\delta_{\tau 3}(\text{nun}+1) - \delta_{\tau 3}(\text{nun})))\times((\delta_{\tau 3}(\text{nn}) - \delta_{\tau 3}(\text{nun}-1)));
\]
end
qq=nn-6;
for nnn=1:qq-1
    nn=qq+1-nnn;
    Nsnewi(nn)=((rho(nn)-Nrho3(nn))^2+(Nz3(nn)-z(nn))^2)^(1/2);
    NQ22i(nn)=-1/(p22i(nn)-Nsnewi(nn));
    NQ22r(nn)=1/(((Nz3(nn)-fx)^2+((Nrho3(nn)-fy)^2)^(1/2));
    incx(nn)=(rho(nn)-Nrho3(nn))/(Nz3(nn)-z(nn));
    incy(nn)=(Nrho3(nn)-fy)/(Nz3(nn)-fx);
end
for x=1:2
    Nmrho3(nn-1,x)=Nrho3(nn)+((1-x)*h);
end
for m=1:20
    for x=1:2
        Nmz3(nn-1,x)=F-(Nmrho3(nn-1,x)/(yint(nn-1)/(F-xint(nn-1))));
        Check(nn-1,x)=((rho(nn-1)-Nmrho3(nn-1,x))^2+(z(nn-1)-Nmz3(nn-1,x))^2)^(1/2);
        Nmtau3(nn-1,x)=((F-Nmz3(nn-1,x))^2+(Nmrho3(nn-1,x))^2)^(1/2);
        Ndttau3(nn,nx)=Ntau3(nn+1)-Nmtau3(nn-1,x))/((delta3(nn+1)-delta3(nn-1));
        Nddtau3(nn,nx)=(Ntau3(nn+1)-2*Ntau3(nn)+Nmtau3(nn-1,x))/((delta3(nn+1)-delta3(nn))*(delta3(nn)-delta3(nn-1)));
        NQ22c(nn,nx)=(Ntau3(nn)^2-
        Ntau3(nn)*Nddtau3(nn,nx)+2*Ndttau3(nn,nx)^2)/(Ntau3(nn)^2+2*Ndttau3(nn,nx)^2)^(3/2);
        Gu(nn,nx)=NQ22r(nn)-NQ22i(nn)-(2*NQ22c(nn,nx)/((1/(1+((incx(nn)+incy(nn)))/(1-
        incx(nn)*incy(nn))))^2)^(1/2))+(1/2)));)
end
Nmrho3(nn-1,3)=(Nmrho3(nn-1,1)+Nmrho3(nn-1,2))/2;
Nmz3(nn-1,3)=F-(Nmrho3(nn-1,3)/(yint(nn-1)/(F-xint(nn-1))));
Nmtau3(nn-1,3)=((F-Nmz3(nn-1,3))^2+(Nmrho3(nn-1,3))^2)^(1/2);
Ndttau3(nn,3)=(Ntau3(nn+1)-Nmtau3(nn-1,3))/((delta3(nn+1)-delta3(nn-1));
Ndtdau3(nn,3) = (Ntau3(nn+1)-2*Ntau3(nn)+Nmtau3(nn-1,3))/((delta3(nn+1)-

delta3(nn))*(delta3(nn)-delta3(nn-1)));
NQ22c(nn,3) = (Ntau3(nn)^2-
Ntau3(nn)*Ndtdau3(nn,3)+2*Ndtau3(nn,3)^2)/(Ntau3(nn)^2+Ndtau3(nn,3)^2)^3/2);
Gu(nn,3) = NQ22r(nn)-NQ22i(nn)-(2*NQ22c(nn,3)/((1/(1+((incx(nn)+incy(nn)))/(1-

incx(nn)*incy(nn))))^2*(1/2))+1/2 )^(1/2) );
if Gu(nn,1)*Gu(nn,3)<0
    Nnmrhor3(nn-1,2) = Nmrho3(nn-1,3);
    Nnmrhor3(nn-1,1) = Nmrho3(nn-1,1);
else
    Nnmrhor3(nn-1,1) = Nmrho3(nn-1,3);
    Nnmrhor3(nn-1,2) = Nmrho3(nn-1,2);
end
Nmrho3 = Nnmrhor3;
end
Nrhor3(nn-1) = Nmrho3(nn-1,1);
Nz3(nn-1) = Nmz3(nn-1,1);
Ntau3(nn-1) = Nmtau3(nn-1,1);
end
break
else
for x=1:2
    Nmrho3(nn-1,x) = Nrho3(nn)+((1-x)*h);
end
for m=1:20
    for x=1:2
        Nmz3(nn-1,x) = F-(Nmrho3(nn-1,x)/(yint(nn-1)/(F-xint(nn-1))));
        Check(nn-1,x) = ((rho(nn-1)-Nmrho3(nn-1,x))^2+(z(nn-1)-Nmz3(nn-1,x))^2)^1/2;
        Nmtau3(nn-1,x) = (F-Nmz3(nn-1,x))^2+(Nmrho3(nn-1,x))^2)^1/2;
        Ndtau3(nn,x) = (Ntau3(nn+1)-Nmtau3(nn-1,x))/(delta3(nn+1)-delta3(nn-1));
    end
end
end
end
end
end
end
Nddtau3(nn,x)=(Ntau3(nn+1)-2*Ntau3(nn)+Nmtau3(nn-1,x))/((delta3(nn+1)-
delta3(nn))*(delta3(nn)-delta3(nn-1)));  
NQ22c(nn,x)=(Ntau3(nn)^2-
Ntau3(nn)*Nddtau3(nn,x)+2*Ndtau3(nn,x)^2)/(Ntau3(nn)^2+Ndtau3(nn,x)^2)^3/2;  
Gu(nn,x)=NQ22r(nn)-NQ22i(nn)-(2*NQ22c(nn,x)/(  
(1/(1+((incx(nn)+incy(nn))/(1-
incx(nn)*incy(nn))))^2)^1/2))+(1/2))+(1/2);  
end
Nmrho3(nn-1,3)=(Nmrho3(nn-1,1)+Nmrho3(nn-1,2))/2;  
Nmz3(nn-1,3)=F-(Nmrho3(nn-1,3)/(yint(nn-1)/(F-xint(nn-1))));  
Nmtau3(nn-1,3)=(F-Nmz3(nn-1,3))^2+(Nmrho3(nn-1,3))^2)^1/2;  
Ndtau3(nn,3)=(Ntau3(nn-1)-Nmtau3(nn-1,3))/((delta3(nn-1)+delta3(nn))-
delta3(nn))*(delta3(nn)-delta3(nn-1));  
NQ22c(nn,3)=(Ntau3(nn)^2-
Ntau3(nn)*Nddtau3(nn,3)+2*Ndtau3(nn,3)^2)/(Ntau3(nn)^2+Ndtau3(nn,3)^2)^3/2;  
Gu(nn,3)=NQ22r(nn)-NQ22i(nn)-(2*NQ22c(nn,3)/(  
(1/(1+((incx(nn)+incy(nn))/(1-
incx(nn)*incy(nn))))^2)^1/2))+(1/2))+(1/2);  
if Gu(nn,1)*Gu(nn,3)<0  
Nmmrhol3(nn-1,2)= Nmrho3(nn-1,3);  
Nmmrhol3(nn-1,1)= Nmrho3(nn-1,1);  
else  
Nmmrhol3(nn-1,1)= Nmrho3(nn-1,1);  
Nmmrhol3(nn-1,2)= Nmrho3(nn-1,2);  
end  
Nmrho3=Nmmrhol3;  
end  
Nrho3(nn-1)=Nmrho3(nn-1,1);  
Nz3(nn-1)=Nmz3(nn-1,1);  
Ntau3(nn-1)=Nmtau3(nn-1,1);  
end  
end
for nn=2:N
    Ngradang(nn)=atan((-Nrho3(nn-1)+Nrho3(nn+1))/(-Nz3(nn-1)+Nz3(nn+1))); end
Ngradang(1)=Ngradang(2);
Ngradang(N+1)=Ngradang(N);
Nnormang=atan(-1./tan(Ngradang));
pol3=polyfit(Nrho3,Nz3,3);
Cp=(input-z)+(rhol-Nrho3).^2+(z-Nz3).^2).^(1/2)+((fy-Nrho3).^2+(fx-Nz3).^2).^(1/2);
xlabel('z axis, height of reflector, F=4');
ylabel('rho axis, diameter of reflector');
title('Dual Reflector Profiles with 5 degree scan, and incoming plane wave');
axis([0,5,4,4]);
Np2r=1./NQ22r;
Nnewangle=delta3;

%Calculating where rays from reflector 2 end up.
Np2r(1)=Np2r(2)+0.05;
Nincang=(atan(inex)+atan(incy))/2;
Nincang(1)=Nincang(2)-0.025;
for n=1:N
    rayh1(n)=Nz3(n)-Np2r(n)*cos(Nincang(n)+Nnormang(n));
    rayv1(n)=Nrho3(n)-Np2r(n)*sin(Nincang(n)+Nnormang(n));
end
rayh1(1)=rayh1(2);
rayh1(N+1)=rayh1(N);
rayv1(1)=rayv1(2);
rayv1(N+1)=rayv1(N);

%Calculating the optimal focal point, using least squares method.

for s=2:N/2
    Nm(s)=(rayv1(s)-Nrho3(s))/(rayh1(s)-Nz3(s));
    Naa(s)=2*Nm(s)^2;
end
\[ \text{Nab}(s) = -2\text{Nm}(s); \]
\[ \text{Nba}(s) = -2\text{Nm}(s); \]
\[ \text{Nbb}(s) = 2; \]
\[ \text{Nca}(s) = 2(Nm(s)^2)Nz3(s) - 2\text{Nm}(s)*\text{Nrho3}(s); \]
\[ \text{Ncb}(s) = -2\text{Nm}(s)*Nz3(s) + 2\text{Nrho3}(s); \]

end

for \( s = (N/2)+2:N \)
\[ \text{Nm}(s) = (\text{rayv1}(s) - \text{Nrho3}(s))/(\text{rayh1}(s) - Nz3(s)); \]
\[ \text{Naa}(s) = 2\text{Nm}(s)^2; \]
\[ \text{Nab}(s) = -2\text{Nm}(s); \]
\[ \text{Nba}(s) = -2\text{Nm}(s); \]
\[ \text{Nbb}(s) = 2; \]
\[ \text{Nca}(s) = 2(Nm(s)^2)Nz3(s) - 2\text{Nm}(s)*\text{Nrho3}(s); \]
\[ \text{Ncb}(s) = -2\text{Nm}(s)*Nz3(s) + 2\text{Nrho3}(s); \]

end

\[ \text{NA} = \text{sum(\text{Naa}) sum(\text{Nab}) sum(\text{Nba}) sum(\text{Nbb})}; \]
\[ \text{NB} = \text{sum(\text{Nca}) sum(\text{Ncb})}; \]
\[ \text{D} = \text{NA}\text{NB}; \]
\[ \text{fx} = \text{D}(1); \]
\[ \text{fy} = \text{D}(2); \]

end

\[ \text{yy} = \text{[input } z(1) \text{ Nz3}(1) \text{ rayh1}(1)); \]
\[ \text{xx} = \text{[rho}(1) + \text{rhoscan}(1) \text{ rho}(1) \text{ Nrho3}(1) \text{ rayv1}(1)); \]
\[ \text{yy1} = \text{[input } z(1+(N/10)) \text{ Nz3}(1+(N/10)) \text{ rayh1}(1+(N/10))]; \]
\[ \text{xx1} = \text{[rho}(1+(N/10)) + \text{rhoscan}(1+(N/10)) \text{ rho}(1+(N/10)) \text{ Nrho3}(1+(N/10)) \text{ rayv1}(1+(N/10))]; \]
\[ \text{yy2} = \text{[input } z(1+(2\times N/10)) \text{ Nz3}(1+(2\times N/10)) \text{ rayh1}(1+(2\times N/10))]; \]
\[ \text{xx2} = \text{[rho}(1+(2\times N/10)) + \text{rhoscan}(1+(2\times N/10)) \text{ rho}(1+(2\times N/10)) \text{ Nrho3}(1+(2\times N/10)) \text{ rayv1}(1+(2\times N/10))]; \]
\[ \text{yy3} = \text{[input } z(1+(3\times N/10)) \text{ Nz3}(1+(3\times N/10)) \text{ rayh1}(1+(3\times N/10))]; \]
\text{xx3}=[\text{rho}(1+(3*N/10)) + \text{rhoscan}(1+(3*N/10)) \text{ rho}(1+(3*N/10)) \text{ Nrho3}(1+(3*N/10)) \text{ rayv1}(1+(3*N/10))];
\text{yy4}=[\text{input z}(1+(4*N/10)) \text{ Nz3}(1+(4*N/10)) \text{ rayh1}(1+(4*N/10))];
\text{xx4}=[\text{rho}(1+(4*N/10)) + \text{rhoscan}(1+(4*N/10)) \text{ rho}(1+(4*N/10)) \text{ Nrho3}(1+(4*N/10)) \text{ rayv1}(1+(4*N/10))];
\text{yy54}=[\text{input z}(-2+(5*N/10)) \text{ Nz3}(-2+(5*N/10)) \text{ rayh1}(-2+(5*N/10))];
\text{xx54}=[\text{rho}(-2+(5*N/10)) + \text{rhoscan}(-2+(5*N/10)) \text{ rho}(-2+(5*N/10)) \text{ Nrho3}(-2+(5*N/10)) \text{ rayv1}(-2+(5*N/10))];
\text{yy55}=[\text{input z}(-3+(5*N/10)) \text{ Nz3}(-3+(5*N/10)) \text{ rayh1}(-3+(5*N/10))];
\text{xx55}=[\text{rho}(-3+(5*N/10)) + \text{rhoscan}(-3+(5*N/10)) \text{ rho}(-3+(5*N/10)) \text{ Nrho3}(-3+(5*N/10)) \text{ rayv1}(-3+(5*N/10))];
\text{yy56}=[\text{input z}(-4+(5*N/10)) \text{ Nz3}(-4+(5*N/10)) \text{ rayh1}(-4+(5*N/10))];
\text{xx56}=[\text{rho}(-4+(5*N/10)) + \text{rhoscan}(-4+(5*N/10)) \text{ rho}(-4+(5*N/10)) \text{ Nrho3}(-4+(5*N/10)) \text{ rayv1}(-4+(5*N/10))];
\text{yy57}=[\text{input z}(-1+(5*N/10)) \text{ Nz3}(-1+(5*N/10)) \text{ rayh1}(-1+(5*N/10))];
\text{xx57}=[\text{rho}(-1+(5*N/10)) + \text{rhoscan}(-1+(5*N/10)) \text{ rho}(-1+(5*N/10)) \text{ Nrho3}(-1+(5*N/10)) \text{ rayv1}(-1+(5*N/10))];
\text{yy58}=[\text{input z}(5*N/10) \text{ Nz3}(5*N/10) \text{ rayh1}(5*N/10)];
\text{xx58}=[\text{rho}(5*N/10) + \text{rhoscan}(5*N/10) \text{ rho}(5*N/10) \text{ Nrho3}(5*N/10) \text{ rayv1}(5*N/10)];
\text{yy5}=[\text{input z}(1+(5*N/10)) \text{ Nz3}(1+(5*N/10)) \text{ rayh1}(1+(5*N/10))];
\text{xx5}=[\text{rho}(1+(5*N/10)) + \text{rhoscan}(1+(5*N/10)) \text{ rho}(1+(5*N/10)) \text{ Nrho3}(1+(5*N/10)) \text{ rayv1}(1+(5*N/10))];
\text{yy51}=[\text{input z}(2+(5*N/10)) \text{ Nz3}(2+(5*N/10)) \text{ rayh1}(2+(5*N/10))];
\text{xx51}=[\text{rho}(2+(5*N/10)) + \text{rhoscan}(2+(5*N/10)) \text{ rho}(2+(5*N/10)) \text{ Nrho3}(2+(5*N/10)) \text{ rayv1}(2+(5*N/10))];
\text{yy52}=[\text{input z}(3+(5*N/10)) \text{ Nz3}(3+(5*N/10)) \text{ rayh1}(3+(5*N/10))];
\text{xx52}=[\text{rho}(3+(5*N/10)) + \text{rhoscan}(3+(5*N/10)) \text{ rho}(3+(5*N/10)) \text{ Nrho3}(3+(5*N/10)) \text{ rayv1}(3+(5*N/10))];
\text{yy53}=[\text{input z}(4+(5*N/10)) \text{ Nz3}(4+(5*N/10)) \text{ rayh1}(4+(5*N/10))];
\[
\begin{align*}
xx53 &= [\text{rho}(4+(5*\text{N}/10)) + \text{rhoscan}(4+(5*\text{N}/10)) \ \text{rho}(4+(5*\text{N}/10)) \ \text{Nrho}(1+(5*\text{N}/10)) \ \text{rayv}(1+(5*\text{N}/10))]; \\
\text{yy6} &= [\text{input z}(1+(6*\text{N}/10)) \ \text{Nz}(1+(6*\text{N}/10)) \ \text{rayh}(1+(6*\text{N}/10))]; \\
xx6 &= [\text{rho}(1+(6*\text{N}/10)) + \text{rhoscan}(1+(6*\text{N}/10)) \ \text{rho}(1+(6*\text{N}/10)) \ \text{Nrho}(1+(6*\text{N}/10)) \ \text{rayv}(1+(6*\text{N}/10))]; \\
\text{yy7} &= [\text{input z}(1+(7*\text{N}/10)) \ \text{Nz}(1+(7*\text{N}/10)) \ \text{rayh}(1+(7*\text{N}/10))]; \\
xx7 &= [\text{rho}(1+(7*\text{N}/10)) + \text{rhoscan}(1+(7*\text{N}/10)) \ \text{rho}(1+(7*\text{N}/10)) \ \text{Nrho}(1+(7*\text{N}/10)) \ \text{rayv}(1+(7*\text{N}/10))]; \\
\text{yy8} &= [\text{input z}(1+(8*\text{N}/10)) \ \text{Nz}(1+(8*\text{N}/10)) \ \text{rayh}(1+(8*\text{N}/10))]; \\
xx8 &= [\text{rho}(1+(8*\text{N}/10)) + \text{rhoscan}(1+(8*\text{N}/10)) \ \text{rho}(1+(8*\text{N}/10)) \ \text{Nrho}(1+(8*\text{N}/10)) \ \text{rayv}(1+(8*\text{N}/10))]; \\
\text{yy9} &= [\text{input z}(1+(9*\text{N}/10)) \ \text{Nz}(1+(9*\text{N}/10)) \ \text{rayh}(1+(9*\text{N}/10))]; \\
xx9 &= [\text{rho}(1+(9*\text{N}/10)) + \text{rhoscan}(1+(9*\text{N}/10)) \ \text{rho}(1+(9*\text{N}/10)) \ \text{Nrho}(1+(9*\text{N}/10)) \ \text{rayv}(1+(9*\text{N}/10))]; \\
\text{yy10} &= [\text{input z}(1+N) \ \text{Nz}(N+1) \ \text{rayh}(N+1)]; \\
xx10 &= [\text{rho}(N+1) + \text{rhoscan}(N+1) \ \text{rho}(N+1) \ \text{Nrho}(N+1) \ \text{rayv}(N+1)]; \\
\text{plot(z, rho, z2, rho2, Nz3, Nrhol, yy, xx, yy1, xx1, yy2, xx2, yy3, xx3, yy4, xx4, yy5, xx5, yy6, xx6, yy7, xx7, yy8, xx8, yy9, xx9, yy10, xx10)]; \\
\text{xlabel('z axis, height of reflector, f=0.5, F=4')}; \\
\text{ylabel('rho axis, diameter of reflector')}; \\
\text{title('Dual Reflector Profiles with 1 degree scan, and incoming plane wave')}; \\
\text{axis([0,4,-4,4])};
\end{align*}
\]

**Appendix B6-1 – Podipoleall**

*Program of flat reflector and dipole analysis.*

Poa;

Pob;
Appendix B6-1-1 – Poa

Matlab script file defining variables

```
lam=1;       % wavelength
k0=2*pi/lam; % wavenumber
Z0=376.7;   % free-space characteristic impedance

%%% secondary reflector parameters

Das=lam;   % subreflector diameter, inches
dysm=.055*lam; % maximum y sampling spacing
dxsm=.055*lam; % maximum x sampling spacing
Nerx=floor(Das/dxsm)+1; % number of x samples
derx=Das/(Nerx-1);   % actual x sampling increment
rex=-Das/2+((1:Nerx)-1)*derx; % x sampling
z0=lam/10;   % location of reflector

%%% dipole3.m is being used instead of horn
Dasd=0.5*lam; % subreflector diameter, inches
zd=0; % location of dipole
Nerxd=floor(Dasd/dxsm)+1; % number of x samples
derxd=Dasd/(Nerxd-1);   % actual x sampling increment
rexd=-Dasd/2+((1:Nerxd)-1)*derxd; % x sampling

%%% pattern angles

thd1=0;       % theta cut start angle, deg
thd2=180;     % theta stop angle, deg
Nth=181;      % number of theta angles
Nthg=5;       % number of simultaneous angles
phd=0;        % phi angle for theta cut
I13=ones(1,3); % 1 by 3 array of ones
```
Appendix B6-1-2 – Pob

Matlab file calling routines to evaluate dipole feed currents and patterns

Dipole; % dipole aperture currents
pod; % feed forward patterns
Ephf=Eph;
Ethf=Eth;

Appendix B6-2 – Dipole

Program for evaluating the dipole source fields

%Initializing matrices.

rexl=[];
rs=[];
DSe=[];
Ne=0; % counter zero
curr=[];
r=[];
y=[];
Evert=[];
une=[];
l=[];
rf=[];

l=(sin((2*pi/lam)*((lam/4)-abs(rexd))))';

for nerx=1:Nerx % iterate over radius
  yrmx(nerx)=Dans/2;
  yrmn(nerx)=-Dans/2;
  Neryr(nerx)=floor(abs(yrmx(nerx)-yrmn(nerx))/dysm)+1;
% array of numbers of y samps
deyr(nerx)=(ymx(nerx)-ymn(nerx))/Neryr(nerx);

% array of actual y sample spacings
Ne=Ne+Neryr(nerx);  \% total number of aperture samples
end

for n=1:Nerxd
rs=[rs;rexd(n) 0 0];  \% array of primed position vectors
end

for nerx=1:Nerx  \% iterate over radius
yP=ymn(nerx)+((1:Neryr(nerx))-0.5)*deyr(nerx);
zF=z0*ones(Neryr(nerx),1);  \% z field positions
rf=[rf;[(rex(nerx)*ones(Neryr(nerx),1)) yP zf]];
end

for n=1:Ne
for x=1:Nerxd
    r=[r;sqrt(((rf(n,3)-rs(x,3)).^2+(rf(n,1)-rs(x,1)).^2+(rf(n,2)-rs(x,2)).^2))]
end
end

rF=reshape(r,Ne,Nerxd);

for n=1:Ne
    Evera=0;
    Hvera=0;
    for x=1:Nerxd
        const(n,x)=j*60*pi*I(x)*dxsm/lam/rF(n,x);
        if rs(x,1)<rf(n,1)
            theta(n,x)=(pi/2)-asin(abs(rf(n,1) rs(x,1))/rF(n,x));
        elseif rs(x,1)>rf(n,1)
            theta(n,x)=(pi/2)+asin(abs(rf(n,1)-rs(x,1))/rF(n,x));
        end
    end
end
else
    theta(n,x)=pi/2;
end

ang(n,x)=(2*pi*rr(n,x)/lam)*180/pi;
phase(n,x)=exp(-j*2*pi*rr(n,x)/lam);
E_dipole(n,x)=const(n,x)*sin(theta(n,x)).*phase(n,x);
E_vert(n,x)=E_dipole(n,x)*sin(theta(n,x));
E_era=E_era+E_vert(n,x);
H_dipole(n,x)=E_dipole(n,x)/Z0;
H_era=H_era+H_dipole(n,x);
end

E_era(n)=E_era;
H_era(n)=H_era;
DSe=[DSe;dy*dx];  % surface areas of samples
end

for n=1:Nerxd, red(n,:)=rexd(n) 0 0; end

une=[une;[0 0 1]];
E_era=E_era(:,[1 0 0]);
E_plot=reshape(E_era(:,Nerx,Neryr(1)));  
E_plot=reshape(E_era(:,Nerx,Neryr(1)));  
E_era=abs(E_era);
H_era=H_era(:,[0 1 0]);
Ma=-cross01(une,E_era);
Ja=cross01(une,H_era);

P_m=1/2/Z0*sum(abs(dot01(Ma,Ma.')).*DSe);  % power
P_e=1/2/Z0*sum(abs(dot01(Ja,Ja.')).*DSe);
M_s=Ma/sqrt(P_m).*(DSe*ones(1,3));  % current in Volt* meter
J_s=Ja/sqrt(P_e).*(DSe*ones(1,3));  % current in Amp* meter

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Appendix B6-3 – Pod

Program for evaluating the radiation patterns

nth=(1:Nth); % theta pattern index array
dthd=(thd2-thd1)/(Nth-1); % theta increment
phpat=phd*pi/180; % phi angle of theta cut, radians
thd=thd1+(nth-1)*dthd; % Nth by 1 array of theta angles
thp=pi/180*thd; % in radians
php=phpat*ones(Nth,1); % Nth by 1 array, phi angles

EM=propff01(rf,Ms,thp,php,k0); % FF electric field from Mn
HJ=-propff01(rf,Js,thp,php,k0); % FF magnetic field from Jn

Eth=EM(:,1)+HJ(:,2)*Z0; % theta electric fields
Eph=EM(:,2)-HJ(:,1)*Z0; % phi electric fields
Er=1/sqrt(2)*(Eth+i*Eph); % RHCP voltage
El=1/sqrt(2)*(Eth-i*Eph); % LHCP voltage
thpd=thp+pi/2;
if thpd==0,
    Edth=0;
else
    Edth=(-j*60*pi/lam)*(cos((pi/2)*cos(thpd)))./(sin(thpd)).*(exp(j*(2*pi*derx*(cos(thpd))/lam));
end
% Eth far field of dipole

Ethtot=((Edth/max(abs(Edth)))+(Eth/max(abs(Eth))))/2;
Epthtot=Eph;

Pth=abs(4*pi/2/Z0*(Eth).^2);
Pph=abs(4*pi/2/Z0*(Eph).^2);
\%PthdB=10*log10(Pth);
\%PphdB=10*log10(Pph);

Pr=abs(4*pi/2/Z0*Er.^2);
Pl=abs(4*pi/2/Z0*El.^2);
PrdB=10*log10(Pr);
PldB=10*log10(Pl);

**Appendix B6-4**

\%Dual curved reflector rectangular aperture
\% Total program of Paraboloid reflector analysis.

Poa1;
Pob1;
Poc1;
Pod;
Poe;
Pof1;

**Appendix B6-5 - Poa1**

**NB: Same as Poa but with the main reflector parameters included as shown below.**

\%main reflector parameters

Darx=0.6; \% elliptical aperture x dimension, m
Dary=0.01016; \% elliptical aperture y dimension, m

xa0=0; \% aperture center x location, m
ya0=0; \% aperture center y location, m
zv0=0; \% reflector vertex z location, m
drxmax=.0025;  % max allowed x sampling spacing
drymax=.0025;  % max allowed y sampling spacing

Appendix B6-6 – Pob1

Matlab files for the horn currents and patterns are called in this routine
pofeedb;  % feed aperture surface definition
pofeedc;  % feed aperture currents
pofeedd1; % feed forward patterns
Ethf=Eth;
Ephp=Eph;

Appendix B6-7 – Poc

Matlab file defining the subreflector
Ne=0;
for nerx=1:Nerx  % iterate over radius
  yrmx(nerx)=Dasy/2;
  yrmn(nerx)=-Dasy/2;
  Neryr(nerx)=floor(abs(yrmx(nerx)-yrmn(nerx))/dysm)+1;
  % array of numbers of circ. samps
deyr(nerx)=(yrmx(nerx)-yrmn(nerx))/Neryr(nerx);
  % array of actual y sample spacings
  Ne=Ne+Neryr(nerx);  % total number of aperture samples
end

res=[];  % initialize arrays
DSe=[];
phep=[];
une=[];
alphs=[];

for nrx=1:Nrx    % iterate over radius
    if(nrx/20==ceil(nrx/20));[nrx Nrx],end
    yP=ymn(nrx)+((1:Neryr(nrx))-0.5)*deyr(nrx);
    re=sqrt(rex(nrx)^2);
    res=[res;rex(nrx)*ones(Neryr(nrx),1)yP*sqrt(0.36re)^2.*ones(Neryr(nrx),1)];
    % array of primed position vectors
    xlm1=rex(nrx)+derx/2;    % integral limits for surf area
    xlm2=rex(nrx)-derx/2;
    ylm1=yP+deyr(nrx)/2;
    ylm2=yP-deyr(nrx)/2;
    pheP=[pheP;atan2(abs(yP),rex(nrx)*ones(Neryr(nrx),1))]; % phi angles
end
alphs=atan2(res(:,3),res(:,1));% alpha angles
gradient=res(:,1)/sqrt(0.36-(res(:,1)^2));
angle=atan(gradient);
DSe=derx*deyr(1)./cos(angle);
une=[une;[-cos(pheP).*cos(alphas) -sin(pheP).*cos(alphas) -sin(alphas)]];
ut2e=norm01(cross01(une,[1 0 0]));% second surf. tangent
ut1e=cross01(ut2e,une);    % first surface unit tangent
    % position vectors

Appendix B6-8 – Poe
This file is very similar to the sub reflector definition and comprises of the main reflector definition and current calculation.
Appendix B6-9 - Pof
MATLAB script file for calculating far-field patterns of Cassegrain reflector

nth=(1:Nth)'; % Nth by 1 theta angle index array
dthd=(thd2-thd1)/(Nth-1);% theta increment, deg
thd=thd1+(nth-1)*dthd; % Nth by 1 array, theta angles
thr=pi/180*thd; % in radians

EM=[];
HJ=[];

for nth=1:Nthg:Nth % break up thd into smaller arrays
    [nth Nth]
    thrs=thr(nnth:min([nth+Nthg-1,Nth])); % theta angles, rad
    phr=pi/180*phd*ones(size(thrs)); % phi angles, rad

    FMs=propff01(rr,Mrr,thrs,phr,k0);
    % FF electric field from Mn
    HJs=-propff01(rr,Jrr,thrs,phr,k0);
    % FF magnetic field from Jn
    EM [FM;FM]; % cascade thd arrays
    HJ [HJ;HJ];
end

Feth=FM(:,1)+HJ(:,2)*Z0; % theta pol E field
Feph=FM(:,2)+HJ(:,1)*Z0; % phi pol E field
Fr=sqrt(0.5)*(Feth*i*Feph); % RHCP E field
Fl=sqrt(0.5)*(Feth-i*Feph); % LHCP E field

Pth=abs(4*pi/2*Z0*Feth.*2);
\[ P_{ph} = \text{abs}(4 \pi i/2/Z0*E_{ph}^2); \]
\[ P_{r} = \text{abs}(4 \pi i/2/Z0*E_{r}^2); \]
\[ P_{l} = \text{abs}(4 \pi i/2/Z0*E_{l}^2); \]
\[ P_{thd} = 10 \cdot \log_{10}(P_{th}); \]
\[ P_{phd} = 10 \cdot \log_{10}(P_{ph}); \]
\[ P_{rd} = 10 \cdot \log_{10}(P_{r}); \]
\[ P_{ld} = 10 \cdot \log_{10}(P_{l}); \]
\[ E_{Mb} = []; \]
\[ H_{Jb} = []; \]
\% add subreflector blockage patterns

for nth = 1:Nth; Nth \% break up thd into smaller arrays
[nnth Nth];

\[ \text{thrs} = \text{th}(\text{nnth:} \text{min((nnth}+\text{Nth}^{-1}, \text{Nth}))); \% \text{theta angles, rad} \]
\[ \text{phr} = \pi/180 \cdot \text{phd} \cdot \text{ones(size(thrs))}; \% \text{phi angles, rad} \]
\[ E_{Mb} = \text{propff01(res,Mdc,thrs,phr,k0)}; \]
\% FF electric field from Mn
\[ H_{Jb} = \text{propff01(res,Jdc,thrs,phr,k0)}; \]
\% FF magnetic field from Jn
\[ E_{Mb} = [E_{Mb}; E_{Mb}]; \% \text{cascade thd arrays} \]
\[ H_{Jb} = [H_{Jb}; H_{Jb}]; \]
end

\[ E_{thb} = E_{Mb(:,1)} \cdot H_{Jb(:,2)} \cdot Z0; \% \text{theta pol E field} \]
\[ E_{phb} = E_{Mb(:,2)} \cdot H_{Jb(:,1)} \cdot Z0; \% \text{phi pol E field} \]

\[ E_{tht} = E_{th}; \]
\[ E_{ph} = E_{ph}; \]
Ert=\sqrt{(0.5) \ast (E_{tht} + i \ast E_{ph})}; \quad \% \text{RHCP E field}
Elt=\sqrt{(0.5) \ast (E_{tht} - i \ast E_{ph})}; \quad \% \text{LHCP E field}

P\text{tht}=\text{abs}(4\ast\pi/2/Z_0*E_{tht}^2);
P\text{pht}=\text{abs}(4\ast\pi/2/Z_0*E_{ph}^2);
P\text{rt}=\text{abs}(4\ast\pi/2/Z_0*E_{rt}^2);
P\text{lt}=\text{abs}(4\ast\pi/2/Z_0*E_{lt}^2);

P\text{thtdB}=10\ast\log(10)(P\text{tht});
P\text{phtdB}=10\ast\log(10)(P\text{pht});
P\text{rtdB}=10\ast\log(10)(P\text{rt});
P\text{ltdB}=10\ast\log(10)(P\text{lt});

\textbf{Appendix B8-4: Matlab code for Chapter 8}

\textbf{Program B8-1-1: Array of point sources – sum of exponentials}

defreq = 1 \ast 10^9;
c = 3 \ast 10^8;
\lambda = c/\text{freq};
d = \lambda / 32;
n = 64;
spacing = 2 \ast \pi \ast d / \lambda;
angle = 60;
beam \text{ dir} = \text{angle} \ast \pi / 180;
x = \cos(\text{beam \text{ dir}});
phase \text{ diff} = -x \ast \text{spacing};
\quad \text{for } \phi = 1:180,
\quad \text{si}(\phi) = \text{spacing} \ast \cos((\phi) \ast \pi / 180) + \text{phase \text{ diff}};
\quad \text{for } m = 1:n
\[ \text{ex}(m) = \exp(i*(m-1)*\text{si}(\text{phi})); \]
end
\[ \text{E}(\text{phi}) = \frac{\text{abs}(\text{sum}(\text{ex}))/n}{n}; \]
end

**Program B8-1-2** : Stepped phase shift - sum of exponentials

\begin{verbatim}
for phi = 1:180,
    si(phi) = spacing*cos((phi)*pi/180);
    for m = 1:n1
        ex1(m) = \exp(i*(m-1)*si(phi));
    end
    sum1 = sum(ex1);
    for mn = 1:n2
        ex2(mn) = \exp(i*(n1*(mn-1)*si(phi)+n1*(mn-1)*\text{phase}._\text{diff}));
    end
    sum2 = sum(ex2);
    E(phi) = \text{abs}(sum2)/(n2);
end
\end{verbatim}

**Program B8-1-3** : Stepped phase shift – sine simplification

\begin{verbatim}
for phi = 1:360,
    si(phi) = spacing*cos(phi*pi/180);
    delta(phi) = 4*si(phi)+\text{phase}._\text{diff};
    E(phi)=
    \text{abs}((\text{sin}(4*si(phi)/2))/(4*(\text{sin}(si(phi)/2))))*\text{abs}((\text{sin}(n*delta(phi)/2))/(n*(si
    n(delta(phi)/2))));
    E1(phi) = \text{abs}((\text{sin}(4*si(phi)/2))/(4*(\text{sin}(si(phi)/2))));
end
\end{verbatim}
Program B8-1-4 : Evaluation of s for varying parabola surface.

\[ a = n1 \times \text{spacing}; \]
\[ b = n2 \times a; \]
\[ p = 20; \]
\[ \lambda = \text{lamda}; \]
\[ \text{for } q = 1:9 \]
\[ \text{angle}(q) = q \times 10; \]
\[ \text{beam}_\text{dir}(q) = \text{angle}(q) \times \pi / 180; \]
\[ x(q) = \sin(\text{beam}_\text{dir}(q)); \]
\[ \delta(q) = \lambda \times x(q) \times b / (4 \times \pi); \]
\[ a(q) = -4 \times \delta(q) - 8 \times p; \]
\[ b(q) = -8 \times p \times p + 2 \times \delta(q) \times \delta(q) - 2 \times x1 \times x1; \]
\[ c(q) = 2 \times p \times \delta(q) \times \delta(q) + 4 \times \delta(q) \times p \times p + \delta(q) \times x1 \times x1; \]
\[ s2(q) = (-b(q) - (b(q) \times 2 - 4 \times a(q) \times c(q)) \times (1/2)) / (2 \times a(q)); \]
\[ \text{end} \]
\[ \text{for } q = 1:10 \]
\[ y(q) = (q \times q / (4 \times (p - s2(5)))) + s2(5); \]
\[ \text{dist}_\text{rad}(q) = 2 \times \pi \times (((p - y(q))^2 + q^2)^{(1/2)} + y(q) + p); \]
\[ \text{end} \]

Program B8-1-5 : Program comparing the phase shift assumed earlier with the phase shift due to the path difference.

\[ \text{for } q = 1:9 \]
\[ \text{angle}(q) = q \times 10; \]
\[ \text{beam}_\text{dir}(q) = \text{angle}(q) \times \pi / 180; \]
\[ x(q) = \sin(\text{beam}_\text{dir}(q)); \]
\[ \delta(q) = \lambda \times x(q) \times b / (4 \times \pi); \]
\[ a(q) = -4 \times \delta(q) - 8 \times p; \]
\[ bb(q) = -8*p*p+2*delta(q)*delta(q)-2*x1*x1; \]
\[ cc(q) = 2*p*delta(q)*delta(q)+4*delta(q)*p*p+delta(q)*x1*x1; \]
\[ s2(q) = (-bb(q)-(bb(q)^2-4*aa(q)*cc(q))^(1/2))/(2*aa(q)); \]

end

for q=1:10
\[ y(q)=(q*q/(4*(p-q*s2(9))))+q*s2(9) \]
\[ dist\_rad(q)=(p+y(q)-((p-y(q))^2+q^2)^(1/2)) \]
end

**Program B8-1-6: Parabolic step reflector**

\[ conv = 2*pi/lamda; \]
\[ aa = lamda/10; \]
\[ bb = 50*lamda; \]
\[ a = aa*conv; \]
\[ b = bb*conv; \]

for q = 1:9
\[ angle(q) = q*10; \]
\[ angle1(q) = (10-q)*10; \]
\[ beam\_dir(q) = angle1(q)*pi/180; \]
\[ x(q)=sin(beam\_dir(q)); \]

for phi = 1:0.01:180,
\[ si1(phi) = a*sin(phi*pi/180)/2; \]
\[ si2(phi) = b*(sin(phi*pi/180)-x(q))/2; \]
\[ E1(phi) = abs(sin(si1(phi))/si1(phi)); \]
\[ E2(phi) = abs(sin(si2(phi))/si2(phi)); \]
\[ E(phi) = E1(phi)*E2(phi); \]
end

\[ EE(q) = E1(angle(q))*E2(angle1(q)); \]

End
A Study of
Antenna Design Concepts
For
Future Large Radio Telescopes

By
Philo Vinita Daniel-Tran

University of Western Sydney.

Submitted for the
Degree of Doctor of Philosophy - Electrical Engineering.

May 2001
PLEASE NOTE

The greatest amount of care has been taken while scanning this thesis,

and the best possible result has been obtained.
For My Parents
Acknowledgement

I would firstly like to thank my chair supervisor Godfrey Lucas for his inspiration, encouragement and guidance throughout my post graduate years. I am sincerely grateful for his dedication, enthusiasm and time contributed towards supporting the development of my research work.

I would also like to thank CSIRO for providing me with the top-up scholarship, and my supervisors at CSIRO, Graeme James and Wim Brouw, for their input at the early stages of my research.

I would like to acknowledge all the technical, administration and workshop staff in the Electrical Engineering Division at the University of Western Sydney for their support over the years especially Rattan Bhandari, Wayne Hatty, Rhonda Gibbons and Robyn Carter. I would also like to thank the academic staff especially Mahmood Nagrial, Robert Mondel and Phillip Madigan for their advice on various issues during my candidature.

I couldn’t have completed this thesis without the emotional support from my family, I thank them for never questioning me, and for always having complete faith in me, especially my wonderful parents Raja and Praba Daniel, I couldn’t thank them enough for everything they have done for me. My brother John Vijay and sister Geraldine Vindhya for putting up with me all these years, I do appreciate it. I am indeed grateful to my in-laws Sang and Maria Tran for being so understanding. And last but not least, I thank my husband Benjamin Tran, who has stood by me through thick and thin, making it all worthwhile.
Statement of Sources

I declare that the work submitted is the result of my investigation and is not submitted in candidature for any other degrees.

Philo Vinita Daniel-Tran
(Candidate)
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List of Frequently used Terms and Abbreviations

Z  Impedance (ohms)
Y  Admittance (mhos)
D  Flux Density
B  Susceptance
H  Magnetic Field Strength (amp m)
E  Electric Field Strength (volt m)
J  Current Density
s  Seconds
t  Time
c  Speed of Light (3.0*10^8 m s)
λ  Wavelength (m)
γ  Propagation constant
α  Attenuation constant
β  Phase constant
σ  Conductivity (mho m)
ε  Permittivity
ω  Angular Frequency (rad sec)
TEM Transverse Electromagnetic
MM Mode Matching
FFT Fast Fourier Transform
FDTD Finite Difference Time Domain
v  Speed (m/s)
PO Physical Optics
GO Geometrical Optics
SKA Square Kilometer Array
S11 Reflection coefficient at port 1
S21 Transmission coefficient from port 1 to port 2
FD Focal length Diameter
Abstract

The work reported here is just one of the many lines of investigation being followed in connection with the proposed ‘Square Kilometer Array’ (SKA) for the next generation Radio telescope. A parallel plate system with a circular reflector has been investigated as a possible future candidate for “SKA” to achieve wideband performance and beam scanning ability. Essentially the area that needed to be developed, was the theoretical analysis of the fields in the parallel plate structure, in order to gain a better understanding of the behavior of the fields within the device and at the discontinuity. The analysis used here is a combination of Finite Difference Time Domain method for the cross section and Physical Optics for the reflector analysis. Code has been developed in ‘Matlab’ for both theories and practical measurements agree well with the theory. Ways of integrating the two analyses to obtain an overall result will also be presented. No research has been found which has successfully analysed this structure previously.

The present effort builds on earlier work and explores the possible limits of extension. It also develops an accurate and robust theoretical analysis of the device, which will allow all possible configurations to be simulated. It is anticipated that the final scanning arrangement will consist of at least a double reflector system, possibly with some re-configurability, where the parallel plates will have two 180° bends with the output being some form of directly radiating aperture.

Other areas that have been investigated are that of re-configurable antennas. Practical results were obtained for a Re-configurable Monopole, where diodes are used as switches to physically vary the length of the antenna element and hence increase the bandwidth of the antenna. Re-configurable reflectors were also investigated, where a frequency sensitive surface can be either reflective or transparent, and hence facilitate the active shaping of the reflector. Geometrical Optics was used for shaping a dual-reflector Cassegrain system for beam scanning. All of these methods have produced satisfactory practical and analytical results and are discussed extensively in the main body of the thesis.