The Effects of Weld-Induced Imperfections on the Stability of Axially Loaded Steel Silos

M. Pircher

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University of Western Sydney
PLEASE NOTE

The greatest amount of care has been taken while scanning this thesis,

and the best possible result has been obtained.
Synopsis

The strength of thin-walled cylindrical shell structures is highly dependent on the nature and magnitude of imperfections. Most importantly, circumferential imperfections have been reported to have an especially detrimental effect on the buckling resistance of these shells under axial load. Due to the manufacturing techniques commonly used during the erection of steel silos and tanks, specific types of imperfections are introduced into these structures, among them circumferential weld-induced imperfections between strakes of steel plates.

The main objective of this thesis was to investigate the exact nature of these circumferential welds and their influence on the buckling resistance of silos and tanks under axial load.

The results of a survey of imperfections in existing silos at a location in Port Kembla / Australia (Ding 1992) were used to develop and calibrate a shape function which accurately describes the geometric features of circumferential weld imperfections. It was found that after filtering out the effects of overall imperfections, three parameters governed the shape of the surveyed imperfections: the depth; the wave length; and the roundness.

A study on several factors influencing the buckling of silos and tanks was carried out using the finite element method. The interaction between neighbouring circumferential weld imperfections was investigated and it was found that the influence on the buckling behaviour depended on the strake height in relation to the linear meridional bending half wave length and the depth of the imperfection. The strengthening effect of weld-induced residual stress fields for a range of different geometries was also studied, and diagrams were derived giving the influence of the newly developed shape function on the buckling behaviour.
A post-buckling analysis was undertaken and a model for the post-buckling behaviour of cylindrical thin-walled shells with circumferential weld imperfections was developed.

The methods used for the analysis of thin-walled cylinders were applied in a study on the buckling behaviour of welded box-sections. It was found that weld-induced residual stress fields governed the buckling behaviour of these columns.
Preface

This thesis is submitted to the University of Western Sydney, Australia, for the degree of Doctor of Philosophy. The work described in this thesis was carried out by the candidate during the years 1997 – 2000 at the School of Civic Engineering and Environment under the supervision of Professor R.Q. Bridge.

The author submits that the work presented in this thesis is original unless otherwise referenced within the text. No part of this thesis has been submitted for a higher degree at any other institution.

The derivation and calibration of the shape function describing a circumferential weld in Chapter 2 was performed as original work in co-operation with my colleague P. Berry. The studies on the buckling behaviour of imperfect thin-walled cylinders in Chapter 4 are claimed as original as well as the study on the post-buckling behaviour in Chapter 5. The analysis work on welded thin-walled box sections presented in Chapter 6, although original in itself, is an extension of previous work done by M. O'Shea and R.Q. Bridge.

Several conference and journal papers were written in conjunction with Prof. R.Q. Bridge and others on the work presented in this thesis. All conference papers listed were presented by the candidate. These papers are:


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Notation

The following notation is used in this thesis. All symbols are defined where they first appear in the text. Generally, only one meaning is assigned to each symbol, but in those cases where more meanings than one are possible, the correct one will be evident from the context in which it is used.

Latin letters

\( b \)  
Width of a box section.

\( D \)  
Elastic axial bending stiffness.

\( E \)  
Modulus of elasticity.

\( f \)  
Ratio of maximum compressive residual stress to maximum tensile residual stress.

\( F \)  

\( F_u \)  
Maximum axial force.

\( k \)  
Elastic reference stiffness.

\( K \)  
Stiffness matrix.

\( K_0 \)  
Stiffness matrix at the base state.

\( K_\Delta \)  
Differential stiffness matrix.

\( l_0 \)  
Distance between the centre of the weld to the place \( x \) where \( w(x) = 0.2 w_0 \) after Steinhardt & Schulz (1970).

\( l_1, l_2, l_3, l_4 \)  
Lengths of straight sections of a shape function.

\( l_d \)  
Imperfection length of a circumferential weld-imperfection after Tennyson & Muggeridge (1969) and Hutchinson (1971).

\( L \)  
Strake height of a silo or tank, or length of a box section.

\( M_0 \)  
Moment at the weld.

\( m \)  
Number of circumferential buckling waves.

\( m_{ci} \)  
Number of circumferential buckling waves based on classical theory.

\( m_{cr} \)  
Critical number of circumferential buckling waves.

\( p \)  
Welding arc efficiency.

\( \mathbf{P}_0 \)  
Load vector leading to the base state.

\( \mathbf{P}_{cr} \)  
Critical buckling load vector.

\( Q \)  
Power input to the welding arc.

\( \mathbf{Q} \)  
Load vector with which to perform the buckling analysis.
\( R \) Radius of a thin-walled cylinder.

\( s \) Rate of welding arc travel, or standard deviation.

\( t \) Wall thickness of a thin-walled cylinder, or wall thickness of a box section.

\( u \) Axial displacements.

\( v \) Tangential displacements.

\( v \) Displacement vector.

\( v_i \) Buckling mode shapes, \( i \) refers to the number of the eigenmode.

\( w \) Radial displacements.

\( w_0 \) Radial displacement at the circumferential weld imperfection.

\( w_1, w_2 \) Imperfection amplitudes at \( x = l_1 \) and \( x = l_2 \)

\( w_i \) Width of the tension zone of the circumferential residual membrane stresses.

\( x \) Axial cylinder coordinate.

\( x_w \) Nominal axial weld position.

\( y \) Tangential cylinder coordinate.

\( z \) Radial cylinder coordinate.

**Greek letters**

\( \alpha \) Shape function parameter after Steinhardt & Schulz (1970), or scaling parameter for the bending stiffness at the weld.

\( \beta \) Overall imperfection parameter.

\( \beta_0 \) Axial rotation at the weld.

\( \Phi \) Shape functions after White & Dwight (1977).

\( \gamma \) Overall imperfection parameter.

\( \phi \) Angular cylinder coordinate.

\( \lambda \) Weld imperfection half wave length.

\( \lambda_0 \) Linear meridional bending half wave length of a thin-walled cylinder.

\( \lambda_i \) Scaling factor for a differential loading pattern.

\( \kappa \) Overall imperfection parameter.

\( \nu \) Poisson’s ratio.

\( \sigma_{ci} \) Axial buckling stress according to classical theory.

\( \sigma_{\phi} \) Circumferential membrane stress.
\( \sigma_{\text{long}} \) Buckling strength of a long thin-walled cylinder.

\( \sigma_{\text{yield}} \) Yield stress.

\( \Omega \) Amount of wrap-up after White & Dwight (1977).

\( \Psi \) Shape functions after White & Dwight (1977).

\( \zeta \) Roundness parameter.
1. Introduction

1.1 Background & Motivation

Storage bins are an important link in the chain of materials handling for a large number of industries. In many cases, the ability of storage facilities to provide a reliable flow of bulk material determines the success of a plant. Many such storage bins are manufactured from steel plates and thus belong to the group of thin-walled shell structures which are renowned for their “imperfection sensitive behaviour”. Small deviations from the nominal, or perfect geometry result in significant loss of strength of such structures where imperfections of just one wall thickness in magnitude can reduce the axial buckling stress to only one third the classical value for perfect cylinders. Many aspects of the imperfections commonly found in silos or tanks have received little or no attention and their influence on the load carrying capacity of these storage facilities is unknown. Babcock (1974) stated that “the most important problem confronting the experimentalist is the influence of initial imperfections on the buckling loads obtained experimentally”. This message was more or less repeated almost 25 years later in an extensive review article where Teng (1996) concluded that “despite extensive research over many decades, our knowledge of many shell buckling problems is still very limited. Consequently, shell stability design criteria contained in design codes for various structures such as tanks, silos, pressure vessels and offshore platforms generally cover only the basic geometries of cylinders, cones, and spheres, and simple loading conditions...”.

Engineers have, over the years, expressed their need for more research in the area of thin-walled structures. In a symposium organised by the Australian Institute of Steel Construction in 1980, Deutsch (1980) recommended that “the engineering industry should, via Standards Association of Australia, develop an Australian Standard for the design of concrete and steel bins and silos” and that “there is also the opportunity to lead the world in developing a standard specifically covering steel bins”. Later, in 1985, a
survey (Pham & Maunsell, 1985) on Research Needs for Metal Structures amongst engineers placed 'bins and tanks' as second only to the general category 'buildings' as an area needing research, consistently exceeding the needs for research on bridges, offshore structures, cranes, towers, scaffolding and long-span roofs.
1.2 Aims

Steel silos and tanks are constructed from plates which are rolled to the correct curvature and welded together to form strakes. Several strakes of curved plates, placed on top of each other then form the completed structure. At each circumferential weld, a slight hourglass depression occurs forming axisymmetric imperfections which are known to be most deleterious. The influence of this particular type of imperfection on the buckling of silos and tanks has been studied in the past (e.g. Steinhardt & Schulz 1970, Häfner 1982, Rotter & Teng 1989, Teng & Rotter 1992) but there are still a number of important aspects that remain to be investigated in detail. The aspects considered in this thesis are as follows.

1.2.1 Shape of the Weld Imperfection

An exact survey of the detailed shape of an actual silo has only been performed in one instance (Ding 1992). Steinhardt (1970) also took imperfection measurements but only to a very limited extent. These measurements in combination with a theoretical approach have been used to derive a new shape function that describes the geometry of the weld.

1.2.2 Weld shape and Buckling Strength

A variety of shape functions describing the local weld depression has been given leading to a range of predictions for the buckling strength of silos and tanks. This suggests that the shape of the weld imperfection influences the buckling behaviour of thin-walled cylindrical structures. A parameter study was performed to clarify the importance of the shape of the weld and to link these findings with the newly defined shape function.

1.2.3 Interaction between Neighbouring Welds

The individual strakes are commonly not high enough to isolate the influence of neighbouring circumferential welds from each other i.e. interaction between the weld imperfections can take place. This interaction between individual localised imperfections can reduce the buckling strength of the structure even further. A parameter study has been carried out to examine the effects of strake height on the buckling strength for a range of weld imperfections. A design procedure has been developed.
1.2.4 Weld-Induced Residual Stresses

Locked-in (residual) stresses resulting from the welding process also affect the axial buckling strength of thin-walled silos. The nature and the extent thereof has been under dispute in the literature (Häfner 1982, Rotter 1996). The strengthening effect of weld-induced residual stresses has been confirmed and studied for a range of $R/t$-values and imperfection amplitudes.

1.2.5 Post-Buckling Behaviour

No previous studies are known to have investigated the post-buckling of thin-walled cylinders with a circumferential weld imperfection. A numerical study was performed and a model for the post-buckling behaviour was developed to explain the results of this study.

1.2.6 Box Sections

The methods used for the analysis of thin-walled silos were adapted for the analysis of other welded thin-walled structures, namely box-sections under compression. Experimental data for weld-induced residual stresses and displacements were reproduced in a Finite-Element analysis. The influence of various parameters on the buckling of these box-sections under compressive load was investigated.
1.3 Methodology

A mixture of analytical and numerical methods has been employed to tackle the problems investigated in this thesis. Existing computer tools were used wherever available and were complemented by software developed for the specific tasks where necessary. Software package GNU-plot was used for most of the data processing and curve-fitting in Chapter 2 and finite element code ABAQUS was used for the numerical analyses of the buckling and post-buckling investigations in the other chapters of this thesis. Additional software-routines were developed to allow efficient data-processing, to build the numerical models and to include residual stress-fields.
1.4 Buckling of Cylinders

1.4.1 Classical Theory

While classical theory works well to predict the stability of flat plates and columns, the situation for circular cylindrical shells (Figure 1-1) is quite different. Taking all the usual simplifications of classical theory into account, the following can be observed. All three of these systems show a linear response until a critical load is reached. Idealised columns cannot develop transverse stresses to restrain out-of-plane displacements. The response after buckling is therefore undefined. Imperfections cause increased lateral displacements and the maximum load is reached asymptotically. The load capacity of flat plates increases even after the critical load has been reached as transverse tensile stresses work against the growth of out-of-plane displacements. Again, the post-buckling response of the perfect plate is approached asymptotically by an imperfect plate, smoothing the kink in the load-deflection curve. The post-buckling response of a perfect cylinder is significantly different. A steep descent in the stress-strain characteristic can be observed after the critical load has been reached. Imperfections can reduce the buckling strength dramatically as the narrow gap between the pre-buckling and the post-buckling segment of the response curve is bridged at a much lower level.

![Figure 1-1. Post-buckling behaviour of elastic flat plates, columns and cylinders.](image-url)
Plasticity affects the behaviour of the three systems in similar ways as the load capacity of each member drops off once elastic limits are exceeded. The severest reductions in strength generally occur when the yield limit and the critical buckling load are close to each other.

1.4.2 Imperfection Sensitivity

Calladine (1995) lists three criteria to define “imperfection-sensitive behaviour”:

(i) Buckling loads which fall short of the predictions of classical buckling theory.
(ii) Unpredictability of buckling strength, as evidenced by the wide scatter of experimentally observed buckling loads.
(iii) Unstable, dynamic behaviour of the shell after the maximum load has been reached - leading to catastrophic failure in some circumstances.

Thin walled circular cylindrical structures have been proven to show all of the above mentioned behaviour patterns. Buckling loads in laboratory tests are usually scattered. Very small differences in the amplitude of imperfections can lead to enormous differences in strength. Buckling typically occurs suddenly and dynamic effects distinguish the behaviour immediately after the critical load has been reached.
1.5 Historical Review

Comprehensive reviews of the work on buckling and post-buckling of thin-walled cylinders under axial compression have been given by Hutchinson & Koiter (1970), Flugge (1973), Yamaki (1984) and more recently Teng (1996) amongst others. Therefore this chapter concentrates mainly on the work which is directly relevant to the problems discussed in this thesis.

The first solutions for a thin-walled cylinder under axial load appeared at the beginning of the last century, namely by Lorenz (1908, 1911), Timoshenko (1910) and Southwell (1913). The derivations in these works were restricted to cylinders of perfect initial geometry, simple supports, elastic material behaviour and membrane stress distribution in the pre-buckling region. An axisymmetric buckling pattern was assumed. The resulting critical buckling strength is commonly referred to as the “classical buckling strength” \( \sigma_{cd} \) and is widely used as a reference value, where

\[
\sigma_{cd} = \frac{E \cdot t}{R \sqrt{3(1 - \nu^2)}} = 0.605 \frac{E \cdot t}{R} \tag{1-1}
\]

in which \( E \) is the modulus of elasticity, \( \nu \) is Poisson’s ratio, \( R \) is the cylinder radius and \( t \) is the wall thickness. The value 0.605 is given for steel with \( \nu = 0.3 \). As this formula for \( \sigma_{cd} \) is based on an axisymmetric buckling shape only a meridional wave number can be derived. Donnell & Wan (1950) give a number of circumferential waves \( m_{cl} \) based on this theory by applying a ratio of 0.75 (meridional to circumferential wave length) which they observed in experiments:

\[
m_{cl} = \sqrt[4]{12(1 - \nu^2)} \sqrt[3]{\frac{R}{t}} \approx 0.873 \sqrt[3]{\frac{R}{t}} \tag{1-2}
\]

In the 1930s, external surfaces of aircraft started to be manufactured from thin sheet metal and were made load-bearing triggering much of the research in the field of shell buckling. Experiments by Lundquist (1933), Donnell (1934) and others on thin-walled cylindrical
shells failed to confirm the stresses suggested by classical theory. Buckling stresses as low as only 20% - 40% of $\sigma_{cl}$ were recorded (Figure 1-2). It was also noted that the load-carrying capacity decreased sharply after the buckles had appeared.

![Diagram](image)

**Figure 1-2. Test results after Donnell & Wan (1950)**

All the restrictions of classical theory had previously been used successfully by Euler to study the buckling of columns and it came as a very unpleasant surprise that they were obviously not applicable in the case of thin-walled cylinders. Four lines of attack were pursued to explain the gap between experimental results and theoretical predictions. Firstly, the effects of boundary conditions other than simply supported conditions were explored (e.g. Thielemann & Esslinger 1964, Hoff & Soong 1965). Secondly, pre-buckling deformations were taken into account (Flügge 1932, Almroth 1966). Thirdly, load eccentricities were included. Fourthly, the influence of initial imperfections were
suspected and investigated. It was the latter explanation which became gradually accepted, especially since neither variations in boundary conditions nor pre-buckling deformations could account fully for the large discrepancies between theory and experiments (Teng 1996).

During the same period of time, a number of empirical formulas were proposed for design purposes (Ballerstedt & Wagner 1936, Kanemitsu & Nojima 1939) but only Batdorf et al (1947) managed to incorporate some degree of theoretical background for the given formulas.

Donnell (1934) came close to explaining the large discrepancies between experimental results and classical theory predictions when he suggested ‘initial deviations’ as their cause. He developed a set of non-linear large displacement equations which permitted initial deviations from the perfect shape to be considered. Unfortunately he emphasised local yielding in his theory and did not come to the correct conclusions. Karman et al (1940) were the first to explain the surprising test results with a model that showed the effects of geometric imperfections, in combination with geometric non-linearity, can have on cylindrical shells. A year later, Karman & Tsien (1941) solved the Donnell (1934) equations using an energy method and found a primary and a secondary equilibrium path for the perfect cylinder. Their work explained the sudden nature of the buckling in laboratory tests. They also derived three different states of equilibrium for the same load and suggested that an appropriate disturbance can cause a cylinder to ‘jump’ to a more deformed state without reaching its full strength.

Building on Donnel’s equations, a number of researchers (Legget & Jones 1942, Michielsen 1948, Kempner 1954) advanced the theory by using more and more complicated assumptions for the buckled shape of the cylinder to determine the post-buckling minimum. The idea of using this minimum for design recommendations had to be discarded for obvious economical reasons when Almroth (1964) used computers to derive a post-buckling strength as low as 0.108 of $\sigma_d$. More significant work based on Donnel’s set of differential equations was done by Pflueger (1963, 1966) who proposed a design formula for cylinders with high $R/t$-ratios.
Koiter (1945) took another step forward when he set up his new “general theory of stability”. His approach concentrated on the stability of shells in the immediate vicinity of a bifurcation point and allowed him to determine the slope of the initial post-buckling curve. He investigated the case of a cylinder under compression and proved that interacting modes can produce highly unstable initial post-buckling behaviour. By including the effects of initial imperfections having the same shape as the axisymmetric buckling mode, he derived a formula giving the buckling loads for imperfect cylinders where

\[(1-\bar{\delta})^2 = \sqrt{\frac{27}{4}} \left(1-v^2\right) \frac{\delta_0}{t} \left. \bar{\sigma} \right|_b \]  

with : \[\bar{\sigma} = \frac{\sigma_b}{\sigma_d}\]  

(1-3)
in which \(\sigma_b\) is the mean axial stress at elastic buckling and \(\delta_0\) is the amplitude of the geometric imperfection. Koiter (1963) expanded his theory for finite displacements, again for the case of imperfections of the shape of the axisymmetric buckling mode. This theory became known as Koiter’s (1963) “special theory” with which, finally, good agreement between experiments and theory over a large range of imperfection amplitudes was achieved. Further work by Amazingo & Budiansky (1972) amongst others expanded Koiter’s “special theory” for more complicated imperfection shapes.

The lack of measured imperfections of test specimens made it impossible to compare these new theories with experimental data. Babcock & Sechler (1962) performed tests using precisely manufactured specimens of a defined initial geometric imperfection and confirmed the strong influence that imperfections have on the buckling strength of thin-walled cylinders. More data including initial geometric imperfections was collected by Arbocz & Babcock (1969) and Arbocz & Sechler (1974) and their experimental results matched theoretical predictions well. Tests performed by Tennyson & Muggeridge (1969) gave buckling loads within 10% of Koiter’s predictions. Finally the contradiction between theory and experiment was now resolved for the prediction of critical loads. However, more questions remained as the post-buckling deformations of thin-walled
cylinders in laboratory tests continued to differ from theoretical predictions. Esslinger (1970) and Tennyson (1969) both used high-speed cameras to document the development of the post-buckling patterns. Yamaki et al. (1973, 1975) documented post-buckling test results in great detail, and Berry (1997) tested and documented cylinders with exactly measured imperfections.

The development of computer methods, namely the finite element method, and the dramatic growth in computing power during the last three decades finally allowed researchers to simulate the post-buckling behaviour of thin-walled shells in detail considering various types of imperfections and loading conditions (eg. Guggenberger 1995, Rotter 1996). Mention must also be made of two promising reports recently published in Germany (Deml 1997, Spohr 1998). Spohr (1998) expands in his work the so-called 'disturbance-energy concept'. This concept uses disturbances in the load vector of an otherwise perfect structural system. The minimal amount of disturbance energy needed to push a stable system into its post-buckling configuration is computed and used as a measure for the 'degree of stability' of the pre-buckling configuration. Deml (1997) investigated a method to find the most detrimental geometrical imperfection for a given shell-structure and found that the approach suggested in the ECCS (1988) does not always lead to the most conservative buckling strength for imperfection-sensitive shell structures.

1.5.1 Axisymmetric Imperfections

During the first half of the twentieth century, practical applications for the research into shell buckling were only found in the aviation industry. Axisymmetric imperfections do not generally occur in aircraft structures and consequently they were dismissed as of little relevance to practical problems (Arbocz 1974). Their influence on the buckling of shells was not investigated in depth until the 1970s despite Koiter's (1941, 1963) results which showed that axisymmetric imperfections are extremely severe. Buckling of thin-walled shell structures only started to become relevant for structural engineers in the 1950s when failures in thin walled steel tanks and silos became an issue. Measurements on silos and tanks (Steinhardt & Schulz 1970, Clarke & Rotter 1988, Ding 1992) revealed that predominantly axisymmetric imperfections do occur in these structures. Esslinger & Geier (1977) investigated cylinders with lapped joints and concluded that the axisymmetric portion of the imperfections governed the buckling of their specimens.
In Amazingo & Budiansky (1972), an asymptotic formula for the buckling stress of cylinders with arbitrary axisymmetric imperfections is given and Hutchinson et al. (1971) applied this formula to a cylinder with a localised axisymmetric imperfection which they called a 'cosine dimple'. Steinhardt & Schulz (1970) measured imperfections on four tanks and performed an analytical investigation on the influence on the buckling strength of these structures. Their work was continued by Häfner (1982) and Bornscheuer et al. (1983) who developed a computer program to analyse imperfect shell structures. Rotter & Teng (1989) and Teng & Rotter (1992) used the finite element method to investigate axisymmetrically imperfect structures which feature imperfections resembling weld-induced grooves found in modern silos and tanks. Rotter (1996) presented more work on circumferential weld-induced imperfections taking into account interaction between neighbouring welds and residual stresses, and Berry (1997) investigated axisymmetric imperfections which extend only over a small portion of the circumference.

Remarkably few tests have been carried out which actually tried to emulate imperfections of structures such as silos or tanks, partly due to the fact that the problem of cylinder buckling was first recognised by aircraft engineers. Steinhardt & Schulz (1970) did some statistical work on measured imperfection data and tried to duplicate some of the characteristics found in their tests. Berry (1997) did more tests on cylinders with imperfections resembling circumferential welds in silos. More recently, laboratory tests on cylinders and conical hoppers by Teng & Zhao (1999) have yielded first results.
1.6 Imperfect Silos and Tanks

Many types of imperfections that occur in silos and tanks have been identified and their influences on the load carrying behaviour are generally well known. Imperfections are introduced at various points: during the erection of the structure; during maintenance work; or by time dependent processes such as corrosion or foundation settlement. Modern silos and tanks are typically made from rolled steel plates. Strakes of such plates are welded together to form the cylindrical shape. Bolted connections are less commonly used and will not be discussed in this thesis. Bottom support is generally realised by one of the following three methods: the structure is connected to the foundation directly; evenly distributed columns provide discrete supports for elevated silos or tanks around the bottom circumference; or in some cases of water supply tanks, the shell structure can be situated on top of a tower-like structure. Discharge facilities may necessitate a mixture of these forms. Roofing and ring stiffeners can provide additional rigidity.

1.6.1 Geometric imperfections

(i) Overall deviations from the cylindrical shape. Comparatively high amplitudes of overall imperfections such as out-of-roundness, out-of-straightness, differential diameters or others are commonly found in any silo or tank. Unfortunately, virtually no measured data exists on these deviations from the nominal cylindrical shape and the assumptions which are commonly made in order to model them are highly unfavourable and may well be far too conservative.

(ii) Depressions at vertical welds. These imperfections generally are not aligned in neighbouring strakes of silos and tanks. They are therefore non-symmetric and can be rated to be far less detrimental than axisymmetric forms of imperfections. However, they are of interest in other fields of engineering such as the design of pressure vessels or aircraft (Ravn-Jensen & Tvergaard, 1987).

(iii) Depressions at circumferential welds. Panel rolling and weld shrinkage both contribute to imperfections of this type. Although of lesser amplitude than the above mentioned imperfections, these depressions lead to great strength reductions due to their circumferential nature (Rotter & Teng, 1989).
(iv) Dents. Dents are introduced at various points during erection and maintenance of the structure. The influence of dents on the overall behaviour of the structure has not attracted much research and should deserve closer attention.

(v) Holes and cut-outs. Stress concentrations occur around such holes and can be controlled by carefully designing the details in such places accordingly. Failures have been reported in cases where holes had to be put into existing structures.

(vi) Corrosion. Many stored materials become highly corrosive when damp. Over the years, the shell weakens in spots where corrosion takes place and stress conditions within the structure may change.

1.6.2 Imperfection in the initial stress distribution

(i) Weld-induced residual stresses. Tensile stresses in the immediate vicinity of welds often reach values up to the yield stress of the material. For equilibrium reasons, these tensile stresses are accompanied by compressive stresses further away from the weld which typically reach between 40% and 60% of the yield stress (Ravn-Jensen & Tvergaard, 1987).

(ii) Rolling induced stresses. Originally flat steel plates need to be rolled into the required radius. This is obviously only possible by straining the plates beyond yielding. The stress situation is further complicated when these plates are welded together and stress relaxation near the weld takes place.

(iii) Residual stresses accompanying dents. Dents introduce localised tensile and compressive stresses. Again, more research is needed to evaluate their influence on the buckling behaviour of cylinders. As with all other problems related to stress fields in structures, there has to be consistency between displacements and stress components (Holst et al., 1996).

1.6.3 Boundary conditions

Real life boundary conditions can vary considerably from those assumed in the structural model. Conditions near support structures and load application points are hard to predict and therefore difficult to model in a structural analysis (Li, 1990). Boundary conditions may be seriously influenced by the stiffness of rings in ring stiffeners. Unexpected load conditions can lead to failures in these support structures. Uplifting forces can occur and have been known to be responsible for the failures of steel silos (Wood, 1997). Changes
over time such as foundation settlement can induce stresses and deformations (Gorenc et al., 1986) that may lead to buckling after long times of usage.

1.6.4 Imperfections in the material properties

Although steel in general behaves very homogenically, imperfections in the material properties can be observed, especially near welds. Heating beyond melting point and subsequent cooling can alter the molecular structure. The filler material for the welds also introduces sections of different properties into the shell. These local changes in the material properties can also affect the distribution of residual stresses if the yield stress in these places is altered.

1.6.5 Imperfect load assumptions

(i) Loads from the stored material. Fluids in tanks only exert hydrostatic loads onto the surrounding shell structure. The nature of the bulk solids stored in silos makes load predictions a lot more complicated. Friction between the silo walls and the stored material creates a combination of radial and axial loading of the shell. Small changes in the stored material (moisture, swelling etc.) or the surface roughness of the shell can alter these loads dramatically (Gorenc et al., 1986). Geometric imperfections or corrosion can also lead to different loading situations than originally planned for.

(ii) Wind and seismic loads. Due to the complicated load pattern of these events, only rough approximations (Trahair et al. 1983, Gorenc et al. 1986) of the real loading can be achieved. Some of these approximations apply only for a limited range of geometries and can be misleading for silos with dimensions that lie beyond these limits (Greiner & Derler, 1995). Great care must be taken with interpreting the results of such calculations based on these approximations.

(iii) Roof loads. Concentrated loads (conveyor belts etc.) are often carried down to the support structure by vertical stiffeners. Special attention must be given to how these loads are applied and to how much of the load has to be carried by the actual shell.

(iv) Loads due to discharge. Unexpected loads can occur during discharge, especially when the flow properties change due to prolonged storage, moisture variations or changes in wall roughness. Eccentric discharge facilities introduce non-axisymmetric stresses into silos and need to be treated accordingly. Even seemingly
symmetric discharge facilities can cause stress distributions that have lead to buckling in some steel storage silos (Buzek & Epstein, 1988).

1.6.6 Imperfections due to usage

All the above-mentioned imperfections in a circular cylindrical shell structure can be considered and taken care of during the design process. New imperfections are often introduced through improper maintenance or usage of the bin. Bearings become “frozen” by dirt or corrosion, rough cleaning regimes can add unnecessary additional dents, extensions to the existing structure may add loads and corrosion can weaken the shell itself or other components of the system. Some bins have to be moved to different locations after a period of service in one spot. These structures are dismantled and re-erected at a different location. New imperfections are impossible to avoid during this procedure and must be accounted for. Corroded or damaged parts of the structure need replacement. Temporary changes in the boundary conditions may lead to considerable changes in the strength of the structure (e.g. jacking at ground level to replace corroded parts of the bottom plate in tanks). Inspections and proper maintenance could have avoided many cases of failure (Ravenet, 1983).

1.6.7 Interaction Between Different Types Of Imperfections

A combination of most of the listed imperfections can be found in any existing steel silo or tank. Many of them have been studied and their effects on the strength of these structures are documented. Even though many types of imperfections always occur in combination with others, their effects have been studied predominantly isolated from the other influences. Every geometric imperfection is usually caused by some action that beside the obvious change in geometry, also creates residual stresses, residual strains and possibly local changes in material parameters. Every imperfection in the boundary conditions results in stress fields in the structure and every additional cut-out changes the stress distribution in the cylinder.

Analyses investigating these interactions have very often neglected many of the contributing factors. Research on geometric imperfections has been emphasised. The more complicated these interactions get, the more important it is to simulate the physical processes that lead to the imperfections in question. Consistent stress fields, strain fields
and deformations can be generated and can serve as imperfections in subsequent structural investigations.

The number of parameters interacting to influence the strength, and the wide range of values that some of these factors can take, create an immense task when investigating the problems in detail. Numerous examples of complex interaction of all kinds of imperfections can be found in any silo or tank. The matter is rendered even more complicated by the fact that very little measured data on most of these imperfections exists. To base the theoretical work on a better foundation two approaches seem possible:

(i) Imperfections in existing silos and tanks need to be measured and recorded. A reliable data base of measurements concerning all factors playing a role in the various imperfections encountered would certainly lead to more rational assumptions concerning the influence of these factors. Structures of various sizes and fabrication techniques need to be considered.

(ii) Secondly, experiments under laboratory conditions need to be performed, modelling imperfections as close as possible to their “real world” counterparts. Experiments on such specimens could provide valuable information so that parameter studies and computer models can be calibrated against them.
1.7 Design Rules

The approach used in many current design rules is to limit the extent of the various imperfections and then design against buckling using so the called “knock down factors”. These factors are based on empirical lower bounds of laboratory tests and represent the ratio between the classical buckling strength and the actual design strength. Unfortunately, the specimens used in many of these tests often show little correspondence with the imperfection forms found in practical structures. More recently developed rules (eg. ECCS, 1988) are based on measured imperfections and their assumed amplitudes. Production tolerances and different levels of strength according to the chosen manufacturing accuracy are given. Emphasis has been put on geometric imperfections but other types will also have to be taken into account as methods advance.

1.7.1 Probabilistic Approach

In order to obtain reliable and practical predictions of buckling loads and to improve structural efficiency, a probabilistic approach will be necessary. More measurements of existing shell structures such as done by Clarke & Rotter (1988) or Ding (1992) are necessary to reach stochastic reliability. This data must then be stored in one standard format in imperfection data banks. Shells of different sizes and fabrication methods must be investigated and measured. Once enough data is accumulated in such a data bank, new design methods can be adopted such as outlined by Singer and Abramovich (1995).

Probabilistic approaches can improve design procedures at several stages:

(i) During the preliminary design, imperfection-degradation characteristics can be evaluated along with other factors such as cost, availability etc. and provide arguments to decide between different design possibilities.
(ii) Probable imperfection patterns can then be introduced into the analysis model and give realistic predictions of buckling modes.
(iii) Construction tolerances can be based on data obtained from the imperfection data base.
(iv) Finally, data gathered from the imperfection data base can be used in frequent serviceability checks and measurements gained from such checks could be used to complement the existing data.

The last point deserves a closer look. While design codes regulate many aspects concerning the design and the erection of silos and tanks, no attention is being paid to imperfections caused by usage and maintenance. Wear and tear over time can introduce considerable imperfections and the importance of frequent serviceability checks should not be underestimated.

1.7.2 Imperfection Measurement in Codes

The deviations from the nominal shape which have to be checked in order to comply with code tolerances for shell structures are manifold. According to a survey by Chryssanthopoulos et al. (1996), out-of-roundness tolerances are the hardest to achieve. Other tolerances to be met include out-of-straightness, local variations of the curvature (dents etc.) and differential diameters among others. The ECCS (1988) and the EC3 (1998), as pointed out recently by Rotter (1996), give imperfection measurement procedures and tolerances for different imperfection classes for silos and tanks.

A full scan of the structure would be desirable in order to check whether geometric tolerances have been met. For obvious economic reasons, this cannot be achieved and other methods have to be employed. Local deviations from a straight line are measured by using a straight rod of a characteristic length. The maximum distance between the shell wall and the rod is measured. A circular template with a nominal radius equal to the outside radius of the shell serves to measure the maximum deviation of the shell wall from this template. Again, the template is of a certain characteristic length which often corresponds to buckling wave lengths. Both these methods only measure local imperfections. Global imperfections of the cylindrical shape are very difficult to measure in large structures and the methods proposed in various codes are more applicable for pressure vessels. A possible way of recording comprehensive imperfection maps of a large shell structures using stereo photogrammetry has been pointed out on a number of occasions (Ethrog 1986, Moss 1990) but this system has yet to be put into practical use.
2. The Circumferential Weld Imperfection

2.1 Introduction

Until the 1970s, axisymmetric imperfections in thin-walled cylindrical structures were dismissed as less important from an engineering point of view. Research in the area of thin-walled shell structures was, until then, centred around problems associated with the aviation industry and axisymmetric imperfections were thought to be unrepresentative of the imperfection geometries measured in aircraft components (Arbocz 1974). In the late 1960s and early 1970s manufacturing techniques advanced to a state which allowed the construction of cylindrical steel containers that could be classified as thin-walled cylinders. When such structures are built, rolled steel plates are usually welded together to form series of individual strakes which are joined together by circumferential welds (Figure 2-1) forming a brickwork-type pattern of welded curved steel plates. A localised axisymmetric imperfection is introduced at each circumferential weld. Deformations at each joint are partly caused by the rolling process, but mostly these deformations are due to shrinkage of the heated metal in the vicinity of the weld. The extent of these deformations depends to a large degree on the manufacturing techniques and on the degree of quality assurance during the erection of the structure. Lacher & Haspel (1980) describe measures to reduce such geometric imperfections during the construction of a large silo in Germany.

![Figure 2-1. Erection of a Circular Silo or Tank](image-url)

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A close look at the area in the immediate vicinity of this axisymmetric imperfection shows that the geometric imperfection is accompanied by various other interacting influences (Figure 2-2). Residual stresses result mainly from weld shrinkage and due to the curvature of that region a complex stress field is introduced which includes membrane and bending stresses. The governing stress component is the circumferential membrane stress which reaches yield level in the tension zone close to the weld. For equilibrium reasons these tension stresses have to be accompanied by compressive stresses further away from the weld. The material properties of the filler material of the weld and also the heat affected zone may have different properties than the material used for the actual shell structure. Overall deviations from the cylindrical shape also play a role in reducing the buckling strength of the structure even further, most notably the deviations caused by the offset vertical welds. Moreover, each individual strake is typically not high enough to isolate the effects of one weld imperfection from its neighbouring ones.

Figure 2-2: Circumferential weld imperfection
A number of studies have modelled the influence of a single purely geometric axisymmetric imperfection on the buckling behaviour of thin-walled circular cylindrical shells (Hutchinson et al. 1971, Steinhardt & Schulz 1970, Esslinger & Geier 1977, Rotter & Teng 1989, Teng & Rotter 1992). The imperfection was treated as a localised geometric deviation from the nominal cylindrical shape and the buckling strength was found to be dramatically reduced compared to the classical buckling prediction of the perfect cylinder. Gellin (1979) investigated plastic buckling of cylinders with an axisymmetric imperfection. Two papers are known to have looked at circumferential welds in circular cylindrical shell structures including the effects of residual stresses (Bornscheuer et al. 1983, Rotter 1996). In Rotter’s study (Rotter 1996), interaction between neighbouring strakes was also taken into account and found to be an important influence reducing the buckling strength of the structure even further.
2.2 Weld Geometry

Several researchers have suggested shape functions to describe the geometry of a circumferential weld in a silo or tank. Most of these suggestions are based on theoretical assumptions and only a few shape functions are loosely based on actual measurements of existing structures. This represents a central problem in modelling the shape of a circumferential weld - or any other imperfection in silos or tanks for that matter. Not enough measurements exist for the assumptions concerning the imperfection geometry to be based on statistically sound data.

Tennyson & Muggeridge (1969) and Hutchinson (1971) performed a theoretical and experimental study on a localised axisymmetric imperfection. They investigated an axisymmetric "cosine dimple" given by

\[ w(x) = \frac{w_0}{2} \left( 1 + \cos \frac{\pi x}{l_d} \right) \]  

(2-1)

in which \( x \) is the axial distance from the centre of the weld, \( w(x) \) is the radial displacement, \( w_0 \) is the amplitude of the imperfection at the weld and \( l_d \) defines the length of the imperfection. Results are given for an imperfection of a length of \( l_d = \lambda_0 \) with \( \lambda_0 \) being the linear meridional bending half wave length given by:

\[ \lambda_0 = \pi \frac{Rt}{\sqrt{12(1-v^2)}} = 2.444\sqrt{Rt} \]  

(2-2)

A similar imperfection shape was used by Gellin (1979) in his study of plastic buckling of rather thick-walled cylinders.

Steinhardt & Schulz (1970) took measurements on empty tank structures in Germany using a measurement device of a length of one metre. They surveyed four different circumferential welds on three different empty tanks at the ESSO-refineries in
Karlsruhe/Germany. Four measurements at the quarter points around the circumference were taken per weld. They found local inward indentations of a magnitude between 0.60 and 1.80 wall thicknesses. The radius to thickness ratios of the cylinders measured ranged between 829 and 1710. Steinhardt & Schulz (1970) approximated the weld geometry in their theoretical study with the following function (Figure 2-5):

\[ w(x) = w_0 \cdot e^{-ax^2} \]  
(2-3)

Parameter \( a \) is given by:

\[ a = 1.73 \cdot \frac{2R}{l_0} \]  
(2-4)

in which \( l_0 \) is the distance between the centre of the weld to the place \( x \) where \( w(x) = 0.2 w_0 \).

Häfner (1982) used the same measured data as Steinhardt & Schulz (1970) to develop two shape functions. The first of these functions is a combination of four straight lines symmetrical on either side of the weld (Figure 2-3). Table 2-1 lists the parameters defining this shape function.

<table>
<thead>
<tr>
<th>( l_1 )</th>
<th>( l_2 )</th>
<th>( l_3 )</th>
<th>( l_4 )</th>
<th>( w_1 )</th>
<th>( w_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.28 t</td>
<td>15.72 t</td>
<td>15.71 t</td>
<td>14.29 t</td>
<td>0.2 ( w_0 )</td>
<td>0.1 ( w_0 )</td>
</tr>
</tbody>
</table>

Table 2-1. Parameters defining the imperfection shape after Häfner (1982).
His second ‘heart-shaped’ function is given by the following formula:

$$w(x) = w_0 \left( \frac{x^2}{l^2} - \frac{2x}{l} + 1 \right)$$  \hspace{1cm} (2-5)$$

in which $l = 35.7 \text{ m}$.

A different approach was chosen by White & Dwight (1977) and again by White et al. (1979) who give comprehensive results of weld-induced deformations and residual stresses for stiffened and unstiffened tubes with an R/t-ratio between 250 and 300. Their “tendon force” approach is similar to Rotter & Teng’s (1989) proposal (Figure 2-4). The “tendon force” $F$ for a single pass weld according to White & Dwight (1977) is given by:

$$F = 0.2 \cdot p \frac{Q}{s}$$  \hspace{1cm} (2-6)$$

where $p$ is the arc efficiency, $Q$ the power input to the arc and $s$ the rate of travel. Adjustments for multi-pass welds are also given. White & Dwight (1977) also take into account shrinkage perpendicular to the weld (transverse shrinkage) and describe an effect called wrap-up which is caused by variations in the transverse shrinkage and is an angular distortion about the line of weld. Their suggestion for the shape of the weld imperfection is based on elastic theory for long cylinders and takes into account the effects of the tendon force and wrap-up. The shape is given by:
\[ w(x) = F \frac{2R \pi}{E \lambda_0} \cdot \Phi + \Omega \frac{\lambda_0}{4\pi} \cdot \Psi \]  \hspace{1cm} (2-7)

in which \( \Omega \) is the amount of wrap-up and \( \Phi \) and \( \Psi \) are given by:

\[
\Phi(x) = e^{-\frac{\pi x}{\lambda_0}} \left( \cos \frac{\pi x}{\lambda_0} + \sin \frac{\pi x}{\lambda_0} \right)
\]

\[
\Psi(x) = e^{-\frac{\pi x}{\lambda_0}} \left( \cos \frac{\pi x}{\lambda_0} - \sin \frac{\pi x}{\lambda_0} \right)
\]  \hspace{1cm} (2-8)

Rotter & Teng (1989) used two different shape functions which are also based on the elastic bending theory for long thin-walled cylinders and which are similar to the suggestions given in White & Dwight (1977). They reasoned that an idealized ring force, similar to the “tendon force”, pulls the weld towards the centre of the cylinder during the cooling of the heated material. No bending stiffness between the two joined strakes was assumed for Shape function ‘Rotter & Teng B’ and full bending capacity was assumed for ‘Rotter & Teng A’ (Figure 2-4). Shape function ‘Rotter & Teng A’ is identical to that by White & Dwight (1977) when the wrap-up effect is neglected.

The two shape functions are given by:

‘Rotter & Teng A’:

\[ w(x) = w_0 \cdot e^{-\frac{\pi x}{\lambda_0}} \left( \cos \frac{\pi x}{\lambda_0} + \sin \frac{\pi x}{\lambda_0} \right) \]  \hspace{1cm} (2-9)

‘Rotter & Teng B’:

\[ w(x) = w_0 \cdot e^{-\frac{\pi x}{\lambda_0}} \cdot \cos \frac{\pi x}{\lambda_0} \]  \hspace{1cm} (2-10)

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Loosely based on measurements carried out on a later occasion (Rotter & Clarke 1988, Ding X.L. 1992) on an elevated steel silo at Port Kembla in New South Wales, Australia, Rotter (1996) revised his shape functions and suggested a set of four functions (Rotter "open", "closed", "best" and "final") composed from combinations of straight lines. He reasoned that the Port Kembla measurements indicated a longer wave length of the weld imperfection than elastic bending theory predicted. An empirical fitting technique was therefore employed by Rotter (1996) resulting in shape functions with much longer wave lengths than previously assumed (Figure 2-5, Rotter "final", Rotter "closed"). Using Figure 2-3 these functions can be described by the following parameters:

Table 2-2. Parameters defining the imperfection shape after Rotter (1996).

<table>
<thead>
<tr>
<th>Function</th>
<th>( l_1 )</th>
<th>( l_2 )</th>
<th>( l_3 )</th>
<th>( l_4 )</th>
<th>( w_1 )</th>
<th>( w_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>closed</td>
<td>4.17 ( t )</td>
<td>37.5 ( t )</td>
<td>25 ( t )</td>
<td>-</td>
<td>0.333 ( w_0 )</td>
<td>-</td>
</tr>
<tr>
<td>open</td>
<td>12.5 ( t )</td>
<td>100 ( t )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>best</td>
<td>8.33 ( t )</td>
<td>75 ( t )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>final</td>
<td>0.0 ( t )</td>
<td>41.67 ( t )</td>
<td>83.33 ( t )</td>
<td>104.17 ( t )</td>
<td>0.375 ( w_0 )</td>
<td>0.083 ( w_0 )</td>
</tr>
</tbody>
</table>

In a paper by Ummelhofer & Knödel (1996), a reference is made to two more shape functions given by Fritschi (1995). Unfortunately, the material leading to these suggestions has not been published and could not be obtained. The two functions are given by:
'Fritschi A':
\[ w(x) = w_0 \left[ \frac{(l_d - x)}{l_d} \right]^5 + 0.15 \left[ \frac{2\pi \left( \frac{l_d - x}{l_d} \right)}{\cos \left( \frac{2}{l_d} \right) + 1} \right] \] (2-11)

'Fritschi B':
\[ w(x) = \frac{w_0}{1.3} \left[ \left( \frac{x - l_d}{l_d} \right)^3 + \cos \left( \frac{\pi x}{l_d} \right) + 1 \right] \] (2-12)

Shape function 'Fritschi A' is similar in character to some of the other proposals but shape function 'Fritschi B' is strangely different. The imperfection does not reach its maximum at the weld but in an area a few wall thicknesses away from the weld (Figure 2-5).

Figure 2-5. Selected shape functions describing a circumferential weld-induced imperfection.
2.3 Weld-Induced Residual Stresses

When two shell segments are welded together, the material along the weld is heated to the melting temperature, and stress relaxation at high temperature is so rapid that the heated structure is essentially stress free. However, when the structure is subsequently cooled to room temperature, large tensile stresses parallel to the weld develop as a result of thermal contraction close to the weld centre line, while compressive stresses develop farther away from the weld to keep equilibrium (Figure 2-6). Typically, circumferential tension stresses along the weld reach yield. For perfectly flat plates welded together this stress field can satisfy equilibrium accurately but in the case of circumferential welds in cylindrical structures these circumferential stresses are accompanied by stresses in the axial direction perpendicular to the weld and shear stresses to maintain equilibrium.

Applicable measured data on weld induced residual stresses in shells is rare and virtually no stress measurements have been conducted near circumferential welds in thin-walled cylindrical shell structures. Ebel (1934) measured residual stresses around a circumferential weld in a pipe with an R/t-ratio of 32. Measurements were taken only on the outside of the pipe. Along with high hoop stresses, considerably high axial stresses were recorded due to high bending moments in the vicinity of the weld. Unfortunately these results don't necessarily apply for thin-walled cylinders. More experimental data has been collected on flat plates and although differences in the stress fields exist between cylinders and plates, these results give some indication on how the stresses in thin walled cylinders are distributed. Dwight & Moxham (1969) suggested a model to approximate the residual stresses parallel to the weld as shown in Figure 2-6 where the tensile stress at the weld is set to yield over a certain length and the compressive stresses are somewhat smaller than yield. Faulkner (1977) measured residual stresses in flat plates and suggested that the width of the tension zone should be between three and six wall-thicknesses. Ravn-Jensen & Tvergaard (1987) presented work on the distribution of residual stresses in meridional welds in thin-walled cylinders and adopted this model. In their paper they give results for $f = 0.2$ to 0.4 (Figure 2-6) where $f$ is the ratio of maximum compressive stress to maximum tensile stress. Häfner (1983) also investigated residual stress patterns and used three different functions to approximate circumferential residual stresses. Unfortunately, errors in the formulas for two of these functions were
found and only the parabolic stress approximation (B) could be used in its corrected form in addition to the linear approximation (A):

\[
A: \quad \sigma_\varphi = \sigma_y \quad \text{for} \quad 0 \leq x \leq l_1 \quad l_s = 30t \quad l_1 = \frac{40}{9} t
\]
\[
\sigma_\varphi = \frac{l_1}{l_2} \sigma_y \quad \text{for} \quad l_1 \leq x \leq l_2 \quad l_2 = l_s - l_1
\]

\[
B: \quad \sigma_\varphi = \sigma_y \left( \frac{3x^2}{l_s^2} - \frac{4x}{l_s} + 1 \right) \quad l_s = 30t
\]

(2-13)

A sketch of these functions is given in Figure 2-6. While function A uses a similar approach to that of Dwight & Moxham (1969), function B employs a parabolic fit-function to approximate the real distribution closer. Häfner (1983) also used elastic theory to determine stresses perpendicular to the weld and residual shear stresses. Holst et al. (1996) cast doubt on Häfner's (1983) results by pointing out that equilibrium was not satisfied for an unloaded shell.

The width of the tension zone varies greatly between these functions for the stress distribution. The stress field depends on the amount of heat input during welding which is roughly a function of the amount of weld metal deposited into the weld. Discussions with manufacturers of silos and tanks revealed that this does not vary proportionally to the thickness of the steel plates. Different techniques of weld preparation lead to relative savings in material deposits for thicker plates, therefore leading to relatively smaller tension zones in thicker shell structures in comparison to thinner shells when normalised to the wall thickness. When thicker steel plates are welded together more welding passes become necessary. The heat input of each pass partially relieves the stresses introduced by subsequent passes. Again, this results in relatively smaller tension zones for thicker steel plates. To account for these differences, a constant tension zone of 50mm above and below the weld was used throughout the work for this thesis unless where otherwise specified. This width satisfies the suggestion given in Faulkner (1977) for all cases investigated in this thesis and conforms with the stress field used by Rotter (1996).
Figure 2-6. Residual stresses along a weld after Dwight & Moxham (1969) and Häfner (1983)

Two papers are known to have investigated circumferential welds in circular cylindrical shell structures including the influence of residual stresses on the buckling strength (Bornsheuer et al. 1983, Rotter 1996). These two papers reach different conclusions. Bornsheuer et al. (1983) report a decrease of the buckling strength of their model by up to 10% while Rotter (1996) concludes that “Circumferential residual stresses in the welded joint, developed by shrinkage of the weld, appear to increase the buckling strength...”.

2.4 Weld Material Properties

Depending on the welding technique used, certain weld preparations have to be carried out before two curved steel plates can be joined together. The edges of the plates need to be shaped at an angle to accommodate the filler metal which is introduced during welding. This is usually done by grinding or flame cutting. White & Dwight (1977) state that the effects of these manufacturing procedures are usually washed out by the subsequent heat input of the butt weld. Once the weld is in place, three distinct zones can be observed (Granjon 1991): the zone where the weld metal is introduced; the heat affected zone where the welding heat cycle has produced one or more transformations of the initial parent material; and the parent metal of the actual structure (Figure 2-7). Welds in silos and tanks are usually overmatched which means that the filler metal has greater strength than the steel used for the actual shell. The material properties of the heat affected zone also differ slightly from the rest of the structure. The modulus of elasticity is usually not affected by the welding process but differences in the yield limit of the individual zones can be expected (Goldak 1995). Buckling typically takes place long before yield stresses are reached and it could be assumed that the imperfection in the material properties will not affect the buckling behaviour. However, residual stress patterns reach yield level in tension near the weld and therefore, tension stresses at the weld can be considerably higher where filler material is introduced into the weld.

![Diagram of weld zones](image)

Figure 2-7. Constituent zones of a weld.
Steel is used in most practical applications of thin-walled cylindrical structures. Some material properties of steel are highly temperature dependent and these dependencies have to be taken into account when studying the weld during cooling. Figure 2-8 illustrates the dependencies of ultimate stress, yield stress, the coefficient of expansion (Lie & Chabot 1990) and the elastic modulus (AS 4100-1990) normalised to the values of the respective properties at 20°C.

![Graph showing the variation of material properties of steel with temperature](image)

**Figure 2-8: Variation of material properties of steel with temperature**

Depending on the type of steel and the particular alloy in use these values will vary but for the purpose of the present work, these variations are of little consequence. Wherever these temperature dependencies were taken into account, the stress-strain relationship for each value of temperature was modelled to be bi-linear: linear from zero to onset of yielding; and linear from the onset of yielding to the point where ultimate stress is reached at a plastic strain of 0.2.
2.5 Interaction of Neighbouring Weld Imperfections

With the exception of Rotter (1996), all research concerning the buckling of cylinders with localised axisymmetric imperfections has been conducted on cylinders with clamped ends or on cylinders of indefinite length with an imperfection in the middle. Due to the restrictions on the dimensions of the steel sheets which ultimately form silos or tanks (Figure 2-1) this is rather unrealistic. Strake heights are generally not high enough to isolate the effects of one weld imperfection from its neighbouring imperfection and interaction between neighbouring circumferential welds needs to be taken into account. This aspect is considered in detail in Chapter 4.

2.6 Interaction with other Imperfections

Interaction with other types of imperfections will occur and the extent of the influence of these interactions will depend on their distribution and magnitude as discussed in Chapter 1. However, axisymmetric imperfections have been shown to be the most significant for thin-walled cylinders and can be extremely detrimental. It is well established that they are the governing imperfection if they are present in thin-walled cylindrical structures (Koiter 1963, Steinhardt & Schulz 1970, Yamaki 1984, Rotter & Teng 1989, Berry et al. 2000). Therefore only axisymmetric imperfections, particularly those produced by circumferential welding, are investigated in this thesis.
2.7 Modelling the Weld Shrinkage Process

Two lines of attack in defining shape functions for the circumferential weld imperfection can be observed throughout all publications on this topic: firstly, the derivation of shape functions by analytical means, based on elastic shell theory; and secondly, the fitting of series of straight lines and curves to measured data.

It can be argued that during the shrinkage process most parts of the structure will deform elastically while the area in the near vicinity of the weld will experience plastic strains, forming a plastic hinge during parts of the cooling. The area where plasticity takes place will still offer a certain resistance to the induced bending moments but to a lesser degree than the rest of the shell. Rotter & Teng (1989) considered the two extreme cases by assuming either full meridional moment continuity (‘Rotter & Teng A’) or no moment continuity at the weld at all (‘Rotter & Teng B’). The degree of this meridional bending stiffness will depend on a number of factors mainly governed by the welding process in each particular case. A Finite Element model of the shrinkage process demonstrates this effect. Temperature dependent material properties (Table 2-3, Figure 2-8) and axisymmetric finite elements were used to model the cooling of the weld from 900°C to room temperature of 20°C. Heat flow was neglected but an area roughly the size of the heat affected zone was assigned a lower starting temperature as shown in Figure 2-9.

Table 2-3: Material properties for steel grade 200

<table>
<thead>
<tr>
<th>Temperature [° Celsius]</th>
<th>Young's Modulus [kN / mm2]</th>
<th>Yield Stress [kN / mm2]</th>
<th>Ultimate Stress [kN / mm2]</th>
<th>Coefficient of Expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>200000.0</td>
<td>200.00</td>
<td>288.00</td>
<td>1.208E-04</td>
</tr>
<tr>
<td>200</td>
<td>188268.1</td>
<td>200.00</td>
<td>370.00</td>
<td>1.280E-04</td>
</tr>
<tr>
<td>400</td>
<td>160458.7</td>
<td>146.38</td>
<td>354.23</td>
<td>1.360E-04</td>
</tr>
<tr>
<td>600</td>
<td>101006.4</td>
<td>88.41</td>
<td>159.13</td>
<td>1.440E-04</td>
</tr>
<tr>
<td>800</td>
<td>36972.5</td>
<td>30.43</td>
<td>33.54</td>
<td>1.520E-04</td>
</tr>
</tbody>
</table>

36
Figure 2-9. Axisymmetric FE-model of the shrinkage process.

While this is by no means a comprehensive study of the cooling process, it demonstrates the mechanisms leading to the circumferential weld imperfection. Plasticity on the surface due to axial bending stresses sets in after less than $20^\circ$C of cooling and the cross section fully plasticises at $30^\circ$C of cooling. The end result is an imperfection shape somewhere between ‘Rotter & Teng A’ and ‘Rotter & Teng B’ (Rotter & Teng 1989) (Figure 2-10). Variations in the starting conditions, namely the width of the heated zones, lead to slightly different shapes but the outcome will always remain between the two extreme cases of ‘Rotter & Teng A’ and ‘Rotter & Teng B’.

The stress pattern which is induced by this shrinkage process also correlates well with the idealised patterns for circumferential residual stresses used in Ravn-Jensen & Tvergaard (1987). The tension zone of six wall thicknesses and the corresponding compressive stresses are shown in Figure 2-11. This shrinkage process also gave good agreement with residual stresses measured in thin-walled box-sections fabricated from plates with longitudinal welds as shown later in Chapter 6.
Figure 2-10. Deformations according to FE-simulation of the cooling process.

Figure 2-11. Weld-induced residual stresses according to FE-simulation of the cooling process.
2.8 Defining a More Realistic Imperfection Shape Function

Elastic thin-walled shell theory allows a simple solution for long cylinders loaded by a ring (or tendon) force for both conditions of moment continuity described above. Moreover, if only partial moment continuity is considered, a simple analytical solution is also possible by employing an elastic spring boundary condition at the weld (Figure 2-12).

![Diagram](image)

**Figure 2-12. Moment continuity at the weld**
Axisymmetric bending of an elastic cylindrical shell is governed by the following differential equation (Timoshenko & Woinowsky-Krieger, 1959):

\[
D \frac{d^4 w}{dx^4} + \frac{Et}{R^2} w = 0
\]

\[
D = \frac{Et^3}{12(1-\nu^2)}
\]

(2-14)

in which \(D\) stands for the flexural rigidity of the shell. Prescribing a radial deformation of \(w_0\) and axial rotation as \(\beta_0 = 0\) at the weld leads to the shape function for ‘Rotter & Teng A’ whereas prescribing the radial deformation of \(w_0\) and setting the moment \(M_0 = 0\) at the weld leads to the shape function ‘Rotter & Teng B’. These two cases are in fact the two extremes of the more general case in which the weld is assumed to have a bending stiffness \(\alpha k\) during cooling. The boundary condition at the weld, along with the radial displacement \(w_0\), is then

\[
M_0 = \alpha k \beta_0
\]

\[
\alpha = 2\pi \frac{D}{\lambda_0}
\]

(2-15)

with \(k\) being a reference stiffness (Figure 2-12). Solving the differential equation 2-14 for the boundary condition given in equations 2-15 leads to the shape function proposed by Berry et al. (2000):

\[
w(x) = w_0 \cdot e^{-\frac{x}{\lambda_0}} \left( \cos \frac{\pi x}{\lambda_0} + \zeta \cdot \sin \frac{\pi x}{\lambda_0} \right)
\]

\[
\zeta = \left( \frac{\alpha}{\alpha - 1} \right)
\]

(2-16)

with parameter \(\zeta\) simplifying the equation. The shape function is plotted for different values of \( \zeta \) in Figure 2-13. \( \zeta = 1 \) means full moment continuity while \( 0 < \zeta < 1 \) stands for different degrees of stiffness of the idealised spring element. For values of \( \zeta > 1 \) the shape function degenerates as shown in Figure 2-13 and for \( \zeta < 0 \) the kink at the weld becomes more and more pronounced. Only for \( 0 < \zeta < 1 \) does the proposed shape function have physical meaning in the context of equation 2-16.
Figure 2-13. Weld imperfection for various values of $\xi$.

Ding (1992) collected comprehensive high-precision imperfection measurements of the outside surfaces of three silos at Port Kembla, Australia. The design radius of these silos was 12 m and was kept constant along the inside wall surface, leading to step changes on the outside between the lower strakes. Strakes were three metres in height. Radial measurements were taken along meridians on the outside of the silos. The elaborate surveying method developed by Ding (1992) allowed him to generate precise imperfection maps of the surveyed silos, including the localised imperfections at the circumferential welds. The measurements of one silo by Ding (1992) were used to calibrate the proposed shape function.
2.8.1 Correcting for Global Imperfection Patterns

Being a comprehensive, three-dimensional map of the imperfect cylinder, the measurements by Ding (1992) also included imperfections of a more global nature which needed to be filtered out before fitting the proposed shape function for the localised weld imperfection to the measurements. The global imperfections considered were:

(i) **Translation.** The term $+\gamma$ in equation 2-17 accounts for the radial offset of the axis of the fitted function to the idealised perfect structure.

(ii) **Rotation.** The term $+(\beta \cdot x)$ corrects for the tilt of the fitted function in relation to a perfectly vertical structure.

(iii) **Angle between neighbouring strakes.** The parabolic term $+(\kappa \cdot x^2)$ finally filters out shape deviations caused by mis-alignments of neighbouring strakes.

\[
w(x) = w_0 \cdot e^{-\frac{x}{\lambda}} \left( \cos \frac{\pi x}{\lambda} + \zeta \cdot \sin \frac{\pi x}{\lambda} \right) + \gamma + \beta \cdot x + \kappa \cdot x^2 \quad (2-17)
\]

While $\gamma$, $\beta$ and $\kappa$ correct for the effects of global imperfections, the values for $w_0$, $\zeta$ and $\lambda$ give the geometric properties of the weld. Parameter $\lambda$ gives the wave length of the individual imperfections, parameter $\zeta$ is a measure for the moment continuity and $w_0$ defines the amplitude of the imperfection shapes. Figure 2-16 to Figure 2-19 show the fitted shape functions for four selected meridional profiles before and after correcting for overall imperfections. Examples are given for inward facing (Figure 2-16 to Figure 2-18) and outward facing weld-imperfections (Figure 2-19). These examples cover different values of $\zeta$ and $\lambda$, and various cases of overall imperfections.

2.8.2 Processing the Measurements

The measurements recorded by Ding (1992) are grouped into 92 series of points along vertical meridians on the outside of the silo. The vertical spacing between points within each surveyed meridian is 50mm and the distance around the circumference between these meridians is 500mm. Measurements were taken between axial coordinates $x = 0$ mm and $x = 20850$ mm. The nominal axial coordinates of the welds $x_w$ were 1800, 4800, 7800, 10800, 13800, 16800 and 19800 mm respectively, consistent with a strake height of
3000 mm. Each profile was partitioned into series of measurements reaching from half a strake height below the nominal weld to half a strake height above the nominal weld. Using the software package GNU-plot, least square fits were performed to find the closest match of the proposed shape function to the measured data.

It was found that a considerable number of imperfections fitted best for values of $\zeta > 1$. The explanation for this rather unexpected result lies in the fact that in many instances measurements were taken very close to the weld-centre where the weld metal deposited at this point forms a small protruding dimple (welding bead) on the shell surface (Figure 2-14). While all other measurements are exact indicators for the geometry of the shell mid-surface, the measurements very close to the weld distort the result for the shape function. Values for $\zeta$ were therefore limited to $\zeta < 1$ which is also a requirement considering the basic theoretical assumptions for equations 2-15 and 2-16. The measurements exactly at the weld centre were ignored. Furthermore, a systematic error was detected in the remaining measurements whereby a disproportionate shift in radial measurements occurred at exactly every 1000 mm in the axial direction. This shift was eliminated by replacing the measurement where the shift occurred with the average of the measurements immediately above and below the shift.

![Diagram](image)

**Figure 2-14.** Detail of welding bead and discrepancy between the geometry of the outer surface and the mid-surface.

### 2.8.3 Fitting the Shape Function

A least square fit of the proposed shape function to the measured data was performed, using $w_0$, $\lambda$, $\alpha$, $\gamma$, $\beta$, and $\kappa$ as free variables. It was found from the measurements that the
position of the actual weld centre did not always coincide with the nominal weld centre position. Therefore the weld centre position was taken to be variable as well. This fitting procedure was performed for each of the 92 recorded profiles and for each of the seven welds. Values for $\alpha$ were limited to a maximum of $\alpha = 1000$ to avoid numerical difficulties. In eight instances, no results could be obtained due to numerical problems during fitting. These problems were caused by grave localised distortions in these profiles, apparently caused during assembly of the silo. These profiles were discarded. The least square fit of a small number of profiles returned very unreasonable results and upon inspecting the corresponding profiles it became clear that these results were also caused by localised dents near the weld. The results were therefore screened for such unrepresentative data and any sets of results that did not satisfy the following two criteria were discarded:

(i) The centre of the weld of the fitted function had to be within 300 mm on either side of the nominal weld.

(ii) The offset of the axis of the function $\gamma$ had to be less than five wall thicknesses.

Results of 91.6% of the profiles fell within these limits and were accepted for further analysis. Figure 2-16 to Figure 2-19 give examples for the fitting of the shape function to the measured data and also demonstrate the versatility of the proposed shape function. In these figures, $x_w$ and $\varphi$ are defined as shown in Figure 2-15.

![Figure 2-15. Silo geometry (Ding, 1992).](image-url)
Figure 2-16. Profile at $x_w = 4800$, $\varphi = 81.7^\circ$

Figure 2-17. Profile at $x_w = 4800$, $\varphi = 28.6^\circ$
Figure 2-18. Profile at $x_w = 16800$, $\varphi = 14.3^\circ$

Figure 2-19. Profile at $x_w = 7800$, $\varphi = 150.4^\circ$
2.8.4 Results

The proposed imperfection shape function was fitted individually to each of the seven welds in each of the measured 92 profiles. Results for the seven free parameters for the shape function were obtained. Table 2-4 gives the mean values and standard deviations for each weld. Weld depth $w_0$ and translation parameter $\gamma$ were normalised against the wall thickness $t$ and $\lambda$ was normalised against $\lambda_0$ (Equation 2-2). Parameter $\zeta$ was derived from the fitted values of $\alpha$. Where plates of different wall thicknesses were joined, the average of the values for $t$ and $\lambda$ were taken as reference values. Graphs showing the detailed distribution of the obtained results around the measured portion of the circumference are given separately at the end of this chapter (Figure 2-28 to Figure 2-83).

The mean amplitude of the depth of the imperfection $w_0$ ranged from 0.176 $t$ to 0.810 $t$. In most cases the imperfection was found to be inward facing but in some rare cases the opposite was the case (Figure 2-19). The standard deviation for the depth of the weld imperfection appeared rather high for some welds, ranging from 0.056 to 1.106. Figure 2-52 and Figure 2-76 show the reason for this high scatter with obvious localised dents at the weld in a small number of places and otherwise consistent values for the amplitude of the imperfection $w_0$. The wave length $\lambda$ was found to be close to $\lambda_0$ for the two lowest welds ($x_w = 1800$ and $x_w = 4800$) and closer to $2\lambda_0$ for the other welds. A reason for this longer than expected wave length can be found in the fact that the strake height is not quite long enough to satisfy the requirements of a long cylinder. Therefore interaction between the two weld-imperfections exists and influences the shape of the imperfections. The mean values for $\zeta$, the parameter for the stiffness of the weld, ranged from 0.527 to 0.832 resulting in weld shapes well between the two extremes given by ‘Rotter & Teng A’ and ‘Rotter & Teng B’ (Rotter & Teng 1989). The position of the centre of the fitted function did not always match the nominal position of the welds. This is partly due to manufacturing tolerances and partly due to the fact that the closest fit of the proposed function was in some cases achieved with a slight offset to the physical weld.
Table 2-4. Means and standard deviations of the weld imperfection parameters for each weld location.

<table>
<thead>
<tr>
<th>weld location x_w</th>
<th>1800</th>
<th>4800</th>
<th>7800</th>
<th>10800</th>
<th>13800</th>
<th>16800</th>
<th>19800</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = \bar{\omega}/t</td>
<td>0.176</td>
<td>0.214</td>
<td>0.329</td>
<td>0.687</td>
<td>0.507</td>
<td>0.480</td>
<td>0.810</td>
</tr>
<tr>
<td>s_A</td>
<td>0.334</td>
<td>0.056</td>
<td>0.551</td>
<td>0.954</td>
<td>0.511</td>
<td>0.465</td>
<td>1.106</td>
</tr>
<tr>
<td>\overline{A} = \overline{\omega}/\lambda_c</td>
<td>1.319</td>
<td>1.106</td>
<td>1.701</td>
<td>2.266</td>
<td>1.876</td>
<td>2.005</td>
<td>2.446</td>
</tr>
<tr>
<td>s_{\overline{A}}</td>
<td>1.222</td>
<td>0.363</td>
<td>1.399</td>
<td>1.543</td>
<td>0.964</td>
<td>1.188</td>
<td>1.808</td>
</tr>
<tr>
<td>\bar{a}</td>
<td>305.6</td>
<td>310.9</td>
<td>521.5</td>
<td>525.7</td>
<td>766.3</td>
<td>655.5</td>
<td>660.3</td>
</tr>
<tr>
<td>s_{\bar{a}}</td>
<td>443.2</td>
<td>447.7</td>
<td>487.3</td>
<td>489.7</td>
<td>414.4</td>
<td>460.1</td>
<td>419.1</td>
</tr>
<tr>
<td>\bar{c}</td>
<td>0.527</td>
<td>0.656</td>
<td>0.653</td>
<td>0.754</td>
<td>0.816</td>
<td>0.832</td>
<td>0.793</td>
</tr>
<tr>
<td>s_{\bar{c}}</td>
<td>0.445</td>
<td>0.349</td>
<td>0.448</td>
<td>0.364</td>
<td>0.381</td>
<td>0.325</td>
<td>0.393</td>
</tr>
<tr>
<td>\bar{x}</td>
<td>1802</td>
<td>4872</td>
<td>7792</td>
<td>10687</td>
<td>13774</td>
<td>16806</td>
<td>19732</td>
</tr>
<tr>
<td>s_{\bar{x}}</td>
<td>123</td>
<td>85</td>
<td>140</td>
<td>122</td>
<td>99</td>
<td>134</td>
<td>120</td>
</tr>
<tr>
<td>\overline{r} = \bar{r}/t</td>
<td>0.279</td>
<td>0.292</td>
<td>0.548</td>
<td>0.730</td>
<td>0.617</td>
<td>0.415</td>
<td>0.677</td>
</tr>
<tr>
<td>s_{\overline{r}}</td>
<td>0.366</td>
<td>0.195</td>
<td>0.677</td>
<td>0.927</td>
<td>0.514</td>
<td>0.544</td>
<td>1.056</td>
</tr>
<tr>
<td>\bar{\beta}</td>
<td>1.163</td>
<td>-0.269</td>
<td>0.193</td>
<td>0.279</td>
<td>-0.570</td>
<td>-0.821</td>
<td>-0.666</td>
</tr>
<tr>
<td>s_{\bar{\beta}}</td>
<td>0.856</td>
<td>1.310</td>
<td>1.484</td>
<td>2.194</td>
<td>1.626</td>
<td>1.685</td>
<td>1.697</td>
</tr>
<tr>
<td>\bar{\kappa}</td>
<td>-0.792</td>
<td>-0.195</td>
<td>-0.670</td>
<td>-0.946</td>
<td>-1.135</td>
<td>-0.909</td>
<td>-1.944</td>
</tr>
<tr>
<td>s_{\bar{\kappa}}</td>
<td>1.012</td>
<td>0.716</td>
<td>1.281</td>
<td>1.779</td>
<td>1.473</td>
<td>1.175</td>
<td>2.882</td>
</tr>
</tbody>
</table>

The individual mean results drawn for all the measured weld imperfections are shown in Figure 2-20 to Figure 2-27. The error bars indicate the extent of the standard deviations on both sides of the mean value.
Figure 2-20. Mean $w_0/t$ for all welds.

Figure 2-21. Mean $\lambda/\lambda_0$ for all welds.
Figure 2-22. Mean $\alpha$ for all welds.

Figure 2-23. Mean $\zeta$ for all welds.
Figure 2-24. Mean location $x$ for all welds.

Figure 2-25. Mean $\gamma$ for all welds.
Figure 2-26. Mean $\beta$ for all welds.

Figure 2-27. Mean $\kappa$ for all welds.
2.8.5 Fitted Results in Detail

In the following, fitted results for each weld are shown in detail (Figure 2-28 to Figure 2-83). The results for each least square fit were collected for each weld and plotted for each individual weld location. It can be seen that for most welds, results are limited to a small range with the exception of a very small number of very localised places where dents caused by the manufacturing of the silos caused the fitting results to deviate from the mean.
Figure 2-28. Least square fit of $w_0/t$ for weld at $x_w = 1800$

Figure 2-29. Least square fit of $\lambda/\lambda_0$ for weld at $x_w = 1800$
Figure 2-30. Least square fit of $\alpha$ for weld at $x_w = 1800$

Figure 2-31. Least square fit of $\zeta$ for weld at $x_w = 1800$
Figure 2-32. Least square fit of $x$ for weld at $x_w = 1800$

Figure 2-33. Least square fit of $\gamma$ for weld at $x_w = 1800$
Figure 2-34. Least square fit of $\beta$ for weld at $x_w = 1800$

Figure 2-35. Least square fit of $\kappa$ for weld at $x_w = 1800$
Figure 2-36. Least square fit of $w_\phi/t$ for weld at $x_\phi = 4800$

Figure 2-37. Least square fit of $\Lambda/\lambda_0$ for weld at $x_\phi = 4800$
Figure 2-38. Least square fit of $\alpha$ for weld at $x_w = 4800$

Figure 2-39. Least square fit of $\zeta$ for weld at $x_w = 4800$
Figure 2-40. Least square fit of $x$ for weld at $x_w = 4800$

Figure 2-41. Least square fit of $\gamma$ for weld at $x_w = 4800$
Figure 2-42. Least square fit of $\beta$ for weld at $x_w = 4800$

Figure 2-43. Least square fit of $\kappa$ for weld at $x_w = 4800$
Figure 2-44. Least square fit of $w_0/t$ for weld at $x_w = 7800$

Figure 2-45. Least square fit of $\lambda/\lambda_0$ for weld at $x_w = 7800$
Figure 2-46. Least square fit of $\alpha$ for weld at $x_w = 7800$

Figure 2-47. Least square fit of $\zeta$ for weld at $x_w = 7800$
Figure 2-48. Least square fit of $x$ for weld at $x_w = 7800$

Figure 2-49. Least square fit of $\gamma$ for weld at $x_w = 7800$
Figure 2-50. Least square fit of $\beta$ for weld at $x_w = 7800$

Figure 2-51. Least square fit of $\kappa$ for weld at $x_w = 7800$
Figure 2-52. Least square fit of $w/lt$ for weld at $x_w = 10800$

Figure 2-53. Least square fit of $\lambda/\lambda_0$ for weld at $x_w = 10800$
Figure 2-54. Least square fit of \( \alpha \) for weld at \( x_w = 10800 \)

Figure 2-55. Least square fit of \( \zeta \) for weld at \( x_w = 10800 \)
Figure 2-56. Least square fit of $x$ for weld at $x_w = 10800$

Figure 2-57. Least square fit of $\gamma$ for weld at $x_w = 10800$
Figure 2-58. Least square fit of $\beta$ for weld at $x_w = 10800$

Figure 2-59. Least square fit of $\kappa$ for weld at $x_w = 10800$
Figure 2-60. Least square fit of $w_0/t$ for weld at $x_w = 13800$

Figure 2-61. Least square fit of $\lambda/\lambda_0$ for weld at $x_w = 13800$
Figure 2-62. Least square fit of $\alpha$ for weld at $x_w = 13800$

Figure 2-63. Least square fit of $\zeta$ for weld at $x_w = 13800$
Figure 2-64. Least square fit of $x$ for weld at $x_w = 13800$

Figure 2-65. Least square fit of $\gamma$ for weld at $x_w = 13800$
Figure 2-66. Least square fit of $\beta$ for weld at $x_w = 13800$

Figure 2-67. Least square fit of $\kappa$ for weld at $x_w = 13800$
Figure 2-68. Least square fit of $w_{o}/t$ for weld at $x_{w} = 16800$

Figure 2-69. Least square fit of $\Lambda/\lambda_{0}$ for weld at $x_{w} = 16800$
Figure 2-70. Least square fit of $\alpha$ for weld at $x_w = 16800$

Figure 2-71. Least square fit of $\zeta$ for weld at $x_w = 16800$
Figure 2-72. Least square fit of $x$ for weld at $x_w = 16800$

Figure 2-73. Least square fit of $\gamma$ for weld at $x_w = 16800$
Figure 2-74. Least square fit of $\beta$ for weld at $x_w = 16800$

Figure 2-75. Least square fit of $\kappa$ for weld at $x_w = 16800$
Figure 2-76. Least square fit of $w_0/t$ for weld at $x_w = 19800$

Figure 2-77. Least square fit of $\lambda/\lambda_0$ for weld at $x_w = 19800$
Figure 2-78. Least square fit of $\alpha$ for weld at $x_w = 19800$

Figure 2-79. Least square fit of $\zeta$ for weld at $x_w = 19800$
Figure 2-80. Least square fit of $x$ for weld at $x_w = 19800$

Figure 2-81. Least square fit of $\gamma$ for weld at $x_w = 19800$
Figure 2-82. Least square fit of $\beta$ for weld at $x_w = 19800$

Figure 2-83. Least square fit of $\kappa$ for weld at $x_w = 19800$
2.9 Summary & Conclusions

Circumferential weld-induced imperfections in thin-walled cylindrical shell structures have been discussed in detail and the properties of such imperfections have been described. Previous research has been reviewed and various shortcomings have been noted.

A new shape function describing the geometry of circumferential weld-induced imperfections has been proposed. This new shape function is based on elastic shell theory and accounts for overall imperfections as well as for the localised imperfection at the weld. A least square fit to measured data has been performed to calibrate this shape function. The function matched the measurements very closely. From the fitted results the following conclusions can be drawn:

1) The mean amplitude was found to lie between 0.5 and 0.8 wall thicknesses confirming observations made by other researchers.

2) The typical wave length of the imperfection proved to be between one and two times the linear meridional bending half wave length.

3) Values for $\zeta$ were found scattered between the two extremes of zero and one, indicating varying bending stiffness at the weld during cooling. This conformed well with a FE-analysis of the cooling process and confirmed the basic theoretical assumptions made to derive the shape function.

4) Overall imperfections were filtered out of the measured data. The magnitude of these imperfections was up to several wall-thicknesses high and deserves further research.

5) The proposed shape function is the first function to combine shell theory with measured data. It is a continuous function and incorporates all the necessary features to represent the geometry of a circumferential weld-induced imperfection.

6) More measured data is needed to correlate imperfection characteristics with welding techniques and environmental parameters.
A parameter study investigating the influence of the shape function on the buckling behaviour under axial load will be presented in Chapter 4. Welding methods, environmental parameters and cylinder geometry all have an influence on the shape of the circumferential weld. More measurements of circumferential weld imperfections are needed to determine characteristic imperfection parameters for individual weld configurations. A data base of measured imperfection data would be a much needed first step into that direction.
3. Modelling the Imperfect Cylinder

"Like cartoons, mathematics is not real. Yet, even in cartoons, some relationship to reality is nearly always intended. To understand either cartoons or mathematics, every individual must define his own relationship to reality." (Goldak 1995)

3.1 Introduction

As outlined in Chapter 2, circumferential weld-induced imperfections consist of many interacting components – eg. weld geometry, residual stresses and changes in the material properties to name just a few. The influence of such an imperfection, as a whole, on the buckling behaviour of a large thin-walled structure and the influence of each of these interacting components individually has been studied in the course of this thesis using the Finite Element Method (FEM). A commercially available computer program called ABAQUS (Hibitt et al., 1998a) which utilises the FEM was employed for the structural modelling. This software has been widely used in past research projects in the area of shell buckling and has been verified on many occasions (eg. Guggenberger 1995, Greiner & Derler 1995, Teng & Song 2000).

3.2 Modelling the Cylindrical Structure

3.2.1 Axisymmetric and 3-Dimensional Shell Elements

A large number of different finite shell elements have been described in the literature (eg. Bushnell 1976, Zienkiewicz 1977, Deml 1997, Teng & Hong 1998) and a good selection of elements for different types of applications are available in ABAQUS (Hibitt et al., 1998b). Structures with axisymmetric loading and axisymmetric structural response can be analysed very economically using 2-noded axisymmetric stress-displacement shell elements (SAX1-elements) with 3 degrees of freedom (radial displacement $u_r$, meridional displacement $u_z$ and rotation about the tangential direction $\varphi$) at each end (Figure 3-1). In the present work, these assumptions were only valid for
a small number of cases (e.g. where the shrinkage process of the circumferential weld was modelled). Full shell elements allow loading and displacements in all directions (Figure 3-1). These elements were used in the actual buckling analyses as bifurcation into non-axisymmetric modes had to be expected. Fully integrated general shell elements called S4-elements were used for all non-axisymmetric analyses. This element type is formulated as a finite-membrane-strain element and gives accurate solutions to the buckling problems of the type investigated in this thesis. S4-elements allow transverse shear deformation and can be used for both thin and thick shell analyses. Many previous investigations that used ABAQUS as a numerical tool used S9R5-elements. Comparisons between the two element types showed that results matched in several benchmark tests but computing was cheaper when S4 elements were used. Moreover, S4 elements have no hourglass modes which is an advantage in comparison to S9R5-elements. Teng & Song (2000) used S9R5 elements for their investigation and they describe various problems they encountered which were directly connected with such hourglass modes. For the recently implemented S4 elements, these problems do not appear to exist.

\[ \text{Nodes} \]
\[ \times \text{Integration points} \]

![Diagram](image)

**Figure 3-1. SAX1, S4 and S9R5 elements.**

### 3.2.2 Geometric Properties

Restrictions in computing capacity did not allow the analysis of whole silos or tanks. A number of simplifications had to be employed to keep the model size within limits
imposed by the computer system. It was therefore decided to use a cylindrical model of one circumferential weld and half a strake above and below this weld (Figure 3-2). It was assumed that variations in the loading are small enough within this limited region to be negligible and loads were assumed to be constant within the model.

![Diagram](image)

**Figure 3-2. Geometric parameters of the analysed model.**

### 3.2.3 Circumferential Symmetries

Perfectly axisymmetric cylinders under axial load are known to bifurcate into non-axisymmetric buckling modes (Yamaki 1984) consisting of periodic waves around the circumference. Previous studies of the circumferential weld imperfection (Tennyson & Muggeridge 1969, Steinhardt & Schulz 1970, Rotter & Teng 1989, Teng & Rotter 1992, Berry 1996) have shown that the buckling patterns are also symmetric about the weld.

A number of different modelling techniques have been used in the past to take advantage of these periodical symmetries (Figure 3-3). The computationally most efficient simplification is a model that uses only one half-wave of one particular buckling pattern. A number of different patterns have to be examined in order to find the critical buckling mode ie. the buckling mode associated with the lowest load. A similar approach is the use of one full buckling wave which essentially doubles the model size with the same number of analyses necessary to determine the critical buckling pattern and load. However, if post-buckling is also of interest during the analysis, the full wave model allows secondary bifurcation into a half wave mode to take place which is not possible for the half-wave model. If a whole structure model is used, no revisions to the FE-model have to be made but computer power might not
always be available for such large models. If only half the structure is modelled with symmetry along the meridians, the results are just as good but symmetry of the deformations along a vertical diametrical plane has to be ensured which means that torsional modes are not possible.

![Different models using circumferential symmetries.](image)

\textbf{Figure 3-3. Different models using circumferential symmetries.}

In practical applications, a mixture of these different models is often used. A thorough discussion of the advantages and disadvantages of these models for a cylinder with a ring load can be found in Teng & Song (2000). Guggenberger (1996) applied a mixture of these models during his analysis of a cylinder under external pressure while Greiner & Derler (1995) used a half-structure model for their analysis of silos buckling under wind load.

\textbf{3.2.4 Interaction between Neighbouring Circumferential Imperfections}

The strake height typically used in silos or tanks is usually not large enough to prevent interaction between adjacent weld imperfections. Rotter (1996) suggested a system of taking interaction between weld imperfections into account which is based on the assumptions that neighbouring imperfections are equally detrimental to the buckling strength. This assumption is rather conservative but it allows the results to be used with confidence until better statistical data is available. Figure 3-4 illustrates possible buckling patterns and their inclusion in a FE-model simply by varying the boundary conditions (symmetry and anti-symmetry) at strake mid-height. Looking at one meridian in the buckled mode, three combinations of boundary conditions can be listed: S-S (symmetry at strake mid-height on both sides); A-A (anti-symmetry at strake mid-height on both sides); and A-S (symmetry at strake mid-height on one side and anti-
symmetry on the other side) as shown in Figure 3-4. Rotter (1996) found that the A-A condition yields the lowest bifurcation loads and the S-S condition the highest. The A-S condition was found to result in bifurcation loads somewhere between these two extremes. As symmetry at the weld has been observed in numerous investigations (e.g. Berry 1997), the A-S condition is not considered further in this thesis.

Figure 3-4. Buckling patterns and boundary conditions.

Figure 3-5. A-A and S-S boundary conditions.
The symmetric S-S boundary condition fixes the axial displacement and the rotations about the radial and the tangential axis while the anti-symmetric A-A boundary condition allows axial displacement but fixes the radial and tangential displacement and the axial rotation (Figure 3-5).

The axial boundary displacement is responsible, to a high degree, for the differences in buckling strength between the two sets of boundary conditions. Enforcement of the symmetry boundary condition in the S-S model means that the model deforms at an equal rate over the circumference which effectively simulates displacement control. On the other hand, with the axial displacement degree of freedom left free to move in the A-A model, displacement control could not be used and a constant load around the circumference was modelled instead. The mean axial displacement was used for load displacement curves for anti-symmetric boundary conditions. Reality must lie somewhere between these two extremes. However, only the A-A and the S-S conditions have been used in the analyses in this thesis.

Figure 3-6. Boundary and loading Conditions for S-S, A-A and elastic spring model.
In real silos or tanks, axial boundary conditions are fixed at the bottom. Ring stiffeners and roof structures also provide restrictions in the axial direction. When looking at one isolated weld imperfection, these restrictions of axial movement will provide some degree of axial stiffness and it makes sense to take this stiffness into account by including additional spring elements as shown at the centre of Figure 3-6. This adds a certain degree of axial stiffness and also leads to more realistic load distributions with the spring elements relaxing the unrealistic assumption of constant loading around the circumference. This aspect requires further research.
3.3 Modelling the Circumferential Weld Imperfection

Including residual stresses into a weld imperfection of a given geometry is a requirement for a number of analyses performed for the present work. Although the task of modelling residual stresses in thin-walled shell structures has not been discussed widely in the literature (Beer 1976, Bornscheuer et al. 1983, Rotter 1996, Holst et al. 1996) two approaches to this problem have been proposed. In the first approach, which has been chosen by Bornscheuer et al. (1983), a given pattern of residual stresses is superimposed onto the structure prior to the buckling analysis. Rotter (1996) on the other hand argued that the residual stresses arise from a physical action (the cooling of the weld and the lack of fit between plates). In this second approach, this physical action was replicated in the analysis and a pattern of residual strains was applied onto the area near the weld. This consequently led to the development of a corresponding stress field. Both strategies are valid and have their advantages and disadvantages.

For the present work it seemed worthwhile to implement both methods and compare the results. Only scarce measurements exist for residual stresses in the vicinity of circumferential welds and these were for rather thick-walled cylindrical shell structures (e.g. Ebel 1934, Dwight & Moxham 1969). Reliable measured data for thin-walled shells is not available. Measurements display a high scatter and influences during welding certainly have an effect on the resulting stress field. Results gained during a FEM simulation (see Chapter 2) of the weld shrinkage compare well with measured results and models suggested by others (e.g. Ravn-Jensen & Tvergaard, 1987). These results will be used for the comparison of the different modelling techniques.

When a pattern of residual stresses is postulated for a structural component, this pattern must satisfy equilibrium throughout the structure. This task is straightforward for structures in which stresses are essentially one-dimensional (i.e. beams, flat plates, columns and many others). However, this is not the case in shells where stresses are two-dimensional and affected by the curvature of the shell. Postulating a two-dimensional stress pattern which is in equilibrium everywhere is a difficult task, especially locally where three-dimensional weld-geometries are involved. The stress patterns for circumferential stresses given by Dwight & Moxham (1969) and Hafner
(1983) were stored into the model. In the vicinity of the circumferential weld, the shell structure is doubly curved. In order to achieve a stress field that is in equilibrium everywhere, including these places, a small nominal axial load was applied and equilibrium was established by solving this load case, thus generating the other components of the stress field. Axisymmetric elements were used in these analyses.

Alternatively, imposing a reasonable residual strain pattern effectively models the process that leads to the residual stress field in the vicinity of the circumferential weld. All components of the stress field are automatically generated as a result of the strain-loading of the structure. Strain loading according to Rotter (1996) was used. Strain loading was applied inside an area of 4.2 wall thicknesses from the weld centre. This area was assumed to experience a shrinkage strain equal to the yield strain.

Figure 3-7 shows the resulting circumferential membrane stresses for three of the described methods and Figure 3-8 illustrates the generated axial stresses on the inside of the shell giving a measure for the axial bending moment.

Figure 3-7. Circumferential residual stresses according to different models.
Figure 3-8. Axial residual stresses on the inside of the cylinder according to different models.

When implemented in this way both methods – the application of residual stresses as well as the application of residual strains – led to similar results. Their effect on the buckling strength of the structure will be investigated in Chapter 4.

Both methods also led to additional deformations in the structure. When stresses are applied and the structure is forced into equilibrium by the nominal load case explained above, small displacements occur. Stresses generated by the residual strain field will also lead to displacements as the structure attempts to relieve some of the ensuing stresses by deforming. These deformations have to be taken into account when modelling cylinders with specific imperfection shapes. The deformations generated by the application of residual stresses have to be subtracted from the initial model so that the model, which later includes the residual stresses, has the desired shape prior to any external loading.
3.4 Buckling and Post-buckling Analysis

3.4.1 Linear and Non-Linear Buckling Loads

Traditional buckling analysis as used for the solution of the classical textbook Euler-column can be formulated as an eigenvalue problem. The load for which the model stiffness matrix becomes singular is determined for

\[ \mathbf{K} \mathbf{v} = 0 \]  \hspace{1cm} (3-1)

in which \( \mathbf{K} \) is the tangent stiffness matrix of the structure when the loads are applied and \( \mathbf{v} \) are non-trivial displacement solutions. However, the same problem can be formulated differently as

\[ (\mathbf{K}_0 + \lambda_i \mathbf{K}_\Lambda) \mathbf{v}_i = 0 \]  \hspace{1cm} (3-2)

in which \( \mathbf{K}_0 \) is the stiffness matrix of a base state, \( \mathbf{K}_\Lambda \) is a differential stiffness matrix due to a differential loading pattern on top of the base state, \( \lambda_i \) is the scaling factor of this differential loading pattern and \( \mathbf{v}_i \) are the buckling mode shapes with \( i \) referring to the number of the eigenmode. If the base state is selected to be the load-free system or the result of a linear analysis, then \( \mathbf{K}_\Lambda \) will not contain any non-linear pre-buckling deformation and the solution will be the 'linear buckling load' for the system. If geometric non-linearities are taken into account during the computation of the base state and the base state is sufficiently close to buckling then \( \mathbf{K}_\Lambda \) will include all relevant non-linear pre-buckling deformations and the solution is called the 'non-linear buckling' load for the system (Figure 3-9). In both cases the critical buckling loads \( P_{cr} \) can be determined by

\[ P_{cr} = P_0 + \lambda_i Q \]  \hspace{1cm} (3-3)

where \( P_0 \) are the loads leading to the base state and \( Q \) are the loads with which the buckling analysis has been performed. Naturally, the lowest eigenvalue is of greatest interest as it determines the critical load of the system. Closely spaced bifurcation
points for different buckling modes generally indicate that the system is imperfection sensitive.

Figure 3-9. Linear and non-linear buckling.

Table 3-1 illustrates the concept of base states further. A series of buckling analyses was performed on a single-wave model for mode $m = 21$ and the eigenvalues of the structure were determined based on different base states along the pre-buckling loading path. It can be seen that the non-linear buckling stress differs considerably from the linear buckling stress. Non-linear buckling analyses were used for the computations in Chapter 4.

Table 3-1: Buckling loads according to different base states

<table>
<thead>
<tr>
<th>Load level at base state</th>
<th>Corresponding buckling load</th>
</tr>
</thead>
<tbody>
<tr>
<td>($\sigma / \sigma_{el}$)</td>
<td>($\sigma / \sigma_{el}$)</td>
</tr>
<tr>
<td>0.0</td>
<td>0.3228</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2815</td>
</tr>
<tr>
<td>0.25</td>
<td>0.2750</td>
</tr>
<tr>
<td>0.27</td>
<td>0.2731</td>
</tr>
</tbody>
</table>
3.4.2 Unstable Collapse and Post-Buckling Analysis

Upon reaching the critical load, thin-walled cylinders under axial load bifurcate into extremely unstable states of equilibrium. The initial post-buckling response is often characterised by negative stiffness and the load carrying capacity drops dramatically before a stable response can be observed again. The high concentration of bifurcation points for different bifurcation modes near the critical load also means that more than one possible post-buckling path is available within close proximity of the critical load. Secondary bifurcation or interaction between these modes is to be expected along the post-buckling branch of the load-displacement response. To solve post-buckling problems of such a severe nature, ABAQUS (Hibbit et al., 1998a) offers the ‘modified Riks method’ (Riks, 1979) which has been developed to produce solutions along the static equilibrium path in load-displacement space for such unstable cases. The load magnitude is used as an additional unknown in this method and loading is taken to be proportional to a set load vector. An artificial parameter, the arc-length, is introduced to measure the progress of the solution.

To continue an analysis using the ‘modified Riks method’ past a bifurcation point, it must be turned into a system with a continuous response. This can be achieved by either introducing small initial geometric imperfections (Figure 3-10 A) or by applying small load disturbances just before buckling which push the system into the post-buckling equilibrium (Figure 3-10 B). In the present case of a cylinder under axial load, a further complication exists due to the close proximity of bifurcation points at critical load. Each of these bifurcation points correlates to a different buckling mode and a separate post-buckling response. When the structure passes from the pre-buckling configuration into a post-buckling equilibrium state, this has to be achieved in a controlled way that ensures that the desired post-buckling path is reached. To determine the different bifurcation modes near the critical load, a non-linear buckling analysis needs to be carried out. For the actual post-buckling analysis, it has to be ensured that the structure’s initial post-buckling response follows the path that corresponds with the critical mode.
Figure 3-10. Changing into post-buckling equilibrium.

When initial geometric overall imperfections are introduced into the model, the kink at the bifurcation point is rounded out and the analysis proceeds asymptotically into the post-buckling path (Figure 3-10 A). The shape of the initial overall imperfections determines which post-buckling response the structure displays and the amplitude of the initial imperfection determines how much of the kink is rounded out. Therefore the choice of the shape and the amplitude of the initial imperfections is of critical importance. The greater the magnitude, the greater the rounding. This method is potentially problematic when mode interactions or secondary bifurcation take place close to the bifurcation point. As shown in Chapter 5 problems also occur when the initial system is changed due to initial imperfections (eg. from a cylinder into a three-dimensional shell).

Potential pitfalls are also present when lateral load disturbances are chosen to proceed past a bifurcation point into the post-buckling region (Figure 3-10 B). Great care must be taken that these lateral loads are sufficiently small and that they trigger the desired bifurcation mode by determining the lowest eigenmode and choosing trigger loads which push the structure into this mode.

Once past the initial bifurcation point, the system has to be monitored continuously because secondary bifurcation can occur and the process might have to be repeated (see Chapter 5).
3.5 Material Model

Buckling of thin-walled cylindrical shell structures made from steel typically occurs well within the elastic range. The modulus of elasticity was set to $E = 200\,000$ MPa for all buckling and post-buckling analyses in this thesis. As shown in Chapter 5 plasticity occurs when large displacements in the post-buckling range lead to large strains. Plasticity was taken into account by implementing an elastic-plastic material model with the onset of plasticity at a yield-stress $\sigma_{\text{yield}}$ given in the text where applicable. In some cases the effects of strain-hardening were taken into account by using a bi-linear material law where the stress-strain relationship was taken to be linear from zero to onset of yielding, and linear from the onset of yielding to the point where ultimate stress is reached at a plastic strain given in the text. Many mechanical properties of steel are highly temperature dependent and had to be taken into account for the simulation of weld-shrinkage in Chapter 2 where the exact data for these dependencies is also given.
3.6 Pre – Post Processing

To facilitate meshing of the imperfect cylinder, a FORTRAN pre-processing program was developed which creates a mesh of nodes on a cylinder with specified local imperfections. Predefined sections of an imperfect cylinder with various types of geometric imperfections can be readily generated by this program and can then be used by FE-packages such as ABAQUS. FE-meshes generated by this pre-processing program can include circumferential and meridional imperfections as well as dents in any combination. All shape functions mentioned in this thesis can be used for imperfection geometries.

The software package ABAQUS is a multi-purpose general FE-code which can be used for many different applications. Input files containing commands similar to a programming language drive ABAQUS analyses. To incorporate specific properties of an analysis problem, ABAQUS allows the user to import external procedures into the analysis. For the present work, a FORTRAN subroutine was written to prescribe residual stress fields near the weld and to investigate the effects of these stress-fields during buckling analyses. This subroutine was used to compare the stress patterns proposed by Dwight & Moxham (1969) and Häfner (1983) with another model proposed by Rotter (1996).

Another series of FORTRAN programs was developed to enable quick extraction and processing of results. These FORTRAN routines extract specified results from ABAQUS output files, track specified results during analyses that require several increments, and formats this data for further use in other applications like spreadsheet programs.

For the processing of the measured imperfection data (Ding 1992) and the development of a new shape function describing the circumferential weld imperfection a software package called GNU-plot was used. A series of programs to manage the large amounts of data and to fit the shape function to the measured data was developed.
4. Buckling under Axial Compression

4.1 Introduction

In Koiter’s original theory (Koiter 1945) and later again in his special theory (Koiter 1963), axisymmetric imperfections in the shape of indefinitely extensive sinusoidal waves were used among other imperfection patterns to determine the imperfection sensitivity of thin-walled cylindrical shells. The fact that axisymmetric imperfections lead to the most dramatic reductions in buckling strength under axial load has been known since then but did not receive much attention as practical applications of this imperfection geometry did not exist at the time. In the late 1960s and early 1970s the erection of ever larger silos and tanks led to a change in this attitude as these structures displayed significant axisymmetric imperfections at circumferential welds. Research by Tennyson & Muggeridge (1969), Hutchinson et al. (1971) and Amazingo & Budiansky (1972) investigated localised axisymmetric imperfections and demonstrated that a single axisymmetric imperfection can still have a significant effect on the buckling strength of thin-walled cylinders. Rotter & Teng (1989) and Teng & Rotter (1992) contributed more research on the topic of circumferential weld-induced imperfections. With the exception of Häfner (1982) only geometric imperfections in indefinitely long cylindrical shells were considered until Rotter (1996) did a limited study which included interaction between neighbouring welds and residual stresses for a small number of examples. This study (Rotter 1996) contradicted the conclusions reached by Häfner (1982) regarding the influence of residual stresses.

Three aspects of axisymmetric imperfections in shell-buckling problems have been investigated and the results will be presented in this chapter. Firstly, the effects of interaction between neighbouring weld imperfections on the buckling behaviour were modelled using the system proposed in Rotter (1996) and described in Chapter 3. Secondly, a parametric study taking into account weld-induced residual stresses was carried out and the results were used to document the influence of weld-induced
residual stress fields on the buckling behaviour. Finally, a study on the parameters affecting the weld imperfection shape was performed considering previously proposed shape functions, and putting these functions into context with the new shape function presented in Chapter 2.

FE-models representing sectors of the full cylinder were used and it was assumed that the strength of the shell structure is limited by bifurcation into a non-axisymmetric mode. Details of the FE-model are given in Chapter 3. Elastic-plastic material properties were considered but buckling always took place at stress levels well below the yield level for the considered examples. A preliminary study was performed to determine the optimal element mesh for each of the studies. Forty elements in the circumferential direction and seventy elements in the axial direction were used with an especially fine mesh near the weld to accommodate the modelling of residual stresses and to accurately represent variations of shape functions in that area.
4.2 Interaction Between Neighbouring Weld Imperfections

4.2.1 Introduction

Strake heights in silos and tanks are dictated by the size of plates commercially available. This typically leads to strake heights which are too small to isolate the effects of neighbouring circumferential weld imperfections from each other. A simple but efficient modelling technique to account for this interaction was proposed by Rotter (1996). The case example given in this particular paper is based on the same silo at Port Kembla, Australia, which was also the basis for the measurements documented by Ding (1992) and used in Chapter 2. Rotter (1996) gives three combinations of boundary conditions for a model encompassing one weld imperfection with half a strake height above and half a strake height below the weld: symmetry on both sides (S-S); anti-symmetry on both sides (A-A); and a mixture of the two (A-S). The results given in Rotter (1996) and Table 4-1 for an imperfection amplitude of \( \lambda = 1.0 \) (shape function “Rotter best”), \( R = 12000 \text{ mm} \), \( R/t = 1000 \) and half strake height \( L = 1500 \text{ mm} \) indicate that the S-S boundary condition yields the highest buckling values of the three conditions and the A-A boundary condition the lowest. The results given in Table 4-1 were originally given by Rotter (1996) and were reproduced using ABAQUS (1997) and the modelling techniques described in Chapter 3. The buckling stress in Table 4-1 and throughout this chapter is given as a ratio of the classical buckling stress \( \sigma_{cl} \) (Equation 1-1).
Table 4-1. Buckling strengths for different boundary conditions after Rotter (1996)

<table>
<thead>
<tr>
<th>Assumption</th>
<th>$\sigma / \sigma_{cl}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amazingo &amp; Budianski (1972)</td>
<td>0.320</td>
</tr>
<tr>
<td>Long cylinder with single weld</td>
<td></td>
</tr>
<tr>
<td>Rotter &amp; Teng (1989)</td>
<td>0.305</td>
</tr>
<tr>
<td>Long cylinder, single weld type ‘A’</td>
<td></td>
</tr>
<tr>
<td>Rotter &amp; Teng (1989)</td>
<td>0.363</td>
</tr>
<tr>
<td>Long cylinder, single weld type ‘B’</td>
<td></td>
</tr>
<tr>
<td>Boundary condition S-S,</td>
<td>0.429</td>
</tr>
<tr>
<td>Half strake height L = 1500 mm</td>
<td></td>
</tr>
<tr>
<td>Boundary condition S-A,</td>
<td>0.260</td>
</tr>
<tr>
<td>Half strake height L = 1500 mm</td>
<td></td>
</tr>
<tr>
<td>Boundary condition A-A,</td>
<td>0.250</td>
</tr>
<tr>
<td>Half strake height L = 1500 mm</td>
<td></td>
</tr>
</tbody>
</table>

4.2.2 Comparison between the A-A and the S-S condition

The A-A, A-S and S-S conditions have been described in Chapter 3 (Figure 3-4). Boundary condition A-S lies between the two extremes and doesn’t result in symmetry at the weld – a condition observed many times in laboratory tests and failures in practice. The A-S condition was therefore not considered any further. A comparison of the A-A condition with the S-S condition over a range of strake heights and for a cylinder with an $R/t$-ratio of 1000 is shown in Figure 4-1. The cylinder used to compute the results for Table 4-1 is marked as “Reference silo” in Figure 4-1 and has the same geometric properties as the silo in Port Kembla, Australia, which was surveyed by Ding (1992) and on which much subsequent research was performed (eg. Rotter 1996) including the derivations of the weld shape performed in Chapter 2.
Clearly the A-A condition leads to significantly lower buckling values than the S-S condition over the whole range of strake heights. The nature of the boundary conditions becomes insignificant for half strake heights $L > 6000$ mm where both solutions equal the result for the indefinitely long cylinder. The buckling strength for the A-A boundary condition reaches a minimum between $L = 1500$ mm and $L = 2000$ mm. Remarkably, this is the region where most of the strakes of the reference silo at Port Kembla are situated. In the following only the A-A boundary condition was considered.

4.2.3 Various $R/t$-ratios

A series of buckling analyses of FE-models for four different $R/t$-ratios ($R/t = 500$, $1000$, $1500$, $2000$) was performed over a large range of strake heights. The shape of the imperfection was modelled after equation 2-17 with parameters $\lambda/\lambda_0 = 1.73$ and $\zeta = 1.0$. Results for the analyses for an imperfection amplitude of $w_0/t = 1.0$ are given in Figure 4-2.
Figure 4-2. Buckling strength for various $R/t$-ratios and half strake heights $L$.

A perfect thin-walled cylinder buckles in a chess-board type of pattern and the A-A condition leads to similar buckling patterns (see Figure 3-4). It comes as no surprise that this particular boundary condition results in a minimum in buckling strength where the buckling pattern induced by the A-A boundary condition comes closest to the chess-board pattern of the perfect cylinder. When the half strake height becomes shorter than $L_{\text{min}}$, the half strake height where the minimum buckling resistance is observed, the structure is forced to buckle into modes with more and more circumferential buckles and the buckling strength rises again. However, it can be safely assumed that once it rises above a certain limit, the A-A boundary condition pattern will be replaced by different patterns. Half strake heights longer than $L_{\text{min}}$ also cause an increase in buckling resistance, eventually reaching the buckling strength of the “long cylinder” where boundary conditions cease to have an influence on the buckling strength of the structure.

For any thin-walled cylinder bifurcation modes in the vicinity of the critical mode have buckling strengths very close to the critical buckling strength. For long cylinders
however, more bifurcation modes in close vicinity of the critical mode were generally observed than for shorter cylinders of equal imperfection characteristics and slenderness. Figure 4-3 illustrates this observation comparing buckling loads for bifurcation modes for a cylinder with $R/t = 1500$ and $L/\lambda_0 = 9.24$ (long) and 1.06 (short) respectively. Both these cylinders buckle at roughly the same load of 0.32 $\sigma_d$ (Figure 4-2) but the long cylinder has almost twice as many circumferential buckles with buckling loads less than 0.35 $\sigma_d$.

![Figure 4-3. Buckling strengths for various bifurcation modes.]

In Figure 4-4 the same results as in Figure 4-2 are plotted again, this time normalised against the elastic meridional half-bending wave length $\lambda_0$ (Equation 2-2). Interestingly, all results were found to fall into one single curve giving the variations of buckling strength ($\sigma/\sigma_d$) with normalised half strake height $L/\lambda_0$ for cylinders with constant $w_0/t = 1.0$. 

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Figure 4-4. Buckling strength for various $R/t$-ratios plotted for normalised half strake heights $L/\lambda_0$.

In another series of bifurcation analyses, other imperfection amplitudes where introduced into the FE-model. Results for imperfection amplitudes of $w_0/t = 0.5$, 1.0 and 1.5 are given in Figure 4-5. The same results were redrawn in Figure 4-6, normalised against the buckling strength of the long cylinder $\sigma_{long}$. Figure 4-7 gives the buckling strength for imperfection amplitudes $w_0/t < 4.0$ for long cylinders. The combination of Figure 4-6 and Figure 4-7 can be used to determine the buckling strength for a wide range of half strake heights $L$ and imperfection amplitudes $w_0/t$ provided the imperfections are of the particular shape chosen for the computations. The difference in the choice of the shape function explains the slight difference between the diagram given in Figure 4-7 and a similar diagram given in Rotter & Teng (1989). However, the significance of the particular shape function chosen for the computations in this chapter will be explained at a later stage in this chapter.
Figure 4-5. Buckling strength for various imperfection amplitudes $w_0/t$ and half strake heights $L$. 
Figure 4-6. Buckling strength for various imperfection amplitudes $w_0/t$ and half strake heights $L$ in relation to $\sigma_{long}$.

Figure 4-7. Variation in buckling strength for different imperfection amplitudes.
4.3 Weld-Induced Residual Stresses

4.3.1 Comparison of Modeling Approaches

Two papers are known to have discussed the effects of residual stresses at circumferential welds on the buckling behaviour of thin-walled cylindrical shell structures (Häfner 1982, Rotter 1996). Ravn-Jensen & Tvergaard (1987) investigated this effect in longitudinal welds. These researchers applied residual stresses on to their numerical models in two ways: Häfner (1982) and Ravn-Jensen & Tvergaard (1987) postulated residual stress fields and Rotter (1996) applied a strain field which subsequently leads to a residual stress field. Ravn-Jensen & Tvergaard (1987) based their stress field on measurements by White & Dwight (1977) and Häfner (1982) postulated two functions describing such a stress field. When a stress field rather than a strain field is stored into the initial FE-model the difficulty lies in achieving a starting model which is in equilibrium and which encompasses all stress components necessary to establish equilibrium in a three-dimensional structure. Häfner (1982) has been criticised by Holst et al (1996) for not abiding by this rule. However, once the main stress component (the circumferential membrane stress) is stored into the numerical model, the other stress components can be found numerically before starting the actual buckling analysis at the price of slightly distorting the original distribution of the circumferential stress components. This approach was adopted in the following when the individual residual stress models were compared with each other for a cylinder with \( R/t = 1000 \), a half strake height \( L = 1.62 \lambda_0 \) and an imperfection amplitude \( w_0/t = 1.0 \). Figure 4-8 shows the buckling strengths for the different models and compares them with the initially stress free model, the rightmost bar in the diagram. All these computations confirm Rotter's (1996) findings showing an increase, to different degrees, in the buckling strength due to the presence of a residual stress field. In the following only one method will be used to analyse the influence of residual stresses on the buckling behaviour, namely the method proposed by Rotter (1996) and described in Chapter 3. The method actually models the process which leads to the presence of residual stresses and results in stress patterns consistent with the displaced shape.
4.3.2 Boundary Conditions

The strengthening effect of residual stresses is not confined to the A-A boundary condition. Increases were also noted for FE-models using the S-S boundary condition, this time using the imperfection shape “Rotter closed” (Table 4-2).

Table 4-2. Buckling strengths and modes for S-S and A-A boundary conditions for $\omega_{clt} = 1.0$.

<table>
<thead>
<tr>
<th></th>
<th>S-S</th>
<th>A-A</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma / \sigma_{cl}$ - initially stress-free</td>
<td>0.452</td>
<td>0.245</td>
</tr>
<tr>
<td>mode - initially stress-free</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>$\sigma / \sigma_{cl}$ - incl. res. stresses</td>
<td>0.467</td>
<td>0.273</td>
</tr>
<tr>
<td>mode - incl. res. stresses</td>
<td>22</td>
<td>21</td>
</tr>
</tbody>
</table>
4.3.3 *Explanation for the Strengthening Effect of Residual Stresses*

Circumferential membrane stresses are by far the most influential component of the stress field induced during the welding process and the development of these stresses during the application of axial loading is shown along one meridian in Figure 4-9. As axial load is applied on the initially stress-free model, circumferential compressive stresses develop near the centre of the weld thus creating an area of two-dimensional compression. When the applied axial load approaches the bifurcation load, infinitesimally small buckles start to form which subsequently cause buckling of the structure. Circumferential compressive stresses accelerate the development of these pre-buckling deformations as is shown in Figure 4-10 (a) (Esslinger 1967).

![Graph showing the effect of residual stresses on tension and compression](image)

**Figure 4-9. Circumferential membrane stresses**

In a welded cylinder, circumferential residual stresses reach yield-level in tension at the centre of the weld and even at the point of buckling under axial stress this area is still under considerable tension (Figure 4-9). Compressive residual stresses further away from the weld typically range from 0.2 to 0.4 of the yield stress and are further increased by axial compression. The stabilising effect of the tension stresses near the
weld is illustrated in Figure 4-10 (b). With the increase in axial load the tensile stresses decrease and the compressive stresses further away from the weld are further increased until a point is reached where the structure buckles. The second contributing factor is the presence of axial stresses and the resulting bending moments. These bending moments can be expected to become relatively more influential in more thick-walled shells and erode the stabilising effect of the circumferential tensile membrane stresses.

![Diagram](image)

(a) compressive circumferential stresses

(b) tensile circumferential stresses

Figure 4-10. Effect of circumferential residual stresses

4.3.4 Varying the Stress Field

Measurements of residual stresses in cylinders are rare and only available for rather thick-walled structures. It can be expected that the nature of the residual stress field varies considerably depending on welding techniques, cylinder geometry and environmental factors. Some parameters of residual stress fields were implemented in the FE-model and studied.

The width of the tensile zone \( w_t \) was varied and buckling analyses performed. Figure 4-11 illustrates the results of this study for cylinders with four different \( R/t \)-ratios \((R/t = 500, 1000, 1500, 2000)\), an imperfection amplitude of \( w_0/t = 1.0 \) and a half strake height of \( L = 1.62 \lambda_0 \). Clearly, weld-induced residual stresses lead to a much greater increase of buckling strength \( \Delta \sigma \) for the more thin-walled cylinders. In all four cylinder geometries however, this trend of increase is reversed for tension fields wider than a
certain maximum width. However, in practical applications the width of the tensile zone \(w_t\) is typically between three and six wall thicknesses wide and for all studied cylinder geometry this leads to an increase in buckling strength.

![Graph showing the increase in buckling strength through residual stress fields of various widths for selected \(R/t\)-ratios](image)

**Figure 4-11. Increase in buckling strength through residual stress fields of various widths for selected \(R/t\)-ratios**

Next the intensity of the stress field was varied. As the stress field is a result of the residual strains applied before the buckling analysis the intensity of the stress field was regulated by varying the intensity of these initial residual strains. Strains were applied with 0.25, 0.50 and 1.00 times the strain \(\varepsilon_{\text{yield}}\) at the assumed yield stress of 250MPa. The width of the tensile zone was six wall thicknesses \((w_t = 6t)\). Figure 4-12 shows the resulting circumferential stresses along one meridian. Buckling strengths of a cylinder with \(R/t = 1000\), an imperfection amplitude \(w_0/t = 1.0\) and a half strake height of \(L = 1.62 \lambda_0\) were determined. The stresses at bifurcation in relation to the classical buckling stress \(\sigma_{cl}\) are listed in Table 4-3 and hold no surprises – the buckling strength was found to increase as the intensity of residual stresses was increased.
Table 4-3. Buckling strengths for different levels of residual stress.

<table>
<thead>
<tr>
<th>$\varepsilon / \varepsilon_{\text{yield}}$</th>
<th>$\sigma / \sigma_{\text{cI}}$</th>
<th>$\Delta \sigma_{\text{cI}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.245</td>
<td>0</td>
</tr>
<tr>
<td>0.25</td>
<td>0.26</td>
<td>0.015</td>
</tr>
<tr>
<td>0.5</td>
<td>0.273</td>
<td>0.028</td>
</tr>
<tr>
<td>1</td>
<td>0.275</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Figure 4-12. Circumferential residual stresses of different magnitudes.

Residual tensile stresses at the weld typically reach yield level. The yield stress of the weld metal which is deposited between two adjoining plates is usually higher than the yield stress of the plates. This consequently leads to residual stress peaks at the weld. This effect was taken into account during a series of analyses where material properties where set in such a way that residual stresses locally at the weld reached up to four times the yield stress of the surrounding structure. The width of this strip was varied from extremely thin to about one wall thickness. The buckling strength of these FE-models was found to be practically unaffected by these variations.
4.3.5 Different R/t-Ratios

The two competing effects of residual stresses – the strengthening tension band of the circumferential residual membrane stresses on the one hand, and the weakening effects of axial bending caused by residual stresses in the axial direction on the other hand - can be expected to lead to different degrees of influence on the buckling strength for cylinders with different R/t-ratios. An extensive study was undertaken to investigate this influence including cylinders with R/t-ratios between 500 and 2000. The half strake height was kept constant to \( L = 1500 \text{ mm} \) and the imperfection amplitude to \( w_0/t = 1.0 \).

Figure 4-17 to Figure 4-32 show the buckling strengths for these FE-models and for bifurcation modes in the vicinity of the critical mode for two scenarios: firstly for a FE-model of a cylinder including a residual stress field and secondly for a cylinder of exactly the same geometry but initially stress-free. The bifurcation stress for the critical modes is plotted again in Figure 4-13. The lower curve in Figure 4-13 seems to contradict the curve given in Rotter & Teng (1989) at first sight. However, this is not the case as the curves given in Figure 4-13 also include the effects of the chosen half strake height \( L = 1500\text{mm} \). This influence can be filtered out quite easily using the diagram given in Figure 4-6 resulting in a curve for the initially stress-free long cylinder. Buckling strengths (\( \sigma/\sigma_{cl} \)) for the “long cylinder” proved to be constant for all R/t-ratios and almost exactly matched the diagram given in Rotter & Teng (1989). The remaining small differences could be explained by the fact that a different shape function for the imperfection geometry was used. Finally, the difference in buckling strength between an initially stress-free cylinder and a model of a cylinder including the effects of weld-induced residual stresses is given in Figure 4-14. The maximum gain for a cylinder with R/t = 2000 is shown to be just under 7% of \( \sigma_{cl} \). This is actually a considerable increase since the buckling strength of the stress-free cylinder was found to be at just over 25% of \( \sigma_{cl} \).
Figure 4-13. Bifurcation stress for critical modes for different $R/t$-ratios

Figure 4-14. Strength gain through weld-induced residual stresses for different $R/t$-ratios.
Figure 4-17 to Figure 4-32 give detailed results for a number of bifurcation modes beside the critical mode for each $R/t$-ratio considered in this investigation. The number of bifurcation modes in close proximity to the critical mode increases considerably for higher values of $R/t$. It can also be seen that the difference in strength between initially stress-free models and models including residual stresses becomes slightly smaller for bifurcation modes less than the critical mode but this difference increases slightly for modes greater than the critical mode. The same results are re-arranged to produce the diagrams given in Figure 4-15 and Figure 4-16. These diagrams give an indication how the curves for given bifurcation-modes flatten out for increased values of $R/t$ to result in clusters of buckling modes in the close vicinity of the critical mode.

![Diagram showing buckling stresses for various bifurcation modes for initially stress-free cylinders.](image)

**Figure 4-15.** Buckling stresses for various bifurcation modes for initially stress-free cylinders.
Figure 4-16. Buckling stresses for various bifurcation modes including the effects of weld-induced residual stresses.
Figure 4-17. Buckling stress for a cylinder with $R/t = 500$

Figure 4-18. Buckling stress for a cylinder with $R/t = 600$
Figure 4-19. Buckling stress for a cylinder with $R/t = 700$

$R/t = 700$
$m_{cr} = 18$
difference at $m_{cr} = 1.9\%$ of $\sigma_{cl}$

Figure 4-20. Buckling stress for a cylinder with $R/t = 800$

$R/t = 800$
$m_{cr} = 19$
difference at $m_{cr} = 2.1\%$ of $\sigma_{cl}$
Figure 4-21. Buckling stress for a cylinder with $R/t = 900$

Figure 4-22. Buckling stress for a cylinder with $R/t = 1000$
Figure 4-23. Buckling stress for a cylinder with $R/t = 1100$.

Figure 4-24. Buckling stress for a cylinder with $R/t = 1200$. 

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**Figure 4-25.** Buckling stress for a cylinder with \( R/t = 1300 \)

\[
R/t = 1300 \\
m_{cr} = 23 \\
difference at \ m_{cr} = 3.8\% \ of \ \sigma_{cl}
\]

- Initially stress-free
- Including residual stresses

**Figure 4-26.** Buckling stress for a cylinder with \( R/t = 1400 \)

\[
R/t = 1400 \\
m_{cr} = 24 \\
difference at \ m_{cr} = 4.2\% \ of \ \sigma_{cl}
\]

- Initially stress-free
- Including residual stresses
R/t = 1500
m_cr = 25
difference at m_cr = 4.7% of $\sigma_{cl}$

$\sigma / \sigma_{cl}$

![Graph](image)

Mode

19 20 21 22 23 24 25 26 27 28 29 30 31 32

Figure 4-27. Buckling stress for a cylinder with R/t = 1500

R/t = 1600
m_cr = 25
difference at m_cr = 4.9% of $\sigma_{cl}$

$\sigma / \sigma_{cl}$

![Graph](image)

Mode

19 20 21 22 23 24 25 26 27 28 29 30 31 32

Figure 4-28. Buckling stress for a cylinder with R/t = 1600

125
Figure 4-29. Buckling stress for a cylinder with $R/t = 1700$

Figure 4-30. Buckling stress for a cylinder with $R/t = 1800$
Figure 4-31. Buckling stress for a cylinder with $R/t = 1900$

Figure 4-32. Buckling stress for a cylinder with $R/t = 2000$
4.3.6 Imperfection Amplitude

The influence of the imperfection amplitude in combination with weld-induced circumferential residual stress fields was investigated for cylinders of a half strake height of \( L = 1500 \text{mm} \), \( R = 12000 \text{mm} \) and various \( R/t \)-ratios (500, 1000, 1500 and 2000). The results for the four different cylinder geometries are shown in Figure 4-33 to Figure 4-36. The strengthening effect of the residual stress field is much greater for the more thin-walled shells (larger \( R/t \)-ratios). Also, the strengthening effect is greater for shells with smaller circumferential imperfections (small imperfection amplitudes \( w_0/t \)). The strengthening effect for the four different shells is plotted in Figure 4-37. The greatest gain of almost 10% of \( \sigma_e \) was recorded for small imperfection amplitudes in the most thin-walled shell investigated (\( R/t = 2000 \)). Again this represents quite a remarkable increase in the buckling strength for shells of such a high slenderness.

![Diagram](image)

Figure 4-33. Buckling strength for \( R/t = 2000 \).
Figure 4-34. Buckling strength for $R/t = 1500$.

Figure 4-35. Buckling strength for $R/t = 1000$. 
Figure 4-36. Buckling strength for \( R/t = 500 \).

Figure 4-37. Influence of residual stresses for various imperfection amplitudes \( w_0/t \).
4.4 Imperfection shape and Buckling Strength

4.4.1 Parameter Study of the new Shape Function

A shape function based on elastic shell theory was introduced in Chapter 2 and calibrated against measurements gathered by Ding (1992). A parameter study was performed to investigate the influence of the two key parameters of this function on the buckling strength of thin-walled cylinders with a circumferential weld-induced imperfection. The two parameters in question are: the wave length of the imperfection $\lambda$ (Figure 4-39); and the parameter $\zeta$ which indicates the degree of ‘roundness’ at the centre of the weld ($\zeta = 0$ for a “pointy” imperfection and $\zeta = 1$ for a round shape, see Figure 4-38). The study was performed at first for cylinders replicating the upper four strakes of the reference silo on which the measurements (Ding 1992) referred to in Chapter 2 were taken ($R = 12000$mm, $R/t = 1000$, $L = 1500 = 1.62 \lambda_0$). The magnitude of the imperfection amplitude was kept at a constant value of $w_0/t = 1.0$. In a second step the same study was performed for long cylinders with $R/t = 1000$. The parameter $\zeta$ was kept constant for $\zeta = 0, 0.25, 0.5, 0.75$ and 1.0 respectively and the wave length was varied and plotted to give the diagram in Figure 4-40. All curves intercept at one point for $\lambda = 1.73\lambda_0$. Before this point, buckling strengths for $\zeta$ close to zero are considerably higher than for shapes with $\zeta$ close to one. However, for values of $\lambda > 1.73\lambda_0$ this is reversed and shape functions with a rather pointed shape at the weld were shown to be weaker than more rounded shapes. To compare these results with the results given in Teng & Rotter (1992), the same series of analyses was performed again for a “long cylinder”. Results for this second investigation are given in Figure 4-41. Again all curves intercept in one point, this time at $\lambda = 1.85\lambda_0$. Results for the influence of the wave length $\lambda$ are also given in Teng & Rotter (1992) where parameter $\zeta$ was kept constant to represent shape function ‘Rotter & Teng A’. The curve in Figure 4-41 for $\zeta = 1$ matches the corresponding curve in Rotter & Teng (1992). The differences between the two extremes ($\zeta = 1$ and $\zeta = 0$) are of considerable magnitude. For wave lengths beyond the intersection point the buckling strength for $\zeta = 1$ lies 50% above the buckling strength for $\zeta = 0$. There is also a distinct minimum between $\lambda = 1.0\lambda_0$ and $\lambda = 2.0\lambda_0$ for all curves.
Figure 4-38. Variations of $\zeta$ for $\lambda = \lambda_0$ and $w_0 = t$.

Figure 4-39. Variations of $\lambda$ for $\zeta = 0.5$ and $w_0 = t$. 

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Figure 4-40. Buckling strength for variations of $\lambda$ and $\zeta$ for a short cylinder.

Figure 4-41. Buckling strength for variations of $\lambda$ and $\zeta$ for a long cylinder.
Curves for constant values of $\lambda$ for variations of $\zeta$ are given in Figure 4-42 for the short cylinder and in Figure 4-43 for the long cylinder. Both diagrams clearly show a nearly linear relationship between the buckling strength and values of $\zeta$. Linear interpolation between buckling strengths for the two extreme values of $\zeta = 0$ and $\zeta = 1$ is therefore appropriate.

The influence of the shape of a circumferential weld-induced imperfection on the buckling strength of a thin-walled cylindrical shell structure is significant. Three parameters determine the buckling strength under axial load: the imperfection amplitude $w_0$; the wave length of the imperfection $\lambda$; and the degree of roundness at the centre of the imperfection $\zeta$. In order to correlate imperfection shapes in existing structures with these three parameters, more measurements must be gathered for structures with a range of sizes and wall-thicknesses. The influence of different welding techniques and environmental factors also deserve more research in the future. Once enough measurements have been gathered and the influence of all these parameters on the imperfection shape is clear, the diagrams given earlier in this section can be used to determine the buckling strength for each individual case.
Figure 4-42. Buckling strength for variations of $\xi$ for a short cylinder.

Figure 4-43. Buckling strength for variations of $\xi$ for a long cylinder.
4.4.2 Buckling Strength for other Shape Functions

Chapter 2 gives an extensive overview on shape functions proposed by other researchers. The buckling strengths for these functions for a short cylinder of \( R = 12000 \text{ mm}, \frac{R}{t} = 1000 \) and \( L = 1500 \text{ mm} \) were determined and are given in Table 4-4 for imperfection amplitudes of \( \frac{w_0}{t} = 1.0 \). Values of buckling strength were found to cover a wide range from \( \frac{\sigma}{\sigma_{cl}} = 0.238 \) to 0.470.

Table 4-4. Buckling strength for existing shape functions.

<table>
<thead>
<tr>
<th>Shape function</th>
<th>( \frac{\sigma}{\sigma_{cl}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotter &amp; Teng B</td>
<td>0.302</td>
</tr>
<tr>
<td>Rotter &amp; Teng A</td>
<td>0.247</td>
</tr>
<tr>
<td>Rotter final</td>
<td>0.295</td>
</tr>
<tr>
<td>Steinhardt &amp; Schulz</td>
<td>0.470</td>
</tr>
<tr>
<td>Häflner A</td>
<td>0.308</td>
</tr>
<tr>
<td>Häflner B</td>
<td>0.318</td>
</tr>
<tr>
<td>Tennyson &amp; Muggeridge</td>
<td>0.238</td>
</tr>
<tr>
<td>Rotter closed</td>
<td>0.251</td>
</tr>
<tr>
<td>Rotter open</td>
<td>0.297</td>
</tr>
<tr>
<td>Rotter best</td>
<td>0.252</td>
</tr>
</tbody>
</table>

These imperfection shapes were then approximated by the new shape function introduced in Chapter 2 using a least square fit. The results of this approximation (values for \( \frac{\lambda}{\lambda_0}, \zeta \) and \( \frac{w_0}{t} \)) are given in Table 4-5 and plotted diagrams showing points along the original functions and the least square fit approximations can be seen in Figure 4-46 to Figure 4-53. These results indicate a rather wide spread of wave lengths \( \frac{\lambda}{\lambda_0} \). Imperfection amplitudes varied slightly but were found to be in close vicinity of \( \frac{w_0}{t} = 1.0 \). In the light of the parametric investigation of measured imperfections given in Chapter 2, all approximations were found to be within reasonable limits except shape function ‘Rotter open’ which displayed a wave length of \( \lambda = 3.32 \lambda_0 \) which seemed rather excessive.
Table 4-5. Result of a least square fit approximation of existing shape functions.

<table>
<thead>
<tr>
<th>Shape function</th>
<th>$\lambda / \lambda_0$</th>
<th>$\zeta$</th>
<th>$w_0 / t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotter &amp; Teng B</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Rotter &amp; Teng A</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Rotter final</td>
<td>2.4</td>
<td>0</td>
<td>0.96</td>
</tr>
<tr>
<td>Steinhardt &amp; Schulz</td>
<td>0.22</td>
<td>1</td>
<td>1.01</td>
</tr>
<tr>
<td>Häfner A</td>
<td>0.53</td>
<td>1</td>
<td>1.05</td>
</tr>
<tr>
<td>Häfner B</td>
<td>0.63</td>
<td>0.42</td>
<td>0.94</td>
</tr>
<tr>
<td>Tennyson &amp; Muggeridge</td>
<td>1.39</td>
<td>1</td>
<td>1.02</td>
</tr>
<tr>
<td>Rotter closed</td>
<td>1.25</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>Rotter open</td>
<td>3.32</td>
<td>1</td>
<td>1.11</td>
</tr>
<tr>
<td>Rotter best</td>
<td>1.53</td>
<td>1</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Buckling strengths for the new shape function were computed using the parameters found by the approximation (Table 4-5). A comparison between the results for the approximated functions with the results for the original functions (Figure 4-44) revealed that buckling strengths for both versions are either identical or extremely close. Not only can the geometry of existing shape functions be approximated very well by the new shape function, this new function also led to near-identical results in buckling analyses. Finally, buckling strengths for short cylinders with the approximated shape functions were re-computed for $w_0 / t = 1.0$ and the results are shown in Figure 4-45. These results confirm the conclusion that the shape function given in Chapter 2 is an excellent tool to describe all relevant parameters of a localised weld-induced circumferential imperfection. The results presented in Chapter 2 showed the ability of the function to adequately represent the geometry of such an imperfection. The results in this chapter demonstrate that the two main features, other than depth $w_0 / t$, of a weld imperfection that influence the buckling behaviour of thin-walled cylinders under axial load are the wavelength of the imperfection $\lambda$ and the roundness of the shape of the weld $\zeta$. 

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Figure 4-44. Buckling strengths for various shape functions and their approximations through the new shape function.

Figure 4-45. Existing shape functions in the context of the new shape function.
Figure 4-46. Approximation of Shape function ‘Rotter final’.

Figure 4-47. Approximation of Shape function ‘Steinhardt & Schulz’.
Figure 4-48. Approximation of Shape function ‘Häfner A’. 

Figure 4-49. Approximation of Shape function ‘Häfner B’.
Figure 4-50. Approximation of Shape function ‘Tennison & Muggeridge’.

Figure 4-51. Approximation of Shape function ‘Rotter closed’.
Figure 4-52. Approximation of Shape function ‘Rotter open’.

Figure 4-53. Approximation of Shape function ‘Rotter best’.
4.5 Shape of the Buckling Mode

The buckled shapes for various cylinder and imperfection geometries were collected and compared. Displacements around the circumference at the weld and along meridians for maximum inwards and outwards displacements were recorded. The displacements at the circumference were found to follow the shape of a sine-wave for all considered cases. The displaced shapes of the observed meridians were the same for inwards and outwards buckles, therefore only the meridian at the peak of the outwards buckle will be shown. The amplitude of outwards buckles was found to be slightly greater than the amplitude of inwards buckles.

Buckled shapes for cylinders with variations in imperfection amplitudes were investigated first. Three amplitudes \((w_0/t = 0.1, 1.0\) and \(4.0\)) were considered for a long cylinder of \(R/t = 1000\) and a short cylinder with a half strake height \(L = 1.62 \lambda_0\). While variations in imperfection amplitude barely changed the buckled shape for the short cylinder (Figure 4-54), significant changes were observed for the long cylinder (Figure 4-55).

The influence of the weld shape parameters was investigated next. For the buckled meridians shown in Figure 4-56, the wave length \(\lambda\) was kept constant at \(\lambda = 1.73 \lambda_0\) and the imperfection amplitude kept constant at \(w_0/t = 1.0\). Buckled shapes for the long and the short cylinder are given for \(\xi = 0.0, 0.5\) and \(1.0\) respectively. Again, differences between the long and the short cylinder are significant whereas variations in \(\xi\) only have a small influence. Meridional buckling mode shapes for variations in the wave length \(\lambda\) are shown in Figure 4-57 for imperfections with a constant \(\xi = 0.5\) and \(w_0/t = 1.0\), illustrating that changes in the wave length trigger only small differences in buckled shapes for both the long and the short cylinder.

Finally the buckling mode shapes for cylinders with various \(R/t\)-ratios are given in Figure 4-58 for \(\lambda = 1.73 \lambda_0\) and \(w_0/t = 1.0\). These confirm the results by Rotter & Teng (1989) for the long cylinder. Interestingly the short cylinder displayed greater changes in buckling mode shapes.
Figure 4-54. Meridional buckling modes for a short cylinder and variations of $w_0/t$.

Figure 4-55. Meridional buckling modes for a long cylinder and variations of $w_0/t$. 

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Figure 4-56. Meridional buckling modes for variations of $\zeta$, $\lambda = 1.73$.

Figure 4-57. Meridional buckling modes for variations of $\lambda$, $\zeta = 0.5$
Figure 4-58. Meridional buckling modes for variations of $R/t$. 
4.6 Conclusions

Some aspects regarding the buckling of thin-walled cylindrical shell structures with circumferential weld-induced imperfections have been investigated and results have been documented in this chapter. The following conclusions can be drawn:

Interaction between neighbouring imperfections takes place. Diagrams have been given to determine the weakening effect of this interaction depending on imperfection amplitude, $R/t$-ratio and strake height.

Residual stresses are invariably introduced during the welding process and have a strengthening effect on the buckling behaviour. An explanation for this effect has been given and diagrams have been provided to determine the degree of this strengthening effect taking into account variations in $R/t$-ratio and imperfection amplitude. The strength gain is greater for more slender structures and also for smaller imperfection amplitudes.

The shape of the imperfection has been found to play an important role in the buckling behaviour. The shape function introduced in Chapter 2 has been shown to be a reliable tool to explain the influence of the shape of the weld imperfection on the buckling behaviour. The roundness of the imperfection at the weld-centre and the wave length of the imperfection have been shown to be the governing shape parameters.

Examples have been given to demonstrate the influence of all these factors on the buckling mode shape. Only small differences in the shape of the buckled structures could be detected for short cylinders. However, significant changes in buckling mode shape with variations in the imperfection amplitude were recorded for long cylinders.
5. Post Buckling Behaviour

5.1 Introduction

The post-buckling behaviour of thin-walled cylinders under axial load has been the subject of large amounts of research – theoretical and experimental – since the early work summed up comprehensively in Hutchinson & Koiter (1970). Experimental documentation for the severely unstable post-buckling behaviour by Yamaki (1984) and Esslinger & Geier (1972) gave detailed insights and Esslinger (1967) offered a simple yet highly descriptive theoretical explanation for the initial post-buckling process. Increased computer capacity during the past two decades has also opened possibilities to study the post-buckling of cylinders numerically leading to interesting results for a wide range of problems which had been inaccessible before (eg. Guggenberger 1995, Goto & Zhang 1999). A review of recent advances is given in Teng (1996).

The post-buckling behaviour of cylinders with axisymmetric imperfections was the subject of the classic theoretical works by Koiter (1945, 1963). Rotter & Teng (1989) and Teng & Rotter (1992) used a numerical approach to determine bifurcation loads of cylinders with axisymmetric weld-induced imperfections. Teng & Song (2000) raised a number of issues connected with the use of three-dimensional finite elements in post-buckling analyses and this chapter will highlight the importance of some of these issues for the cylinder with circumferential weld imperfections. The post-buckling behaviour of the studied thin-walled cylinders under axial load is highly unstable. This chapter concentrates on giving a qualitative model for the post-buckling load-carrying mechanism and examining the influence that some parameters studied in the earlier chapters of this thesis have on this mechanism. A brief comparison between different modeling approaches will be given, namely a comparison between the results gained by sector-models as opposed to the results gained by half-structure models. The influence of geometric overall imperfections in addition to the localised circumferential imperfection will be considered and documented.
5.2 Sector Model

The post-buckling analyses of models covering one buckling wave were used to investigate the post-buckling behaviour of a cylinder with a circumferential weld-induced imperfection. The overall response of the full structure was then pieced together utilising the post-buckling response of the wave models and following the load-deformation path corresponding to a minimum of total energy of the system. This technique has been used by other researchers in the past (e.g. Guggenberger 1995) and is believed to be an efficient way to analyse the post-buckling of structures with sectorial symmetries in the post-buckling displacements. Again, the geometry of the upper strakes of a silo at Port Kembla (Ding 1992) with \( R = 12000\text{mm}, R/t = 1000 \), and a half strake height of \( L = 1500 \) mm was used as a starting point. Boundary conditions ‘A-A’ were imposed and perfectly elastic material behaviour was used except where otherwise specified. Imperfection shape ‘Rotter closed’ with an amplitude of \( w_{o}/t = 1.0 \) except where otherwise specified was used.

5.2.1 Post-Buckling Behaviour of Sector Models

A large number of wave models corresponding to different circumferential buckling patterns were analysed and the load-displacement response under axial load was recorded well into the post-buckling region. Small lateral loads were applied just before bifurcation points were reached to push the model into post-buckling equilibrium. The lateral loads were then taken off and the load displacement response of the model was followed further once post-buckling equilibrium was achieved. If secondary bifurcation occurred this process was repeated. Secondary bifurcation always involved a change from a whole wave model into a half wave model ie. halving of the buckling mode as shown in Figure 5-1.
The analysed FE-models fell into two groups with distinctly different post-buckling behaviour. The first group covered FE-models of all modes with a number of circumferential buckling waves $m$ less or equal to the critical mode $m_{cr}$. The second group consisted of FE-models of modes with $m > m_{cr}$. The critical buckling mode for the chosen geometry was 21 circumferential buckling waves ($m_{cr} = 21$). Bifurcation occurred at 0.246 $\sigma_{cl}$ and the load displacement characteristics displayed a slight decrease in load thereafter (Figure 5-2). A post-buckling minimum was reached at 0.238 $\sigma_{cl}$ after which the equilibrium became stable again. No secondary bifurcation occurred for the critical buckling mode. The behaviour of models for modes $m < m_{cr}$ was similar except for the more pronounced drop in load-bearing capacity after bifurcation into the non-axisymmetric mode. The load-displacement characteristics after buckling became more unstable with decreasing number of circumferential
buckling waves and even though the critical load for these models increased, the post-buckling minimum decreased. As a consequence of these differences in initial post-buckling behaviour, the load-displacement curves for modes between \( m_{cr} = 21 \) and \( m = 17 \) intersected each other in the post-buckling region. The load-displacement characteristic for \( m = 17 \) serves as an example in Figure 5-2. Bifurcation occurs at \( 0.314 \sigma_{el} \) and the post-buckling minimum is reached at \( 0.170 \sigma_{el} \). Again, the curve becomes stable after this minimum is reached. Critical loads and post-buckling minimums for all relevant modes \( m < m_{cr} \) are listed in Table 5-1.

Table 5-1. Critical loads and post-buckling minimums for \( m < m_{cr} \).

<table>
<thead>
<tr>
<th>Mode</th>
<th>Bifurcation</th>
<th>Post-buckling minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>0.314</td>
<td>0.170</td>
</tr>
<tr>
<td>18</td>
<td>0.283</td>
<td>0.191</td>
</tr>
<tr>
<td>19</td>
<td>0.260</td>
<td>0.207</td>
</tr>
<tr>
<td>20</td>
<td>0.249</td>
<td>0.223</td>
</tr>
<tr>
<td>21</td>
<td>0.246</td>
<td>0.238</td>
</tr>
</tbody>
</table>

The behaviour for all modes \( m > m_{cr} \) was found to be significantly different from the behaviour of modes \( m < m_{cr} \) as illustrated for mode \( m = 22 \) and \( m = 30 \) in Figure 5-2. The load-bearing capacity increased even after buckling into a full wave although at an increased rate of displacement compared to the pre-buckling behaviour although this is less obvious for higher modes (compare \( m = 22 \) and \( m = 30 \) in Figure 5-2). Secondary bifurcation into a half wave occurred after which the load was observed to drop off until a post-buckling minimum was reached. The load-displacement path becomes stable again after this minimum. Table 5-2 contains a detailed list of critical loads, secondary bifurcation loads and post-buckling minimums for these models. Figure 5-3 illustrates these values for all models that were investigated.
Table 5-2. Critical loads and post-buckling minimums for $m > m_{cr}$.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Secondary bifurcation</th>
<th>Post-buckling minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>0.297</td>
<td>0.217</td>
</tr>
<tr>
<td>24</td>
<td>0.336</td>
<td>0.189</td>
</tr>
<tr>
<td>26</td>
<td>0.371</td>
<td>0.168</td>
</tr>
<tr>
<td>28</td>
<td>0.414</td>
<td>0.145</td>
</tr>
<tr>
<td>30</td>
<td>0.403</td>
<td>0.132</td>
</tr>
<tr>
<td>32</td>
<td>0.352</td>
<td>0.147</td>
</tr>
</tbody>
</table>

Figure 5-2. Load displacement characteristics for selected buckling modes.
5.2.2 Combining the results

The mean axial displacement \( u \) and the axial load for the FE-models were used to derive non-dimensional load-deflection curves where the load is expressed as a stress ratio of \( \sigma/\sigma_{el} \) and the displacement \( u \) is normalised against the wall thickness. The results for the individual FE-sector models for all modes covering one buckling wave after initial bifurcation were combined under the assumption that the complete structure will follow the post-buckling path of least energy and that sectorial symmetry will be maintained throughout. The quasi-static nature of the analysis neglects any dynamic effects and is therefore only an approximation of the actual process. It can be seen typically in Figure 5-4 that the post-buckling response of the whole structure consists of small portions of the post-buckling response for a number of different modes \( m \).
5.3 Initial Post-Buckling Behaviour

5.3.1 Imperfection Amplitude

The imperfection amplitude was varied \((w_0/t = 0.3, 0.5, 0.8 \text{ and } 1.0)\) with the overall geometry kept constant. The combined load deflection curves for the four investigated series of FE-models are shown in Figure 5-4.

For all four cases the behaviour of the structure was found to be very unstable after reaching the critical load. The initial buckling mode of \(m = 21\) to \(m = 24\) (depending on the imperfection amplitude, see Table 5-3) undergoes a number of transitions until it reaches the post-buckling minimum at \(m = 15\) to \(m = 16\) where the load path finally becomes stable again.

Small imperfections were found to lead to higher pre-buckling stiffness of the structure. The axial load is converted to inward deflection locally at the axisymmetric imperfection. The greater the imperfection amplitude, the greater the loss in pre-buckling stiffness. For all four cases the load bearing capacity drops off quite significantly after maximum load is reached. However, the slope of the initial post-buckling curve is much steeper for cylinders with imperfections of smaller amplitudes. After buckling and the initial drop-off in load bearing capacity, the load-displacement curves for the four cases quickly come very close to each other. The values of \(\sigma/\sigma_{pl}\) at initial buckling for \(w_0/t = 0.3\) was twice that for \(w_0/t = 1.0\) but at the post-buckling minimum they are within 12%. Results for each of the four analyses are given in Table 5-3.
Table 5-3. Post-buckling data for various imperfection amplitudes

<table>
<thead>
<tr>
<th>$w_0/t$</th>
<th>Load $(\sigma/\sigma_{cl})$</th>
<th>Maximum $(\sigma/\sigma_{cl})$</th>
<th>Minimum $(\sigma/\sigma_{cl})$</th>
<th>Post-buckling Mode</th>
<th>Buckling Mode</th>
<th>Mode at Post-buckling Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.492</td>
<td>0.203</td>
<td>0.165</td>
<td>24</td>
<td>24</td>
<td>16</td>
</tr>
<tr>
<td>0.5</td>
<td>0.424</td>
<td>0.190</td>
<td>0.159</td>
<td>22</td>
<td>22</td>
<td>16</td>
</tr>
<tr>
<td>0.8</td>
<td>0.320</td>
<td>0.181</td>
<td>0.149</td>
<td>22</td>
<td>22</td>
<td>15</td>
</tr>
<tr>
<td>1.0</td>
<td>0.246</td>
<td>0.167</td>
<td>0.144</td>
<td>21</td>
<td>21</td>
<td>15</td>
</tr>
</tbody>
</table>

Figure 5-4. Initial post-buckling response for variations in imperfection amplitude.
Upon reaching the post buckling minimum the response for all four cases was very similar. There were a number of buckling modes \( m \) less than the critical mode \( m_{cr} \) that came very close to each other in the vicinity of the post-buckling minimum suggesting that these modes, particularly modes between \( m=12 \) and \( m=16 \), are all possible and interaction between these modes is probable (Figure 5-5). At this stage, outward-facing folds between the dominating inward buckles form. These folds carry the majority of the compressive load and can be interpreted as secondary columns, essentially changing the system from a shell to a folded plate structure. These folds become more pronounced with increasing radial deformation, thus increasing the second moment of area of these columns. The load carrying capacity of the structure rises again. If elastic material properties are assumed, the load can be further increased until these columns buckle and final collapse of the structure takes place in conjunction with the formation of secondary buckles (Figure 5-6).

![Figure 5-5. Increase in load-bearing capacity after the post-buckling minimum.](image-url)
Figure 5-6: Change of load bearing system

Figure 5-7. Axial membrane stresses illustrating the changed load-bearing system in the post-buckling region.
5.4 Weld-Induced Residual Stresses

The inclusion of weld-induced residual stresses into the FE-model has been shown to increase the buckling strength of the structure in Chapter 4. Figure 5-8, Figure 5-9 and Figure 5-10 compare the load-displacement response for FE-models including a weld-induced residual stress field with geometrically identical initially stress-free FE-models. Comparisons were made for cylinders with weld imperfection amplitudes of $w_0/t = 0.3$ and $w_0/t = 1.0$.

It can be seen that the pre-buckling stiffness is not affected by the presence of residual stresses and critical loads for the initially stress-free FE-models are lower. Soon after buckling, the load displacement paths become indistinguishable and the responses are identical.

![Graph showing load-displacement response]

Figure 5-8. Effect of weld-induced residual stresses for $w_0/t = 0.3$. 
Figure 5-9. Effect of weld-induced residual stresses for $w_0/t = 1.0$.

Figure 5-10. Combined results for $w_0/t = 0.3$ and 1.0.
5.5 Boundary Conditions

For an imperfection amplitude of $w_0/t = 1.0$, the two boundary conditions A-A and S-S were compared. As shown in Chapter 4, the S-S boundary condition results in a much higher buckling load. As for the A-A boundary condition, the initial post-buckling behaviour for the S-S boundary condition is characterised by a sharp drop in load carrying capacity which levels out at a stress level similar to the initial buckling load of the A-A model. For the S-S condition buckles at the individual welds are aligned and grow larger with increasing axial deformation. Eventually these buckles merge allowing no further increase in load. The load-displacement diagrams for both boundary conditions are shown in Figure 5-11.

The influence of weld-induced residual stresses on the post-buckling behaviour of FE-models with the S-S boundary condition was found to be similar to the A-A boundary condition. The load-displacement curves for FE-models including residual stresses and for initially stress-free FE-models merged soon after buckling and no difference between the two could be detected thereafter (Figure 5-12).
Figure 5-11. Comparison of the A-A and S-S boundary condition.

Figure 5-12. Residual stresses and S-S boundary condition.
5.6 Plasticity

Thin-walled cylindrical shell structures made from steel generally buckle elastically. The initial post-buckling behaviour of these structures as described in this chapter leads to large deformations and large local strains which eventually cause the shell to yield. Taking into account plasticity therefore does not affect the buckling strength of the structure but it reduces the strength gain in the post-buckling region (Figure 5-13) as plastic hinges form along the outward folds (Figure 5-6). Figure 5-13 compares the post-buckling responses of two geometrically identical FE-models – one assumed to be fully elastic and the other one assumed to be made from elastic-plastic material with a yield stress of 200 MPa. Only small gains in load bearing capacity beyond the post-buckling minimum can be observed for the elastic-plastic version of the FE-model with plasticity starting to affect the behaviour just before the post-buckling minimum is reached as is indicated by the deviation between the two curves at that point (Figure 5-13).

![Graph of elastic and elastic-plastic material behaviour](image)

Figure 5-13. Elastic and elastic-plastic material behaviour.
5.7 Half-Structure Model

A FE-model encompassing half a strake above and below a circumferential weld imperfection and spanning around half the circumference was generated and the response of this half-structure model was compared to the response of the sector model. Symmetry boundary conditions were imposed at the meridians at 0° and 180°.

5.7.1 Axisymmetric FE-model

Firstly, the buckling and post-buckling behaviour under axial load was studied for a perfectly axisymmetric half-structure model with a weld imperfection of an amplitude of \( w_0/t = 1.0 \). A non-linear buckling analysis was performed and the first nine bifurcation modes and associated bifurcation loads were recorded. Table 5-4 gives the buckling strengths of these first nine bifurcation modes and a comparison with the equivalent sector model. Buckling loads for the half-structure model are slightly higher than for the equivalent sector models, an effect that has also been noted by Teng & Song (2000) for a different load case.

Table 5-4. Comparison Sector Model and Half Structure Model.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Half Structure Model ((\sigma/\sigma_cl))</th>
<th>Sector Model ((\sigma/\sigma_cl))</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>0.314</td>
<td>0.299</td>
</tr>
<tr>
<td>19</td>
<td>0.282</td>
<td>0.268</td>
</tr>
<tr>
<td>20</td>
<td>0.264</td>
<td>0.248</td>
</tr>
<tr>
<td>21</td>
<td>0.259</td>
<td>0.244</td>
</tr>
<tr>
<td>22</td>
<td>0.263</td>
<td>0.249</td>
</tr>
<tr>
<td>23</td>
<td>0.275</td>
<td>0.261</td>
</tr>
<tr>
<td>24</td>
<td>0.294</td>
<td>0.281</td>
</tr>
<tr>
<td>25</td>
<td>0.316</td>
<td>0.302</td>
</tr>
<tr>
<td>26</td>
<td>0.341</td>
<td>0.329</td>
</tr>
</tbody>
</table>
A small lateral load was then used to ‘push’ the half-structure model into the closest post-buckling configuration and the post-buckling response was recorded. Extremely small load steps had to be prescribed during this part of the analysis to ensure that the analysis follows the primary load deflection path. The diagrams in Figure 5-14 show the load-deflection curves for the half-structure model in comparison with the curve derived for the sector model. Immediately after the maximum load had been reached, the half-structure model developed 21 circumferential buckles (Figure 5-15) and the load bearing capacity dropped off at a much greater rate than the sector model.

![Graph showing load-deflection curves for the half-structure model and the sector model.](image)

**Figure 5-14.** Load-deflection curves for the half-structure model and the sector model.

The buckling mode started to change at point “A” in Figure 5-14 when one buckling wave started to become dominant (Figure 5-16). This dominant inward buckle subsequently became much larger than the other buckles (Figure 5-17) and reached a maximum at point “B” in Figure 5-14 (Figure 5-18). After this point other buckles gradually became bigger (Figure 5-19) until a point was reached where a buckling pattern of 12 circumferential buckles could be identified (point “C” in Figure 5-14,
Figure 5-20). This point roughly coincided with the post-buckling minimum of the sector model. After this point, the dominant buckle formed again and a buckling pattern of 15 circumferential buckles formed. The analysis then circled between 15 and 12 circumferential buckles at a more or less constant load-level. Lateral loads around the circumference could stabilise any mode between 12 and 15 buckles leading to an increase in load bearing capacity similar to the response that had been observed for the sector model.

Fourier analyses of the radial displacements at the weld were performed at a number of points along the loading path to understand the changes in buckling modes better. Figure 5-15 to Figure 5-20 show the deflections around the circumference at the weld normalised against the wall thickness $t$ and the results of a Fourier-decomposition of the buckling pattern normalised against the greatest Fourier factor. Figure 5-15 shows the buckling mode of precisely 21 circumferential buckles at bifurcation. Very soon thereafter, other buckling modes start to contaminate the pattern and one dominant buckle starts to appear (Figure 5-16 to Figure 5-18). The results of the corresponding Fourier decompositions show the change in the dominating buckling mode from the initial 21 buckling waves down to 12 buckling waves in Figure 5-20.

A great number of small analysis increments was necessary to ensure that the primary path was followed which led to an accumulation of round-off errors. As also noted in Teng & Song (2000), there is no guarantee that the deformation path that includes these accumulated round-off errors follows the correct post-bifurcation path. By repeating the analysis on the same FE-model, the response was found to be similar until around point C in Figure 5-14. However, the load-displacement paths started to differ after this point indicating that the results were starting to depend on the extent of the accumulated round-off error. This represents a serious disadvantage for the use of the half-structure model for post-buckling analyses.
Figure 5-15. Fourier decomposition and radial displacements immediately after buckling.

Figure 5-16. Fourier decomposition and radial displacements at point A in Figure 5-14.

Figure 5-17. Fourier decomposition and radial displacements between point A and point B in Figure 5-14.
Figure 5-18. Fourier decomposition and radial displacements at point B in Figure 5-14.

Figure 5-19. Fourier decomposition and radial displacements between point B and point C in Figure 5-14.

Figure 5-20. Fourier decomposition and radial displacements at point C in Figure 5-14.
5.7.2 Overall Imperfections

A second way of reaching a post-buckling equilibrium was explored by superimposing overall imperfections onto the initial FE-model. In addition to the localised axisymmetric weld imperfection with an amplitude of $w_0/t = 1.0$, an overall imperfection in the shape of the first bifurcation mode was introduced. This overall imperfection was scaled to different magnitudes $f_a$ and axial loads were applied. The load deflection curves for FE-models with a range of different overall imperfection amplitudes $f_a = 0.0t$ to $f_a = 1.0t$ are shown in Figure 5-21 and Table 5-5 gives the ultimate strengths for these cylinders. Surprisingly, three entirely different types of behaviour could be observed within the range of imperfection amplitudes shown in Figure 5-21.

![Graph showing load deflection curves for various amplitudes of overall imperfections.](image)

Figure 5-21. Load deflection curves for various amplitudes of overall imperfections.
FE-models with an overall imperfection with a scaling factor $f_a < 0.05t$ behaved similar to the axisymmetric half-structure model. The linear pre-buckling behaviour was followed by a short section where 21 circumferential buckles formed and the gap into the post-buckling path of the model with no overall imperfections ($f_a = 0.0$) was bridged. The ultimate strength was found to be slightly lower than for the axisymmetric half-structure model, the degree depending on the magnitude of the scaling factor $f_a$.

The pre-buckling behaviour for FE-models with a scaling factor of $0.05t < f_a < 0.5t$ was found to be entirely different. An initial period of linear behaviour with the same stiffness as the previous FE-models was followed by a distinct change in stiffness where 21 circumferential buckles formed but the load could be further increased. Ultimate load was eventually reached at a much higher level of axial displacements. The ultimate load was also found to increase slightly for larger values of $f_a$.

For scaling factors $f_a > 0.5$ the initial response of the FE-models was found to be of a lesser stiffness gradually becoming softer as the point of ultimate strength was reached. However, ultimate strength was found to increase again for larger values of $f_a$.

Table 5-5. Ultimate strength $\sigma/\sigma_{ct}$ for different levels of overall imperfections.

<table>
<thead>
<tr>
<th>Amplitude $f_a / t$</th>
<th>$\sigma / \sigma_{ct}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.259</td>
</tr>
<tr>
<td>0.05</td>
<td>0.243</td>
</tr>
<tr>
<td>0.1</td>
<td>0.240</td>
</tr>
<tr>
<td>0.2</td>
<td>0.243</td>
</tr>
<tr>
<td>0.3</td>
<td>0.249</td>
</tr>
<tr>
<td>0.5</td>
<td>0.261</td>
</tr>
<tr>
<td>1.0</td>
<td>0.293</td>
</tr>
</tbody>
</table>

The reason for this rather unexpected behaviour can be found in the fact that overall imperfections actually change the structural system. The superimposed pre-buckling deformations allow buckles to form at an early point along the load-displacement path.
These buckles then quickly become large enough to form a structural system similar to the system shown in Figure 5-7. This is illustrated in Figure 5-22 where the radial displacements at the peak of the inward-overall imperfection at $\phi = 0.0$ are plotted for the region before ultimate load is reached. For values of $f_a > 0.05t$ the analysis of a cylindrical shell structure turns into an analysis of a three dimensional shell-structure with quite different properties. It is therefore considered that it is not appropriate to use the shape of a bifurcation mode as the superimposed overall imperfections for the post-buckling analysis of a cylinder with a circumferential weld imperfection.

![Figure 5-22. Radial displacements at $\phi = 0$ for different overall imperfection amplitudes $f_a$](image-url)
5.8 Conclusions

Sector models were used to develop an understanding of the post-buckling behaviour of thin-walled cylinders with a localised circumferential weld imperfection. The response of numerous sector models was studied and the response of the full structure was pieced together from these results. A post-buckling load bearing model was developed and the influence of weld imperfection amplitude, weld-induced residual stress-fields, different boundary-conditions and elastic-plastic material behaviour was studied.

It was found that weld-induced residual stresses cease to influence the load bearing behaviour of the cylinder shortly after buckling. The drop in load bearing capacity was steeper for smaller weld imperfection amplitudes but there was convergence of the post-buckling minimum for all tested imperfection amplitudes and similar behaviour thereafter. The A-A boundary condition resulted in an entirely different post-buckling response than the S-S boundary condition. Taking into account elastic-plastic material behaviour resulted in a less pronounced rise in capacity after the post-buckling minimum compared to FE-models with linear elastic material behaviour.

A half-structure model was developed for comparison with the sector-model. It was found that the axisymmetric half-structure model buckled at a slightly higher load level than the sector model. The initial post buckling response was found to be significantly different with several interacting buckling modes. Half-structure models with overall imperfections in the shape of the scaled lowest bifurcation-mode led to significant changes in the pre-buckling response depending on the amplitude of the overall imperfection. A high number of small loading increments was necessary to obtain results along the primary deformation path leading to the accumulation of round-off errors along the way. Therefore reliable results for half-structure models could only be obtained for the initial post-buckling response.
6. Buckling of Welded Box-Sections

6.1 Introduction

Thin-walled steel tubes filled with concrete form an economical solution for columns subjected to primarily axial loading. For economy reasons, a thin-walled steel section sufficient to carry the construction loading is provided while relatively inexpensive concrete is used as the major component to carry the design loading.

The strength of the steel tube is influenced by local buckling of the tube walls which is a function of the slenderness of the plate elements forming the tube. For rectangular or square bare steel tubes, the local buckling pattern can consist of inward and outward buckles and the influence of this local buckling on the column strength has been included in all major steel design specifications.

From the results of tests on thin-walled concrete filled tubes, Grimault & Janss (1977) proposed an empirical effective area approach to account for observed local buckling effects. A theoretical study by Wright (1993) of the elastic and inelastic buckling of plates in contact with a rigid medium, e.g. concrete, was empirically modified to determine plate slenderness limits for plastic, compact, semi-compact and slender plates. Nakai et al. (1985) proposed an empirical relationship for the local buckling strength of steel tubes filled with concrete by assuming the plate elements to have all edges clamped. Some tests by Ge and Usami (1992) indicated that this relationship was conservative. However, the plates were relatively stocky and the strength of the steel tube was determined by subtracting the strength of the concrete from the strength of the composite concrete-steel tube. Considering the variability of concrete and its compatibility with the steel tube, particularly where local buckling occurs, this procedure could be inaccurate.
Uy and Bradford (1994) examined the elastic and inelastic local buckling of cold-formed profiled steel sheeting as used in composite profiled beams. They used the finite strip method to model the behaviour of thin plates against a rigid medium. This method is also applicable to concrete filled tubes.

In a paper by Bridge & O'Shea (1998) the results of a series of tests on thin-walled square steel tubes with varying plate slenderness and varying length to width ratios are reported. Two types of tests were performed for a range of combinations of plate slenderness and length: axial load on the bare steel tube; and axial load on the steel alone with un-bonded concrete infill to provide only lateral restraint. Residual stress measurements and plate imperfections resulting from the manufacturing process were also measured. A number of these measurements indicated that some specimens displayed much greater deformations after fabrication than others. These imperfections were obviously introduced when the four flat plates representing the four sides of the box section were welded together to form the box. Stresses induced during the cooling of the four welds were enough to exceed the buckling resistance of the steel plates. Due to the great care that was taken to record the deformations and residual stress patterns after welding, a Finite-Element model of one of these specimens could be built where the recorded data was used for calibration. This FE-model and the influence of various parameters on the local buckling under axial load was investigated making use of the methods described in the previous chapters of this thesis.
6.2 The Finite Element Model

6.2.1 Tube Geometry

The measurements indicated that deformations and stress patterns were symmetric about the longitudinal plate centre-lines. Therefore only one quarter of the specimen had to be represented in the FE-model as shown in Figure 6-1. Figure 6-1 also illustrates the dimensions of specimen ‘B27’ (O’Shea & Bridge 1997, Bridge & O’Shea 1998) which was used for the case study presented in this chapter.

![Figure 6-1. Welded Square Steel Tube – Cross section and FE-model.](image)

6.2.2 Modelling the Welds

The modelling technique described in Chapter 3 was adopted for the task of modelling the longitudinal welds joining the four sides of the box-section. Strains were applied along the welded corners of the section leading to the subsequent development of a residual stress field. Measured residual stresses were found to be in close accordance with stresses computed with the FE-model (Figure 6-2).
6.2.3 Material Properties

To determine the steel material properties in Bridge & O'Shea (1997) and O'Shea & Bridge (1998), tension coupons were cut from a randomly selected steel sheet. These coupons were then tested in accordance with Australian Standard AS 1391 (1991). The final averaged steel properties were found to be $t = 2.142$ mm, $\sigma_{\text{yield}} = 282$ MPa and $E = 199400$ MPa. In the computer analysis, two different simplified models were used and compared: firstly an elastic-plastic model; and secondly a bi-linear model which took strain hardening into account.

![Graph](image)

**Figure 6-2.** Measured and computed weld-induced residual stresses.
6.3 Test Specimens

6.3.1 Manufacture of Test Specimen and Imperfection Measurement

All tubes in O'Shea & Bridge (1998) were manufactured from mild steel sheets with a nominal thickness of 2 mm. Four plates were cut from the sheet, tack welded into a box shape and then welded with a single bevel butt weld at the corners as shown in Figure 6-1. The out-of-plane geometric imperfections of the tube walls after welding were measured on a grid on each face of the tubes using a Wild NA2 automatic level with a parallel plate micrometer. The results for the four faces of specimen B27, 282x928BS are shown in Figure 6-3 to Figure 6-6. Residual stresses induced by the welding process were measured on a representative box using the sectioning technique. Strain gauges were placed at mid-height (cross section through point “A” in Figure 6-1) on the internal and external faces. The complete set of measurements for geometric imperfections and residual stresses has been reported in the research report by O'Shea & Bridge (1998).

6.3.2 Testing Procedure

For the axial compression tests described in Bridge & O'Shea (1997) and O'Shea & Bridge (1998), the ends of the specimen were rotationally and laterally fixed by low temperature metal in a milled groove in custom built end plates. The specimens were tested under stroke control in a DARTEC 2000 kN testing machine. Axial shortening of the specimen was measured between the thick machined plates using four linear displacement transducers evenly spaced around the specimen.
Figure 6-3. Plate Imperfection Side 1, B27, 282x928BS

Figure 6-4. Plate Imperfection Side 2, B27, 282x928BS
Figure 6-5. Plate Imperfection Side 3, B27, 282x928BS

Figure 6-6. Plate Imperfection Side 4, B27, 282x928BS
6.4 Numerical Study

6.4.1 Purely Geometric Imperfection

Only geometric imperfections were considered in a first series of analyses to separate out the influences of geometric imperfections and weld-induced residual stresses on the buckling behaviour. The shape of the first eigenmode under axial load was determined and superimposed onto the perfect shape of the box section. The amplitude of this imperfection was scaled to up to 2.0 wall-thicknesses and a buckling and post-buckling analysis under axial load was performed. The initial stiffness and the ultimate strength depended on the amplitude of the imperfection as shown in Table 6-1. Figure 6-7 shows the load-displacement curves for these FE-models.

Table 6-1. Results for various amplitudes of geometric Imperfections

<table>
<thead>
<tr>
<th>Imperfection amplitude in relation to wall-thickness</th>
<th>Max. Axial Force $F_u$ [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 %</td>
<td>247.580</td>
</tr>
<tr>
<td>50 %</td>
<td>242.084</td>
</tr>
<tr>
<td>100 %</td>
<td>234.316</td>
</tr>
<tr>
<td>200 %</td>
<td>224.708</td>
</tr>
</tbody>
</table>
Figure 6-7. Load deflection curves - purely geometric imperfections
6.4.2 Weld Shrinkage

In a second series of analyses residual stresses were taken into account. To produce a residual stress field matching measured results, strains were applied gradually along the welded zones. As the applied strains increased, the box sections responded linearly – tension stresses developed at the welded edges and pulled these edges outwards, inducing compressive stresses and small bending moments in the areas away from the weld which subsequently caused the sides of the box section to buckle. Figure 6-8 shows the lateral deflection of Point “A” (Figure 6-1) in relation to the applied strains at the weld. During the pre-buckling phase only small deflections could be observed while the residual stress field developed to levels up to yield in tension. Strains applied after buckling had a great influence on the lateral displacements of the buckles but did not alter the residual stress patterns.

![Graph showing load-displacement curve for box sections under weld-induced strain loading](image)

**Figure 6-8.** Load – displacement curve for box sections under weld-induced strain loading
6.4.3 Residual Stresses and Geometric Imperfection

The amplitudes of the measured weld induced buckles averaged 1.2 wall-thicknesses. This value is reached at "Result Point 2" (Figure 6-8). At "Result Point 1" (Figure 6-8), the stress field is already fully developed but the amplitude of the geometric imperfection is only 10% of a wall-thickness. These two points were used as a basis for ensuing analyses of the box under axial compression. In Figure 6-9 the influence of the residual stress field becomes apparent. The stiffness and the ultimate strength of the models including residual stresses are considerably lower than in the initially stress-free models. However, the amplitude of the geometric imperfection hardly influences the results when residual stresses are considered.

![Graph showing load deflection curves for different stress conditions.](image)

Figure 6-9. Load deflection curves – Influence of residual stresses.
6.4.4 Boundary conditions

The results shown in Figure 6-9 are for clamped boundary conditions modelling the boundary conditions during the experiments in O'Shea & Bridge (1997). The same analyses were repeated for simple supports. As illustrated in Figure 6-10, using simply supported instead of clamped boundary conditions led to slightly lower ultimate loads and less stiffness in the pre-buckling phase.

![Graph showing load deflection curves]

*Figure 6-10. Load deflection curves – Influence of boundary conditions.*
6.4.5 Strain Hardening

Elastic-plastic material behaviour was compared to a bi-linear material law which assumed first yield at 282 MPa and an ultimate stress of 374 MPa at a plastic strain of 0.6. These values correspond to the tension tests performed by O'Shea & Bridge (1997). Both material models resulted in the same weld-induced residual stress fields and displacements. Differences occurred in the post-buckling behaviour of the box where plastic hinges are developed and strain-hardening reserves result in a significant gain in post-buckling strength (Figure 6-11). Again the amplitude of the initial geometric imperfection had hardly any effect on the load-deflection path of the structure.

![Graph showing comparison of material models](Figure 6-11. Comparison of material models.)
6.5 Comparison with laboratory test results

As can easily be seen in Figure 6-7 and Figure 6-9 to Figure 6-10 some differences exist between the measured results and the results gained from the computer models. The closest match was achieved when using the bi-linear material model. Pre-buckling stiffnesses still differed. An explanation can be found in the boundary conditions used in the tests. Rotations might not have been fully restricted and the low temperature metal holding the specimen in place might have had a softening effect. Ultimate loads and post-buckling behaviour in the FE-model closely resembled the results gained from the experiments in O'Shea & Bridge (1997). The residual stress patterns and the deflections caused by the cooling of the welds could be generated in the computer models in very close accordance with the measurements given in O'Shea & Bridge (1997).
6.6 Conclusions

A numeric study on the imperfections induced during manufacturing of bare steel tubes with a box cross-section was performed. Measured imperfections from a specimen of a laboratory test were reproduced in a computer analysis. This model was then used to perform a parameter analysis studying the influence of these imperfections on the buckling behaviour of this box under axial load.

It was shown that the amplitude of the imperfections effectively has no effect on the buckling strength, stiffness and post-buckling behaviour of the box when residual stresses are considered. However, when residual stresses are ignored and the buckling analysis is based on purely geometric imperfections, the amplitude of these imperfections has a strong influence on the response of the studied box.

Clamped end-supports were compared to simple supports and it was shown that clamping the ends of the box increases the stiffness and the ultimate strength by a small degree. Elastic-plastic material behaviour was compared to a bi-linear material model which accounts for strain-hardening and it was shown that using the bi-linear material law increased the post-buckling strength of the box. The response of the box in the laboratory test was matched closest by this computer model. Small differences between the numerical results and the test results could be explained by the softening effect of the implemented boundary conditions during the test program.
7. Conclusions

7.1 Summary

7.1.1 Introduction
Thin walled cylindrical storage bins are commonly manufactured from welded steel plates and are renowned for their "imperfection sensitive behaviour". Small deviations from the nominal, or perfect geometry generally result in significant loss of strength of such structures. Imperfections that occur at each circumferential joint between two strakes of steel plates have been recognised as extremely detrimental to the buckling resistance. Most of the work presented in this thesis focuses on the effects of this particular type of imperfection.

7.1.2 Shape Function
A new shape function describing the geometry of a circumferential weld was developed. The shape function was derived using linear elastic shell bending theory. Extensive measurements taken on a silo near Port Kembla / Australia (Ding 1992) were then used to calibrate the new shape function. Global overall imperfections were filtered out of the measured data and the new shape function was fitted to the remaining measurements of the localised seven welds of the silo. Three parameters were found to govern the exact shape of the weld: the depth of the weld imperfection, the wave length of the imperfection and the roundness at the centre of the weld.

7.1.3 Finite Element Analyses
The finite element code ABAQUS was used for the buckling and post-buckling analyses which led to the results documented in Chapters 4 to 6. Modelling techniques used by other researchers were reviewed and compared and a modelling system was developed. The software package ABAQUS was complemented by a system of
FORTRAN modules which facilitated the pre- and post processing and the definition of residual stress fields.

7.1.4 Buckling Analysis

Non-linear buckling analyses on finite element models were performed to determine the influence of a number of parameters on the buckling behaviour of thin-walled cylindrical shells.

Firstly, the influence of interaction between neighbouring weld imperfections was investigated. A large range of strake heights, $R/t$-ratios and imperfection amplitudes was considered and diagrams were developed to give the buckling strength for shells within the range of these parameters.

Secondly, weld-induced residual stress fields were considered and their strengthening effect on the buckling was documented for a range of cylinder and imperfection geometries.

Thirdly, the influence of the shape of a circumferential weld on the buckling strength of thin-walled cylinders was determined. The newly developed shape function was used for this investigation. It was found that the three parameters mentioned earlier—depth, wave length and roundness—are important factors governing the buckling strength of silos or tanks. Shape functions previously suggested by other researchers were fitted to the new shape function and fluctuations in previously reported buckling strengths could be explained by the differences in the shape parameters.

7.1.5 Post-Buckling Analysis

A post-buckling analysis on finite element models of cylinder sectors encompassing one circumferential buckling wave was performed and a model for the post-buckling behaviour of thin-walled cylinders with circumferential weld imperfections was developed. These results were compared to results gained from finite element models spanning over half the circumference of the cylinder. The use of these much larger finite element models resulted in a number of modelling problems which were described in detail.
7.1.6 Box Sections

The modelling techniques which were used for the analysis of thin-walled cylinders can readily be applied in various other problems. Welded box sections have been known to buckle under the residual stress fields introduced during welding. This effect was reproduced in a finite element analysis and a parameter study was performed to determine the strength of the buckled box section columns under axial load. Accurate measurements of displacements and residual stresses existed from a series of laboratory tests (O'Shea & Bridge 1997). These measurements could be accurately reproduced during the numerical work in Chapter 6, further validating the modelling techniques used throughout this thesis. The influence of two different sets of boundary conditions, strain hardening, geometric overall imperfections and weld-induced residual stresses on the buckling of these box sections under axial load was investigated.
7.2 Discussion

7.2.1 Weld Shape

Measured data of the silo in Port Kembla / Australia (Ding 1992) was available for 92 meridians over seven welds. Measurements were organised into sets spanning from strake mid-height below a weld to strake-mid height above a weld. The new shape function was designed to filter out distortions caused by overall imperfections and was fitted to the measured data. A small number of measured meridians caused numerical problems due to highly localised distortions but 91.6% of the measurements were accepted for the fitting process.

The proposed shape function is the first function to combine shell theory with measured data. It is a continuous function and incorporates all the necessary features to represent the geometry of a circumferential weld-induced imperfection.

From the fitted results the following conclusions can be drawn:

Three parameters determine the shape of a circumferential weld imperfection: the depth of the weld; the wavelength of the imperfection and the roundness at the weld. The mean amplitude of the weld imperfection (weld depth) was found to lie between 0.5 and 0.8 wall thicknesses confirming observations made by other researchers. For some welds the standard deviation of the weld depth was found to be rather high, ranging from 0.056 to 1.106. This can be explained by a small number of result points around the circumference where localised dents at the welds intersperse the otherwise almost constant imperfection depth. The typical wave length of the imperfection proved to be between one and two times the linear meridional bending half wave length. Values for the roundness at the centre of the weld were found scattered between the two extremes of zero and one, indicating varying bending stiffness at the weld during cooling. This conformed well with a finite element analysis of the cooling process and confirmed the basic theoretical assumptions made to derive the shape function.
Overall imperfections were filtered out of the measured data. The three types of overall imperfections considered for each meridian were a parallel offset between the silo surface and the surface of the idealised cylinder, a rotation of the measured surface compared to the perfectly vertical meridian and a kink between two neighbouring strakes. The magnitude of these imperfections was up to several wall-thicknesses high in some places and deserves further research.

More measured data is needed to correlate imperfection characteristics with welding techniques and environmental parameters. This could then lead to design procedures where buckling strengths for silos or tanks can be given dependent on the specific weld parameters used in the fabrication process.

7.2.2 Buckling Analyses

Three aspects regarding the buckling of thin-walled cylindrical shell structures with circumferential weld-induced imperfections have been investigated: the interaction between neighbouring weld imperfections; the influence of weld induced residual stress fields on the buckling behaviour; and the influence of the shape of the weld on the buckling behaviour. Parameter studies were performed to study these problems for a range of cylinder geometries.

The following conclusions can be drawn:

Interaction between neighbouring imperfections is an important factor. Strake heights commonly used in silos and tanks do not isolate the effects of circumferential welds on the buckling behaviour. Diagrams have been given to determine the weakening effect of this interaction depending on imperfection amplitude, $R/t$-ratio and strake height.

Residual stresses are invariably introduced during the welding process and have a strengthening effect on the buckling behaviour. Circumferential residual membrane stresses are responsible for this strengthening and can be viewed as internal tension bands around the circumference at the weld. The second contributing factor is the presence of axial stresses and the resulting bending moments. These bending moments can be expected to become relatively more influential in more thick-walled shells and erode the stabilising effect of the circumferential membrane stresses. Diagrams have
been provided to determine the degree of this strengthening effect taking into account variations in $R/t$-ratio and imperfection amplitude. The strength gain is greater for more slender structures and for smaller imperfection amplitudes.

The shape of the imperfection has been found to play an important role in the buckling behaviour. The new shape function has been shown to be a reliable tool to explain the influence of the shape of the weld imperfection on the buckling behaviour. The buckling strength was shown to fluctuate greatly depending on the particular shape of the weld. The roundness of the imperfection at the weld centre and the wave length of the imperfection have been shown to be the governing parameters.

Examples have been given to demonstrate the influence of all these factors on the buckling mode shape. Only small differences in the shape of the buckled structures could be detected for short cylinders. However, significant changes in buckling mode shape with variations in the imperfection amplitude were recorded for long cylinders.

7.2.3 Post-Buckling Analysis

A post-buckling load bearing model was developed and the influence of weld imperfection amplitude, weld-induced residual stress-fields, different boundary-conditions and elastic-plastic material behaviour was studied. A sharp drop-off in load bearing capacity immediately after buckling was recorded. This drop-off was found to be more severe for smaller amplitudes of the weld imperfection. The pre-buckling stiffness of the structure also depended on the weld imperfection amplitude. After buckling and the initial drop-off in load bearing capacity, the load-displacement curves for all considered cases quickly came very close to each other. After reaching the post-buckling minimum outward-facing folds between the dominating inward buckles appear. These folds carry the majority of the compressive load and can be interpreted as secondary columns, essentially changing the system from a shell to a folded plate structure. These folds become more pronounced with increasing radial deformation, thus increasing the second moment of area of these columns. The load carrying capacity of the structure rises again. If elastic material properties are assumed, the load can be further increased until these columns buckle and final collapse of the structure takes place in conjunction with the formation of secondary buckles between these columns.
It was found that weld-induced residual stresses cease to influence the load bearing behaviour of the cylinder shortly after buckling. The A-A boundary condition resulted in an entirely different post-buckling response than the S-S boundary condition. Taking into account elastic-plastic material behaviour resulted in a less pronounced rise in capacity after the post-buckling minimum compared to FE-models with linear elastic material behaviour because of the formation of plastic hinges along the lines of the folds which softened the secondary load carrying system.

Sector models were used to develop this model of the post-buckling behaviour of thin-walled cylinders with a localised circumferential weld imperfection. The response of numerous sector models was studied and the response of the full structure was pieced together from these results. A half-structure model was developed for comparison with the sector-model. It was found that the axisymmetric half-structure model buckled at a slightly higher load level than the sector model. The initial post buckling response was found to be significantly different with several interacting buckling modes.

Half-structure models with overall imperfections in the shape of the scaled lowest bifurcation-mode led to significant changes in the pre-buckling response depending on the amplitude of the overall imperfection. A high number of small loading increments was necessary to obtain results along the primary deformation path leading to the accumulation of round-off errors along the way. Therefore reliable results for half-structure models could only be obtained for the initial post-buckling response.
7.2.4 Box Sections

Laboratory tests by O'Shea & Bridge (1997) showed that some box sections buckle due to residual stresses introduced during welding. The computer methods used for the analysis of silos and tanks were also used to reproduce this effect in finite element models of box sections. The results gained from the numerical analyses compared very well to measurements of displacements and residual stresses taken during the experiments by O'Shea & Bridge (1997). A parameter study on these box sections was undertaken to investigate their behaviour under axial load. It was found that the weld-induced residual stress field is the governing factor determining the strength of these box sections under axial load. Strain hardening was shown to have a strengthening influence on the post-buckling response of these sections.
7.3 Future Research

Cylindrical silos and tanks made from steel display specific imperfection patterns and more data about these imperfections needs to be gathered in order to make more specific predictions on the buckling strength of these structures. Differences in manufacturing techniques may lead to significant differences in imperfection patterns which could lead to different results for the strength of cylindrical shell structures. A survey of a sufficient number of existing silos resulting in a representative imperfection database is a much needed task.

Axisymmetric imperfections are predominantly introduced by the welding process. The shape function introduced in the second chapter of this thesis is based on measurements taken on one particular silo at Port Kembla / Australia. A survey of circumferential weld imperfections on a number of different silos and tanks could lead to valuable insights into the relationship between the weld shape on the one hand and silo geometry, fabrication and construction techniques on the other hand. This could then lead to characteristic values for the shape function parameters for given welds resulting from particular silo geometries and fabrication and construction techniques. Design buckling strengths could then be predicted based on these characteristic values of the shape function.

Besides refining the strength predictions for circumferential weld imperfections, predictions on the shape of overall imperfections and their influence on the buckling behaviour are much needed to replace the simple but potentially unrealistic use of bifurcation shapes. Full imperfection maps of silos and tanks are needed for this task and advanced surveying techniques will have to be developed.

Perfectly vertical and evenly distributed loads were assumed for the work in this thesis. Future work on this topic should also include the use of imperfect loading patterns that invariably result from actual silo loads.

The modelling techniques used in this thesis only accounted for one single circumferential weld imperfection with half a strake height above and below the weld.
Interaction between neighbouring welds was taken into account by imposing certain boundary conditions at strake mid-height. With increasing computing capacities it will soon be feasible to model series of strakes with several weld-imperfections. Such a comprehensive model would automatically take interaction between neighbouring welds into account and will lead to more detailed insights into the overall buckling behaviour of silos and tanks.

Circumferential weld imperfections govern the strength predictions for buckling under axial load. Interaction with other imperfection types is very likely to occur in real structures and requires more research in the future. A comprehensive review of imperfections which are commonly encountered in silos and tanks has been given in Chapter 1 and only a small number of the imperfections listed there has received much research.

The design of silos and tanks is mostly based on knock-down factors which relate the design strength to the classical buckling strength. Unfortunately, the factors are very often based on laboratory tests on unrepresentative specimens. Recently developed design rules tend to emphasise the importance of strength predictions based on measured imperfections and fabrication tolerances. This step away from a deterministic approach and towards a probabilistic design approach must be continued. Again, this will only be possible if research leading in this direction can be based on reliable imperfection databases.
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