APPLICATIONS OF MICROWAVE HOLOGRAPHY TO THE ASSESSMENT OF ANTENNAS AND ANTENNA ARRAYS

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I hereby certify that the work in this thesis is the result of original research and has not been submitted for a higher degree to any other University or institution.

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SUMMARY

Gabor’s original holography, which is the basic theory of modern microwave holographic techniques, is introduced. By computer simulations, it is demonstrated that the conventional microwave holographic approach can be used as a tool to reconstruct aperture field distributions of an antenna with some constraints.

Computer simulations of the theory and technique of the improved microwave holographic approach originally introduced by Rahmat-Samii et al., are carried out. The results show that it can be used for surface distortion diagnosis of large reflector antennas. The physical optics integral formulation is derived by general solutions of the vector wave equations.

The necessary theory, which is needed to reconstruct the aperture field from near-field measurements both in a rectangular coordinate system and in a cylindrical coordinate system is developed. It is based on the plane wave spectrum and the vector wave modal expansion of an electromagnetic field. By using a simple dipole and other well-defined antennas, computer simulations have been performed. The results show that the technique is rigorous and applicable. It is also demonstrated that the sampling intervals and the number of sampling points should be chosen carefully in order to obtain a satisfactory resolution of the reconstructed aperture field.

Furthermore, the simulations carried out in this work reveal that the real aperture field distribution of a dipole antenna has a maximum point at each end of
the antenna. This characteristic can only be obtained at a very close distance to the antenna.

This study also reveals the significant contributions of the evanescent waves to the aperture field reconstruction. A simple but effective method for examining the evanescent waves from the measured near-field is also presented.

By using dipoles and other well-known antennas and antenna arrays, the experiments were carried out.

The experimental results provide reasonable good agreements with the simulations. The technique proposed is effective and accurate.
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To My Parents
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PREFACE

There is a very real requirement to be able to monitor the performance of large complex antenna arrays. A typical application is found in Secondary Surveillance Radar Antenna Systems. Microwave holographic measurement approach often provides the best solution.

There are quite a variety of microwave holographic techniques. The earlier work on this field was directly derived from the optical holography, in which the phase information is indirectly obtained from the measured intensity (hologram).

In order to show how the microwave holographic technique is evolved from optical holography, Chapter 1 introduces optical holography. The applications of the microwave holographic techniques are also reviewed.

Chapter 2 presents the basic theory of the conventional microwave holography which is directly derived from the optical holography. Computer simulations, by using a dipole antenna, show that the feasibility and limitation of the technique.

Using Huygens' principle, a mathematical presentation of the physical optics integral is introduced in Chapter 3. It is a basic mathematical tool to solve electromagnetic wave problems, especially in the representation of the field radiated by a large reflector antenna.

Based on the physical optics integral, Rahmat-Samii et al have developed a microwave holographic approach for surface diagnosis of large reflector antennas. Compared with the conventional microwave holographic technique, this approach
has improved accuracy and performance of large reflector antennas dramatically. Chapter 4 presents the details of the technique. The simulation results are also given in this chapter.

Although Rahmat-Samii’s improved microwave holographic technique is widely used for reflector antennas, it can not be applied to general antennas, especially to large complex arrays, for which the theory in this thesis is developed. A more general approach, which is based on plane wave spectrum method is developed and presented in Chapter 5.

In Chapter 5, the theory of plane wave spectrum is described first. Then the application of the theory to the aperture field reconstruction is developed from near-field measurement. Using this technique, simulations with well-known antennas and antenna arrays are carried out. The results show that the technique can be applied to aperture field reconstruction for general antennas and antenna arrays without constraints to a particular antenna type. Furthermore, the importance of evanescent waves in the aperture field reconstruction using near-field measurement is revealed.

In order to prove the theory and simulations developed in Chapter 5, experiments were carried out in the Fleurs Field Station of the University. The results are presented in Chapter 6. Reasonable agreements between simulations and experiments have been achieved.

For circularly symmetrical antennas or antenna arrays, it is more desirable to use near-field measurement on a cylinder. Chapter 7 presents the development of aperture field reconstruction by cylindrical near-field measurement. The
technique can be applied to aperture field analysis and the diagnosis of antennas or antenna arrays, where it is more convenient to use a cylindrical near-field measurement. Simulations using a dipole antenna are also carried out. The results show that the technique is equally promising as the plane wave spectrum method.

To give a more complete description of the applications of microwave holographic techniques, Chapter 8 presents the theory of near-field to far-field transformation by cylindrical probe-compensated near-field measurement.

Chapter 9 presents conclusions and further research suggestions.
CHAPTER 1

EARLY MICROWAVE HOLOGRAPHY AND ITS APPLICATIONS

1.1 A BRIEF INTRODUCTION TO OPTICAL HOLOGRAPHY

As an introduction to the concept of microwave holography, this chapter mainly gives the description of the principles of the conventional microwave holographic technique, which was directly derived from the optical holography. It is the basic theory of both earlier and modern microwave holographic techniques. The image processing techniques from microwave holograms of passive objects are summarised from the earlier work on this field. Further improved techniques will be detailed in the following chapters.

Optical holography first gained the attention of scientists and engineers when it was demonstrated [41] as a two-step imaging process. It later gained widespread attention because it produced remarkably lifelike three-dimensional images. Subsequent research and development led to the application of holographic techniques for data processing and non-destructive testing. Combining holographic methods with acoustic waves and microwaves, has provided new insights into wave phenomena with significant potential for measurement, information collection, wave propagation, and object identification. This chapter surveys research into microwave holography and also earlier work on this field, its techniques, and its applications.
In the first of the two steps which are basic to holography, a coherent wave illuminates an object and scatters a field onto a detector, such as a photographic film for light waves. A portion of the illuminating wave that bypasses the object also illuminates the film to produce an interference pattern, which the film records. Images are then formed in a second step when the developed film transparency, a hologram, is illuminated with coherent light. The hologram diffracts two waves, one proportional to the object-scattered field and another proportional to its complex conjugate. Both waves form images, but they can overlap and obscure each other. This overlap was a disadvantage to holography for several years until Leith and Upatneiks [85] introduced a new experimental technique; since then, holography has grown rapidly. Gabor originated holography when he proposed the method to increase the resolution of electron microscopes [41]. Although Gabor demonstrated the feasibility of holography with light, it has not yet improved electron microscopes. However, the two-step method has significantly influenced optics. The method provides an indirect means of recording the phase of optical wavefronts, since photographic film responds to intensity but not to complex amplitude. Holography is remarkable because it provides an interval between the two steps. The interval is useful for processing and analysing the data stored in the hologram. Some terminology is now common. The wave that bypasses the object is the reference wave; it is coherent with the wave that illuminates the object. The first of the two steps is the hologram formation step; the second, the wavefront reconstruction step.
Holograms and wavefront reconstruction have become significant for optics since an experimental technique was introduced, in which the reference wave is a distinct beam, produced by a beam splitter and inclined to the beam from the object [85]. The technique separates the overlapping images produced in Gabor’s original arrangement, in which the reference and object waves were nearly parallel. Aside from imaging, optical holography has many applications and the field is well documented [90].

1.2 PRINCIPLE OF MICROWAVE HOLOGRAPHY

In this section, the principle of microwave holography is described which is based on scalar diffraction theory and it is directly derived from the optical holographic theory.

According to Huygens’ principle (or the equivalence theorem) each point on a wave front acts as a secondary source, and the sum of the radiations from this surface sheet of secondary sources reproduces the radiated wave at points beyond the surface. Accordingly, if the magnitude and phase of the field from primary source is recorded at every point on the same surface, e.g., a plane, and then each point on the surface is made to generate a signal having that magnitude and phase, the wave will be reconstructed beyond the surface, and appear to be coming from the original source (uniqueness theorem). At optical frequencies recording the magnitude of a wave is readily accomplished with a photographic plate, but recording phase is a more difficult task. For doing so, the addition of a reference
coherent light is needed to convert phase variations into amplitude variations, which in turn may be recorded on a photographic plate, *i.e.*, a hologram.

An experimental arrangement for preparing a microwave hologram is shown in Fig 1.1. The reference wave is, for simplicity, considered as a spherical wave (ideal point source), and can be expressed in the hologram plane $\alpha$-$\beta$ as

$$R(\alpha, \beta) = A \exp\left(-j\phi_R\right)$$  \hspace{1cm} (1.1)

where $A$ is a constant and $\phi_R$ is

$$\phi_R(\alpha, \beta) = k \sqrt{\alpha^2 + \beta^2 + D^2}$$  \hspace{1cm} (1.2)

where $D$ is the distance from the source plane to the hologram plane, and $k=(2\pi/\lambda)$

![Fig 1.1 Geometry of hologram formation](image)

is wavenumber in free space. Suppose that the object wave distribution is $P(x,y)$
which lies in x-y plane. The scattered field by the object P(x, y) in the hologram can be obtained by using the Fresnel-Kirchhoff integral

\[ Q(\alpha, \beta) = \iint_{-\infty}^{\infty} P(x, y) \exp(-j\Phi_P) \, dx \, dy \]  

(1.3)

where \( \Phi_P \) is

\[ \Phi_P(\alpha, \beta) = k \sqrt{(x - \alpha)^2 + (y - \beta)^2 + D^2} \]  

(1.4)

It is this total wave field of both the reference and that scattered by the object which acts on the microwave recording device. Suppose this device responds to energy rather than field strength and thus the effect upon the recording is proportional to the square of the total wave field. The superposed reference wave and the object wave are recorded and form the fringe-like pattern which is the microwave hologram and can be written as

\[ h(\alpha, \beta) = (R(\alpha, \beta) + Q(\alpha, \beta))(R(\alpha, \beta) + Q(\alpha, \beta))^* \]  

(1.5)

\[ = |R|^2 + |Q|^2 + R^*Q + QR^* \]

where the asterisk (*) denotes complex conjugate.

In practice, the wavefront reconstructions from microwave holograms are conducted either optically or computationally. For the optical reconstructions, the microwave holograms must be reduced in size, because the wavelengths of the microwave and light waves are very different. The process will result in a
transparency (optical hologram) having a light-transmission coefficient of $T(\alpha, \beta) = h(M\alpha, M\beta)$, where $M$ is the down-scale factor. Reconstructions of the original scattered-field images are then accomplished by shining a beam of coherent light on the transparency.

For simplicity, the theoretical description of the reconstruction procedure

![Fig 1.2 Reconstruction of the original object from hologram](image)

will be discussed without reducing the size of the microwave holograms. It can be referred to as computational. As illustrated in Fig 1.2, Let the reconstruction microwave be $C(u, v, D_\alpha)$. According to Huygens’ principle the wave passing through the hologram may be regarded as setting up equivalent sources over the surface of the hologram, the strength of these sources being proportional to the transmission coefficient in (1.5). Then this wave field on the hologram plane is
\[ C(\alpha, \beta) = C(u, v) \exp \left( -j \phi_c \right) \quad (1.6) \]

where \( \phi_c \) is

\[ \phi_c(\alpha, \beta) = k \sqrt{(u - \alpha)^2 + (v - \beta)^2 + D_c^2} \quad (1.7) \]

Thus the wave field on the surface of the hologram can be expressed in the form

\[ F(\alpha, \beta) = \tilde{h}(\alpha, \beta) C(\alpha, \beta) \quad (1.8) \]

In another plane \( x_1 - y_1 \), at a distance \( D_1 \) from the hologram plane, the following diffracted field from the hologram is obtained

\[ I(x_1, y_1) = \int_\infty^{-\infty} \int_\infty^{-\infty} F(\alpha, \beta) \exp \left( j \phi_f \right) d\alpha d\beta \quad (1.9) \]

where \( \phi_f \) is

\[ \phi_f(\alpha, \beta) = k \sqrt{(x_1 - \alpha)^2 + (y_1 - \beta)^2 + D_1^2} \quad (1.10) \]

Substituting (1.5), (1.6), and (1.8) into (1.9), it follows that

\[ I(x_1, y_1) = \int_\infty^{-\infty} \int_\infty^{-\infty} C(u, v) \left[ |R(\alpha, \beta)|^2 + |Q(\alpha, \beta)|^2 + R^* Q + R Q^* \right] \exp \left( j \phi_c \right) \exp \left( j \phi_f \right) d\alpha d\beta \quad (1.11) \]
Examination of this expression reveals that the first two terms in the brackets have no contributions to the image of the original object. The third term is one of the images of the object \( P(x, y) \), and it can be expressed as

\[
I_1(x_1, y_1) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C(u, v) \left( R^* (\alpha, \beta) Q(\alpha, \beta) \right) \exp(-j\phi_c) \exp(j\phi_t) da db
\]

(1.12)

Substitute the reference wave (1.1) and the object wave (1.3) into (1.12) results in:

\[
I_1(x_1, y_1) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C(u, v) \left( A \exp(j\phi_R) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x, y) \exp(-j\phi_P) dx dy \right) \exp(-j\phi_c) \exp(j\phi_t) da db
\]

(1.13)

Under the assumption of paraxial rays, the four phase terms of \( \phi_R, \phi_P, \phi_C, \) and \( \phi_t \) can be expressed as

\[
\phi_R(\alpha, \beta) = k \left( D + \frac{\alpha^2 + \beta^2}{2D} \right), \quad \alpha \wedge \beta << D
\]

(1.14)

\[
\phi_P(\alpha, \beta) = k \left( D + \frac{(x - \alpha)^2 + (y - \beta)^2}{2D} \right), \quad (x - \alpha) \wedge (y - \beta) << D
\]

(1.15)

\[
\phi_C(\alpha, \beta) = k \left( D_C + \frac{(u - \alpha)^2 + (v - \beta)^2}{2D_C} \right), \quad (u - \alpha) \wedge (v - \beta) << D_C
\]

(1.16)
\[ \phi_j(\alpha, \beta) = k \left( D_1 + \frac{(x_1 \cdot \alpha + y_1 \cdot \beta)^2}{2D_1} \right), \quad (x_1 \cdot \alpha + y_1 \cdot \beta) \ll D_1 \quad (1.17) \]

Substitute (1.14) through (1.17) into (1.13), and choose \( D_c = D_1 \), (1.13) can be simplified as

\[
I_1(x_1, y_1) = AC(u,v) \exp \left( -j \frac{k}{2D_1} (u^2 + v^2) \right) \exp \left( j \frac{k}{2D_1} (x_1^2 + y_1^2) \right) \\
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x, y) \exp \left( -j \frac{k}{2D} (x^2 + y^2) \right) dx dy \\
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left( j \left( \frac{k}{D} x + \frac{k}{D_1} u \cdot \frac{k}{D_1} x_1 \right) \alpha \right) \\
\exp \left( j \left( \frac{k}{D} y + \frac{k}{D_1} v \cdot \frac{k}{D_1} y_1 \right) \beta \right) d\alpha d\beta 
\]

(1.18)

The second integral of (1.18) can be evaluated by using the following property of the delta function

\[
\int_{-\infty}^{\infty} \exp(jxy) dx = 2\pi \delta(y) 
\]

(1.19)

Then, (1.18) is derived as follows:

\[
I_1(x_1, y_1) = 4\pi^2 AC(u,v) \exp \left( -j \frac{k}{2D_1} (u^2 + v^2) \right) \exp \left( j \frac{k}{2D_1} (x_1^2 + y_1^2) \right) \\
P \left( \frac{D}{D_1} (x_1 \cdot u), \frac{D}{D_1} (y_1 \cdot v) \right) \exp \left( -j \frac{kD}{2D_1^2} (x_1 \cdot u)^2 + (y_1 \cdot v)^2 \right) 
\]

(1.20)
If the condition of $D=D_1$ is chosen, (1.20) can be further reduced as

$$I_1(x_1, y_1) = 4 \pi^2 A C(u, v) \exp \left( -j \frac{k}{D} (u^2 + v^2) \right) \exp \left( j \frac{k}{D} (x_1 u + y_1 v) \right) P \left( (x_1 \cdot u), (y_1 \cdot v) \right)$$  \hspace{1cm} (1.21)

In most cases, the illuminating reconstruction wave is either a uniform plane wave or a point source, so that $u$ and $v$ are constants. Therefore, in (1.21), the diffracted field $I_1$ reproduces the object function; that is, the image of the original object is reconstructed from the hologram. A similar result can be obtained by using the fourth term of $R=Q^*$ in the integral (1.11). It is the conjugate image of the original object.

1.3 THE APPLICATIONS OF MICROWAVE HOLOGRAPHY

Microwave holography has many diverse applications. Some involve long distances; others involve small scale. Some applications require complex apparatus or controlled laboratory conditions, but holograms involving objects such as terrain or troposphere are also possible. The best known application is terrain mapping by synthetic aperture radar (SAR) systems. Natural holograms occur, for example, when the troposphere or the Earth’s surface scatters radio waves that interfere with waves directly from a transmitting antenna. The angles to the scatterers from the observing site are computed from the measured interference pattern. In addition to the applications that involve long distances, microwave holography can image nearby objects in optically opaque dielectric regions. Examples include
weapons concealed by clothing and anomalies in soil or solid propellants for rockets. Holographic methods have also been applied to measure fields near radiating antennas. Holograms can evaluate internal reflections in radomes and can be used in the synthesis of antennas.

1.3.1 Airborne Microwave Holographic Radar

Larson et al. [79, 80] developed an airborne microwave holographic radar which differs from SAR because it emits continuous waves rather than pulses. However, like SAR it requires relative displacement of the antenna and object. The radar has 100 receiving antennas in a linear array and a coherent receiver for each antenna. The signals from all antennas are time multiplexed and displayed by an oscilloscope and photographed to generate one dimension of a hologram. The orthogonal dimension is generated by moving the film during the aircraft’s motion. Images are generated by optically processing the hologram. This system utilises 16.8-GHz waves and has a field of view from directly below the aircraft to 45° on each side. Resolution in the direction of motion depends on the length of synthesised aperture. Resolution in the transverse direction is \((R\lambda)/L\) where \(R\) is the range, \(\lambda\) is the wavelength, and \(L\) is the array length. Typical values of range are 300m to 400m.

The system has formed high quality images of objects and features on the Earth’s surface. Unlike conventional cameras it can form images through fog, inclement weather, and at night. The system generates terrain height contours
when it operates at two frequencies. A similar system in which the coherent array is stationary has been used successfully in the imaging of rotating objects.

1.3.2 Dielectrics: Structures and Anomalies

Because microwaves propagate with slight attenuation through many optically opaque dielectric structures, microwave holographic imaging can locate internal anomalies. For example, relatively high microwave frequencies can be used to image imperfect bonds in laminated plastic components. The lower microwave frequencies can be used in geophysical and archaeological explorations to locate voids buried in soil. Iizuka imaged thin flat voids of finite breadth inside flat dielectric layers with 34.26-GHz waves [57, 58] employing a fixed antenna illuminator and a liquid crystal panel as area detector. Radiated power was high (15W) and reflection at smooth outer boundary generated the waves. Reconstruction with laser light from size-reduced holograms produced images. However, the images were somewhat obscured by additional overlapping images because the angle between the object and reference waves was small. Iizuka also has imaged a plasma and removed the obscuration caused by the glass bottle that contained the plasma by means of holographic subtraction. Again the detector was a liquid crystal panel, but the frequency was 94-GHz; the radiated power was 5 W. Reconstructions were made optically from a reduced size copy of the millimetre-wave hologram.

Ogura and Iizuka have developed a helicopter-carried holographic radar designed to determine the thickness of ice layers [98, 99]. This radar is remarkable
because it uses continuous wave radiation rather than pulse and because it employs two linear antenna arrays. The antenna elements in the array are spaced quadratically, rather than uniformly, to transform the quadratically spaced fringes in a Fresnel zone plate to a linear spacing in a mathematical coordinate system. One of the linear antenna arrays receives, while the antennas of second array transmit sequentially. Data were gathered by all the receiving antennas during radiation by each transmitting antenna to generate a matrix of data. The matrix, called a hologram matrix, is analogous to a scattering matrix. The data are computationally processed to form detection peaks which give ranges to the boundaries of the ice.

In another work [149], Yue et al. imaged dielectric anomalies buried in soil using a frequency of 2.5-GHz. A single antenna that received and transmitted simultaneously scanned the area of interest. Reconstructions were made in two ways. One was with laser light illuminating phase holograms. In this case the logarithm of the measured amplitude was encoded to suppress the reflection from the air-soil interface. The second method was computational with algorithms based on the angular spectrum concept. A coherent background was measured in the region without the void; subtracting this background from the data in the region of the object enhanced images. Images from both reconstruction methods were improved when the soil surface was slightly roughened with surface deformations of random shape and depth less than about one-tenth of a wavelength.

More recently, an object which was underground [101] has been imaged three-dimensionally. In this case, the multifrequency microwave holographic
approach was employed. The propagation velocity of wave in soil, which varies from soil to soil, is estimated. Also the depth of the object from a pulse-echo image based on the minimum squared error is proposed. The result of underground object image reconstruction from real pulse-echo data was reported to demonstrate high estimation accuracy of the propagation velocity and fine resolution of the reconstructed image.

1.3.3 Imaging Metallic Objects in Optically Opaque Dielectric Regions

Airline hijacking stimulated the development of methods for detecting concealed weapons. Although laboratory experiments demonstrated the feasibility of microwave holographic imaging for this application, no systems have been developed for use at airports. Radio frequency metal detectors and X-ray machines are the accepted devices for screening passengers and luggage because they are cheaper and more effective. Nevertheless the thrust for concealed weapons detection stimulated the development of microwave holography. It seems that outdoor applications exist that require relatively low frequencies and that permit only backscattering measurements. Examples of this sort are buried pipes and cables.

The earliest work on imaging concealed weapons appears to be that of Farhat and Guard, who imaged a pistol concealed by clothing and leather [38]. They utilised 70-GHz continuous waves and phase-locked receiver to form phase-only holograms. Images were formed with laser light from scale-reduced holograms. Data were collected by spiral scanning the receiving antenna over the
recording aperture. An example of the test hologram is shown in [38]. The elaborate hologram was recorded through a layer of heavy clothing. The shape of the weapon can be recognised clearly from the reconstructed images.

Orme and Anderson imaged metallic objects in dielectric surrounding with 34.88-GHz wave [100]. Their novel apparatus used one antenna to illuminate the object and a rotating dipole antenna to receive the object field. The dipole and illuminator were scanned together to improve resolution. The phase of the reference wave was linearly changed with probe position to simulate an offset reference beam so that images are displaced laterally from undiffracted light during reconstruction optically. The phase shift was produced by radiating the reference beam from an antenna on the probe carriage toward a large fixed reflector and mixing the reference wave with the object scattered wave.

A three-dimensional imaging of objects buried in snow, again using multifrequency microwave holography has been discussed [123]. An experimental system using X-band microwaves was constructed and the image reconstruction was accomplished mathematically by a computer, with the image displayed on a CRT using a specially designed circuit. The 3-D images of metallic cylinders and a mannequin buried in snow were obtained. The method has the potential to identify the position and shape of targets in snow.

1.3.4 Radar Target Identification

Dooley described one of the earliest experiments in which images of a microwave illuminated target was produced [33]. In this experiment, Gabor type
holograms were formed with 3-cm waves. The fields reflected from a radar target were measured by scanning probe over the hologram aperture and displaying the hologram data on an oscilloscope. Holograms were formed by photographing the display and scale reducing the photograph. Reconstruction was made with laser light.

An electronic method for imaging radar targets was developed by Waters and Eikenberg [139]. They utilised a transmitting antenna and an array of 128 receiving antennas. The system electronically scanned the pattern of the array and combined the received signals. Farhat suggested imaging with a large circular array. The method would use target motion to synthesise an imaging aperture and correct for irrotational target shift [36].

Recently, 3D images of radar cross-section (RCS) have been generated [22]. An extensive 3D radar cross-section (RCS) data set has been acquired. The data set includes diversity in azimuth and elevation through antenna placement as well as frequency diversity. The addition of elevation diversity to standard Inverse Synthetic Aperture Radar (ISAR) imaging allows 3D object reconstruction, as opposed to the 2D defocused projections obtained by conventional ISAR. The method, based on spherical collection and a paracentric approximation for the target space, relates the reconstruction to the acquired data by a 3D Fourier transform. Resulting images were presented for full- and quasi-3D reconstructions, including 2D tomographic slices of the 3D target and a focused projection of the 3D target data onto a single horizontal plane.
1.3.5 Radio Wave Propagation

One of the applications of microwave holography was in locating reflecting regions of the troposphere or Earth’s surface [59]. It was used to make measurements of the angles of arrival of 4-GHz waves. Holograms were formed by the interference of waves that are scattered by the troposphere, reflected by the Earth, and propagated directly from the transmitter. The fringe pattern was measured by an antenna that moved vertically on a mast 85 m tall. The hologram pattern was computationally analysed to determine angles of arrival and hence the directions of scatterers.

More recently, a study of radio wave propagation in a wireless LAN environment was carried out [73]. A dual-frequency continuous wave (CW) radio holography method was proposed for evaluating radio wave propagation environments for LANs. The method images the propagation environment with higher resolution (1 ns or less) and higher sensitivity than conventional techniques which use frequency-modulated CW, pseudonoise (PN) code-modulated waves, or pulse-modulated waves. A narrow-band technique virtually eliminates the measurement inaccuracy caused by frequency response dispersion, which is often a problem in conventional methods. This study introduced a new radio wave propagation observation method by presenting examples of indoor propagation images and delay profiles observed at around 18 GHz.
1.3.6 Radomes

Most antennas are protected by the dielectric shells, radomes. Although necessary, radomes produce undesirable effects. They attenuate waves, cause errors in measurements, and distort antenna patterns. These effects can be recognised, reduced, and corrected by microwave holographic approach.

A practical technique was developed to determine the electric and/or magnetic field on objects and sources inside a spherical measurement surface [48]. The technique, known as spherical microwave holography (SMH), provides a nondestructive, nonintrusive method of point-by-point evaluation of radomes over their spatial extent. The resolution capability of SMH was developed and demonstrated by measurements. Resolution in SMH is only limited by the measurement system’s capabilities. Dielectric and metallic obstacles on the surface of a radome were located and identified. A resolution of $0.33\lambda$ was reported.

1.3.7 Microwave Holography Applications in Antenna Development

Two examples of applications of microwave holography during antenna development and testing were presented in a study [35]. The first example was that of an antenna for aeronautical communications, where actual antenna measurements coupled with the use of microwave holography were used to enhance the theoretical model and hence improve the design and performance of the antenna. The second example illustrated a typical production type antenna for satellite news gathering applications where moulding accuracies have been carefully checked using microwave holography. The effect of data window size has
been discussed and it was concluded that for most reflector diagnostics applications small measured data window sizes are adequate.

1.3.8 Far-field Pattern Predictions of Millimetre-Wave Reflector Antennas

With trends towards utilisation of millimetre wavelengths, the metrology of reflector antennas via holographic-based techniques will be expensive, and perhaps unreliable, because of the requirement for a phase reference channel and the difficulty of achieving accurate phase measurements. An alternative measurement strategy, requiring only intensity, has recently gained credibility with the successful application of phase-retrieval algorithm [67]. In its most convenient form, phase-retrieval requires only a single intensity pattern and the extension of this objective to planar near-field scanning was demonstrated.

1.3.9 Surface Diagnosis of Large Reflector Antennas

This will be described in detail in chapter 4.

1.3.10 Antenna Far-Field Patterns

The far-field radiation pattern of an antenna can be determined holographically. The holographic approach is to measure the field radiated by the antenna in the region of Fresnel diffraction and to compute the far-field patterns.

This application will be described in chapter 8 with more details.
1.3.11 Antenna Near-Field Diagnostics, Aperture Field Reconstruction

As the main subject of this dissertation, this issue will be detailed in the following chapters. For the different antennas, the different holographic techniques will be discussed.
CHAPTER 2

SIMULATIONS BY CONVENTIONAL MICROWAVE HOLOGRAPHIC TECHNIQUE

The conventional microwave holographic approach is directly derived from early optical holographic techniques. The wavelength of a light wave is much shorter than microwaves. The phase information on the holograms in the optics scene, can only be recorded (measured) indirectly. Because it was difficult, in the earlier microwave age, to directly measure phase of an electromagnetic field, the concept of optical holography was employed to the microwave measurements. The phase information was not directly measured. Rather, it is implicitly acquired by way of interference with a second reference radiation source contained in the holograms. Because it was easy to record intensity (magnitude only), it was still a very useful tool for surface distortion diagnosis of highly directive antennas, such as parabolic reflector antennas.

The concept of the microwave holography has been evolved so that both intensity and phase information are obtained by way of direct measurements. The word “holography” is still used for the modern age of microwave measurements. But the concept of measuring “intensity only” of electromagnetic fields in microwave holographic technique is no longer used in the microwave community.

Modern microwave techniques, such as vector network analysers, have made microwave measurements much easier with high accuracy, especially the
measurements of phase. The conventional microwave holography is no longer as useful as it was. But it is still worthwhile to describe here the principle and the concept of the early conventional microwave holography.

2.1 HOLOGRAM FORMATION PROCESS

The basic microwave system used to form the hologram of an antenna is shown in Fig 2.1. The antenna is located at the origin $O$, and the field intensity is scanned at a plane $z=D$. An ideal point source which is static provides the reference wave which is obtained from the same signal generator via a directional coupler. The hologram signal is sampled at regular intervals during the scanning and the demodulated signal is stored for subsequent processing. The distance $d$ between test and reference antennas is chosen carefully to form the so-called off-axis
reference wave and to ensure adequate separation of the reconstructed image from the obscuration of reference background.

As shown in Fig 2.1, the hologram coordinates are \( x_1 \) and \( y_1 \). Hence the spherical off-axis reference wave will be of the form

\[
F_R = B \exp \left( -jk \sqrt{(x_1 - d)^2 + y_1^2 + D^2} \right)
\]

(2.1)

where \( B \) is a constant and \( k \) is the wavenumber. The above expression can be approximated, if the hologram plane is far away from the source plane, i. e.

\[
F_R = B \exp \left( -j k \left( D + \frac{(x_1 - d)^2 + y_1^2}{2D} \right) \right), \quad D >> (x_1 - d), y_1
\]

(2.2)

It can be further simplified to

\[
F_R = B_1 \exp \left( -j k \left( \frac{x_1^2 + y_1^2}{2D} \right) \right) \exp (jkc x_1)
\]

(2.3)

where

\[
c = \frac{d}{D}
\]

(2.4)

is a constant denoting the linear phase change in the \( x_1 \) direction, and \( B_1 \) is another constant

\[
B_1 = B \exp (-jk(D + \frac{d^2}{2D}))
\]

(2.5)
The field radiated by the antenna on the hologram plane is denoted $F_O(x_1, y_1)$. The resulting hologram distribution is therefore

$$H(x_1, y_1) = |F_O|^2 + |F_R|^2 + F_O^* F_R + F_O F_R^*$$

$$= |F_O|^2 + |B_1 \exp \left(-j \frac{k x_1^2 + y_1^2}{2 D}\right) \exp(j k c x_1)|^2$$

$$+ B_1 F_O^* \exp \left(-j \frac{k x_1^2 + y_1^2}{2 D}\right) \exp(j k c x_1)$$

$$+ B_1 F_O \exp \left(j \frac{x_1^2 + y_1^2}{2 D}\right) \exp(-j k c x_1)$$

(2.6)

where $F_O(x_1, y_1)$ is the field distribution of the antenna under test in the far-field hologram plane, and $^*$ denotes conjugate of the complex function.

### 2.2 IMAGE RECONSTRUCTION

The processing of data in the form of (2.6) can be achieved by using the fast Fourier transform (FFT) algorithm. The second term of (2.6) is the power distribution of the reference wave, and can be removed computationally, giving

$$H(x_1, y_1) = |F_O|^2 + B_1 F_O^* \exp \left(-j \frac{k x_1^2 + y_1^2}{2 D}\right) \exp(j k c x_1)$$

$$+ B_1 F_O \exp \left(j \frac{x_1^2 + y_1^2}{2 D}\right) \exp(-j k c x_1)$$

(2.7)

If an image distribution is required at the output, then the input to the FFT must be proportional to the Fourier transform of the image. The complex conjugate of the Fourier transform of image is also an acceptable function since this only produces a spatial reversal of the image.
Because the far field pattern of most directive antennas is the Fourier transform of aperture distribution (This will be proved in later chapters), it is necessary to change the coordinates \(x_1\) and \(y_1\) in the hologram plane to the angular coordinates. From Fig 2.1, it is seen that

\[
\begin{align*}
\begin{cases}
x_1 &= r \sin \theta \cos \phi \\
y_1 &= r \sin \theta \sin \phi \\
D &= r \cos \theta
\end{cases}
\end{align*}
\tag{2.8}
\]

Let

\[
\begin{align*}
\begin{cases}
k_x &= k \sin \theta \cos \phi \\
k_y &= k \sin \theta \sin \phi
\end{cases}
\end{align*}
\tag{2.9}
\]

then

\[
\begin{align*}
\begin{cases}
x_1 &= \frac{r}{k} k_x \\
y_1 &= \frac{r}{k} k_y \\
r &\approx D
\end{cases}
\end{align*}
\tag{2.10}
\]

The exponential part of (2.7) becomes

\[
\exp \left( j k \frac{x_1^2 + y_1^2}{2 D} \right) \exp(-j k c x_1) = \exp \left( j D \frac{k_x^2 + k_y^2}{2 k \cos^2 \theta} \right) \exp (-j d \frac{k_x}{\cos \theta}) \tag{2.11}
\]

If the hologram plane is in far-field zone of the antenna, then \(\cos \theta \approx 1\). Substitution of (2.10) and (2.11) back into (2.7) gives
\[ H(k_x, k_y) = \left| F_O \right|^2 + B_1 F_O^* \exp \left( -jD \frac{k_x^2 + k_y^2}{2k} \right) \exp (jd k_x) \]
\[ + B_1 F_O \exp \left( jD \frac{k_x^2 + k_y^2}{2k} \right) \exp(-jd k_x) \]  \hspace{1cm} (2.12)

Premultiplication by the negative exponent

\[ \exp \left( -jD \frac{k_x^2 + k_y^2}{2k} \right) \]  \hspace{1cm} (2.13)

gives a new distribution

\[ H_1(k_x, k_y) = \left| F_O \right|^2 \exp \left( -jD \frac{k_x^2 + k_y^2}{2k} \right) + B_1 F_O \exp (jd k_x) \]
\[ + B_1 F_O^* \exp \left( -j \frac{D}{k} (k_x^2 + k_y^2) \right) \exp (jd k_x) \]  \hspace{1cm} (2.14)

Taking the inverse Fourier transform of the above expression, gives

\[ F^{-1} \{H_1(k_x, k_y)\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left| F_O(k_x, k_y) \right|^2 \exp \left( -jD \frac{k_x^2 + k_y^2}{2k} \right) \]
\[ + B_1 F_O^* (k_x, k_y) \exp \left( -j \frac{D}{k} (k_x^2 + k_y^2) \right) \exp (jd k_x) \]
\[ + B_1 F_O (k_x, k_y) \exp (-jd k_x) \exp (j(k_x \xi + k_y \eta)) \exp (j(k_x \xi + k_y \eta)) d_{k_x} d_{k_y} \]  \hspace{1cm} (2.15)

By using the convolution property of the Fourier transform, and the following identities:
\[ F^{-1} \{ \exp (\pm j\omega) \} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp (\pm j\omega) \exp (j\omega t) d\omega = \delta(t \pm a) \quad (2.16) \]

\[ F^{-1} \{ \frac{A \cdot (1 \pm j)}{a} \exp \left( \frac{\pm j\omega^2}{4a^2} \right) \} = A \exp (\pm ja^2 t^2) \quad (2.17) \]

the equation (2.15) can be further simplified as

\[ F^{-1} \{ H_1(k_x, k_y) \} = A P(\xi, \eta) \otimes P^* (-\xi, -\eta) \otimes \exp \left( j \frac{k}{2D} (\xi^2 + \eta^2) \right) + B P^* (-\xi, -\eta) \otimes \exp \left( j \frac{k}{4D} (\xi^2 + \eta^2) \right) \otimes \delta(\xi + d) + C P(\xi, \eta) \otimes \delta(\xi - d) \quad (2.18) \]

where \( A, B, \) and \( C \) are constants, and where \( \otimes \) denotes the convolution process and \( P(\xi, \eta) \) is the Fourier transform of the distribution of the far-field pattern \( F_d(k_x, k_y) \) radiated by the antenna and it is the original object field distribution. The first

![Image distribution on \( \xi \)-axis](image)

Fig 2.2 Image distribution on \( \xi \)-axis

term of (2.18) contributes an auto-correlation function of the aperture distribution which is centred about \( \xi=0 \). The second term is an image of the aperture.
distribution situated at $\xi=-d$, whereas the desired image is contained in the third term and positioned at $\xi=+d$, as shown in Fig 2.2.

In practice, when the FFT is applied to solve the Fourier transform, the input spatial frequencies of the FFT are band-limited. By application of the sampling theorem [112] (this will be discussed in more detail in Chapters 4 and 5), the required spatial range $S$ in the $\xi-\eta$ plane is given

$$S = \frac{2\pi}{\Delta k_x} \tag{2.19}$$

where $\Delta k_x$ is the sampling interval in the $k_x-k_y$ plane. From (2.19) the following relationship between $\Delta \xi$ and $\Delta k_x$ can also be derived

$$\Delta \xi = \frac{S}{N_x} = \frac{2\pi}{\Delta k_x N_x} \tag{2.20}$$

where $\Delta \xi$ is sampling interval of $\xi$, and $N_x$ is total sampling number on $\xi$–axis in the spatial domain $\xi-\eta$. It is important to consider the positions of the reconstructed aperture distribution. As indicated by expression (2.18), the output of the FFT includes three functions. Only the third term is the desired aperture distribution which is separated by making the appropriate choice of $d$. According to (2.19) and (2.20), the values for $d$ should be in the range of $(S/2)$. For the given sampling interval of $\Delta k_x$ and sampling number of $N_x$, the $S$ is determined by (2.19). Then the choice of $d$ should be made to avoid image overlapping or violation. Fig 2.3 illustrates the effects of three different values of $d$. In Fig 2.3(a) the value of $d=d_1$ is too small and the two images of the aperture distribution are degraded by
the auto-correlation response. If a large value of $d=d_2$ is used, then the violation of
the sampling theorem occurs, and image degradation is incurred through

![Diagram](image)

**Fig 2.3 Effect of sampling interval and the position of reference source on image separation**

interference with the adjacent output planes as shown in Fig 2.3(b). A correct
choice of $d=d_3$ ensures good image separation as in Fig 2.3(c).

The conventional microwave holographic aperture field reconstruction
procedure can be now summarised as:

1. Scanning and recording the field intensity (magnitude only) radiated by
   both test and reference antennas to form the intensity hologram;

2. Subtracting the reference field intensity distribution from the hologram
distribution;
(3). Transferring the coordinate system from $x_i-y_i$ to $k_x-k_y$;

(4). Multiplying hologram distribution with the negative exponential function of (2.13);

![Diagram of holographic aperture field reconstruction procedure]

Fig 2.4 Block diagram of conventional holographic aperture field reconstruction procedure

(5). Taking the inverse Fourier transform by FFT algorithm;

(6). Separating and recognising the aperture field image in the output of step (5).

The above procedure is shown as block diagram in Fig 2.4.
2.3 SIMULATION RESULTS

In order to demonstrate that the aperture field distribution can be recovered from measured set of data by the conventional microwave holographic method, it is necessary to perform a simulation. In this section, a simulation of a half wave dipole antenna is given.

2.3.1 Far-field Pattern of the Dipole Antenna

A typical far-field expression of a dipole antenna, in spherical coordinates system as shown in Fig 2.5(a) is written as

\[
\begin{cases}
E_{\phi'} = \frac{e^{jkr'}}{r'} \frac{\cos(kl') \cos(\phi') - \cos(\phi))}{\sin(\theta')} \\
E_{\psi} = 0 \\
E_{\psi'} = 0
\end{cases}
\]  

(2.21)

where \( k \) is the wavenumber and \( l \) is the length of the dipole. The \( E \)-field pattern may be transformed to the Cartesian components by following transformation equations:

\[
\begin{align*}
E_x' &= E_{r'} \sin\theta' \cos\phi' + E_{\theta'} \cos\theta' \cos\phi' - E_{\psi} \sin\phi' \\
E_y' &= E_{r'} \sin\theta' \sin\phi' + E_{\theta'} \cos\theta' \sin\phi' + E_{\psi} \cos\phi' \\
E_z' &= E_{r'} \cos\theta' - E_{\psi} \sin\theta'
\end{align*}
\]

(2.22)

Thus, from (2.21) and (2.22), the Cartesian components of the pattern can be written
\[
\begin{align*}
    E_x' &= E_0' \cos \theta' \cos \phi' \\
    E_y' &= E_0' \cos \theta' \sin \phi' \\
    E_z' &= -E_0' \sin \theta'
\end{align*}
\] (2.23)

In practice, the scanning plane is always perpendicular to the direction of maximum radiation. If the dipole antenna is placed in alignment with the \(y\)-axis as shown in Fig 2.5(b), the necessary coordinate transformation is needed. In Fig 2.5(a), the rectangular coordinates can be written in spherical system:

\[
\begin{align*}
    x' &= r' \sin \theta' \cos \phi' \\
    y' &= r' \sin \theta' \sin \phi' \\
    z' &= r' \cos \theta'
\end{align*}
\] (2.24)

From Fig 2.5(b), the following relationships are obtained:

\[
\begin{align*}
    x &= -x' \\
    y &= z' \\
    z &= y'
\end{align*}
\] (2.25)

Substituting (2.24) into (2.25), and considering \(r=r'\), the final transform of trigonometric function in these two spherical systems of Fig 2.5 (a) and (b) is written as:
\[
\begin{align*}
\cos \theta' &= \sin \theta \sin \phi \\
\sin \theta' &= \pm \sqrt{\sin^2 \theta \cos^2 \phi + \cos^2 \theta} \\
\cos \phi' &= \sin \theta \cos \phi \\
\sin \phi' &= \pm \sqrt{\sin^2 \theta \cos^2 \phi + \cos^2 \theta} \\
\end{align*}
\]

(2.26)

**Fig 2.5 Dipole antenna in two different coordinates systems**

By using (2.23), (2.25), and (2.26), the Cartesian components of \(E\)-field in the coordinate system of Fig 2.5(b) are finally expressed as

\[
\begin{align*}
E_x &= e^{jkr} \frac{\cos \left( \frac{k}{2} \sin \theta \sin \phi \right)}{r} \frac{\cos \left( \frac{k}{2} \cos \phi \right)}{\sin^2 \theta \cos^2 \phi + \cos^2 \theta} \sin \theta \sin \phi \sin \theta \cos \phi \\
E_y &= e^{jkr} \frac{\cos \left( \frac{k}{2} \sin \theta \sin \phi \right)}{r} \left( \cos \left( \frac{k}{2} \right) \cos \left( \frac{k}{2} \sin \theta \sin \phi \right) \right) \\
E_z &= e^{jkr} \frac{\cos \left( \frac{k}{2} \sin \theta \sin \phi \right)}{r} \frac{\cos \left( \frac{k}{2} \cos \phi \right)}{\sin^2 \theta \cos^2 \phi + \cos^2 \theta} \sin \theta \sin \phi \cos \theta \\
\end{align*}
\]

(2.27)
2.3.2 Aperture Field Reconstruction of a Half Wave Dipole

For simplicity, consider the y–component in (2.27) as an example and rewrite as

\[
E_y = \frac{e^{jkr}}{r} \left( \cos\left(\frac{kl}{2}\right) - \cos\left(\frac{kl}{2}\sin\theta\sin\phi\right) \right) \tag{2.28}
\]

From this expression, the far-field distribution of the half-wavelength dipole on the plane of \( D=20\lambda \) is calculated. The contour plot of intensity only (magnitude) is shown in Fig 2.6. The sampling interval in this case is 0.25\( \lambda \) in both the \( x \) and \( y \) directions. The label numbers in both axes are sampling points. The total sampling points \( N_x \times N_y \) are 256\( \times \)256. Fig 2.7 and Fig 2.8 are the \( x=0 \), and \( y=0 \) cuts. It is seen that the maximum value of the distribution is in the centre (\( x=y=0 \)). But on the edge of the distribution, the values are reduced by only about 5 and 17 dBs in the
\( x \)- and \( y \)- directions, respectively, which means that the hologram range is not big enough. Fig 2.9 shows the microwave hologram of the half wave dipole with an ideal point source as reference located at \( d=1\lambda \). The distance of hologram from the antenna plane is \( 20\lambda \).

Following the procedure described in Fig 2.4, the aperture field reconstruction of the dipole antenna has been calculated from the hologram above. The reconstructed holographic image of expression (2.18), which is the mixed
pattern of the convolution of aperture field and its conjugate, and also the reference distribution, is plotted in Fig 2.10. The centre region in Fig 2.10 is the auto-correlation of reference source. The bright region on the right side is the recovered antenna aperture distribution. The aperture field distribution is similar to the near-field distribution of a point source with a broader extent in the y-direction as shown in Fig 2.11 of y=0 cut.

In order to improve the reconstruction of aperture field, various simulations were also carried out with different sampling intervals, number of sampling points, and hologram distances. But the recovered aperture distributions were not improved significantly even with larger hologram range.
2.4 CONCLUSIONS

It is demonstrated that the conventional microwave holographic techniques can be used to recover the aperture field distributions for some antennas [12, 13, 14]. The resolution was poor for the dipole antenna. This is caused by a stringent condition of the Fourier transform relationship between far-field pattern and aperture field for such antenna could not be met satisfactorily.
The application of the technique is restricted to some circumstances. The distance from the transmission antenna to the measurement plane should be far enough to satisfy the condition \( r \rightarrow D \) as shown in equation (2.10). If the main beam of the radiation pattern is narrow, the condition of \( \cos \theta \approx 1 \) can be easily realised with relatively small measurement region. But it is difficult to satisfy this restriction with a dipole antenna due to its broader pattern. In general, the technique is suitable for the reflector antennas or other similar antennas with narrow beamwidth and high gains. It is difficult to use this technique for broad beam antennas or antenna arrays.
CHAPTER 3

GENERAL FORMULATION OF HUYGENS’ PRINCIPLE FOR THE ELECTROMAGNETIC FIELD

3.1 GREEN’S THEOREM IN VECTOR FORM

Green’s theorem is a basic mathematical tool to solve a variety of physical problems. Especially, it is widely used for modelling electromagnetic and antenna problems. It is necessary to describe it here in the clear mathematical approach for the usage of the following chapters.

To start with, following identity is introduced:

\[ \nabla \cdot (M \times \nabla \times N) = \nabla \times (M \cdot \nabla \times N) - M \cdot \nabla \times \nabla \times N \]  \hspace{1cm} (3.1)

where \( M \) and \( N \) are vector functions. Applying Gaussian (divergence) theorem to the above equation, the left-hand side becomes

\[ \iiint_{V} \nabla \cdot (M \times \nabla \times N) \, dv = \iiint_{S} M \times \nabla \times N \cdot dS \]  \hspace{1cm} (3.2)

Then, the first Green’s identity of vector form is obtained:

\[ \iiint_{V} (\nabla \times M \cdot \nabla \times N - M \cdot \nabla \times \nabla \times N) \, dv = \iiint_{S} M \times \nabla \times N \cdot dS \]

Swapping \( M \) with \( N \) in equation (3.3), another identity follows:
\[ \iiint_{V} (\nabla \times \mathbf{N} \cdot \nabla \times \mathbf{M} - \mathbf{N} \cdot \nabla \times \nabla \times \mathbf{M}) \, dV = \iiint_{S} \mathbf{N} \times \nabla \times \mathbf{M} \cdot dS \quad (3.4) \]

Subtracting (3.4) with (3.3) results in the Green’s theorem in vector form

\[ \iiint_{V} (\mathbf{M} \cdot \nabla \times \nabla \times \mathbf{N} - \mathbf{N} \cdot \nabla \times \nabla \times \mathbf{M}) \, dV = \iiint_{S} (\mathbf{N} \times \nabla \times \mathbf{M} - \mathbf{M} \times \nabla \times \mathbf{N}) \cdot dS \quad (3.5) \]

Equation (3.5) states that the volume integration of the vector qualities can be obtained from the surface integration, where the surface “S” encloses the volume “V”.

### 3.2 INTEGRAL EQUATION FORMULATION

The boundary value problem of a conducting or dielectric body immersed in a specified electromagnetic field may be formulated by expressing the scattered field in terms of an integral of a source distribution over the surface or throughout the volume of the scatterer. The boundary condition at every point on the scatterer then leads to an integral in terms of the source function. The integral equation formulation, unlike that of differential equations, contains the boundary conditions of the problem implicit in the formulation, and the mathematical problem of finding a solution to the integral equation is the last step in the problem. The approach to scattering problems forms the principal basis for modern approximation methods.

The formal development of the integral equations for scattering dielectric bodies and conductors will now be carried out.
Consider a homogeneous, isotropic, source-free dielectric body over volume $V$ bounded by a surface $S$. Let $\mathbf{E}$ and $\mathbf{H}$ represent an electromagnetic field interior to the volume. Then throughout $V$, the vectors $\mathbf{E}$ and $\mathbf{H}$ satisfy the vector wave equation

$$\nabla \times \nabla \times \mathbf{F} - k^2 \mathbf{F} = 0 \quad (3.6)$$

where $k=\omega(\mu\varepsilon)^{1/2}$ is the wavenumber in the dielectric medium.

Next, consider a vector function $\mathbf{G}$, the Green’s function, satisfying equation (3.6) except at an observation point $p$ interior to $V$ where $\mathbf{G}$ is singular. If a small sphere of surface $\sigma$, with $p$ at the centre is removed from $V$ as shown in Fig 3.1, the new volume $V_1$ has $S$ and $\sigma$ as boundary surfaces. Then throughout $V_1$, the vectors $\mathbf{E}, \mathbf{H}$ and $\mathbf{G}$ satisfy equation (3.6).

If the vector form of Green’s theorem equation (3.5) is applied to the quantities $\mathbf{F}$ and $\mathbf{G}$, where $\mathbf{F}$ is any vector satisfying equation (3.6), the result is
\[ \iiint_{V_1} (F \cdot \nabla' \times \nabla' \times G - G \cdot \nabla' \times \nabla' \times F) \, dv = \iiint_{S+\sigma} (G \times \nabla' \times F - F \times \nabla' \times G) \cdot \mathbf{n}' \, dS' \]  

(3.7)

where \( \mathbf{n}' \) is outward unit normal vector of surface \( S+\sigma \).

The volume integral vanishes by virtue of equation (3.6) and the result may be written

\[ \iiint_{\sigma} (G \times \nabla' \times F - F \times \nabla' \times G) \cdot \mathbf{n}' \, dS' = \cdot \iiint_{S} (G \times \nabla' \times F - F \times \nabla' \times G) \cdot \mathbf{n}' \, dS' \]  

(3.8)

On \( \sigma \) and \( S \) the normal unit vectors \( \mathbf{n}' \) are outward from \( V_1 \) as shown in Fig 3.1. \( G \) is now selected to have a singularity at \( p \) so that

\[ \lim_{\sigma \to 0} \iiint_{\sigma} (G \times \nabla' \times F - F \times \nabla' \times G) \cdot \mathbf{n}' \, dS' = \text{Finite limit} \neq 0 \]  

(3.9)

To do this, choose

\[ G = \nabla' \times (a \, g) \]  

(3.10)

where

\[ g = \frac{e^{-j k R}}{4 \pi R} = \frac{e^{-j k |r \cdot r'|}}{4 \pi |r \cdot r'|} \]  

(3.11)
\( \mathbf{a} \) is a constant unit vector; and \( R \) is radial distance between \( p \) and the point where \( \mathbf{G} \) is evaluated.

Clearly \( \mathbf{G} \) satisfies equation (3.6) for \( R>0 \), in fact, \( \mathbf{G} \) is proportional to the magnetic field intensity due to an electric dipole of vector potential \( (a \mathbf{g}) \).

With \( g \) defined, the Green’s function takes the form

\[
\mathbf{G} = -\left( j \mathbf{k} + \frac{1}{R} \right) \frac{e^{jkR}}{4 \pi R} (\hat{\mathbf{R}} \times \mathbf{a}) \tag{3.12}
\]

where “\( \mathbf{R} \) hat” is the \( R \)-directed unit vector. These vectors are conveniently represented in spherical coordinates in Fig 3.2. The system is oriented so that the vector \( \mathbf{a} \) lies along the positive \( z \)-axis. From Fig 3.2

\[
\hat{\mathbf{R}} \times \mathbf{a} = -\hat{\phi} \sin \theta \tag{3.13}
\]

Then \( \mathbf{G} \) has only a \( \phi \) component and may be expressed as
\[ G = \hat{\phi} \left( jk + \frac{1}{R} \right) \frac{e^{jkR}}{4\pi R} \sin \theta \]  

(3.14)

Since only the radial component of \( G \times \nabla' \times F \) will contribute to the integral over \( \sigma \), and \( G \) has only a \( \phi \) component, only the \( \theta \) component of \( \nabla' \times F \) will contribute to the integral.

In the limit as \( \sigma \to 0 \), \( \nabla' \times F \) will approach a constant direction and magnitude throughout \( \sigma \) since \( F \) satisfies the wave equation at \( p \). The radial component of \( G \times \nabla' \times F \) on \( \sigma \) is

\[ (G \times \nabla' \times F)_r = -\left( jk + \frac{1}{R} \right) \frac{e^{jkR}}{4\pi R} \sin \theta \]  

(3.15)

The integration of this term over the spherical surface \( \sigma \), after expressing the constant vector \( \nabla' \times F \) in rectangular components gives

\[ \lim_{\sigma \to 0} \iiint_{\sigma} (G \times \nabla' \times F \cdot \hat{n}' \, dS' = \lim_{\sigma \to 0} \iiint_{\sigma} (G \times \nabla' \times F \cdot \hat{R} \, dS' = \frac{2}{3} (\nabla' \times F) \cdot \hat{z} \]  

(3.16)

where,

\[ \hat{n}' = \hat{R} \]  

(3.17)

has been used.

To get the contribution from the term \( F \times \nabla' \times G \), note that only the \( \phi \)- and \( \theta \)-components of \( \nabla' \times G \) can contribute to the integral. From equation (3.14)
\[(\nabla' \times G)_\theta = 0 \quad (3.18)\]

and

\[(\nabla' \times G)_\theta = \frac{e^{jkR}}{4\pi R^3} \sin \theta + \frac{jk}{4\pi R} \left( \frac{1}{R} + jk \right) \sin \theta \quad (3.19)\]

Then, since (3.17),

\[
\int_{\sigma} (F \times \nabla \times G) \cdot \hat{R} \, dS' = \int_{\sigma} (\nabla' \times G)_\phi \, F_{\phi} \, dS'
\]

\[
= -\frac{e^{jkR}}{4\pi} \left( \frac{1}{R^2} + \frac{jk}{R} - k^2 \right) \int_{0}^{\pi} \sin \theta d\theta \int_{0}^{2\pi} F_{\phi} R \sin \theta \, d\phi
\]

(3.20)

From Stokes’ theorem and the first law of the mean

\[
\int_{0}^{2\pi} F_{\phi} R \sin \theta d\phi = \int_{0}^{2\pi} F_{\phi} \, (R \sin \theta) = \pi (R \sin \theta) \frac{\nabla \times F}{\hat{z}}
\]

(3.21)

where

\[\quad (\nabla' \times F) \cdot \hat{z} \quad (3.22)\]

is evaluated at a point in the circle of radius \(R \sin \theta\). Then as \(R \to 0\)

\[
\lim_{\sigma \to 0} \int_{\sigma} F \times \nabla' \times G \cdot \hat{n}' \, dS' = \frac{1}{3} \left( \nabla' \times F \right) \cdot \hat{z}
\]

(3.23)
Combining this result with equation (3.16) gives

\[
\lim_{\sigma \to 0} \iint_{\sigma} (G \times \nabla' \times F - F \times \nabla' \times G) \cdot \hat{n}' \, dS' = (\nabla' \times F) \cdot \hat{z} \quad (3.24)
\]

Then, from equation (3.8), as \(\sigma \to 0\)

\[
\cdot (\nabla' \times F) \cdot a = \iint_{S} (G \times \nabla' \times F - F \times \nabla' \times G) \cdot \hat{n}' \, dS' \quad (3.25)
\]

This equation expresses the component of \(\nabla' \times F\) in the direction of \(a\) in terms of the distributions \(F\) and \(\nabla' \times F\) over the bounding surface \(S\). Moreover, only the tangential components of \(F\) and \(\nabla' \times F\) contribute to the surface integral regardless of the choice of the constant vector \(a\). Since the direction of \(a\) is arbitrary, equation (3.25) expresses, in fact, the total vector \(\nabla' \times F\) at the point \(p\) in terms of the tangential components of \(S\).

The vector \(F\) in equation (3.25) may represent either the field vector \(E\) or \(H\). Hence the electric or magnetic field intensity at an interior point of any homogeneous source-free volume \(V\) is expressed in terms of the tangential components of \(E\) and \(H\) on the boundary surface.

It should be remarked that the choice of the form of the Green's function is not restricted to the one used here. Any solution of the vector wave equation (except at the point \(p\)) which behaves like a Dirac Delta function \(\delta\) at \(p\) may be used as a Green's function.
An important consequence of equation (3.25) is that the point \( p \) must be interior to \( V \) for the surface integral to have a non-zero value. If \( p \) is external to \( V \), \( G \) satisfies the vector wave equation throughout \( V \) and the surface integral is zero, i.e.

\[
\iint_S (G \times \nabla' \times F - F \times \nabla' \times G) \cdot \hat{n}' \, dS' = 0 \tag{3.26}
\]

3.3 PHYSICAL OPTICS INTEGRALS

The physical optics approach is a very good mathematical tool for the solution of scattering problems. This method of solution yields enormously increased accuracy in many important cases without the introduction of excessive analytical difficulties.

The basis for the use of physical optics is Huygens’ principle, which was stated in previous chapters. For unpolarised light waves this principle is employed with a scalar wave function and yields diffraction patterns through apertures and around obstacle without the complication of vector fields. The principle is easily extended to vector fields by means of the Green’s function technique. This may be done to yield the electric or magnetic field intensity at a point in vector form, or an arbitrary component of either field intensity, depending on the choice of the form of the Green’s function. The latter representation is obtained by application of equation (3.25) and (3.26). In terms of the electric field intensity along the fixed vector \( \mathbf{a} \) at \( p \), by letting \( \mathbf{F} \) represent \( \mathbf{H} \), the equation (3.25) becomes
\[ -E \cdot a(p) = \frac{1}{j \omega E} \iint_S \left( G \times \nabla' \times H - H \times \nabla' \times G \right) \cdot \hat{n}' \, dS' \] (3.27)

The surface \( S \) in equation (3.27) is the surface bounding a source-free volume containing the point \( p \). In other words, the closed surface separates the region containing \( p \) from the region containing the field source.

If, in particular, \( S \) is chosen to coincide with a wave front surrounding the source, equation (3.27) may be regarded as an extension of Huygens' principle to vector fields. It should be emphasized that equation (3.27) is exact. However, in applications, it is usually necessary to approximate the field component appearing in the integrand, resulting in an approximated value for the quantity \( E \cdot a \).

Although the quantities in the integrand in equation (3.27) are related by Maxwell's equations, and are not independent, it is convenient to discuss the integral in two parts. One part presents the partial field at point \( p \) due to the tangential component of electric field intensity on \( S \), and the other part contributing to partial field due to the tangential magnetic field on \( S \). For the first partial field

\[ -E \cdot a^{(1)}(p) = \frac{1}{j \omega E} \iint_S \left( G \times \nabla' \times H \right) \cdot \hat{n}' \, dS' \] (3.28)

Using the relation of \( \nabla' \times H \) to \( E \) for the harmonic time dependent case, the above equation may be written

\[ -E \cdot a^{(1)}(p) = \iint_S \left( G \times E \right) \cdot \hat{n}' \, dS' \] (3.29)

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Since, by definition, equation (3.10),

\[ G = \nabla \times (a \times g) = \nabla' g \times a + g \times \nabla' a = -a \times \nabla' g \]  

(3.30)

where \( g \) is the Green’s function in scalar form (3.11), the integrand of equation (3.29) may be written

\[ G \times E \cdot \hat{n}' = a \cdot (\nabla' g \times \hat{n}' \times E) \]  

(3.31)

Then equation (3.29) can be written

\[-E \cdot a^{(1)}(p) = \iint_S a \cdot (\nabla' g \times \hat{n}' \times E) dS' \]  

(3.32)

Since the arbitrary constant vector \( a \), taken of unit length for convenience, may be brought outside the integral sign, equation (3.29) finally becomes

\[-E^{(1)}(p) = \iint_S (\nabla' g \times \hat{n}' \times E) dS' \]  

(3.33)

Equation (3.33) states that a distribution of electric field \( E \) over a surface \( S \) contributes a radiated field of electric \( E^{(1)} \) (a vector) at the point \( p \).

The contribution to the \( a \) component of the electric field at \( p \) due to the surface distribution of magnetic field is given by remainder of the integral in equation (3.27).

\[-E \cdot a^{(2)}(p) = -\frac{1}{j_{0}\varepsilon} \iint_S (H \times \nabla' G) \cdot \hat{n}' dS' \]  

(3.34)

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It would be desirable to express the integrand in the above equation in the form of a scalar product of unit vector $a$ and another vector. To do this, a dyadic is introduced and the following relation is obtained:

$$H \times (\nabla' \times G) \cdot \hat{n}' = a \cdot \underline{T'} \cdot (H \times \hat{n}')$$  \hspace{1cm} (3.35)

where $\underline{T'}$ is a dyadic. By vector manipulation it is found that $\underline{T'}$ is symmetric, and is given by

$$\underline{T'} = \underline{I}' \cdot \nabla'^2 g \cdot \nabla' \nabla' g$$  \hspace{1cm} (3.36)

where $\underline{I}'$ is the identity (unit) dyadic.

Then equation (3.34) gives, in this notation

$$- E \cdot a^{(2)}(p) = - \frac{1}{f \omega \varepsilon} \iint_{S} a \cdot \underline{T'} \cdot (H \times \hat{n}') dS'$$  \hspace{1cm} (3.37)

and, since $a$ is arbitrary, the vector partial field $E^{(2)}$ is

$$- E^{(2)}(p) = \frac{j}{\omega \varepsilon} \iint_{S} \underline{T'} \cdot (H \times \hat{n}') dS'$$  \hspace{1cm} (3.38)

Combining equations (3.33) and (3.38), the total electric field intensity at point $p$ is

$$- E(p) = - \iint_{S} \left( \nabla' g \times (\hat{n}' \times E) + \frac{j}{\omega \varepsilon} \underline{T'} \cdot (H \times \hat{n}') \right) dS'$$  \hspace{1cm} (3.39)
It should be noted that the differentiation of gradient or dyadic operates on $g$ only and must not operate on the vectors $E$ and $H$ in equation (3.39). Since $H$ is a function of the surface coordinates and not of the coordinates of $p$, this can give rise to ambiguity in the application of equation (3.39), since the surface vector $H$ is placed on the right-hand side of $T'$ in the integrand. This difficulty can be avoided by making use of the symmetry of $T'$.

First note that in the expression

$$g = \frac{e^{jkR}}{4\pi R}$$

(3.40)

$R$ is measured from the point $p$ to the element $dS'$ of the surface, and $p$ is the observation point. The differential operations as they appear in equation (3.39) are taken on the surface $S$. If, instead, the operations are modified to apply to $g$ at the point $p$, that is, the direction of positive $R$ is reversed,

$$\begin{align*}
\nabla g &= -\nabla' g \\
\nabla^2 g &= \nabla'^2 g \\
T &= T'
\end{align*}$$

(3.41)

where $\nabla$ is the operation on the observation coordinates $p$. Then equation (3.39) is modified to become

$$E(p) = \int_S \left( \nabla g \times (\hat{n}' \times E) + \frac{j}{\omega \epsilon} T \cdot (\hat{n}' \times H) \right) dS'$$

(3.42)
By using the vector identities,

\[
\begin{align*}
\nabla \psi \times c &= \nabla \times (c \psi), \text{ if } c \text{ is a constant vector} \\
\nabla \times \nabla \times M &= \nabla (\nabla \cdot M) - \nabla^2 M
\end{align*}
\]  

(3.43)

and noting that the operator \( \nabla \) only acts on the \( g \) function and can be brought out of the integral sign, and the tangential fields \( (\hat{n}' \times E) \) and \( (\hat{n}' \times H) \) can be regarded as constant vectors for the differentiation operations, second form of equation (3.42) becomes

\[
E(p) = \nabla \times \int_S \hat{n}' \times E \, g dS' + \frac{j}{\varepsilon} \nabla \times \nabla \times \int_S \hat{n}' \times H \, g dS'
\]

(3.44)

If \( p \) is external to \( V \), \( G \) satisfies the vector wave equation (3.6) throughout \( V \) and the surface integral (3.26) is zero, i.e.

\[
\nabla \times \int_S \hat{n}' \times E \, g dS' + \frac{j}{\varepsilon} \nabla \times \nabla \times \int_S \hat{n}' \times H \, g dS' = 0
\]

(3.45)

Since equation (3.44) and (3.45) contain both electric and magnetic field components on the surface \( S \), they are completely general and exact, however complicated.

The magnetic field expression can be derived in the similar may as in electric field. Alternatively, it can be obtained by applying equation (3.44) to the Maxwell’s equations. For brevity, the procedure is omitted here, and the expression is directly written:
$$H(p) = \nabla \times \iint_{S} \hat{n} \times H \, gdS' - \frac{i}{\omega \mu} \nabla \times \nabla \times \iint_{S} \hat{n} \times E \, gdS' \quad (3.46)$$

If $p$ is outside of volume $V$, the right hand side of equation (3.46) will be zero,

$$\nabla \times \iint_{S} \hat{n} \times H \, gdS' - \frac{i}{\omega \mu} \nabla \times \nabla \times \iint_{S} \hat{n} \times E \, gdS' = 0 \quad (3.47)$$

These developments will be used in the next chapter.
CHAPTER 4

MICROWAVE HOLOGRAPHIC METHOD FOR SURFACE DIAGNOSIS OF LARGE REFLECTOR ANTENNAS

4.1 INTRODUCTION

Large reflector antennas are commonly used for space technologies, radio astronomy, and satellite communications. Reflector surface errors can considerably affect the beam width, gain, sidelobe structure of an antenna's radiation pattern. Today's stringent system requirements demand that the losses due to surface inaccuracies be minimized. This can be done by identifying the location and amount of distortion on the reflector surface and then by correcting this. The conventional microwave holographic technique can be used for this task. Recently, Rahmat-Samii [113-116] has modified the conventional microwave holography and developed a more accurate approach, which is based on the physical optics integrals. This chapter describes the theoretical model and gives examples of this approach.

4.2 APPLICATION OF PHYSICAL OPTICS INTEGRAL TO REFLECTOR ANTENNAS

Consider in Fig 4.1 the illuminated surface $S_r$ of a perfectly conducting reflector entirely surrounded in free space by the imaginary aperture surface $S_a$. 
The surface $S_r+S_a$ encloses the free-space volume $V$. The feed sources of radiation illuminating the reflector are assumed to lie outside $V$. Applying the equations (3.44) and (3.45), developed in Chapter 3, to the total electric and magnetic fields over the closed surface $S_r+S_a$, following identities for the observation points $p(r)$ inside and outside the source-free volume $V$ can be obtained

$$\frac{j}{\omega \varepsilon_0} \nabla \times \nabla \times \int_{S_r+S_a} (\hat{n}' \times H(r')) g(r, r') dS'$$

$$+ \nabla \times \int_{S_r+S_a} (\hat{n}' \times E(r')) g(r, r') dS' = \begin{cases} E(r), & r \in V \\ 0, & r \notin V \end{cases}$$

(4.1)

where

$$g(r, r') = \frac{\exp \left(- j k |r - r'| \right)}{4\pi |r - r'|}$$

(4.2)

is Green’s function in spherical coordinate system. As usual, $\varepsilon_0$ and $\mu_0$ denote the permittivity and permeability, respectively, and $k=\omega(\mu_0\varepsilon_0)^{1/2}$ is the propagation

![Fig 4.1 Geometry of reflector surface $S_r$ capped by aperture surface $S_a$](image)

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constant (wavenumber) in free space. The zero result for \( r \) outside \( V \) is commonly called the extinction theorem, which has been derived for the Franz formulas in Chapter 3.

Because the reflector is perfectly conducting, the tangential component \( \hat{n} \times E \) equals zero on \( S_r \), and \( -\hat{n} \times H \) equals the surface current \( J \) on \( S_r \). Thus the extinction theorem of (4.1) can be rewritten as

\[
\frac{j}{\omega \varepsilon_0} \nabla \times \nabla \times \int_{S_r} J(r') \, g(r, r') \, dS' = \nabla \times \int_{S_a} \hat{n}' \times E(r') \, g(r, r') \, dS' + \frac{j}{\omega \varepsilon_0} \nabla \times \nabla \times \int_{S_r} \hat{n}' \times H(r') \, g(r, r') \, dS'
\]

(4.3)

The expression on the left side of (4.3) is merely the electric field \( E \) radiated by the current on the illuminated side of the reflector. Thus (4.3) can be written as

\[
E(r) = \frac{j}{\omega \varepsilon_0} \nabla \times \nabla \times \int_{S_r} J(r') \, g(r, r') \, dS' = \nabla \times \int_{S_a} \hat{n}' \times E(r') \, g(r, r') \, dS' + \frac{j}{\omega \varepsilon_0} \nabla \times \nabla \times \int_{S_r} \hat{n}' \times H(r') \, g(r, r') \, dS'
\]

(4.4)

Similarly, for the magnetic field \( H \) radiated by the illuminated side of the reflector, the dual equation is

\[
H(r) = \nabla \times \int_{S_r} J(r') \, g(r, r') \, dS' = \nabla \times \int_{S_a} \hat{n}' \times H(r') \, g(r, r') \, dS' + \frac{j}{\omega \varepsilon_0} \nabla \times \nabla \times \int_{S_r} \hat{n}' \times E(r') \, g(r, r') \, dS'
\]

(4.5)

Equation (4.4) and (4.5) state that the electric and magnetic fields determined by integration of current on the illuminated surface \( S_r \) of the reflector are identical to the fields determined by an aperture integration with the Franz formulas of the
total tangential fields over any surface $S_a$ which caps the reflector referring to Fig 4.1.

The far fields are found from (4.4) by letting $\mathbf{r}$ approach infinity, as follows. Let $R = |\mathbf{r} - \mathbf{r}'|$, then (4.2) can be written

$$g(R) = \frac{1}{4\pi} \frac{\exp(-jkR)}{R}$$  \hspace{1cm} (4.6)

Using vector identity:

$$\nabla \times (f \mathbf{c}) = \nabla f \times \mathbf{c}$$  \hspace{1cm} (4.7)

where $f$ is any scalar function and $\mathbf{c}$ is a constant vector, it can be seen that

$$\nabla \times \int_{S_s} \mathbf{J}(\mathbf{r}') g(R) \, dS' = \int_{S_s} \nabla \times (\mathbf{J}(\mathbf{r}') g(R)) \, dS'$$

$$= \int_{S_s} \nabla g(R) \times \mathbf{J}(\mathbf{r}') \, dS'$$  \hspace{1cm} (4.8)

where $\mathbf{J}(\mathbf{r}')$ is constant with respect to the operation on the points of observation.

The gradient of $g(R)$ in (4.8) is

$$\nabla g(R) = \frac{1}{4\pi} \frac{d}{dR} \frac{\exp(-jkR)}{R} \hat{R}$$

$$= \frac{1}{4\pi} \left( -\frac{\exp(-jkR)}{R^2} + \frac{jk \exp(-jkR)}{R} \right) \hat{R}$$  \hspace{1cm} (4.9)

Substituting (4.9) into (4.8), it becomes

$$\nabla \times \int_{S_s} \mathbf{J}(\mathbf{r}') g(R) \, dS' = \frac{1}{4\pi} \int_{S_s} \left( -\frac{\exp(-jkR)}{R^2} + \frac{jk \exp(-jkR)}{R} \right) \hat{R} \times J(\mathbf{r}') \, dS'$$  \hspace{1cm} (4.10)
Taking the Curl operation on both sides of above expression and applying identity (4.7) again, the second term in (4.4) can be expressed

\[ E(r) = \frac{j}{4 \pi \varepsilon_0} \int_{S_0} \left( \frac{1}{R^3} + \frac{j 2k}{R^2} \cdot \frac{k^2}{R} \right) \exp(-j k R) \mathbf{\hat{R}} \times (\mathbf{\hat{R}} \times \mathbf{J}(r')) dS' \]  \hspace{1cm} (4.11)

where \( R \) is

\[ R = \left( r^2 + r'^2 - 2 \mathbf{r} \cdot \mathbf{r}' \right)^{\frac{1}{2}} = r \left( 1 - \frac{2 \mathbf{\hat{r}} \cdot \mathbf{r}'}{r} + \frac{r'^2}{r^2} \right)^{\frac{1}{2}} \]  \hspace{1cm} (4.12)

When \( r \) approaches infinity, then the first two terms in (4.11) vanish. From (4.12) \( R \) can be written:

\[ R \approx \begin{cases} 
\begin{align*} 
r, & \text{for the amplitude} \\
r \cdot \mathbf{\hat{r}} \cdot \mathbf{r}', & \text{for the phase}
\end{align*}
\end{cases} \hspace{1cm} (4.13)

Thus, (4.11) becomes

\[ E(r \to \infty) = -\frac{j k \eta_0}{4 \pi r} \exp(-j k r) \mathbf{\hat{r}} \times \left( \mathbf{\hat{r}} \times \int_{S_0} \mathbf{J}(r') \exp(j k \mathbf{\hat{r}} \cdot \mathbf{r}') dS' \right) \]  \hspace{1cm} (4.14)

where \( \eta_0 = (\mu_0/\varepsilon_0)^{\frac{1}{2}} \) is the impedance of free space. Similarly, the right side of (4.4) can be written

\[ E(r \to \infty) = -\frac{j k}{4 \pi r} \exp(-j k r) \mathbf{\hat{r}} \times \int_{S_0} \left( \mathbf{\hat{n}}' \times E(r') \right) dS' \]  \hspace{1cm} (4.15)

\[ + \eta_0 \mathbf{\hat{r}} \times \left( \mathbf{\hat{n}}' \times H(r') \right) \exp(j k \mathbf{\hat{r}} \cdot \mathbf{r}') dS' \]
Both (4.14) and (4.15) can be used to calculate the far-field radiation from the reflector antenna. Equation (4.15) can be approximated by geometrical optics fields.

In this study, the expression (4.14) is used. In such case, the surface current is usually approximated by assuming that the feed radiation incident upon the reflector surface \( S_r \) reflects locally as a plane wave. This so called physical optics current is given by

\[
J(r') \approx 2 \hat{n} \times H_{in}
\]

where the subscript “\( in \)” denotes the incident fields from the feed. Contributions from fringe currents are neglected. And, by integrating over only the current on the illuminated side \( S_r \) of the reflector, (4.14) ignores the contribution from currents on the shadow side of the reflector.

### 4.3 fourier transform relationship

In this section the development of the radiated pattern of a reflector based on the application of the physical optics integral is discussed. The geometry of a parabolic reflector with diameter \( D=2a \) (radius \( a \)) and focal length \( F \) is shown in Fig 4.2. It is assumed that the reflector is illuminated by a feed located at the focal point and that the reflector surface may have some irregularities. Using the integral (4.14) and the vector identity of \( \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c}) \mathbf{b} - (\mathbf{a} \cdot \mathbf{b}) \mathbf{c} \), the radiated electric field can be expressed as

\[
E = j k \eta_0 \frac{\exp(-jkr)}{4\pi r} (T_{\hat{\theta}} \hat{\theta} + T_{\hat{\phi}} \hat{\phi})
\]
where

\[ T(\theta, \phi) = \int_{S'} J(r') \exp(j k r' \cdot \hat{r}) \, ds \]  \hspace{1cm} (4.18)\]

and \( J \) is the induced surface current defined by (4.16).

\[ \int \cdot \cdot \phi \theta \]

**Fig 4.2 Geometry of a parabolic reflector**

Integration in (4.18) is performed on the curved surface \( S \) with integration parameters defined on it. However, this integral can be performed in terms of the aperture coordinates \((\rho', \phi') \) or \((x', y') \) by using the concept of the surface projection Jacobian [115]. Then (4.18) can be written as

\[ T(\theta, \phi) = \int_S J(r') \exp(j k r' \cdot \hat{r}) J_s \, dx' \, dy' \]  \hspace{1cm} (4.19)\]

where the Jacobian transformation \( J_s \) is
\[ J_s = \sqrt{1 + \left( \frac{\partial f}{\partial x'} \right)^2 + \left( \frac{\partial f}{\partial y'} \right)^2} \]  

(4.20)

\( f \) describes the reflector surface

\[ z = f(x, y) \]  

(4.21)

and \( s \) designates the area of the projection of the reflector surface \( S \), onto the plane \((x - y)\).

Expression (4.19) can further be simplified by employing the following definitions and identities:

\[
\begin{align*}
J'(x', y') &= J(r') J_s \\
r' \cdot \hat{r} &= z' \cos \theta + u x' + v y'
\end{align*}
\]  

(4.22)

where

\[
\begin{align*}
u &= \sin \theta \cos \phi \\
v &= \sin \theta \sin \phi
\end{align*}
\]  

(4.23)

When (4.22) is substituted into (4.19) the following can be obtained

\[ T(u, v) = \int_0^1 J'(x', y') [e^{j k z' \cos \theta}] \exp(j k (u x' + v y')) \, dx' \, dy' \]  

(4.24)

For a flat reflector, \( z' = \text{constant} \), the term in the square brackets will not be a function of the integration variable, and therefore \( T \) will be an exact Fourier transform of the induced current. However, for curved surfaces this will not, in general, be the case.

If, however, (4.24) is re-expressed as
\[
T(u, v) = \int_{x} J'(x', y') e^{i k z'} \left[ e^{-j k z(1 - \cos \theta)} \right] \exp(j k (ux' + vy')) dx'dy'
\]  
(4.25)

(4.25) can then be expanded in terms of the Taylor series for small values of \( \theta \), namely,

\[
T(u, v) = \sum_{p=0}^{p=\infty} \frac{1}{p!} (-j k (1 - \cos \theta))^p T_p
\]  
(4.26)

where

\[
T_p = \int_{x} z'^p J'(x', y') e^{i k z'} \exp(j k (ux' + vy')) \ dx'dy'
\]  
(4.27)

It should be noted that (4.26) is now a sum of Fourier transforms with its dominant term expressed as

\[
T(u, v) = \int_{x} J'(x', y') e^{i k z'} \exp(j k (ux' + vy')) \ dx'dy'
\]  
(4.28)

The contribution of higher order terms in the series expansion of (4.26) becomes significant for wide-angle observations (large \( \theta \)) and laterally displaced feeds [115]. However, in this study, the higher order terms for reconstructing the aperture distributions of a reflector antenna do not need to be considered.

Once \( T \) is determined, the far-field pattern can then be obtained from (4.17).

In many cases, \( T \) may be determined by rectangular components \( T_x, T_y, \) and \( T_z \), which can be transformed to the spherical components by
\[
\begin{align*}
T_0 &= T_x \cos \theta \cos \phi + T_y \cos \theta \sin \phi - T_z \sin \theta \\
T_\phi &= -T_z \sin \phi + T_y \cos \phi
\end{align*}
\] (4.29)

Furthermore, application of Ludwig's [88] third definition allows the definition of the co-polar and cross-polar components of the far-field pattern as

\[
\begin{align*}
T_{co-pol} &= T_0 \sin \phi + T_\phi \cos \phi \\
T_{cross-pol} &= T_0 \cos \phi - T_\phi \sin \phi
\end{align*}
\] (4.30)

In (4.30) it is assumed that the radiated field is predominantly y-polarised. For the x-polarised field case, the rows of the equation (4.30) must be interchanged. From (4.29) and (4.30), it can be shown that for the cases where \( \theta \) is small, the following holds:

\[
\begin{align*}
T_{co-pol} &\equiv T_y \\
T_{cross-pol} &\equiv T_x
\end{align*}
\] (4.31)

The application of the reciprocity theorem implies that the transmitting and receiving patterns are identical. Therefore, if the reflector antenna is used to receive a plane wave, the output at the feed port will be proportional to the antenna receiving pattern for a given direction of incidence and plane wave polarisation. For the sake of simplicity, it is assumed that the dominant
polarisation is in the y direction, and therefore, by using (4.31) and (4.28), the following expression for the pattern is obtained:

\[
T(u,v) = \int J'(x', y') e^{jkz'} \exp(jk(ux' + vy')) \, dx'dy' \tag{4.32}
\]

Notice that the vector notations are removed in (4.32), in order to emphasize that only one polarisation is considered.

### 4.4 CONSTRUCTION OF SURFACE PROFILE

Expression (4.32) relates the Fourier transform of a function, which is related to the induced current, to the far-field pattern. Obviously, the domain of integration in (4.32) only extends to the region where current exists, *i.e.*, the reflector surface. Note that although (4.32) resembles the aperture integration formulation, it is very different, in that the induced current is still on the reflector surface, not on the projected aperture. In order to demonstrate that the surface profile information can be extracted from (4.32), assume that the phase centre of the feed is located at the focal point and that the surface irregularities are described by function \(e(x,y)\) in the normal direction. An enlarged view of an irregularity is depicted in Fig 4.3. It is observed in this figure that for small surface distortions, the following holds:

\[
P' P + P Q = \frac{e}{\cos \xi} + \frac{e}{\cos \xi} \cos 2\xi = 2e \cos \xi \tag{4.33}
\]

where for a parabolic reflector,
\[
\cos \xi = \frac{1}{\sqrt{1 + \frac{\rho^2}{4 F^2}}} = \frac{1}{\sqrt{1 + x^2 + y^2}}
\]

(4.34)

Fig 4.3 Illustration of enlarged surface distortion

The term inside (4.32) may now be expressed as

\[
J'(x', y') e^{jkz'} = \left| J'(x, y) \right| e^{jk r'} e^{j k z'}
\]

(4.35)

Note that the phase term \( \exp(-jkr') \) is obtained from (4.6) because the phase centre of the feed is located at focal point.

The exponents in (4.35) can be expressed as

\[
-r' + z' = -r' - PP^* - r' - PP^* - PQ + PP + PQ
\]

(4.36)

For a parabolic reflector,

\[
-r' - PP'' - PP - PQ = -2F
\]

(4.37)
which finally results in

\[- r' + z' = - 2 F + 2 e \cos \xi \]  \hspace{1cm} (4.38)

If the total distortion phase error is defined as

\[\delta = 4\pi \left( \frac{E}{\lambda} \right) \cos \xi \]  \hspace{1cm} (4.39)

(4.32) can then be expressed as

\[T(u, v) = e^{-j2kF} \int e^{i\delta} \exp(jk(ux' + vy')) \, dx'dy' \]  \hspace{1cm} (4.40)

The constant phase term \(\exp(-j2kF)\) in (4.40) is the result of defining the centre of the coordinate system at the focal point. Different phase constants can result if the centre of coordinate system is displaced from this focal point. It should also be noticed that both co-polar and cross-polar patterns can be used to determine \(\delta\) from (4.40).

If both the amplitude and the phase of the reflector pattern \(T\) are measured (or simulated), the surface distortion can be determined from (4.40) via an inverse Fourier transform, \(i.e.

\[|J'(x', y')| e^{j\delta} = e^{jkF} F^{-1}[T(u, v)] \]  \hspace{1cm} (4.41)

where \(F^{-1}\) designates the inverse Fourier transform.

By using (4.41), (4.39), and (4.34), The surface distortion \(e(x, y)\) in terms of wavelength is finally obtained as
\[
e(x, y) = \frac{1}{\lambda} \left[ 1 + \frac{x^2 + y^2}{4F^2} \right] \text{Arg} \left\{ e^{jkF} F^{-1}[T(u, v)] \right\} \tag{4.42}
\]

where \(\text{Arg}\) designates the phase of the operand.

In constructing \(e(x, y)\) from (4.42), the constant and linearly dependent phase terms in the braces must first be extracted. These phase terms account for the ambiguity in defining the phase reference and the beam displacement due to lateral feed displacement.

### 4.5 Sampling Principles

Since the integrand of (4.32) has a finite range (reflector surface), its Fourier transform \(T\) is an analytic function, which extends to infinity. Conversely, since the transform of \(T\) is a function with a finite range, the sampling theorem can be invoked to express \(T(u, v)\) by only its values at the sampling points. For a reflector with diameter \(D\), the largest sampling interval is [115]

\[
\Delta u_{\text{max}} = \frac{1}{D/\lambda} \tag{4.43}
\]

which is closely related to the reflector beam width. However, since the fast Fourier transform (FFT) algorithm is used to evaluate (4.41), intervals smaller than the largest sampling interval must be used to overcome the overlapping problem. For this reason, a factor of 0.8 (in fact, any fraction number between 0.5 and 1 can be used) is introduced. The sampling interval is then given by

67
\[ \Delta u = \Delta v = \frac{0.8}{D/\lambda} \]  

(4.44)

The value \( e(x, y) \) from (4.42) can be determined almost exactly, provided the pattern \( T(u, v) \) is known in its entire range (infinitely extended) and with an acceptable level of signal-to-noise ratio. In practice this is not the case, and \( T(u, v) \) is measured or simulated only in a finite range. If the total number of measured data in each of the \( u \) and \( v \) directions, is assigned by \( N_M \), then the total number of measured data is

\[ N_M N_M = (2N + 1)(2N + 1) \]  

(4.45)

The variable \( N \) is introduced in (4.45) to denote the number of measured data to one side of the measurement. Similarly, the total number of points used to perform the FFT can be defined as \( N_F \times N_F \). For the cases when \( N_F > N_M \), the domain of the measured pattern must be extended, as discussed in the next section.

Since (4.42) is evaluated using FFT, the values of \( e(x, y) \) are determined at the intervals to be

\[ \Delta x = \Delta y = \frac{D}{0.8 \times (N_F \cdot 1)} \]  

(4.46)

However, as far as the actual surface resolutions for identifying the surface errors are concerned, the parameter \( N_M \) is a more dominant factor than \( N_F \).

It is worthwhile mentioning that in many measurement configurations it may not be possible to measure \( T(u, v) \) at rectangular grid points in \( (u, v) \) space, but rather in other convenient coordinates. For these configurations, the FFT
algorithm can not be directly used until the data are rearranged with an interpolation. An alternative would be to evaluate the double sum resulting from the discrete Fourier representation of (4.32) in a numerical manner.

4.6 ITERATIVE ALGORITHM

In many cases, it may be necessary to extend the domain of the measured data $T(u, v)$ in order to possibly improve the achievable resolution. There are many different algorithms available which use a variety of concepts to extend the measured pattern beyond the measurement range. One of these methods, which appears to be very promising, uses an iterative approach similar to the one discussed by Papoulis [105]. The steps of this iterative procedure as applied to this study are presented here.

First, (4.32) is rewritten as

$$T(u, v) = F[Q] = \int Q(x', y') \exp(j k (ux' + vy')) \, dx' \, dy'$$  \hspace{1cm} (4.47)

Since $Q$ is zero outside the reflector boundary (assuming neither a ground nor any outside obstacles), the domain of integration in (4.47) can be extended to infinity, and hence the Fourier transform pair holds. The function $Q$ is constructed from $T$ using the relation

$$Q(x, y) = F^{-1}[T]$$  \hspace{1cm} (4.48)

Since $T$ is only given in the measurement domain, the iterative procedure is used to extend this domain with the steps depicted in Fig 4.4. The procedure starts
by placing zeros in the desired extended domain to give the zeroth iterated extended pattern \(T_e^0\) such that it takes the same value as \(T\) inside the original domain and zero value outside, \(i.e.\)

\[
\begin{align*}
T_e^0 &= T \quad \text{in the original domain} \\
T_e^0 &= 0 \quad \text{in the extension}
\end{align*}
\]

(4.49)

Next, the inverse Fourier transform (using FFT) is used to find \(Q^0(x, y)\), the zeroth iteration of \(Q(x, y)\). Obviously, since \(T_e^0\) is a truncated function, its transform will extend to infinity. However, it is known from the physics of the problem that \(Q^0(x, y)\) should only be extended within the region where the antenna surface exists. This information is next used to truncate \(Q^0(x, y)\) to the domain of the antenna. Once the truncated version of \(Q^0\), which is expressed as \(Q_t^0\), is constructed, then a forward transform is taken to obtain \(T^0\). From \(T\) and \(T^0\) the first iterated form of \(T\) is constructed

\[
\begin{align*}
T_e^1 &= T \quad \text{in the original domain} \\
T_e^1 &= T^0 \quad \text{in the extension}
\end{align*}
\]

(4.50)

It is clear that \(T_e^1\) is an improvement over \(T_e^0\), since its extended domain is filled with a non-zero function. The inverse Fourier transform of \(T_e^1\) is denoted by \(Q^1\), and this process is followed for higher orders of iterations. Parseval's theorem can be used to show that the tail end of function \(Q(x, y)\) gradually diminishes outside the physical domain of the antenna, which confirms that the procedure is
converging. Once the $n$th iterative value of $Q$ is determined, $Q(x, y)$ can then be determined from (4.42).

Fig 4.4 Steps of the iterative. $T$ is measured (or simulated) far field pattern and $Q^0$ is the amplitude profile

4.7 APERTURE SIMULATION MODEL

The purpose of this model is to simulate an aperture distribution with a prescribed amplitude taper and phase irregularities, in accordance with the reflector illumination and surface irregularities. The model assumes that both the amplitude and the phase are circularly symmetric. The far-field pattern is then
sampled and constructed using this simulated aperture field. The application of FFT to these sampled data finally results in the recovery of the simulated surface distortion. A comparison of the recovered and simulated surface distortion is the

**Fig 4.5 Flow chart of numerical simulations**

basis for judging the effects of parameters such as sampling intervals, number of the samples, etc., on the accuracy of the technique. A block diagram of the steps used in this section is shown in Fig 4.5.

The geometry of a circular aperture with different annular regions is shown in Fig 4.6, where the radii of these regions are designated by \( a_0, a_1, a_2, \ldots, a_N = a \) (in Fig 4.6, \( a_0 \) is used to designate the central blockage region). Let assume that the amplitude and phase distributions across the aperture are given by \( Q \) and \( \delta \), respectively, and that these functions are circularly symmetric. The far-field
pattern of this aperture distribution is constructed using the Fourier transform in polar coordinates to obtain

$$T = \int_{a_0}^{a} \int_{0}^{2\pi} Q(\rho') e^{i\delta(\rho')} \exp(jk\rho' \sin\theta \cos(\phi' - \phi)) \rho' \, d\rho' \, d\phi' \quad (4.51)$$

with the inner region having radius $a_0$ considered as the blockage region ($a_0 = 0$, no blockage).

Fig 4.6 Circular aperture with annular phase distortions

It is further assumed that $\delta(\rho')$ takes the constant value $\delta_n$ in the $n$th annular region, which can be interpreted as the phase irregularity (surface irregularity) in the $n$th annular region. With this assumption, (4.51) may be expressed as
\[ T = \sum_{n=1}^{N} T_{n,n-1} e^{i\delta_n} \quad (4.52) \]

with

\[
\begin{align*}
T_{n,n-1} &= T_n \cdot T_{n-1} \\
T_n &= 2\pi \int_{0}^{a} Q(\rho') J_0(k\rho' \sin \theta) \rho' d\rho'
\end{align*}
\]  

(4.53)

and \( J_0 \) being the zero-order Bessel function.

The aperture amplitude distribution is defined as

\[
Q(\rho) = B + C \left( 1 - \left( \frac{\rho}{a} \right)^2 \right)^p 
\]  

(4.54)

where \( B+C = 1 \), and parameter \( B \) is used to control the edge taper. It has been found that for values of \( 1 \leq p \leq 2 \), (4.54) is an adequate representation of the aperture amplitude distribution for many typical reflectors. In this study, the value of \( p = 1 \) is used, as (4.53) can be integrated in a closed form. To arrive at this, the following identity is used:
\[
I_p(\alpha) = 2 \int_0^1 x^{2p+1} J_0(\alpha x) \, dx
\]

\[=rac{2}{\alpha} J_1(\alpha) \cdot p \left( \frac{2}{\alpha} \right)^2 J_2(\alpha) + p(p - 1) \left( \frac{2}{\alpha} \right)^3 J_3(\alpha) - \cdots \quad (4.55)
\]

\[I_p(0) = \frac{1}{p+1}
\]

Substituting (4.54) into (4.53), using (4.55), and introducing the notation

\[
u_n = k a_n \sin \theta \quad (4.56)
\]

then following result can be finally obtained:

\[
\begin{align*}
T_n(\theta) &= \pi a_n^2 \left( B \frac{2}{u_n} J_1(u_n) + C \left( \frac{2}{u_n} J_1(u_n) \cdot \left( \frac{a_n}{a} \right)^2 \left( \frac{2}{u_n} J_1(u_n) \cdot \left( \frac{2}{u_n} \right)^2 J_2(u_n) \right) \right) \right) \\
T_n(0^\circ) &= \pi a_n^2 \left( B + C \left( 1 - \frac{a_n^2}{2 a^2} \right) \right) \\
\end{align*}
\quad (4.57)
\]

Expression (4.57) can be used to construct the far field from (4.52). In order to further relate the aperture phase error to the reflector surface distortion, the following value for \( \delta_n \) is used:

\[
\delta_n = 4\pi \frac{e_n}{\lambda} \left( 1 + \frac{b_n^2}{4 F^2} \right)^{\frac{1}{2}}
\quad (4.58)
\]
where \( e_n \) and \( b_n \) are the surface error in the normal direction and the average radius of the \( n \)th zone, respectively,

\[
b_n = \frac{a_n + a_{n+1}}{2}
\]

(4.59)

and \( F \) is the focal length of the parabola as shown in Fig 4.3.

**4.8 SIMULATION RESULTS AND DISCUSSIONS**

The simulations have been constructed on the basis of the previous reflector model. A few representative results are presented, which demonstrate the usefulness of the aperture model and the procedure which is used for recovering the surface profile. The operating frequency is 3 GHz, with a corresponding wavelength of \( \lambda = 0.1 \) m. At this frequency the reflector parameters are:

\[
\begin{align*}
\frac{a}{\lambda} & = 320 \quad \text{radius;} \\
\frac{F}{\lambda} & = 1000 \quad \text{equivalent focal length;} \\
\frac{a_0}{\lambda} & = 25 \quad \text{blockage radius;} \\
B & = 0.316, \quad \text{edge taper equal to -10 dB;} \\
P & = 1
\end{align*}
\]

(4.60)

A computer program has been generated on the basis of the mathematical developments of the previous section. In one case, far-field patterns are constructed for the reflector with no surface profile errors and, in another case, with surface profile errors. The amplitude and phase distributions of far-field pattern without
surface distortions are shown in Fig 4.7 and Fig 4.8 respectively. In Fig 4.7, the amplitude values have been normalised to 0 dB. Using this simulated far-field pattern, the aperture profile is calculated by the FFT. The amplitude and phase of the reconstructed aperture profile are shown in Fig 4.9 and Fig 4.10 respectively. The apparent phase discontinuity at the centre of the antenna in Fig 4.10 is due to the plotting function and is not a discontinuity.

In this simulation, there is a feed blockage in the centre of the antenna as shown in Fig 4.9.
In the surface profile model, the distance between \( a \) and \( a_0 \) has been divided by 10. In such case, the width of each annular ring is \( 29.5\lambda \). Some of the simulated surface profile errors are as follows:

\[
\begin{align*}
    e_3 &= 0.02\lambda \quad \text{at annular ring 3;} \\
    e_7 &= 0.1\lambda \quad \text{at annular ring 7;} \\
    e &= 0 \quad \text{at other annular rings}
\end{align*}
\]  \hspace{1cm} (4.61)

**Fig 4.9 Amplitude of the aperture profile reconstructed**

![Amplitude plot](image)

**Fig 4.10 Phase of the aperture profile reconstructed**

The amplitude and phase patterns are shown in Fig 4.11 and Fig 4.12 respectively. For comparison, the amplitude and phase patterns without
surface errors have been reproduced in Fig 4.11 and Fig 4.12 with thicker curves. These results demonstrate the effects of the surface profile distortions on the patterns, specially on the sidelobes.

**Fig 4.11 Amplitude patterns with and without surface**

**Fig 4.12 Phase patterns with and without surface errors**

On the basis of these simulated patterns, the accuracy of holographic technique is evaluated by trying to recover the simulated surface profile. The results of reconstructed aperture profile are shown as aperture amplitude and phase distributions Fig 4.13 and Fig 4.14. In this case, the sampling interval is

$$\Delta u = 0.8 \frac{\lambda}{2 \alpha} = 0.8 \times \frac{0.1}{2 \times 32} = 0.00156$$  \hspace{1cm} (4.62)
The result clearly indicates that for the employed number of sampling points of $N_M=58$ and $N_F=128$, the simulated surface profile error at annular ring 7 has been

![Normalized Aperture Profile (Amp) with Surface Errors](image1)

**Fig 4.13 Recovered aperture profile (amplitude) with simulated errors**

![Normalized Aperture Profile (phase) with Surface Errors](image2)

**Fig 4.14 Recovered aperture profile (phase) with simulated errors**

recovered very well. The smaller distortion error at the annular ring 3 could not be recovered. Again, the apparent phase discontinuity at the centre of the antenna in Fig 4.14 is due to the plotting function in the Mathematica program and is not a discontinuity.
When the sampling points are doubled i.e. \( N_M=116 \) and \( N_T=256 \), and the sampling interval remains the same, the smaller distortion errors at the annular rings 3 and 7 (\( e_3=e_7=0.02\lambda \) in this case) can be recovered. Fig 4.15 shows the recovered distortion error function of equation (4.42). In this figure, two error distortions are clearly identifiable. Note that the distortion at the left end of the curve which represents the centre blockage.

For convenience, all the scales of the size (horizontal axis) in aperture profile and error function graphics are reduced by a factor of 10.

![Error Function (Amp) with Surface Errors](image)

**Fig 4.15 Error function distribution**

It is noticed that the plots of phase displays have rapid variations shown in Fig 4.8, Fig 4.10 and Fig 4.14. It is a 180 degrees phase variation and is caused by the straight line plotting.

It is worthwhile mentioning that for a prescribed surface distortion the constructed far-field patterns are exact, since the aperture model is a closed-form mathematical expression (4.54).
CHAPTER 5

APERTURE FIELD RECONSTRUCTION BY PLANE WAVE SPECTRUM METHOD AND THE EFFECT OF EVANESCENT WAVES

5.1 INTRODUCTION

The modern planar near-field measurement technique has been widely used as a powerful tool to monitor the performance of an antenna or an antenna array. It offers an accurate and economical approach to obtain far-field patterns or other important parameters of an antenna under test. The technique is based on the wave model expansion representation of an arbitrary field, and can be traced back to Stratton [126] who presented the current form in a rectangular coordinate system. The simple relationship between the plane wave spectrum (PWS) and the far-field pattern of an antenna was later described by Booker and Clemmow [29]. A comprehensive review of near-field measurement techniques was conducted by Yaghjian [147].

For most applications, the near-field data in a rectangular coordinate system are processed through the Fourier transform to get a spectrum which is the far-field pattern of the antenna. In this case, it is known that the exclusion of the evanescent waves is the basis for selection of the measurement plane, because the evanescent waves make no contribution to the far-field pattern.
On the other hand, the near-field data can also be used, through a backward transform process, to reconstruct the aperture distribution of an antenna or an antenna array. Such a technique can be employed to find antenna problems such as defective elements in an array antenna or antenna feed problems. The details of this application and its theoretical derivation are to be presented in this Chapter.

When this application of the near-field measurement is involved, it is necessary to investigate the effects of the evanescent waves on the reconstructed aperture distributions. As will be demonstrated in this chapter, aperture distribution reconstruction processes are simulated on a computer. By using a dipole and other well-known antennas and comparing the results with the exact solutions, the simulation provides insight into the effects of evanescent waves on aperture distributions. It will be shown that a higher resolution reconstruction can be achieved by adjusting some of the measurement parameters such as sampling intervals, and the position of the measurement plane. A simple method is also presented for accurately examining the evanescent waves. This chapter also reveals that the traditional view of the aperture distribution of a dipole antenna is probably extremely questionable.

5.2 FUNDAMENTAL THEORY

Any monochromatic arbitrary wave can be represented as a superposition of plane waves, all of the same frequency, travelling in different directions and with different amplitudes. The object of a plane wave spectrum analysis is to determine the amplitudes and directions of propagation of the plane waves in the
superposition. The superposition expression constitutes what is conventionally known as a modal expansion of an arbitrary field.

In a linear, homogeneous, and isotropic medium which is free of sources, and for harmonic time variations, Maxwell’s equations can be written as the vector wave (Helmholtz) equations:

\[ \nabla^2 E + k^2 E = 0 \]  \hspace{1cm} (5.1)

\[ \nabla^2 H + k^2 H = 0 \]  \hspace{1cm} (5.2)

\[ \nabla \cdot E = \nabla \cdot H = 0 \]  \hspace{1cm} (5.3)

where

\[ k = \sqrt{\omega^2 \mu \varepsilon} \]  \hspace{1cm} (5.4)

is the scalar wavenumber in free space. It can be proved that any plane wave

\[ E(x,y,z) = A(k_x, k_y) \exp(-j \cdot k \cdot r) \]  \hspace{1cm} (5.5)

is a solution to the equation (5.1) for \( z \geq 0 \), where \( k_x \) and \( k_y \) are components of the vector wavenumber \( k \) and defined by

\[ k = k_x \hat{x} + k_y \hat{y} + k_z \hat{z} \quad \text{and} \quad k^2 = k \cdot k = k_x^2 + k_y^2 + k_z^2 \]  \hspace{1cm} (5.6)

and \( r \) is the position vector

\[ r = x \hat{x} + y \hat{y} + z \hat{z} \]  \hspace{1cm} (5.7)

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From (5.6) at a fixed frequency, only two of the components of \( \mathbf{k} \) can be independently specified. Let these be \( k_x \) and \( k_y \); then the third component is:

\[
k_z = \begin{cases} 
\left( k^2 - k_x^2 - k_y^2 \right)^{1/2}, & \text{if } k_x^2 + k_y^2 \leq k^2 \\
-j\left( k_x^2 + k_y^2 - k^2 \right)^{1/2}, & \text{otherwise}
\end{cases}
\tag{5.8}
\]

An imaginary \( k_x, k_y \) or \( k_z \) corresponds to an evanescent plane wave spectrum which is very rapidly attenuated away from the source.

The general solution to the equation (5.1) can be obtained by integrating (5.5) with respect to \( k_x \) and \( k_y \). That is

\[
\mathbf{E}(x,y,z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{A}(k_x, k_y) e^{jkr} \, dk_x \, dk_y \tag{5.9}
\]

The magnetic field intensity \( \mathbf{H} \) is given from Maxwell’s equations as

\[
\mathbf{H}(x,y,z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{k} \times \mathbf{A}(k_x, k_y) e^{jkr} \, dk_x \, dk_y \tag{5.10}
\]

Vector coefficients \( \mathbf{A}(k_x, k_y) \) are called the plane wave spectrum (PWS) and can be determined from the particular antenna excitation or the boundary conditions in the plane \( z=0 \), where the source is located. From equation (5.3), the following equation can be derived:

\[
\mathbf{k} \cdot \mathbf{A}(k_x, k_z) = 0 \tag{5.11}
\]

i.e.

\[
k_x A_x + k_y A_y + k_z A_z = 0 \tag{5.12}
\]
It can be seen that only two components of vector $A(k_x,k_y)$ are independent and the third one can be determined from the other two. In general, the selection of two components depends on the components of the field vectors. In this case, the transverse components ($x$ and $y$) are considered.

5.3 CALCULATION OF THE PWS FROM NEAR-FIELD MEASUREMENT

From equations (5.9) and (5.10), if the plane wave spectrum $A(k_x,k_y)$ is known, the field at any point in space can be calculated through these equations. In practice, a near-field measurement at a distance $z=d$ from the radiation source is made. In this section, the formulation and the procedure to obtain the plane wave spectrum $A(k_x,k_y)$ through near-field measurement is described.

With planar near-field measurements, the antenna is placed on a plane in the region $z=0$, as shown in Fig 5.1. Planar scanning is conducted on the plane, specified by $z=d$, near the antenna. From (5.9), at $z=d$, the electric field $E$ can be expressed as

$$E(x,y,d) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A'(k_x,k_y) e^{jk_x x + ik_y y} dk_x dk_y$$  \hspace{1cm} (5.13)

where

$$A'(k_x,k_y) = A(k_x,k_y) e^{jk_x d}$$  \hspace{1cm} (5.14)

The transverse components of vector $E$ are
\[
E_x(x,y,d) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_x'(k_x, k_y) e^{jk_zd} \, dk_x \, dk_y \quad (5.15)
\]
\[
E_y(x,y,d) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_y'(k_x, k_y) e^{jk_zd} \, dk_x \, dk_y \quad (5.16)
\]

where

\[
A_x'(k_x, k_y) = A_x(k_x, k_y) e^{jk_zd} \quad (5.17)
\]
\[
A_y'(k_x, k_y) = A_y(k_x, k_y) e^{jk_zd} \quad (5.18)
\]

From (5.13), it can be seen that \(E(x,y,d)\), which is the \(E\) field measured at the plane \(z=d\), and \(A'(k_x,k_y)\), corresponding to the PWS for that measured \(E\) field, are a Fourier transform pair. When \(z=0\), \(i.e.\) on the antenna aperture plane, (5.13) becomes

\[
E(x,y,0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(k_x, k_y) e^{jk_zd} \, dk_x \, dk_y \quad (5.19)
\]

So that, \(E(x,y,0)\) and \(A(k_x,k_y)\) are another Fourier transform pair. The connection between \(A\) and \(A'\) is given by (5.14).

Clearly, if the spatial dependence of the transverse field over a plane surface is known \(i.e.\) measured at \(z=d\), the plane wave spectrum of the wave can be calculated through (5.15) to (5.18). Conversely, if the plane wave spectrum \(A(k_x,k_y)\) is known, the field is determined everywhere in space.

In the evanescent wave region (close to the antenna) \(k_z\) in equations (5.17) and (5.18) becomes imaginary (referring to (5.8)), which implies that the evanescent waves die off very quickly with the measurement plane distance \(d\). Beyond the evanescent waves area \(k_z\) becomes real again.
In the evanescent wave region the measured field at distance \((d)\) can be transformed through a spatial low pass filter to reconstruct the field even closer to the antenna (this is the digital signal processing approach [63, 112]). This approach applies strictly within the evanescent wave region.

Thus the reconstruction of the aperture distribution from a planar near-field measurement can be performed by the following procedure:

1. Measure the near-field over the plane \(z=d\) to get \(E_s(x,y,d)\) and \(E_y(x,y,d)\);
2. Calculate \(A'_s(k_x,k_y)\) and \(A'_y(k_x,k_y)\) by taking the inverse Fourier transforms of \(E_s(x,y,d)\) and \(E_y(x,y,d)\) using the FFT (5.15) and (5.16);
3. Calculate \(A_s(k_x,k_y)\) and \(A_y(k_x,k_y)\) using (5.17) and (5.18);
4. Take the forward Fourier transform of \(A_s(k_x,k_y)\) and \(A_y(k_x,k_y)\) to obtain the antenna aperture distribution \(E_s(x,y,0)\) and \(E_y(x,y,0)\) (5.19).

It is worthwhile mentioning that in calculating the far-field pattern, the evaluation of the double integral in (5.9) is rather involved, with \(A(k_x,k_y)\) known from the near-field measurement in (5.13) and (5.14). Generally, the integration is difficult when \(r\) is very large. When the method of stationary phase can be used in evaluation of (5.9) (Appendix A), the resulting expression is

\[
E(r) = \frac{j k z}{r} A(k_x, k_y) e^{jkr}
\]  

(5.20)

5.4 SAMPLING AND ITS CRITERIA

In calculations of the far-field from near-field measurements, it is generally accepted that the sampling rate in the spatial \((x,y)\) domain should be equal to or
less than half-wavelength [15, 21]. However, for aperture reconstruction from near-field measurements, it is necessary to discuss this issue and clarify it. The following discussion will show that the sampling interval of a half-wavelength is not really small enough, so far as the evanescent waves are concerned. In general, the sampling intervals will decrease closer to the antenna, i.e. in the evanescent wave region.

Further simulations in the next section will show that for electrically small antennas or small elements of an array, the smaller sampling interval is needed for accurate recovery of aperture fields.

The first step in the near-field measurement is to sample the field at the plane $z=d$. Since only a finite amount of data can be measured and processed, the infinite continuous Fourier transforms (CFT) must be approximated by finite Fourier transforms, which are also called discrete Fourier transforms (DFT). The

![Diagram of antenna and measurement plane](image)

**Fig 5.1 Sampling intervals in a measurement coordinate system**

antenna near-field distribution and its plane wave spectrum are both of finite
extent, so that the DFT or, FFT for equally spacing sampled data, can be employed for approximating the CFT.

Let $\Delta x$ and $\Delta y$ denote the sampling intervals as shown in Fig 5.1. The evenly sampled data with $\Delta x$ and $\Delta y$ in x-y domain will lead to the data in $k_x$-$k_y$ domain being equally spaced as well, when applying FFT techniques. This can be shown by considering $k_{x0}$ and $k_{y0}$ to be the limits of sampling in $k_x$-$k_y$ domain, as shown in Fig 5.2.

![Fig 5.2 Sampling intervals in spectral domain](image)

Then the simple relationship between the sampling intervals and spectral limits of sampling are:

$$\Delta x = \frac{2\pi}{k_{x0}} \quad (5.21)$$

$$\Delta y = \frac{2\pi}{k_{y0}} \quad (5.22)$$

A similar result can be obtained with spectral sampling intervals $\Delta k_x$ and $\Delta k_y$, and the spatial limits of sampling $x_0$ and $y_0$: 

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\[ \Delta k_x = \frac{2\pi}{x_0} \]  
(5.23) 
\[ \Delta k_y = \frac{2\pi}{y_0} \]  
(5.24) 

If the numbers of sampling points on the \( x \)-\( y \) (or \( k_x \)-\( k_y \)) coordinates are \( N_x \) and \( N_y \) respectively, the following equations are obtained:

\[
\begin{align*}
  x_0 &= N_x \Delta x \\
  y_0 &= N_y \Delta y \\
  k_{x0} &= N_x \Delta k_x \\
  k_{y0} &= N_y \Delta k_y 
\end{align*}
\]  
(5.25)

Now it is possible to set up a sampling scheme by selecting intervals in the \( x \)-\( y \) domain and in the \( k \) domain, such that the size of the FFT and the inverse FFT are large enough to contain practically all the spatial and the spectral energies, respectively. Theoretically, the smaller the sampling intervals \( \Delta x \) and \( \Delta y \) (or \( \Delta k_x \) and \( \Delta k_y \)), the closer is the approximation of the CFT by FFT. Considering the time and capacity constraints of the computer, the Nyquist sampling rate is normally the best choice for economy, efficiency and accuracy in practice. According to the Nyquist theorem, for any band-limited signal with maximum frequency \( \omega \) \((-\omega \leq f \leq +\omega\)), the sampling rate (frequency) \( f_s \) should be greater than or equal to twice the maximum frequency \( \omega \), \textit{i.e.}:

\[ f_s \geq 2\omega \]  
(5.26)

In general, the maximum frequency of a spatial spectrum is unknown, so that, it is impossible to determine the sampling rate \( f_s \). However, under some circumstances,
the spatial spectrum of a given signal can be assessed, and the sampling rate can be determined either in the visible or evanescent region in the following way.

Assuming that near-field measurement is employed to compute the far-field pattern, then the plane wave spectrum $A(k_x,k_y)$, which represents the far-field pattern (5.20), should be in the visible region, because $k_x$ and $k_y$ are virtually the function of observation angle ($k\sin\theta\cos\phi$) (refers to Appendix A). In this case, the $z$-

![Diagram](image)

**Fig 5.3 The visible/evanescent spectrum regions**

component $k_z$ of $k$ is real number. This means that the following condition holds:

$$k_x^2 + k_y^2 \leq k^2$$  \hspace{1cm} (5.27)

Fig 5.3 shows the maximum frequencies of $\omega_x$ and $\omega_y$ are within the circle of radius $k=2\pi/\lambda$. From Fig 5.3, it can be seen that the maximum frequency of PWS is

$$\omega_x = \omega_y = \frac{2}{\lambda}$$  \hspace{1cm} (5.28)

Applying the Nyquist theorem equation (5.26), the following is obtained
\[
f_s = \frac{1}{\Delta x} = \frac{1}{\Delta y} \geq 2 \frac{\omega_y}{2\pi} = \frac{2}{\lambda}
\]

(5.29)

i.e.

\[
\begin{align*}
\Delta x & \leq \frac{\lambda}{2} \\
\Delta y & \leq \frac{\lambda}{2}
\end{align*}
\]

(5.30)

From this derivation, it is clear that for most far-field calculations, it is reasonable to make a choice for sampling intervals as one half-wavelength or perhaps as small as one quarter of a wavelength. However, in order to investigate the evanescent waves, a wider band of the spatial spectrum is needed to include significant evanescent components. For example, \( \omega_x = \omega_y = 4\pi/\lambda \) can be considered as a first estimate of the maximum frequency. In this case, the sampling intervals are \( \Delta x = \Delta y = \lambda/4 \), as is demonstrated by simulations in the following sections. But as discussed above, this sampling interval is also subject to the existence of significant evanescent waves.

Moreover, if the antenna or the element of an array is electrically small, for example, less than (or equal to) a half-wavelength, then the details of the aperture distribution can not be recovered properly by choosing the sampling intervals \( \Delta x = \Delta y = \lambda/2 \). This issue will be demonstrated in the next section as well.

When sampling in the evanescent wave region it is recommended that an initial sampling interval of \( \lambda/4 \) be used, followed by an interval of \( \lambda/8 \) and compare the two results in the spectrum domain. This continued halving of the sampling
interval process should be continued until succession results are sufficiently similar.

It is also worthwhile mentioning that if the sampling rate is too low, it may cause evanescent modes to be aliased into the visible region. And this alias may be mistakenly assumed to contribute to the far field when dealing with the sampling rates.

Next the number of sampling points on $x$-$y$ (or $k_x$-$k_y$) coordinate $N_x$ and $N_y$ must be determined so as to include all the important fields in the near-field measurements. From the equations (5.21) to (5.25), the sampling intervals $\Delta k_x$ and $\Delta k_y$ also depend on the sampling points $N_x$ and $N_y$. The $N_x$ and $N_y$ should be large enough to make $\Delta k_x$ and $\Delta k_y$ in the spectral domain sufficiently small to describe the details of the spectrum.

### 5.5 SIMULATION RESULTS AND ANALYSIS

In order to assess how accurately the aperture distributions can be recovered from a measured set of data, and how much the evanescent waves can affect the recovered aperture distributions, simulations have been made. A simple dipole antenna and other well-known antennas are used as test antennas to provide near-field data and aperture field distributions. Some interesting findings in this simulation will be presented. It is worth mentioning that in this ideal simulation process, probe compensations or corrections are not involved, i.e., ideal probes are assumed.
5.5.1 Reconstruction of the Aperture Distribution of A Dipole

The geometry for the field at the point $p$ from a symmetrically centre-fed dipole of length $L$ with sinusoidal current distribution is presented in Fig 5.4. The maximum current is $I_0$. It can be shown that the $y$ component of electric field at a point $p$ is given by [77]:

$$E_y = \frac{jI_0\eta}{4\pi} \left[ \frac{e^{-j\frac{k_0 s_1}{2}} + e^{-j\frac{k_0 s_2}{2}}}{s_1} \cdot 2 \cos \frac{k_0 L}{2} \cdot e^{j\frac{kr}{r}} \right]$$  \hspace{1cm} (5.31)

And correspondingly, the $\phi$ component of magnetic field at the point $p$ is:

$$H_\phi = \frac{jI_0}{2\pi r \sin \theta} \left[ e^{j\frac{k_0 s_1}{2}} + e^{j\frac{k_0 s_2}{2}} \cdot 2 \cos \frac{k_0 L}{2} \cdot e^{j\frac{kr}{r}} \right]$$  \hspace{1cm} (5.32)

$k_0 = 2\pi/\lambda$ is the wavenumber in free space, and $\eta = 377\Omega$ is intrinsic impedance of free space. Equations (5.31) and (5.32) can be used to calculate the exact aperture field

![Diagram of a field at the point p from a centre-fed dipole antenna](image)

**Fig 5.4** The geometry of a field at the point $p$ from centre-fed dipole antenna

distributions and near-field distributions without near-field distance restrictions.
In this simulation, planar scanning techniques were used, so that the parameters \( r, s_1, \) and \( s_2 \) need to be expressed in the rectangular coordinate system, \( i.e. \)

\[
\begin{align*}
    r &= \left( x^2 + y^2 + z^2 \right)^{\frac{1}{2}} \\
    s_1 &= \left[ x^2 + \left( y + \frac{L}{2} \right)^2 + z^2 \right]^{\frac{1}{2}} \\
    s_2 &= \left[ x^2 + \left( y + \frac{L}{2} \right)^2 + z^2 \right]^{\frac{1}{2}}
\end{align*}
\]

(5.33)

In a given plane \( z=d \), the electric and magnetic field can be obtained by equations (5.31) to (5.33). In this simulation, only electric field distributions are examined. By

\[\text{Fig 5.5 Calculated aperture field distributions of a half-wave dipole with different sampling intervals} \]

\[\text{---} \Delta y=0.25\lambda \quad \text{---} \Delta y=0.125\lambda \quad \text{---} \Delta y=0.5\lambda\]

using equations (5.31) to (5.33), with sampling intervals \( \Delta x=\Delta y=\lambda/4 \), and sampling
points $N_x=N_y=128$, the $E$-field distribution of the half-wavelength ($L=\lambda/2$) dipole antenna is computed at a plane $z=0.001\lambda$, which is considered to be the aperture distribution (due to the singularity at $z=0$). The graph of the $y$-component of the electric field at $x=0$ is as shown in Fig 5.5 with solid line. The value of the distribution was normalised.

It can be seen that the distribution has a maximum at each end of the antenna, i.e. at $y=\pm\lambda/4$. When the sampling intervals are changed to $\lambda/2$, these two maximum points can not be recognised. Instead, a maximum point at the centre of the antenna is shown. Since these two maximum values at each end of the antenna are very sharp rising, if the sampling points are off from these points, these values will not be sampled. Obviously, this poor resolution can be avoided by using smaller sampling intervals. The aperture distribution with these sampling intervals is also plotted in Fig 5.5 for comparison. Since the maximum value is different from the previous distribution, the normalised distribution is offset to about 58 dB.

When the sampling intervals are changed to $\lambda/8$, the aperture distribution has some similarities to that with $\lambda/4$ sampling intervals, as shown in Fig 5.5 with dashed curve. The differences in these two curves between the two sampling rates are partly due to the plotting errors of the straight lines connections. Obviously, better results would be achieved with smaller sampling intervals. But, in dealing with much larger problems, i.e., large complex antenna arrays, a trade-off with the time and the capacity of the computer has to be considered.
In the simulation of near-field measurement (data) \( E_r(x,y,d) \) by using (5.31), the measurement plane \( z=d \) must be chosen first. In order to demonstrate how the distance \( d \) affects the reconstruction of the aperture distributions, different values of \( d \) of 0.5\( \lambda \), 1\( \lambda \), and 3\( \lambda \) were considered. The simulated (calculated) near-field distributions at \( x=0 \) cut for different measurement planes \( d \) are shown in Fig 5.6.

The sampling intervals \( \Delta x \) and \( \Delta y \) are \( \lambda/4 \), and the sampling range is \( N_x \times N_y = 128 \times 128 \) in all three simulated measurements. From this simulated near-field data, applying the procedure described in section 5.2, three reconstructed aperture distributions are depicted in Fig 5.7. It is seen that the reconstructed \( E \)-field around the antenna (\( y \) from -3\( \lambda \) to +3\( \lambda \)) by near-field measurements simulated at planes \( d=0.5\lambda \) and \( d=1\lambda \) are a little different. Compared with calculated aperture distribution (at the plane \( d=0.001\lambda \)), as shown in Fig 5.5, the aperture fields are clearly recovered by the PWS method with the measurement plane at \( d=0.5\lambda \) and \( d=1\lambda \), especially these two maximum points at each end of the antenna. But when the measurement plane \( d \) is increased to 3\( \lambda \), the aperture fields can not
be recovered correctly. By examining the spectrum distribution $A_y'$ (shown later in this chapter), it is found that the evanescent components are very small and should be ignored. In this case, the large multiplying factor $\exp(k_d)$, in equation (5.14), distorts evanescent components in the spectral domain. The reconstructed aperture fields will also be distorted. If by assigning this factor $\exp(k_d)$ to be 1 in the evanescent region ($|k_y| > 2\pi/\lambda$) and the computation process in other region remains unchanged, the aperture fields can still be reconstructed. However, those two maximum points located at the ends of the antenna can not be properly recovered, as shown in Fig 5.7 with dashed line curve. In such case, the evanescent components, which can be recognised in spectral domain as shown in Fig 5.11, are so small and below the level of computation or measurement errors, so that the above modified computation process is still valid.

![Fig 5.7](image)

**Fig 5.7** Reconstructed aperture distributions at $d=0.001\lambda$ with the measurements at different planes $z=d$

- - - - - $d=0.5\lambda$
- - - - - - - $d=1\lambda$
- - - - - - - - - - $d=3\lambda$
5.5.2 Simulation Results With Extremely Small Sampling Intervals of $\lambda/100$

To examine effect of the sampling intervals, further simulations have been made by still using the dipole antenna model described in the previous section.

![Graph showing the $E_y$ distribution at a plane $d=0.5\lambda$.]  

This time, much smaller sampling intervals of $\lambda/100$ has been randomly chosen. The sampling points $N_x$ and $N_y$ must be increased accordingly to cover all necessary field extents. After a few test trials, $N_x=N_y=6400$ is considered to be sufficient, as the field strength has dropped to $-75\text{dB}$ show in Fig 5.8. The $E$-field distribution of the half-wavelength ($L=\lambda/2$) dipole antenna is simulated at a plane $d=0.5\lambda$. The y-component of the electric field in a region of $\pm 30\lambda$ is shown in Fig 5.8. The value of the distribution was normalised.

Comparing with the curves in Fig 5.6 of simulated fields at the larger sampling intervals, the resolution of the field distribution in Fig 5.8 is better. And also the field coverage has extended to $-75$ dB range. The shapes of the distributions are similar in the region of $\pm 3\lambda$.  

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Applying the procedure described in section 5.2, the aperture field at the plane $d=0.025\lambda$ has been reconstructed. The reconstructed aperture field distribution is depicted in Fig 5.9.

For a comparison, the field at the plane of $d=0.025\lambda$ was also calculated

![Fig 5.9 Reconstructed $E_y$ field distribution from simulated field at the plane $d=0.5\lambda$ with the sampling intervals of $0.01\lambda$](image)

![Fig 5.10 Reconstructed and simulated field distributions at the same plane (very close to the antenna)](image)

directly from the modelling equation (5.31). Two curves of the reconstructed and directly calculated field at the same plane has been put together as shown in Fig 5.10. It is seen that the reconstructed field around the antenna region ($y$ from $-1\lambda$ to $+1\lambda$) is very close to the calculated. Beyond this region, the reconstructed is
rippling around and this rippling effect is partly a result of computation errors of large numbers in the processing of FFT. It is also partly due to the computational errors in the theoretical model itself outlined in the previous section.

Compared to the previous simulation which used sampling intervals of $\lambda/4$, a higher resolution is achieved with this at the sampling intervals of $\lambda/100$. But it is not much better comparing to the sampling intervals of $\lambda/8$. So that, this much smaller sampling intervals of $\lambda/100$ may not be necessary. A trade-off choice of $\lambda/8$ may be sufficient.

5.5.3 Examination of Evanescent Waves in the Spectral Domain

As pointed out earlier, in the spectral domain, the PWS with an imaginary $k_z$ represents the evanescent waves that are rapidly attenuated by the factor $\exp(-|k_z|d)$ as the measurement plane moves away from the plane $z=0$ in the

![Diagram of plane wave spectrum distributions](image)

Fig 5.11 Plane wave spectrum distributions of a half-wave dipole with measurement planes at $z=d$

- $d=0.5\lambda$
- $d=3\lambda$

direction of positive $z$. The exclusion of evanescent components has been the basis
for the selection of the measurement plane at a sufficiently large distance $d$ where the evanescent waves are well attenuated. To position the scanning plane at least a few wavelengths (typically ten wavelengths) away has been the general practice in planar near-field measurement for the purpose of calculating far-field pattern,

Fig 5.12 Spectrum distribution of a half-wave dipole at plane $d=0.5\lambda$ with sampling interval of $\lambda/100$

because the evanescent waves make no contributions to the far-field. However, this study shows that the existence of evanescent waves can not be easily determined by examining the near-field data in the spatial ($x$-$y$) domain. From the near-field
distributions of Fig 5.6 (or Fig 5.8), it is not obvious which distribution contains evanescent wave information. But, by analysing the spectrum distributions in $k_x$-$k_y$ domain, it is easy to identify the evanescent components. The spectrum distributions ($k_z=0$ cut) of the near-field measured at planes $z=\lambda/2$ and $z=3\lambda$ from the half wavelength dipole antenna, with sampling intervals $\Delta x=\Delta y=\lambda/4$, and sampling points $128\times128$, are plotted in Fig 5.11. When $k_y>k_0=2\pi/\lambda$, the spectrum represents the evanescent components which is the region beyond the two dashed vertical lines. It is seen that the near-field measured at the plane $z=\lambda/2$ contains evanescent waves, and the maximum evanescent spectrum is about 25 dB less than the maximum spectrum at the centre. However, the spectrum distribution calculated from the fields at plane $z=3\lambda$ contains very small evanescent waves and the evanescent spectrum is dropping rapidly with respect to $k_y$.

This decaying effect of evanescent waves can also be demonstrated by the simulations, described in the previous section, with a much smaller sampling interval of $\lambda/100$ as shown in Fig 5.12 and Fig 5.13. Fig 5.12 shows a broad range of the spectrum covering $-50k_0$ to $+50k_0$. It is seen that most of the values are below $-100$dB and can be ignored. The most significant values lie in the centre region. The more detailed distribution in the centre region is selected and shown in Fig 5.13. Again, it is clearly shown that an approximately 15dB jump of the evanescent waves occurs near $k_y=\pm k_0$.

Two other examples, one with an ideal point source and one with a uniformly distributed aperture field are also simulated in this study. With the ideal point source, it is well known that the aperture distribution at the source is a
Delta function and its far-field pattern is a spherical wave. The Fourier transform of the Delta function is the unit function. Consequently, the spectrum of the near-field at the source (antenna) for the ideal point source antenna is uniformly distributed which means that the evanescent wave components are the same as radiated components. This is of course a theoretical result to illustrate the similarities and is impossible to reproduce practically. If the scanning plane is moved away slightly from the source, for example, \( d=0.001 \lambda \), then the evanescent waves of this near-field are reduced, as shown in Fig 5.14.

For directive antennas, such as those having a uniform aperture distribution, the simulation shows that the evanescent waves are very small, even right in the aperture plane. Fig 5.15 shows the spectrum calculated with a uniform \( E \)-field aperture distribution. It contains very small evanescent components in the PWS representation. In fact, for most highly directive antennas, the evanescent waves can be ignored.
5.5.4 The Effects of the Evanescent Waves on the Reconstruction of Aperture Distributions

In the above section, it has been shown that the evanescent waves can be easily examined in the spectral domain. In the following section, it is intended to demonstrate the effects of evanescent waves on the aperture distributions. As indicated earlier in this chapter, most applications of near-field measurements are to calculate the far-field patterns. In such cases, the measurement plane should be chosen to exclude the evanescent waves. However, for the reconstruction of aperture distributions from near-field measurements, the evanescent waves, can play an important role in the aperture distributions.

The effect of evanescent waves can be demonstrated directly by examining the reconstructed aperture distribution of the half wavelength dipole. The exact (calculated) distribution at plane \( d = 0.01\lambda \) is shown in Fig 5.5 with solid line curve, and the reconstructed distribution, with and without the consideration of evanescent components are shown in Fig 5.7 with solid and dashed curves.
respectively. It is clearly seen that the aperture field reconstructed from the PWS including the evanescent field is closer to the exact distribution. Examination of the PWS distribution (in the $k_x$-$k_y$ domain) shows that the aperture field contains significant evanescent components, in Fig 5.11 with solid curve. So that the evanescent waves can not be ignored in this case.

A five elements ideal point source array with spacing $\lambda/2$ between the elements was also studied. Assuming that there is no mutual coupling between elements, the exact aperture fields at plane $d=0.01\lambda$ are depicted in Fig 5.16. The aperture fields shown in Fig 5.17 are reconstructed, from the near-field measured (simulated) at plane $d=0.5\lambda$, with and without evanescent components, and again indicate that for antenna arrays, the evanescent waves also contribute to the aperture distributions.

The sharply looking curves in Fig 5.16 and Fig 5.17 are due to the straight lines have been used to join the points in plotting.
Fig 5.17 Reconstructed aperture field distributions from the field simulated at a plane of d=0.5λ with and without evanescent components.

--- with evanescent  --- without evanescent

Fig 5.18 Spectrum distribution of the five-element array. Each element has a uniform aperture (E-field) distribution

As an additional supporting argument, a practical antenna, such as a five element open-end waveguide array, with E-field uniformly distributed on each element aperture was also simulated. The PWS distribution of the aperture field is
shown in Fig 5.18. It can be seen that the aperture field contains significant

![Graph showing aperture distribution and reconstructed distribution](image)

**Fig 5.19** Comparison of aperture distribution and reconstructed aperture distribution with the evanescent components filtered out. 
--- Exact distribution (calculated) ------ Reconstructed

![Graph showing spectrum distribution](image)

**Fig 5.20** Spectrum distribution of a half-sinusoid distributed aperture field 
evanescent waves. If these components are filtered out, the reconstructed aperture
distribution in Fig 5.19 shows some distortions.
For a single large aperture such as a reflector antenna, the simulation shows that the PWS, even right on the aperture of the antenna, contains very small evanescent components. They have virtually no effect on the aperture distribution. Fig 5.20 shows the PWS distribution of the aperture field of a reflector antenna, with an aperture size of 10λ. It is seen that the maximum evanescent components at \( k_r = \pm k_0 = (2\pi/\lambda) \) are below -45 dB.

5.6 CONCLUSIONS

The aperture field reconstruction by plane wave spectrum method has been developed. The technique is based on the wave model expansion representation of an arbitrary field in a rectangular coordinate system. The measured near-field is used, through a backward transform process by the FFT method, to reconstruct the aperture distribution of an antenna or an antenna array. Such a technique can be employed to find antenna problems such as defective elements in an array antenna or antenna feed problems.

Computer simulations using various antennas and antenna arrays have been conducted. These simulations have demonstrated the feasibility of the technique.

The sampling criteria and other measurement issues such as measurement distance and plane range are also studied. It concludes that the sampling interval of \( \lambda/4 \) is a starting point. A further smaller sampling interval with appropriately increased sampling numbers should be tested until satisfactory results are
achieved. And this can be done easily by examining the evanescent waves in the spectrum domain.

The simulation on the well-known dipole antenna has revealed that two maximums at each end of the antenna exist in its aperture field distribution. This means that the traditional view of the aperture distribution of one maximum located in the centre, is inaccurate. The field distributions around these two maximum points have been well recovered (reconstructed) by the method using the simulation.

Using these simulations, the effect of evanescent waves on the reconstruction of the aperture distribution has been investigated by the plane wave spectrum method. Although the evanescent waves make no contributions to the far-fields, they do contribute to the aperture distribution, if they are significantly large. The evanescent waves can be readily examined in the spectrum domain.

The technique of the PWS method developed in this chapter is rigorous and can be used in reconstructing aperture distributions of generic antennas or arrays.

In the next chapter, experimental results will be presented which verify these conclusions.
CHAPTER 6

EXPERIMENTAL RESULTS OF THE PLANE WAVE SPECTRUM METHOD

Experiments in one dimension have been carried out using the Near-Field Range Facilities at Fleurs Field Station, University of Western Sydney. The measurement system was set up using available equipment. Most of the test antennas and detecting probes were specially designed and developed for this work.

6.1 SYSTEM DESCRIPTION

The Near-Field Range at Fleurs Field Station, University of Western Sydney, Nepean is an open space measurement site on a ground plane. The equipment consisted of a signal generator (i.e. a synthesizer), a vector network analyser (VNA) as a receiver, and a field scanner driven by a step-motor. A directional coupler provided a reference signal to the network analyser. A probe, which was located on top of a wooden pole as a receiving antenna, was used to detect the field. The scanning motion and the VNA were controlled by a personal computer. The scanned field data and position information were digitised and stored in the computer. A block diagram in Fig 6.1 shows details of the measurement configurations.
The antenna under test was used as a transmitting antenna. The position of

![Diagram: Block diagram of near-field measurement system]

Fig 6.1 Block diagram of near-field measurement system

the probe was adjustable. The antenna and the probe were located about 6 meters above the ground. The reflections from the ground are ignored, as all the measurements discussed here were taken close to the antenna (or array). The near-field was scanned along the horizontal direction. The measured field data, both amplitude and phase, was acquired by the computer through the IEEE-488 Bus.
6.2 NEAR-FIELD DISTRIBUTIONS

First, a centre-fed dipole antenna with length of 1.12 metres was used as a transmitting antenna. The distance \( d \) between the dipole and the probe was set to 0.06 metres. The \( E \)-field was probed parallel to the axis of the antenna. The probe was a 0.2-m small dipole.

The probed \( E \)-field distributions of both amplitude and phase at 110 MHz,
135 MHz and 270 MHz are shown in Fig 6.2, Fig 6.3 and Fig 6.4 respectively. The amplitude is normalized to 0 dB; the unit of phase is a degree; and the unit of the x-coordinate is a metre. From Fig 6.2 and Fig 6.3, it can be seen that the patterns of the field distributions are quite similar compared with Fig 6.4 which shows about a 3 dB increase in the centre and about a 3 dB drop around one end of the antenna. This is partially due to the balun (which was designed for 135 MHz) feed of the transmitting antenna was unchanged for all three frequencies, and partially due to the electrical length of the probe. At a frequency of 135 MHz, the antenna under test is about a half wavelength long. The current distribution of a half-wavelength dipole is well defined and evenly distributed about the centre. This distribution characteristic is well reflected in the $E$-field distributions of Fig 6.3. But, when the frequency is increased to 270 MHz, the antenna becomes one wavelength long. In such a case, the narrow band balun circuit becomes defective, and the electrical length of the probe becomes too large. Therefore the probe picks up more signal at the centre of the transmitting antenna. Still at this frequency, when the transmitting antenna is rotated by 180 degrees, the 3 dB down pattern
(Fig 6.4) is changed to the corresponding side, which proves an unbalanced feed circuit.

In fact, it is clearer to see these differences of field distributions in the spectral domain. As shown in Fig 6.5 to Fig 6.7, the distributions of the plane wave spectrum $A(k_y)$ as defined in previous chapters have been plotted. Note that the amplitude of a spectrum $A(k_y)$ has been normalized to 0 dB. The horizontal
coordinate \( k_y \) is the direction of the wave propagation. For convenience, its unit is the wavenumber in free space \( k_0 = 2\pi/\lambda \). At 135 MHz, the spectrum is uniformly distributed with respect to the centre \( k_y = 0 \), while the spectrum distribution of Fig 6.7 shows significant changes from an even distribution. All distributions show noticeable evanescent wave components which lie in the region of \( |k_y| > k_0 = 2\pi/\lambda \). At a higher frequency, \( i.e. f = 270 \) MHz, the evanescent wave components are smaller (Fig 6.7) than the others, because the distance of the measurement plane to the antenna is relatively longer in terms of wavelength. The first trough is around \( k_y = 0.4k_0 \) and the first peak is around \( k_y = -0.5k_0 \) which is far from a symmetric distribution, and suggests that the \( E \)-field distribution will also not be symmetric. And indeed it is true, as is clearly seen from Fig 6.4.

Some of these near-field distributions in the region where the peaks are located, are in very reasonable agreement with the simulations conducted in Chapter 5. The maximum \( E \)-field strength occurs at both ends of the antenna. The \( E \)-field in the centre of the antenna is in the order of 2 dB down from the peaks at both ends, as it is caused by the radiation leaking from the antenna feed, which in practise is not infinitely small. This effect cannot be obtained from the simulation presented in Chapter 5, because the precise antenna feed structure could not be simulated.

### 6.3 Aperture Field Reconstructions

To experimentally verify the algorithm proposed in Chapter 5, field measurements have been made. By using the algorithm developed in Chapter 5,
the aperture fields at a plane $z=d$ have been reconstructed. There are a few criteria which limit the choice of the plane $d$ at which the field is to be reconstructed. Firstly, it cannot be at $d=0$, because a singularity may occur (theoretically) for some antennas, and of course it is not practical. Secondly, it should be close to the antenna so that the evanescent components (if any) can be recovered. And thirdly, it should be practical to conduct a measurement so that the reconstructed field can be compared with the measured field.

$E$-field measurements have been made at the planes $z=0.29m$ and $z=0.50m$ at three frequencies of 110, 135 and 270 MHz using the facility shown in Fig 6.1. Both amplitudes and phases for these three frequencies are presented in Fig 6.8, Fig 8.9 and Fig 6.10, respectively. The thinner line curves and the thicker line curves are the fields at planes of $z=0.29m$ and $z=0.50m$, respectively. The sampling interval for these measurements was 0.1m, which is well below 0.25$\lambda$ even at highest frequency of 270 MHz ($\Delta y=0.09\lambda$ for this frequency). The amplitude is
normalized to 0 dB and the unit for phase is a degree. The unit for x-coordinate (horizontal) is a metre.

**Fig 6.9 Amplitude and phase of measured fields at z=0.29, 0.50 m at 135 MHz**

From Fig 6.8(a), it is clearly seen that as the measurement plane moves away from the antenna, the peak which occurs at each end of the antenna is disappearing (thinner curve); and the single peak at the centre is emerging (thicker curve). This can also be observed at a slightly higher frequency of 135 MHz as shown in Fig 6.9(a). But when the frequency increases to 270 MHz, this effect no
longer holds, as shown in Fig 6.10(a). The distances of measurement planes to the antenna for both curves are longer in terms of wavelength.

Furthermore, from the phase distributions of Fig 6.8(b) and Fig 6.9(b), the patterns of the curves are similar and the phase delay effect can be easily observed.

![Graphs of |E| and E-phase at f=110 MHz](image)

**Fig 6.11** Field distributions measured at z=0.06m and reconstructed field distributions from fields measured at z=0.29m, f=110 MHz

On the other hand, from Fig 6.10(b), the phase distributions at both measurement planes are more complex for this one-wavelength antenna. As stated before, the distributions are not symmetric about the centre of the antenna, because of the balun circuit.

The fields measured at plane z=0.29m for all three frequencies of 110, 135 and 270 MHz have been reconstructed (transformed) back to the plane of z=0.06m, as shown in Fig 6.11 to Fig 6.13. For comparison purposes, the fields measured at the plane of z=0.06m, as shown in Fig 6.2 to Fig 6.4, are also re-plotted on the same graphs for each frequency. In all three figures of 6.11 to 6.13, the measured fields
are represented by the thicker line curves and the reconstructed fields are represented by the thinner line curves.

Both Fig 6.11(a) and Fig 6.12(a) show that two peaks of the amplitude at both ends of the antenna have been recovered very reasonable in terms of positions. But the shapes of the reconstructed curves are broader.

Fig 6.12 Field distributions measured at \( z=0.06 \)m and reconstructed field distributions from fields measured at \( z=0.29 \)m, f=135 MHz

There is a smaller peak, which is about 5 dB below the end peaks, in the centre of the antenna in the measured field distributions. This is again a leaking wave from the centre feed circuit. And this middle peak could not be recovered in the reconstructed distribution. This leaking wave must be very small and it has died off at the measurement plane. The reconstructed field in the centre is somehow buried with the trough. This defect might be due to the accuracies in the measurements, as it did not occur in the simulations in Chapter 5.
The shapes of the reconstructed phase distributions in both Fig 6.11(b) and Fig 6.12(b) are closer to the measured, comparing with the amplitude discussed above, especially in the antenna region.

At a higher frequency of 270 MHz, as shown in Fig 6.13, the right hand-side peak at about $y=+0.5\text{m}$ (one end of the antenna) is recovered well, and even the trough at $y=+0.15\text{m}$ is also recovered. But the two peaks at $y=-0.7\text{m}$ and $y=-0.1\text{m}$ are merged into a single peak at about $y=-0.4\text{m}$. On the other hand, in the phase distributions of Fig 6.13(b), these two peaks at $y=-0.7\text{m}$ and $y=-0.1\text{m}$ are recovered and are recognizable.

It is also noticed that the measured field distribution is not as symmetrical as it should be. As mentioned before, it is partially due to the design of the test antenna and the design of the probe and partially due to the limited accuracy of the test system.

The dipole used as the test antenna was originally designed as a half-wavelength antenna at 135 MHz and so was the probe. The test conducted at
270MHz above (i.e. Fig 6.13) is to see whether the technique can be used in a relaxed condition. From this experiment, it has been observed that a highly asymmetric distribution has occurred at this frequency. It is most likely due to this larger and out-of-balance probe and also incorrect balun used at this frequency. The feeding point at the middle of the antenna also becomes more radiative.

To examine the capability of the technique, the measurement plane was moved further away from the antenna. The fields are also reconstructed from the measurements made at the plane of \( z=0.06 \text{m} \). Although the distance of the measurement plane to the antenna is nearly doubled, the reconstructed field at 110MHz is almost identical to the field reconstructed from the plane \( z=0.29 \text{m} \). In Fig 6.14 and Fig 6.15, reconstructed and measured fields for the two frequencies of 110 and 270 MHz are presented graphically. Again, the thicker and thinner lines are the measured and reconstructed field distributions at the plane of \( z=0.06 \text{m} \) respectively.

Fig 6.14 Field distributions measured at \( z=0.06 \text{m} \) and reconstructed field distributions from fields measured at \( z=0.50 \text{m} \), f=110 MHz
Comparing these two exercises with different measurement planes, it can be seen that the results of reconstructing aperture distributions made from the measurement plane closer to the antenna are better. This is due to the contributions from the evanescent waves.

From the measured field distributions measured at planes of z=0.29m and z=0.50m, it is very difficult to recognize the true distributions at very close to the antenna i.e. at z=0.06m. As shown in Fig 6.2 to Fig 6.4, the peaks at both ends of

![Graph 1](image1)

**Fig 6.15** Field distributions measured at z=0.06m and reconstructed field distributions from fields measured at z=0.50m, f=135 MHz

the antenna are hardly identifiable from the measurements in Fig 6.8 to Fig 6.10. But the reconstructed field distributions shown in the reconstructed graphics, are much closer to the real distributions.

At two frequencies i.e. 110 and 135 MHz, the reconstructed fields at two ends of the antenna are recognizable. This Plane Wave Spectrum method (PWS) and its associated algorithm developed in the previous chapters can be used to investigate the field distributions in the very near-field regions. Furthermore, the
technique can be used to diagnose some antenna problems such as feed and mechanical structure failures.

6.4 FIELD ANALYSIS IN SPECTRAL DOMAIN

As stated in previous chapters, the first step in reconstructing a field using

Fig 6.16 Spectrum distributions of measured fields at planes of z=0.29, 0.50m, at 110 MHz

this algorithm is to calculate the plane wave spectrum from the measured field.

Fig 6.17 Spectrum distributions of measured fields at planes of z=0.29, 0.50m, at 135 MHz
Fig 6.16 to Fig 6.18 show the spectrum distributions of the measured fields at 110 MHz, 135 MHz and 270 MHz respectively. The thinner and thicker lines represent the spectra calculated from fields measured at planes of \( z=0.29 \)m and \( z=0.50 \)m, respectively.

It can be seen that all these spectra have shown significant evanescent wave components, \( i.e. \) at the region of \( |k_y|>k_0 \). And these components do have contributions to reconstruct the aperture fields. It is also clear that the magnitude of the evanescent waves have reduced at least 5 dB at the plane of \( z=0.50 \)m compared with the plane at \( z=0.29 \)m.

![Graph showing spectrum distributions](image)

**Fig 6.18 Spectrum distributions of measured fields at planes of \( z=0.29, 0.50 \)m, at 270 MHz**

### 6.5 MAGNETIC (H) FIELD MEASUREMENT AND ANALYSIS

To gain a full picture of the field distributions very close to the antenna, the magnetic fields were also investigated experimentally.
The measurement configuration was the same as before, as shown in Fig 6.1. The transmitting antenna was still the 1.12m dipole, but the probe was changed to a small loop antenna. The sampling rate and other measurement parameters remained unchanged. The $H$ fields were measured at planes of

![Graph](image1)

**Fig 6.19** Measured and reconstructed $H$ fields at $f=110$ MHz

![Graph](image2)

**Fig 6.20** Measured and reconstructed $H$ fields at $f=135$ MHz

$z=0.07\text{m}$ and $z=0.56\text{m}$. The field was then reconstructed at the plane of $z=0.07\text{m}$, using the field measured at $z=0.56\text{m}$. Fig 6.19 and Fig 6.20 show the reconstructed

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and measured $H$-field distributions at 110 MHz and 135 MHz respectively. Again the thinner curves are the distributions of reconstructed fields.

In contrast with the $E$ field, it is seen that the strongest field in the $H$ field distribution occurs at the centre of the antenna. The reconstructed and the measured $H$ fields are in good agreement within the antenna region.

6.6 ANTENNA ARRAY MEASUREMENT AND ANALYSIS

In previous sections, it has been shown that the technique can be applied to

![Diagram of 2-element dipole array]

**Fig 6.21 2-element dipole array**

single antennas. In fact the main purpose of this study is to deal with array antennas. This section shows some experimental results on array antennas.

Firstly, a 2-element dipole array antenna was examined. Two half-wave dipole antennas, at 1400 MHz, were aligned along their axes as shown in Fig 6.21. The distance between the centres of the 2 dipoles was 0.22m. The $E$-field probe was a small dipole. The measurement system settings were similar as shown in Fig 6.1.
The distance from the measurement plane to the antenna array was \( d=0.075\text{m} \). The array was scanned along the antenna axial direction. When these two dipoles were fed with the same amplitude and in phase, the \( E \)-field distribution was as shown in Fig 6.22. As can be predicted, it was a superposition of the fields from two dipoles. There are 4 peaks in the distribution. Each dipole has a peak at each end, as shown in previous sections. But this 4-peak distribution is not evenly distributed. Due to the symmetrical structure arrangement, in general, the two peaks in the middle should be the same and the two peaks in both sides should be the same. The fact of this uneven distribution of measured field may be caused by the misalignment of the antennas and other factors such as defect in detecting probe, antenna feeds and the test conditions.

When one of the dipoles was fed with -3dB amplitude and the same phase, the field distribution appears changed noticeably, as shown in Fig 6.23.
But when the feeding signals were changed to a 90 degree difference in phase, and the amplitude remained the same, the field distribution changed dramatically, as shown in Fig 6.24.

![Field distribution of 2-element dipole array fed with same amplitude but different phase](image)

**Fig 6.24 Field distribution of 2-element dipole array fed with same amplitude but different phase**

From these two examples, the field of a dipole array is more sensitive to the phase anomalies than the amplitude.

To test how the technique can be used for array antennas, a 4-element helix antenna array was examined. The array was comprised of 4 elements with 0.25m apart. Each element has two and a half turns and was installed in a cap-shaped cavity.

All the $E$-field discussed below is the vertical-polarized component only. The probe used was a small dipole.

Firstly, the $E$-field was measured at a plane of $d=0.57$m. This field was then used to reconstruct the aperture field at a plane of 0.16m. For comparison purposes, the $E$-field was also measured at the plane of $d=0.16$m. The frequency for these tests was 1.6 GHz. The sampling interval for both measurements was 0.05m.
Fig 6.25 shows both reconstructed and measured fields at the plane of $d=0.16 \text{m}$. The amplitude and the phase fed to each of the 4 elements were the same. The thicker line represents the measured field distributions.
When the amplitude of the feed signal of far-left element was -3dB with respect to the rest of the element, the distributions are as shown in Fig 6.26. To examine the effect of amplitude change to an element, the second left element was also fed with a -3dB with respect to the other elements. The field distributions are shown in Fig 6.27. In both cases, there are some differences in the distributions compared to the same amplitude fed to all 4 elements as in Fig 6.25.

But it is clearly shown, in Fig 6.28, that one of the element, in this case the second right element, was fed with a 90 degree phase difference with respect to the other elements.
Fig 6.28 2nd right element fed with a 90 degree difference in phase

In all cases, four peaks in the reconstructed fields in the locations of each element are clearly identifiable. Some of the uneven distributions are partly due to the feed cabling and partly due to the structure of the individual elements.

Due to the limits in accuracies of the experiments and the mutual coupling errors, some of the amplitude anomalies in this helix array exercise could not be detected satisfactorily. A more complex modelling in array antennas should be researched. The more accurate experiment system is also needed for these more complicated problems.
CHAPTER 7

APERTURE FIELD RECONSTRUCTIONS BY CYLINDRICAL NEAR FIELD MEASUREMENTS

7.1 INTRODUCTION

The technique of aperture field reconstruction from near-field data measured on a plane has been fully described in previous chapters. The planar near-field scanning technique is widely used, because its theoretical model is simple. However, in some cases, where the antennas or the antenna arrays have mechanical structures which have some axial symmetry, the cylindrical near-field measurements are preferable.

A method is developed here for determining the aperture field distribution of an antenna from near-field measurements over the surface of a right circular cylinder enclosing the antenna. This technique has been widely employed to determine the far-field patterns of antennas under test, but it seems that no reports have been found where this technique is used to reconstruct aperture fields. This chapter describes determination of the aperture field of an antenna or an array from measurements made in the near-field on a cylindrical surface.

The method proposed in this chapter is derived from the wave modal expansion theory which has already been employed in near-field to far-field transformation techniques. It is based on the three-dimensional vector cylindrical wave expansion of an electromagnetic field.
7.2 SOLUTIONS OF VECTOR WAVE EQUATIONS

In linear, isotropic, and source free regions, the vector wave equation is homogeneous and can be expressed as:

$$\nabla^2 C + k^2 C = 0 \quad (7.1)$$

or

$$\nabla \cdot C - \nabla \times \nabla \times C + k^2 C = 0 \quad (7.2)$$

where $C$ represents any one of the vectors $E$ and $H$, and $k$ is the wavenumber. In the rectangular coordinate system, the above vector equation can be decomposed into three (or sometimes two) independent scalar equations. In most cases, the scalar forms in such a coordinate system are easily solved. However, in general coordinate systems, it is difficult or impossible to decompose the vector equation into scalar forms, so that it is necessary to find a solution to the vector equation (7.1).

Suppose that a solution to the following scalar wave equation

$$\nabla^2 \psi + k^2 \psi = 0 \quad (7.3)$$

is known. Let $a$ be a constant unit vector. Using vector $a$, the following three vector functions can be defined

$$L = \nabla \psi, \quad M = \nabla \times a \psi, \quad N = \frac{1}{k} \nabla \times M \quad (7.4)$$
Now, the following proves that these three vector functions satisfy the vector wave equation (7.2). To do this, substituting $L=\nabla\psi$ for $C$, the equation (7.2) holds. From the vector identity $\nabla\cdot\nabla\times F=0$, and the definitions of $M$ and $N$, it follows that:

$$\nabla\cdot M = 0, \quad \nabla\cdot N = 0 \quad (7.5)$$

Consider the expression

$$\nabla\times\nabla\times M - k^2 M \quad (7.6)$$

Since

$$\nabla\times M = \nabla\times\nabla\times(\psi a) = \nabla\nabla\cdot(\psi a) + a k^2 \psi \quad (7.7)$$

then,

$$\nabla\times M - k^2 \psi a = \nabla\nabla\cdot(\psi a) \quad (7.8)$$

Taking the curl of both sides of this equation:

$$\nabla\times\nabla\times M - k^2 \nabla\times(\psi a) = \nabla\times\nabla\nabla\cdot(\psi a) \quad (7.9)$$

The right hand-side is identically zero upon using $\nabla\times\nabla F=0$, where $F$ is any scalar function. That is

$$\nabla\times\nabla\times M - k^2 M = 0 \quad (7.10)$$

Replacing $C$ with $M$ defined by (7.4) and combining (7.5) and (7.10), it can be seen that $M$ satisfies the vector wave equation (7.2).
Using the definition of $N$ in (7.4), (7.10) can be expressed as

$$\nabla \times N - k M = 0 \quad (7.11)$$

Take curl on both sides and replace $M$ with $N$ as defined by (7.4), then

$$\nabla \times \nabla \times N - k^2 N = 0 \quad (7.12)$$

From (7.5) and (7.12), $N$ also satisfies the vector wave equation (7.2).

Since $a$ is a constant vector, $M$ can be expressed as

$$M = \nabla \times (\psi a) = \nabla \psi \times a \quad (7.13)$$

From the definition of $L=\nabla \psi$, (7.13) may be written as

$$M = L \times a \quad (7.14)$$

From (7.11), $M$ can also be expressed as

$$M = \frac{1}{k} \nabla \times N \quad (7.15)$$

The dot product of $L$ and $M$ is zero, so that they are orthogonal vector functions.

Let $\psi_n$ be any solution to the scalar wave equation (7.3). It has been proven that $L_n$, $M_n$, and $N_n$ obtained from using $\psi_n$ in (7.4) are solutions of the vector wave equation (7.2). Assuming that $\psi_n$ is a continuous, nonsingular, and single-valued function, then any of $L_n$, $M_n$, and $N_n$ is an independent solution to (7.2) due to their orthogonality from (7.14) and (7.15). The linear superposition of $L_n$, $M_n$, and $N_n$, then constitute the complete solutions to the vector wave equation (7.2).
For example, let the Hertz magnetic vector potential $A$ replace $C$ in (7.2). The solution for $A$ can be written as

$$ A = \frac{1}{j\omega} \sum_{n=-\infty}^{\infty} \left( a_n M_n + b_n N_n + c_n L_n \right) $$  \hspace{1cm} (7.16) $$

where the coefficients $a_n$, $b_n$, and $c_n$ are determined by the source and the boundary conditions. From $H=(1/\mu)\nabla \times A$ and Ampere’s law $E=(1/j\omega \varepsilon)\nabla \times H$, solutions of electric field $E$ and magnetic field $H$ can be obtained as

$$ E = \sum_{n=-\infty}^{\infty} (a_n M_n + b_n N_n) $$  \hspace{1cm} (7.17) $$

$$ H = \frac{-k}{j\omega \mu} \sum_{n=-\infty}^{\infty} (b_n M_n + a_n N_n) $$  \hspace{1cm} (7.18) $$

where (7.5) has been used. It is seen that in a source free region, only $a_n$ and $b_n$ need to be determined.

\section*{7.3 CYLINDRICAL WAVE EXPANSION}

In a linear, isotropic region containing no sources, a solution $\psi_n^i$ to the scalar wave equation (7.3) in the cylindrical coordinate system is

$$ \psi_n^i (r, \phi, z) = Z_n^i (\Lambda r) \exp(jn\phi) \exp(-jhz) $$  \hspace{1cm} (7.19) $$

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where \( n \) is any integer, \( h \) is any real number, and where \( \Lambda \) is

\[
\Lambda = \begin{cases} 
\sqrt{k^2 - h^2} & h \leq k \\
-j\sqrt{h^2 - k^2} & h > k 
\end{cases}
\]  

(7.20)

Substitute \( \psi_n^i \) back to the expressions (7.4) and let the vector constant \( a \) equal the unit vector in the \( z \) direction, then vectors \( M_n \) and \( N_n \) are written as

\[
M_{nh}^i(r, \phi, z) = \left( r^2 \frac{jn}{r} Z_n^i(\Lambda r) \cdot \frac{\partial}{\partial r} Z_n^i(r) \right) \exp(j\phi) \exp(-jhz) 
\]  

(7.21)

\[
N_{nh}^i(r, \phi, z) = \left( -r \frac{jh}{k} \frac{\partial}{\partial r} Z_n^i(r) + \frac{nh}{kr} Z_n^i(\Lambda r) + \frac{\Lambda^2}{k} \frac{\partial}{\partial r} Z_n^i(\Lambda r) \right) \exp(j\phi) \exp(-jhz) 
\]  

(7.22)

The function \( Z_n^i(\Lambda r) \) is any one of the four cylindrical Bessel functions given by

\[
Z_n^1(\Lambda r) = J_n(\Lambda r), \quad Z_n^2(\Lambda r) = Y_n(\Lambda r), \quad Z_n^3(\Lambda r) = H_n^{(1)}(\Lambda r), \quad Z_n^4(\Lambda r) = H_n^{(2)}(\Lambda r). 
\]

In the present case, a solution for \( E \) is required, which is valid in the region external to a small cylinder containing all sources defined by \( r \geq r_0 \). In order for \( E \) to satisfy the radiation condition at infinity, the large argument asymptotic expansion of the cylindrical Bessel function must represent surfaces of constant phase which propagates in the positive radial direction. For time variations of the form \( \exp(j\omega t) \), the only one of these functions which satisfies this condition is a Hankel function of the second kind \( H_n^{(2)}(\Lambda r) \). The large argument asymptotic expansion of this function [1] is
$$H_n^{(2)}(\Lambda r) \rightarrow j^{n+\frac{1}{2}} \sqrt{\frac{2}{\pi \Lambda r}} \exp(-j\Lambda r); \quad r \rightarrow \infty$$  \hspace{1cm} (7.23)

The general linear combination of the vectors \( \mathbf{M} \) and \( \mathbf{N} \) will involve an integral over all real \( h \) and a sum over all integer \( n \). Thus, in this case, the general solution for \( \mathbf{E} \) can be written as

$$\mathbf{E}(r, \phi, z) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \left( a_n(h) \mathbf{M}_{nh}^4(r, \phi, z) + b_n(h) \mathbf{N}_{nh}^4(r, \phi, z) \right) dh$$  \hspace{1cm} (7.24)

where \( a_n(h) \) and \( b_n(h) \) are the complex amplitude weighting functions of the vectors \( \mathbf{M} \) and \( \mathbf{N} \), respectively. The corresponding solution for \( \mathbf{H} \) follows from Maxwell’s equation \( \mathbf{H} = (-1/j\omega \mu) \nabla \times \mathbf{E} \) and is given by

$$\mathbf{H}(r, \phi, z) = \frac{-k}{j\omega \mu} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \left( b_n(h) \mathbf{M}_{nh}^4(r, \phi, z) + a_n(h) \mathbf{N}_{nh}^4(r, \phi, z) \right) dh$$  \hspace{1cm} (7.25)

In general, the evaluation of vector equations of (7.24) and (7.25) is a tedious task. However, the calculation can be simplified by decomposing the vector expressions to their scalar forms. Substituting (7.21) and (7.22) into (7.24) and (7.25), the following expressions for electric and magnetic field components can be written as

$$\mathbf{E}_z(r, \phi, z) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\Lambda^2}{k} b_n(h) H_n^{(2)}(\Lambda r) \exp(jn\phi) \exp(-jhz) dh$$  \hspace{1cm} (7.26)
\begin{equation}
E_{\phi}(r, \phi, z) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \left( -a_n(h) \frac{\partial H_n^{(2)}(\Lambda r)}{\partial r} + b_n(h) \frac{n h}{k r} H_n^{(2)}(\Lambda r) \right) \cdot \exp(jn\phi) \exp(-jhz) \, dh
\end{equation}

(7.27)

\begin{equation}
E_{r}(r, \phi, z) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \left( a_n(h) \frac{j n}{r} H_n^{(2)}(\Lambda r) - b_n(h) \frac{j h}{k} \frac{\partial H_n^{(2)}(\Lambda r)}{\partial r} \right) \cdot \exp(jn\phi) \exp(-jhz) \, dh
\end{equation}

(7.28)

\begin{equation}
H_z(r, \phi, z) = \frac{-1}{j \omega \mu} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \Lambda^2 a_n(h) H_n^{(2)}(\Lambda r) \exp(jn\phi) \exp(-jhz) \, dh
\end{equation}

(7.29)

\begin{equation}
H_\phi(r, \phi, z) = \frac{1}{j \omega \mu} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \left( b_n(h) \frac{k}{r} \frac{\partial H_n^{(2)}(\Lambda r)}{\partial r} - a_n(h) \frac{n h}{r} H_n^{(2)}(\Lambda r) \right) \cdot \exp(jn\phi) \exp(-jhz) \, dh
\end{equation}

\end{equation}

(7.30)

\begin{equation}
H_r(r, \phi, z) = \frac{1}{j \omega \mu} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \left( -b_n(h) \frac{j n k}{r} H_n^{(2)}(\Lambda r) + a_n(h)(jhk) \frac{\partial H_n^{(2)}(\Lambda r)}{\partial r} \right) \cdot \exp(jn\phi) \exp(-jhz) \, dh
\end{equation}

(7.31)

where the Bessel function \( Z_n \) has been replaced by a Hankel function of the second kind \( H_n^{(2)}(\Lambda r) \). If the cylindrical wave amplitude weighting functions \( a_n(h) \) and \( b_n(h) \) are known, the fields in everywhere can be evaluated from (7.26) to (7.31).

The examination of equation (7.26) reveals that the \( z \) component of electric field \( E_z(r, \phi, z) \) is in the form of a Fourier series in \( \phi \) and a Fourier integral in \( z \). Thus the equation has an inverse which is given by

\begin{equation}
b_n(h) = \frac{k}{\Lambda^2 H_n^{(2)}(\Lambda r)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_z(r, \phi, z) \exp(-jn\phi) \exp(jhz) \, d\phi \, dz
\end{equation}

(7.32)

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Similarly, the relationship between $a_n(h)$ and the z component of magnetic field, $H_z(r,\phi,z)$, can be obtained

$$a_n(h) = \frac{-j\omega}{\Lambda^2 H_n^{(2)}(\Lambda r)} \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} H_z(r,\phi,z) \exp(-j\phi) \exp(jhz) \, d\phi \, dz \quad (7.33)$$

Expressions (7.32) and (7.33) are the desired results. If an ideal probe is used to measure the near-fields of $E_z$ and $H_z$ on a cylinder $r=r_o$ which encloses the antenna under test, the amplitude weighting functions $a_n$ and $b_n$ can be calculated from (7.32) and (7.33), respectively, using the FFT algorithm.

7.4 NUMERICAL CONSIDERATIONS

In this section, a numerical solution for $a_n(h)$ and $b_n(h)$ is developed which is based on the results of Chapter 5. The numerical evaluation of (7.26) through (7.31) for the fields of an antenna then described.

Let the measurement cylinder be divided into a lattice of points with coordinates $(r_o, n\Delta\phi, m\Delta z)$ where $0 \leq n \leq N-1$, $0 \leq m \leq M-1$, and $M$ and $N$ are positive integers. To exactly evaluate (7.32) and (7.33) from the output voltage of the probe at these points, two conditions must be satisfied. First, $E_z$ and $H_z$ must be zero when $z<0$ and $z>(M-1)\Delta z$. Second, the sampling intervals $\Delta\phi$ and $\Delta z$ in $E_z$ and $H_z$ must follow the Nyquist sampling theorem which has been discussed in the previous chapters. The first condition can not be met with any radiating structure. However, if the test antenna is aligned in a cylinder so that it does not radiate appreciably in the $\pm z$ directions, the output voltage can be evaluated approximately if $M$ is chosen large enough.
Practical upper bounds on $\Delta \phi$ and $\Delta z$ can be obtained by using the Nyquist sampling theorem which has been discussed in Chapter 5 on planar near-field measurements. In this case, the starting point on $\Delta z$ remains the same as $\lambda/4$. For the angular sampling interval $\Delta \phi$, the starting point can be deduced as follows. The arc length with respect to $\Delta \phi$ is $(r_c \Delta \phi)$. It should be less than or equal to $\lambda/4$, according to the sampling criterion developed in earlier work. And the radius of the measurement cylinder should be greater than $a$ which is the radius of the smallest cylinder with centre coinciding with the z axis which completely encloses the antenna. Thus it follows that practical starting bounds on the sampling intervals are

$$\Delta \phi \leq \frac{\pi}{2ka} \quad (7.34)$$

and

$$\Delta z \leq \frac{\lambda}{4} \quad (7.35)$$

It is worthwhile mentioning that if the evanescent waves in the radiating structure are significant, the sampling intervals $\Delta \phi$ and $\Delta z$ should be chosen appropriately to reflect their contributions to the aperture fields. This issue will be discussed in detail in the following sections.

Assuming that the preceding conditions are met, the integrals of $a_n(h)$ and $b_n(h)$ can be evaluated easily with a two-dimensional fast Fourier transform (FFT) algorithm. After obtaining the amplitude weighing functions $a_n(h)$ and $b_n(h)$, the fields at any point $(r, \phi, z)$ can be calculated using (7.26) through (7.31).
7.5 SIMULATIONS WITH A CENTRE-FED DIPOLE

The computer simulated program has been developed using the centre-fed dipole as the test antenna. A series of results which confirm previous work will be presented in this section.

\[ E_z = \frac{-jI_0\eta}{4\pi} \left( e^{jk\theta_1} + e^{jk\theta_2} \cdot 2 \cos\left(\frac{kL}{2}\right) e^{jkR} \right) \]  \hspace{1cm} (7.36)

And correspondingly, the $\phi$ component of magnetic field at the point $p$ is:

\[ H_\phi = \frac{jI_0}{2\pi R \sin \theta} \left( e^{-jk\phi_1} + e^{-jk\phi_2} \cdot 2 \cos\left(\frac{kL}{2}\right) e^{-jkR} \right) \]  \hspace{1cm} (7.37)

where $k=2\pi/\lambda$ is the wavenumber, $\eta=377\Omega$ is intrinsic impedance of free space and $I_0$ is maximum current. The equations (7.36) and (7.37) can be used to calculate the
exact aperture distributions and near-field distributions without any distance restrictions [77].

In this case, the near-field data are simulated on a cylinder \( r=r_0 \) using these equations, so that the parameters \( R \), \( s_1 \), and \( s_2 \) should be expressed in the cylindrical coordinate system, i.e.

\[
R = \sqrt{r^2 + z^2} \tag{7.38}
\]

\[
s_1 = \sqrt{r^2 + (z - \frac{L}{2})^2} \tag{7.39}
\]

\[
s_2 = \sqrt{r^2 + (z + \frac{L}{2})^2} \tag{7.40}
\]

For the given measurement cylinder \( r=r_0 \), the \( z \) component of electric field \( E_z \) is independent of \( \phi \). Therefore the expression (7.32) can be simplified as follows.

Rewrite (7.32):

\[
b_n(h) = \frac{k}{\Lambda^2 H_0^\omega(\Lambda r)} \int_{-\infty}^{\infty} E_z(r_0, z) \exp(-j\phi) \exp(jhz) d\phi dz \tag{7.41}
\]

Let

\[
I = \int_{-\pi}^{\pi} \exp(-j n\phi) d\phi = \frac{2 \sin(n\pi)}{n} \begin{cases} 
2 \pi & n = 0 \\
0 & n \neq 0
\end{cases} \tag{7.42}
\]

then, \( b_n \) becomes \( b_0 \) (\( b_n = 0 \) for \( n \neq 0 \)) only:

\[
b_n(h) = b_0(h) = \frac{2\pi k}{\Lambda^2 H_0^\omega(\Lambda r_0)} \int_{-\infty}^{\infty} E_z(r_0, z) \exp(jhz) dz \tag{7.43}
\]

Let

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\[ b'_0(h) = \int_{-\infty}^{\infty} E_z(r_o, z) \exp(jhz) dz \tag{7.44} \]

It can be seen that \( b'_0 \) and \( E_z(r_o, z) \) is a Fourier transform pair. From (7.43), it follows that

\[ b_0(h) = \frac{2\pi k}{\Lambda^2 H_0^{(2)}(\Lambda r_o)} b'_0(h) \tag{7.45} \]

Since the \( z \) component of the magnetic field, \( H_z(r_o, z) \), is zero in this case, it follows from (7.33) that the other amplitude weighting function \( a_0(h) \) is correspondingly zero. Thus the normalised fields radiated by the dipole on any cylinder can be expressed in terms of \( b_0(h) \) as

\[ E_z(r, z) = \int_{-\infty}^{\infty} b_0(h) \Lambda^2 H_0^{(2)}(\Lambda r) \exp(jhz) dh \tag{7.46} \]

\[ E_r(r, z) = 0 \tag{7.47} \]

\[ E_\theta(r, z) = 0 \tag{7.48} \]

\[ H_z(r, z) = 0 \tag{7.49} \]

\[ H_r(r, z) = 0 \tag{7.50} \]

\[ H_\theta(r, z) = \int_{-\infty}^{\infty} \left[ b_0(h) \frac{\partial H_0^{(2)}(\Lambda r)}{\partial r} \right] \exp(-jhz) dh \tag{7.51} \]

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Let the length of the dipole be $L=\lambda/2$, the sampling interval $\Delta z=\lambda/4$, and the number of sampling points $N=64$. Fig 7.2 shows the near-field $E_z(r_0,z)$ distribution.

![Graph of $E_z(z)$ simulated](image)

**Fig 7.2** Simulated near-field distribution at $r=0.5\lambda$.

![Graph of $b_0'(h)$](image)

**Fig 7.3** Magnitude distribution of the amplitude weighting function $b_0'(h)$.
with respect to $z$ on the cylinder $r_0=0.5\lambda$ which is calculated from (7.36) and (7.38) to (7.40). It can be seen that the field distribution is similar to that described in Chapter 5.

Using this simulated field data, the amplitude weighting function $b_0(h)$ can be obtained from (7.44). The normalised distribution of $b_0(h)$ is shown in Fig 7.3. Therefore $b_0(h)$ can be calculated by (7.45). Using the amplitude weighting function $b_0(h)$ so obtained, the fields on any given cylinder can be evaluated from (7.46) through (7.51).

For example, The electric field $E_z(r_0,z)$ on a cylinder $r_0=0.01\lambda$ which is almost right on the surface of the dipole antenna is reconstructed by the calculated $b_0(h)$ from (7.46). The distribution is depicted in Fig 7.4. For comparison, Fig 7.4 also shows the distribution of $E_z(r_0,z)$ on the same cylinder $r_0=0.01\lambda$ directly calculated from the expression (7.36) with thicker curve. They agree well along the region of

![Calculated and reconstructed field (E_z) distributions on r=0.01\lambda](image)

Fig 7.4 Calculated and reconstructed field ($E_z$) distributions on $r=0.01\lambda$
the antenna’s length except in the middle. The trough from reconstructed field is not as deep as the calculated. This seems a limitation of the technique. Nevertheless the two peaks at each end of the antenna are recovered well. Indeed, this has been demonstrated experimentally in Chapter 6.

Fig 7.5 Calculated and reconstructed field ($H_\theta$) distributions on $r=0.01\lambda$

Following the same procedure, $H_\theta(r_o, z)$ is also evaluated by using the amplitude weighting function $b_\theta(h)$ and is shown in Fig 7.5. The thicker curve represents the directly calculated distribution of $H_\theta(r_o, z)$ on $r_o=0.01\lambda$, from equation (7.37), and the thinner curve is the distribution reconstructed by using equation (7.51). Again, the two curves agreed well in the antenna region. The curve of the reconstructed in the centre region of $\pm\lambda$ is slightly broader. Unlike the $E$ field distributions, there is only one maximum, which occurs at the centre of the antenna. This has been proved by the experiments illustrated in Chapter 6.
From these graphs, it is seen that both electric and magnetic fields can be reconstructed properly over the antenna region by the known amplitude weighting functions $a_n(h)$ and $b_n(h)$.

*Fig 7.6 Simulated $E$-field on the cylinder $r=2\lambda$*

*Fig 7.7 Calculated amplitude weighting function $b_0'(h)$ from simulated field on $r=2\lambda$*
As indicated in the earlier chapters, the numerical value when $h$ is beyond wavenumber $k$ (i.e. $|h| > k$) in the distribution of $b_0'(h)$, referred to Fig 7.3, represents the evanescent waves. Fig 7.6 is the near-field distribution of $E_z(r_o, z)$ calculated on the measurement cylinder of radius $r_o = 2\lambda$. Correspondingly, its amplitude weighting function $b_0'(h)$ is shown in Fig 7.7 in which the evanescent waves are very small compared with the evanescent waves in Fig 7.3. Following the same procedure, using the so obtained $b_0'(h)$ from simulated field on $r_o = 2\lambda$, the field of $E_z$ is reconstructed on the cylinder $r_o = 0.01\lambda$, which is shown in Fig 7.8. The maximum points at each end of the antenna of the distribution shown in Fig 7.4, can not be

![Graph](image.png)

Fig 7.8 Reconstructed field on $r=0.01\lambda$ from the amplitude weighting function $b_0'(h)$ of Fig 7.7

recovered. It clearly shows that such reconstructed field distribution does not exactly represent the details of the real aperture field. This error is due to the evanescent waves being lost when the radius of the measurement cylinder is too large. This effect of the evanescent waves can be equally demonstrated, when the

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measurement cylinder is close enough to the test antenna, but with the evanescent waves filtered out computationally.

This simulation illustrates that the evanescent waves do contribute to the near-field reconstruction, if they exist for some antennas. The question is how these evanescent waves can be measured. The answer to this question is not a simple one, because by examining the measured near-field data, it is simply impossible to know whether they exist or not. This can be illustrated by looking at different near-field distributions measured on the different cylinders (in this case, \( r_v=0.5\lambda \) and \( r_v=2\lambda \)). These two distributions of Fig 7.2 and Fig 7.6 do not differ much except for the scales, but one contains more evanescent waves than the other. By examining the distributions of the amplitude weighting functions, in this case, \( b_0'(h) \), it is easy to recognise the evanescent waves, as seen in Fig 7.3 and Fig 7.6.

It is these evanescent waves that make contributions to the very near fields, i.e. to the aperture fields. As demonstrated in this chapter, they can also be examined in a cylindrical coordinate system.
CHAPTER 8

DETERMINATION OF FAR FIELD FROM PROBE COMPENSATED NEAR-FIELD MEASUREMENTS

8.1 INTRODUCTION

The technique of near-field measurements made on a cylinder has been developed and employed to determine the far-field patterns of the antennas under test. It is particularly relevant to the case of antennas or antenna arrays whose mechanical structures have some axial symmetry.

In chapter 7, the aperture field reconstruction by near-field measurements made on a cylinder has been discussed. It was indicated that the measurement probe is made essentially very small. It should not disturb the field and it should measure the field at a point. Experimental results have shown that such treatment could not produce significant errors for the near-field reconstruction. However, when the near-field technique is used to determine the far-field pattern, the measurement probe should be placed further away from the test antenna to avoid the evanescent waves. Thus, the measurement probe should have a reasonable gain to pick up the weak signals of the test antenna. In such case, the probe compensated near-field technique should be employed.

The method presented is based on three-dimensional modal expansions of the antenna field, which is the plane wave spectrum expansion in rectangular
coordinate system and the cylindrical wave expansion in cylindrical coordinate system. By using this technique, it removes the ideal probe assumption.

The method discussed in this chapter is deduced from the wave modal expansion theory in a cylindrical coordinate system. With a new application to near-field measurement, the approach is similar to that originally used by Brown and Jull [20, 65] in that it is based on an application of the Lorentz reciprocity integral. It is shown that both complete vector aperture field and complex vector far-field pattern of an arbitrary antenna can be obtained from near-field data measured over the surface of a cylinder enclosing the antenna. It is also shown rigorously that the effects of the measurement probe can be compensated for if the complex amplitude weighting functions in the cylindrical wave expansion of field radiated by the probe, when used as a transmitter, are known.

8.2 BASIC THEORY OF PROBE COMPENSATION

Start from reviewing vector expressions (7.21) and (7.22). If the fields are to be evaluated in the far-field region of the source, the Hankel function and its first derivative can be replaced by their large argument asymptotic expansions [1]:

\[
H_n^{(2)}(\Lambda r) \approx j^{n+\frac{1}{2}} \left( \frac{2}{\pi \Lambda r} \right)^{\frac{1}{2}} \exp(-j \Lambda r)
\]  
(8.1)

\[
\frac{\partial H_n^{(2)}(\Lambda r)}{\partial r} \approx j^{n+\frac{1}{2}} \left( \frac{2}{\pi \Lambda} \right)^{\frac{1}{2}} r^{\frac{3}{2}} \left(1 - j \Lambda r\right) \exp(-j \Lambda r)
\]  
(8.2)
The resulting integrals (7.24) and (7.25) can then be evaluated by the method of stationary phase which is similar to the development in Appendix A. Although the evaluation is straightforward, it is tedious. The results are:

\[
E(R, \theta, \phi) = \frac{-2k \sin \theta \exp(-jkR)}{R} \sum_{n=-\infty}^{\infty} j^n \exp(jn\phi) \left( \hat{\phi} a_n(k \cos \theta) + \hat{\theta} j b_n(k \cos \theta) \right) \tag{8.3}
\]

\[
H(R, \theta, \phi) = \frac{\hat{R} \times E}{\eta} \tag{8.4}
\]

where \((R, \theta, \phi)\) are the spherical coordinates of the far-field point and \(\eta = (\mu \epsilon)^{1/2}\).

It was already stated in the previous chapters that the far-field is determined only by those values of \(a_n(h)\) and \(b_n(h)\) for which \(-k \leq h \leq k\), since \(|k \cos \theta| \leq k\), where \(h = k \cos \theta\).

In Fig 8.1, let \(\Sigma_a\) be a cylinder of radius \(r_a\) that contains an arbitrary test antenna connected to signal generator \(A\). Denote the field radiated by the test antenna by \(E_a(r)\) and \(H_a(r)\). Let this field be incident on a probe antenna whose reference origin \(O'\) is located at the point \((r_o, \phi_o, z_o)\). Let the probe be connected via a waveguide feeder to generator \(B\). Denote the field radiated by the probe when generator \(B\) is activated by \(E_b(r')\) and \(H_b(r')\), where \(r'\) is measured with respect to \(O'\). Let the field scattered by the test antenna when generator \(B\) is activated be denoted by \(E_{ad}(r)\) and \(H_{ad}(r)\) and the field scattered by the probe when generator \(A\) is activated by \(E_{bd}(r')\) and \(H_{bd}(r')\). In the following analysis, it will be assumed that

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there are no multiply scattered fields between the test antenna and the probe so that the total scattered field is given by these terms.

It is desired to solve for the signal induced across the terminals of generator $B$ when only generator $A$ is activated. If generator $B$ is then replaced by a linear detector having an input impedance equal to the output impedance of generator $B$, it will be shown that the amplitude weighting functions $a_n$ and $b_n$ in expressions (7.24) and (7.25) can be calculated from the detector’s output voltage if its amplitude and phase are known as functions of $\phi_0$ and $z_o$ over the cylinder of radius $r_o$. The result only holds under the assumption that the amplitude weighing

Fig 8.1 Geometry for probe compensation derivation
functions in the cylindrical wave expansion of field radiated by the probe when generator \( B \) is activated are known.

In Fig 8.1, let \( V \) be the volume bounded by the surfaces \( \Sigma_1, \Sigma_b, \) and \( \Sigma_a \) where \( \Sigma_1 \) is cylinder of radius \( r_1 \), \( \Sigma_b \) is the closed surface lying just outside the probe antenna and shield enclosing generator \( B \) which cuts the waveguide feeder for the probe at \( S_b \), and \( \Sigma_a \) is the sphere of infinite radius. Since \( V \) contains no sources, it follows from the Lorentz reciprocity theorem that

\[
\oint_{\Sigma_1+S_b+S_a} \left( (E_a + E_{ba}) \times (H_b + H_{aa}) \cdot (E_b + E_{ab}) \times (H_a + H_{ba}) \right) \cdot \hat{n} \, da = 0 \tag{8.5}
\]

where all multiply scattered terms have been neglected. The integrand of this expression vanishes identically over \( \Sigma_a \) and over \( \Sigma_b \) except for the area \( S_b \). Also, \( E_{ba} = H_{ba} = 0 \) over \( S_b \) by the fact of their definition. Thus it follows that (8.5) reduces to

\[
\oint_{\Sigma_1} (E_a \times H_b - E_b \times H_a) \cdot (\hat{r}) \, da + \oint_{\Sigma_1} (E_a \times H_{aa} - E_{aa} \times H_a) \cdot (\hat{r}) \, da \\
+ \oint_{\Sigma_1} (E_{ba} \times H_b - E_b \times H_{ba}) \cdot (\hat{r}) \, da + \oint_{S_b} (E_a \times H_b - E_b \times H_a) \cdot (\hat{\mathbf{k}}) \, da = 0 \tag{8.6}
\]

where the terms involving products of scattered fields have been neglected and where it has been assumed that \( E_b + E_{aa} \equiv E_b \) and \( H_b + H_{aa} \equiv H_b \) over \( S_b \). These assumptions are valid if the scattered fields over \( \Sigma_1 \) and \( S_b \) are small compared to the incident fields.

Let the four integrals in (8.6) be denoted by \( I_1, I_2, I_3, \) and \( I_4 \), respectively. The integral for \( I_4 \) can be evaluated by applying Lorentz reciprocity again which is
\[ I_4 = \oint_{S_b} \left( E_a \times H_b - E_b \times H_a \right) \cdot \hat{n}_b \; da = \int_{V_b} \left( E_b \cdot J_a - E_a \cdot J_b \right) dv \]  

(8.7)

where \( J_a \) and \( J_b \) are current density vector in region \( A \) and \( B \), respectively and \( V_b \) is the volume enclosed by \( S_b \). It is obvious that \( J_a = 0 \) in the volume \( V_b \). Thus, \( I_4 \) can be written

\[ I_4 = \oint_{S_b} \left( - E_a \cdot J_b \right) \; da \]  

(8.8)

From this expression, it is clear that the integral \( I_4 \) is the energy induced in the probe antenna. It is a function of position of the probe antenna with respect to the test antenna. If generator \( B \) is replaced by a matched linear detector when only generator \( A \) is activated, the detector output voltage will be proportional to the integral \( I_4 \). Thus the expression for \( I_4 \) can be written

\[ I_4 = -a u(r_o, \phi_o, z_o) \]  

(8.9)

where \( a \) is a constant of proportionality and \( u(r_o, \phi_o, z_o) \) is the detector output voltage.

To evaluate the integral for \( I_4 \) in (8.6), the cylindrical wave expansions of the fields over \( \Sigma_1 \) will be written initially in the forms

\[ E_a(r) = a_n(h) M_{nn}^i(r) + b_n(h) N_{nn}^i(r) \]  

(8.10)

\[ H_a(r) = -\frac{k}{j\omega \mu} \left( b_n(h) M_{nn}^i(r) + a_n(h) N_{nn}^i(r) \right) \]  

(8.11)
\[ E_b(r') = c_m(g) M_{mg}^4(r') + d_m(g) N_{mg}^4(r') \] (8.12)

\[ H_b(r') = -\frac{k}{j\omega \mu} \left( d_m(g) M_{mg}^1(r') + c_m(g) N_{mg}^1(r') \right) \] (8.13)

After \( I_1 \) is evaluated for these fields, the result must be summed in \( m \) and \( n \) and integrated in \( g \) and \( h \) to obtain the final value of the integral.

With the vector translation theorems described in Appendix B, the origin for \( E_b \) and \( H_b \) is first changed from \( O' \) to \( O \). The results are

\[ E_b(r) = \sum_{l=-\infty}^{\infty} (-1)^l H_{m,l}^{(2)}(\chi r_o) \exp(j l \phi_o + j g z_o) \] (8.14)

\[ \left( c_m(g) M_{-lg}^1(r) + d_m(g) N_{-lm}^1(r) \right) \]

\[ H_b(r) = -\frac{k}{j\omega \mu} \sum_{l=-\infty}^{\infty} (-1)^l H_{m,l}^{(2)}(\chi r_o) \exp(j l \phi_o + j g z_o) \] (8.15)

\[ \left( d_m(g) M_{-lg}^1(r) + c_m(g) N_{-lm}^1(r) \right) \]

where \( \chi = (k^2 - g^2)^{1/2} \). Substitution of these expressions and those for \( E_o \) and \( H_o \) into the integrand for \( I_1 \) gives
\[ I_1 = \frac{k}{j \omega \mu} \sum_{l=-\infty}^{\infty} (-1)^l H_{m+l}^{(2)}(r_o \chi r_o) \exp(j l \phi_o + j g z_o) \]

\[
\int_{-\pi}^{\pi} \int_{\infty}^{\infty} \left( a_n(h) d_m(g) + b_n(h) c_m(g) \right) \left( M_{mh}^4 \times M_{lg}^4 + N_{mh}^4 \times N_{lg}^4 \right) \cdot \hat{r} r_1 d\phi dz
\]

(8.16)

where the vectors \( \mathbf{M} \) and \( \mathbf{N} \) are functions of \((r, \phi, z)\).

From the orthogonality properties of cylindrical wave vectors described in Appendix C, it follows that the terms involving the products

\[ M_{mh}^4 \times M_{lg}^4 \cdot \hat{r} \]

and

\[ N_{mh}^4 \times N_{lg}^4 \cdot \hat{r} \]

have zero contribution to (8.16). The remaining terms can be evaluated with the aid of (C.4) in Appendix C. Thus \( I_1 \) becomes

\[ I_1 = \frac{kr_1}{j \omega \mu} \sum_{l=-\infty}^{\infty} (-1)^l H_{m+l}^{(2)}(r_o \chi r_o) \exp(j l \phi_o + j g z_o) \]

\[ \left( a_n(h) c_m(g) + b_n(h) d_m(g) \right) \left( \frac{4 \pi^2 \Lambda^3}{k} \delta(g + h) \right) \]

\[ \cdot \left( J_{-n}^\prime (\Lambda r_1) H_n^{(2)}(\Lambda r_1) - J_{-n} (\Lambda r_1) H_n^{(2)\prime}(\Lambda r_1) \right) \]

(8.17)

where \( \delta(g+h) \) is the Dirac Delta function and \( \delta_{nl} \) is the Kronecker Delta function defined by

\[ \delta_{nl} = \begin{cases} 1 & ; n = l \\ 0 & ; otherwise \end{cases} \]

(8.18)

By using the following identity of Bessel functions,
\[ J'_n(\Lambda r_1) H_n^{(2)}(\Lambda r_1) - J_{-n}(\Lambda r_1) H_{-n}^{(2)}(\Lambda r_1) = \frac{j^2(-1)^n}{\pi \Lambda r_1} \]  

(8.19)

the expression (8.16) can be simplified as

\[
I_1 = \frac{8\pi \Lambda^2}{\omega \mu} H_{m+n}^{(2)}(\Lambda r_o) \exp(jn\phi_o + jgz_o)\delta(g + h) 
(8.20)
\]

\[
\left( a_n(h)c_m(g) + b_n(h)d_m(g) \right)
\]

which is independent of \( r_1 \) and where the index \( l \) has been replaced by \( n \) for the property of Kronecker delta function. When this expression is summed over all \( m \) and \( n \), and integrated in \( g \) and \( h \), the result is

\[
I_1 = \frac{8\pi \Lambda^2}{\omega \mu} \sum_{n=-\infty}^{\infty} \exp(jn\phi_o) \int \left( a_n(h) \sum_{m=-\infty}^{\infty} c_m(-h) H_{n+m}^{(2)}(\Lambda r_o) \right.

(8.21)

\[
+ b_n(h) \sum_{m=-\infty}^{\infty} d_m(-h) H_{n+m}^{(2)}(\Lambda r_o) \right) \exp(-jhz_o) \, dh
\]

where the variable \( g \) has been replaced by \( -h \), due to the selective property of the Dirac delta function.

Since multiple scattering is neglected, \( E_o, H_o, E_{as}, \) and \( H_{as} \) satisfy the homogeneous wave equation out of \( \Sigma_o \). Applying the Lorentz reciprocity theorem again to the integrals of \( I_2 \) and \( I_3 \) in (8.6), It follows that \( I_2 \) and \( I_3 \) are identically zero. Now submitting \( I_1 \) and \( I_4 \) into equation (8.6), it follows that

\[
v(r_o, \phi_o, z_o) = \frac{\Lambda^2}{k^2} \sum_{n=-\infty}^{\infty} \exp(jn\phi_o) \int \left( a_n(h) \sum_{m=-\infty}^{\infty} c_m(-h) H_{n+m}^{(2)}(\Lambda r_o) \right.

(8.22)

\[
+ b_n(h) \sum_{m=-\infty}^{\infty} d_m(-h) H_{n+m}^{(2)}(\Lambda r_o) \right) \exp(-jhz_o) \, dh
\]

where the probe output \( v(r_o, \phi_o, z_o) \) has been normalised by properly choosing the proportional constant \( \alpha \).
Examination of this equation reveals that \( v(r_o, \phi_o, z_o) \) is in the form of a Fourier series in \( \phi_o \) and a Fourier integral (transform) in \( z_o \). Thus the equation has an inverse which is given by

\[
a_n(h) \sum_{m=-\infty}^{\infty} c_m(-h) H_{n+m}^{(2)}(\Lambda r_o) + b_n(h) \sum_{m=-\infty}^{\infty} d_m(-h) H_{n+m}^{(2)}(\Lambda r_o) = \frac{k^2}{\Lambda^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} v(r_o, \phi_o, z_o) \exp(-j n \phi_o) \exp(j h z_o) \, d\phi_o \, dz_o
\]

(8.23)

This is the desired result. It relates the cylindrical wave amplitude weighting functions \( a_n(h) \) and \( b_n(h) \) of an arbitrary test antenna to the two-dimensional Fourier transform of the output voltage of a probe antenna when the measurement surface is a cylinder of radius \( r_o \). If the cylindrical wave amplitude weighting functions for the probe antenna are known, the equation (8.23) can be solved for \( a_n(h) \) and \( b_n(h) \) provided that two independent measurements of \( v(r_o, \phi_o, z_o) \) are made.

Let \( v'(r_o, \phi_o, z_o) \) represent the voltage output of the probe antenna when it is rotated 90° about its axis. An equation identical to (8.23) can be written with which \( c_m(-h) \) and \( d_m(-h) \) must be replaced by the amplitude weighting functions for the rotated probe. If these are denoted by \( c'_m(-h) \) and \( d'_m(-h) \), then this equation and (8.23) can be solved simultaneously for \( a_n(h) \) and \( b_n(h) \) as following

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\[ a_n(h) = \frac{k^2}{\Lambda^2 \Delta_n(h)} \left( I_n(h) \sum_{m=-\infty}^{\infty} d'_m(-h) H_{n+m}^{(2)}(\Lambda r_o) \right) \]
\[ \cdot I'_n(h) \sum_{m=-\infty}^{\infty} d'_m(-h) H_{n+m}^{(2)}(\Lambda r_o) \]  
\[ b_n(h) = \frac{k^2}{\Lambda^2 \Delta_n(h)} \left( I'_n(h) \sum_{m=-\infty}^{\infty} c_m(-h) H_{n+m}^{(2)}(\Lambda r_o) \right) \]
\[ \cdot I_n(h) \sum_{m=-\infty}^{\infty} c'_m(-h) H_{n+m}^{(2)}(\Lambda r_o) \]  
(8.24)  
(8.25)  

where  
\[ I_n(h) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} v(r_o, \phi_o, z_o) \exp(-in\phi_o) \exp(jhz_o) d\phi_o dz_o \]  
(8.26)  
\[ I'_n(h) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} v'(r_o, \phi_o, z_o) \exp(-in\phi_o) \exp(jhz_o) d\phi_o dz_o \]  
(8.27)  
\[ \Delta_n(h) = \left( \sum_{m=-\infty}^{\infty} c_m(-h) H_{n+m}^{(2)}(\Lambda r_o) \right) \left( \sum_{m=-\infty}^{\infty} d'_m(-h) H_{n+m}^{(2)}(\Lambda r_o) \right) \]
\[ \cdot \left( \sum_{m=-\infty}^{\infty} c'_m(-h) H_{n+m}^{(2)}(\Lambda r_o) \right) \left( \sum_{m=-\infty}^{\infty} d_m(-h) H_{n+m}^{(2)}(\Lambda r_o) \right) \]  
(8.28)  

It has been assumed that the probe responds predominantly to one polarisation component so that a solution for \( a_n(h) \) and \( b_n(h) \) exists, i.e., \( \Delta_n(h) \) is certainly not zero.  

Equations (8.24) through (8.28) form the basis of the method for determination of the cylindrical wave amplitude weighting functions of an arbitrary antenna from measurements made with a probe on a cylinder containing
the antenna. By using these equations of $a_n(h)$ and $b_n(h)$, the field radiated by the test antenna in far field region can be determined from (8.3) and (8.4).

### 8.3 PROBE COEFFICIENTS

It has been shown that, over the surface of a sphere of radius $R$, the far field electric field intensity radiated by an antenna can be written in the form:

$$
E_\theta(\theta, \phi) = j \sin \theta \sum_{-\infty}^{\infty} j^n b_n(k \cos \theta) \exp(jn\phi) \quad (8.29)
$$

$$
E_\phi(\theta, \phi) = \sin \theta \sum_{-\infty}^{\infty} j^n a_n(k \cos \theta) \exp(jn\phi) \quad (8.30)
$$

where the constant factor $(-2k \exp(-jkR)/R)$ has been suppressed in each equation. In these equations, $a_n(h)$ and $b_n(h)$, where $h=\cos \theta$, are the amplitude weighting functions of the cylindrical wave vectors $M$ and $N$, respectively, in the cylindrical wave expansion of the field radiated by the antenna.

In this section, a numerical computation for $a_n(h)$ and $b_n(h)$ is discussed which is based on the result of the previous section and then the numerical evaluation of (8.29) and (8.30) for the far field of the antenna is described.

Since in most cases, the probe is located far enough from the test antenna to avoid any evanescent waves, it follows that the cylindrical wave amplitude weighting functions of the probe needs to be known only for arguments inside the interval $-k \leq h \leq k$.

The practical upper bounds on the sampling intervals of $\Delta \phi$ and $\Delta z$ established in the previous chapters still hold and are rewritten here:
\[ \Delta \phi \leq \frac{\pi}{ka} \tag{8.31} \]

and

\[ \Delta z \leq \frac{\lambda}{2} \tag{8.32} \]

To evaluate (8.24) and (8.25) for the amplitude weighting functions in the cylindrical wave expansion of the test antenna, it is necessary to know the amplitude weighting functions in the expansions of field radiated by the probe. Since it is necessary to know these functions only for the wavenumbers \( h \) such that \( |h| \leq k \), it is possible to obtain them from a knowledge of the far field radiated by probe when it is connected to a source.

Since it is necessary to resolve the measured data into a Fourier series in the azimuth angle \( \phi \), some upper limit on the maximum angular harmonic for the probe must be established. This can be done by using the criterion discussed in the previous chapters which concluded that the maximum significant angular harmonic is \( N=ka \), where \( a \) is the radius of the smallest sphere which completely encloses the aperture of the probe. Let the probe antenna be oriented to measure the \( z \) component of electric field. The far-field electric field intensity radiated by the probe can be expressed by (8.24) and (8.25) with \( b_n(h) \) and \( a_n(h) \) replaced by \( d_n(h) \) and \( c_n(h) \) respectively, with \( h= k \cos \theta \). If the probe is then rotated \( 90^\circ \) about its axis in which the \( \phi \) component of \( E \)-field to be measured, the far-field components \( E_0(\theta, \phi) \) and \( E_4(\theta, \phi) \) can be expressed similarly using the cylindrical wave
weighting functions $d_n(h)$ and $c_n(h)$. Thus the probe coefficients are obtained by employing the FFT, if the far-field radiation pattern of the probe is known.
CHAPTER 9

CONCLUSIONS

It has been described that Gabor’s original holography is the basic theory of modern microwave holographic techniques. By using computer simulations, it has been demonstrated that the conventional microwave holographic approach can be used as a tool to reconstruct aperture field distributions of an antenna. However, the constraint, which requires a hologram plane (measurement plane) to be in the far-field region of the antenna in order to satisfy the Fourier transform relationship between the aperture field and the far-field, limits its applications. It can only be used for highly directive antennas. In the modern microwave age, accurate measurements of both magnitude and phase of an electromagnetic field have become a much easier task, so that the “magnitude only” conventional holographic technique is no longer preferred for the application of aperture field reconstruction.

Interestingly, when the Terahertz frequency is concerned, Gabor’s original holography may find its applications to the aperture field reconstructions, since at such a high frequency, the direct measurement of phase continues to be difficult with existing technology.

The improved microwave holographic approach described in Chapters 3 and 4 is very useful and well established. The measurement of the field is conducted in the far-field region and both magnitude and phase are directly measured. It is mainly used for the surface distortion diagnosis of large reflector antennas. The
physical optics integral formulation is derived by general solutions of the vector wave equations. The theory is well established and the technique is robust. Further research may be carried out to apply the technique to more generic antennas or antenna arrays.

The application of a near-field measurement technique to far-field patterns and other far-field related parameters was developed a decade ago. Very few researchers have considered applying this technique to aperture field reconstructions. The theory which reconstructs the aperture field from a near-field measurement has been developed in Chapter 5 for planar near-field measurements in a rectangular coordinate system. By using dipole and other well-defined antennas, computer simulations have been performed. The results show that the technique is rigorous and worthy of application. It is also demonstrated that the sampling intervals and the number of sampling points should be chosen properly in order to obtain a satisfactory resolution of the reconstructed aperture field.

Furthermore, the simulations which have been carried out reveal that the real aperture field distribution of a dipole antenna, which have two maximum points at each end of the antenna. This distribution characteristic can only be obtained at a very close distance to the antenna. The traditional view of the aperture field of a dipole antenna, in which the maximum $E$ field strength is considered to be at the centre of the antenna is inaccurate. The significant contribution of the evanescent waves to the aperture fields is also revealed. A simple but effective method to examine the evanescent waves from the measured near-fields is presented.

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The experiments were carried out by using dipoles and other well-known antennas and antenna arrays. The experimental results show that good agreements with the simulations have been achieved and the technique is practically effective and accurate.

For antennas or antenna arrays whose mechanical structures have some axial symmetry, the cylindrical coordinate system is more convenient. A method has been developed for determining the aperture field distribution of an antenna from near-field measurements over the surface of a right circular cylinder which encloses the antenna. It is derived from the wave modal expansion theory which has already been employed in the development of near-field to far-field transformation techniques. It is based on the three-dimensional vector cylindrical wave expansion of an electromagnetic field.

If the antenna under test is circularly symmetric, for example, a monopole antenna, the mathematical expressions of the transformation theory can be simplified to one dimension. It is then easy to solve and to calculate the field in one dimension. In practice, however this may not be always the case. Even in such circumstances it may still be easier to make measurements on a cylinder than a plane for some antennas or antenna arrays.

Still using the dipole antenna, the simulation and experimental results agreed well.

To fully employ the merit of the cylindrical near-field measurement technique, the probe-compensated near-field to far-field transformation is described. It is also deduced from the wave modal expansion theory in a cylindrical
coordinate system, in that it is based on an application of the Lorentz reciprocity integral. It is shown that the complex vector far-field pattern of an arbitrary antenna can be obtained from near-field data measured over the surface of a cylinder enclosing the antenna. It is also shown rigorously that the effects of the measurement probe can be compensated for, if the complex amplitude weighting functions in the cylindrical wave expansion of field radiated by the probe, when used as a transmitter, are known.

There are some constraints on the results of aperture field reconstructions from both planar near-field measurement and cylindrical near-field measurement. For example, in a cylindrical near-field measurement, it must be assumed that the field radiated in the $z$-direction is very small outside the measurement region $|z|>d$. Similar condition in both $x$- and $y$- directions are assumed in the planar near-field measurement. The applicability of the technique to a particular antenna must be considered with care.

If the near-field measurement can be made on the surface of a sphere which completely encloses the antenna under test, then the above constraint can be removed. This will implicitly involve further research on vector wave modal expansion theory in a spherical coordinate system and is highly recommended for future research. In this case, it would be more complex and challenging.
APPENDIX A

ASYMPTOTIC EVALUATION OF THE FAR-FIELD INTEGRAL USING THE STATIONARY PHASE METHOD

The integral to be evaluated for large value of \( r \) is

\[
E(r) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(k_x, k_y) e^{-jk\cdot r} \, dk_x \, dk_y \quad \text{(A.1)}
\]

The technique that will be used is Rayleigh’s method of stationary phase. The main point of this method is as follows: When \( r \) is very large, \( \exp(-jk\cdot r) \) is very rapidly oscillating function. Thus the contributions to the integral from various points in the \( k_x-k_y \) plane tend to cancel because there is a lack of in-phase addition from the various regions. An exception is a point where \( r=k \), which is a function of \( k_x \) and \( k_y \), to first order does not vary with small changes in \( k_x, k_y \). Such a point is called a stationary phase point and is characterised by the vanishing of the first derivatives of \( k\cdot r \) with respect to \( k_x \) and \( k_y \); that is,

\[
\frac{\partial (k\cdot r)}{\partial k_x} = 0 \quad \frac{\partial (k\cdot r)}{\partial k_y} = 0 \quad \text{(A.2)}
\]

At a stationary phase point the phase of \( \exp(-jk\cdot r) \) does not vary rapidly, and a nonzero contribution to the integral would be obtained from this region of the \( k_x-k_y \) plane. In the small region surrounding the stationary phase point, which denoted by \( k_x=k_1, k_y=k_2 \), the slowly varying function \( A(k_x, k_y) \) is equal to its value at the stationary phase point. The integral that remains then only involves the function \( \exp(-jk\cdot r) \) and can be evaluated.
In order to facilitate the evaluation it is necessary to express \( k \cdot r = k_x x + k_y y + k_z z \) in spherical coordinates by using \( x = r \sin \theta \cos \phi \), \( y = r \sin \theta \sin \phi \), \( z = r \cos \theta \); thus

\[
k \cdot r = r \left( k_x \sin \theta \cos \phi + k_y \sin \theta \sin \phi + \sqrt{k_0^2 - k_x^2 \cdot k_y^2 \cos \theta} \right)
\]  
(A.3)

The stationary phase point is the point where it satisfies the equation (A.2), that is, where

\[
k_x = k_1 = k_0 \sin \theta \cos \phi
\]  
(A.4)

\[
k_y = k_2 = k_0 \sin \theta \sin \phi
\]  
(A.5)

A Taylor series expansion of \( k \cdot r \) in the vicinity of \( (k_1, k_2) \) gives

\[
k \cdot r = k_0 r + \frac{1}{2} \frac{\partial^2 (k \cdot r)}{\partial k_x^2} (k_x - k_1)^2 + \frac{1}{2} \frac{\partial^2 (k \cdot r)}{\partial k_y^2} (k_y - k_2)^2 + \frac{\partial^2 (k \cdot r)}{\partial k_x \partial k_y} (k_x - k_1)(k_y - k_2)
\]

\[= k_0 r \cdot (Au^2 + Bv^2 + Cuv)
\]  
(A.6)

where \( u = k_x - k_1 \), \( v = k_y - k_2 \) and \( A, B, \) and \( C \) are constants.

The asymptotic solution for \( E(r) \) is thus

\[
E(r) \approx e^{\frac{jk_0 r}{2\pi}} A(k_0 \sin \theta \cos \phi, k_0 \sin \theta \sin \phi) \iint_{\Delta s} e^{j(Au^2 + Bv^2 + Cuv)} dudv
\]  
(A.7)

where the function \( A \) is given by its value at the stationary phase point and \( \Delta s \) is a small region centred on the stationary phase point which is at \( u = v = 0 \) in the \( u-v \) plane. Now the stationary phase argument is used again to note that the integrand function \( \exp(j(Au^2 + Bv^2 + Cuv)) \) will oscillate very rapidly when \( u \) and \( v \) are not zero, since the constant \( A, B, \) and \( C \) are proportional to \( r \) and hence very large for large
values of \( r \). Thus the integral can be extended to cover the whole \( u\cdot v \) plane, since in the limit as \( r \) becomes infinite the contributions from \( u \) and \( v \) outside of \( \Delta s \) will cancel from phase interference. Hence the following expression:

\[
\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{i(Au^2 + Bv^2 + Cu)} \, du \, dv
\]  
(A.8)

needs to be evaluated. By simplifying

\[
Au^2 + Bv^2 + Cu = \left( \sqrt{Au} + \frac{Cv}{2\sqrt{A}} \right)^2 \cdot \frac{C^2 v^2}{4A} + Bv^2
\]  
(A.9)

and letting

\[
w = \sqrt{Au} + \frac{Cv}{2\sqrt{A}}
\]  
(A.10)

the integral becomes

\[
I = \frac{1}{\sqrt{A}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{i\varepsilon^2} e^{iA\varepsilon^2} e^{iB\varepsilon^2} \, dw \, dv
\]  
(A.11)

Next the well-known Fourier transform is used

\[
\int_{-\infty}^{\infty} e^{-\pi t^2} e^{i\eta t} \, dt = e^{-\pi \eta^2}
\]  
(A.12)

by letting \( \eta = 0 \), the above becomes

\[
\int_{-\infty}^{\infty} e^{-\pi t^2} \, dt = 1
\]  
(A.13)

Using equation (A.13) to evaluate the integral in (A.11) over \( w \) and \( v \), the following result is obtained:

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\[ I = \frac{j2\pi}{\sqrt{AB-C^2}} = 2\pi j \frac{k_0}{r} \cos \theta \]  \hspace{1cm} (A.14)

where

\[ A = \frac{r}{2} \left( \frac{1}{k_0} + \frac{k_1^2}{k_0 \cos^2 \theta} \right), \quad B = \frac{r}{2} \left( \frac{1}{k_0} + \frac{k_2^2}{k_0 \cos^2 \theta} \right), \quad C = \frac{k_1 k_2 r}{k_0^3 \cos^2 \theta} \]  \hspace{1cm} (A.15)

Thus the final far-field pattern equation is

\[ E(r) = \frac{i e^{-jkr}}{r} \cos \theta \ A(k_x, k_y) \]  \hspace{1cm} (A.16)
APPENDIX B

VECTOR TRANSLATION THEOREMS FOR CYLINDRICAL VECTOR WAVE FUNCTIONS

Let the coordinates \((r_o, \phi_o, z_o)\) be the location of the origin of the coordinate system \((r', \phi', z')\) in the coordinate system \((r, \phi, z)\) as shown in Fig 8.1. It is necessary to express the cylindrical vector wave functions \(M_{nh}^4(r', \phi', z')\) and \(N_{nh}^4(r', \phi', z')\) as functions of coordinates \((r, \phi, z)\).

In the system \((r', \phi', z')\) the generating function \(\psi_{nh}^4\) for these vectors is given, referred to Chapter 7, by

\[
\psi_{nh}^4 (r', \phi', z') = H_n^{(2)}(\Lambda r') \exp(jn\phi') \exp(-jhz') \tag{B.1}
\]

This can be expressed as a function of \((r, \phi, z)\) by using addition theorem for the Hankel function, which states

\[
H_n^{(2)}(\Lambda r) = \exp(-jn\phi) \sum_{m = -\infty}^{\infty} H_{n+m}^{(2)}(\Lambda r_o) J_m(\Lambda r) \exp(jm(\phi - \phi_o)) \tag{B.2}
\]

When this is substituted into (B.1), the generating function is transformed into

\[
\psi_{nh}^4 (r', \phi', z') = \sum_{m = -\infty}^{\infty} \left( H_{n+m}^{(2)}(\Lambda r_o) \exp(jm\phi_o) \exp(jhz_o) \right) \cdot J_m(\Lambda r) \exp(-jm\phi) \exp(-jhz) \tag{B.3}
\]

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The vectors $\mathbf{M}_{nh}^4$ and $\mathbf{N}_{nh}^4$ are obtained by the following operations on $\psi_{nh}^4$:

\begin{equation}
\mathbf{M}_{nh}^4 = \nabla \times \hat{z} \psi_{nh}^4 \tag{B.4}
\end{equation}

\begin{equation}
\mathbf{N}_{nh}^4 = \frac{1}{k} \nabla' \times \mathbf{M}_{nh}^4 \tag{B.5}
\end{equation}

where the primes denote operations on the primed coordinates $(r', \phi', z')$. Since the del operator is invariant to a coordinate transformation, these become

\begin{equation}
\mathbf{M}_{nh}^4 (r', \phi', z') = \sum_{m=-\infty}^{\infty} H_{n+m}^{(2)}(\Lambda r_o) \exp(-jm\phi_o) \exp(jhz_o) \cdot \nabla \times \hat{z} \mathbf{J}_m(\Lambda r) \exp(-jm\phi) \exp(-jhz) \tag{B.6}
\end{equation}

\begin{equation}
\mathbf{N}_{nh}^4 (r', \phi', z') = \frac{1}{k} \sum_{m=-\infty}^{\infty} H_{n+m}^{(2)}(\Lambda r_o) \exp(-jm\phi_o) \exp(jhz_o) \cdot \nabla \times \nabla \times \hat{z} \mathbf{J}_m(\Lambda r) \exp(-jm\phi) \exp(-jhz) \tag{B.7}
\end{equation}

From the definitions

\begin{equation}
\mathbf{M}_{nh}^1 = \nabla \times (\hat{z} \mathbf{J}_m(\Lambda r) \exp(-jm\phi) \exp(-jhz)) \tag{B.8}
\end{equation}

\begin{equation}
= \nabla \times \hat{z} \psi_{nh}^1 \tag{B.8}
\end{equation}

\begin{equation}
\mathbf{N}_{nh}^1 = \frac{1}{k} \nabla \times \mathbf{M}_{nh}^1 \tag{B.9}
\end{equation}

it follows that (B.6) and (B.7) reduce to
\[
M_{nh}^{1}(r',\phi', z') = \sum_{m=-\infty}^{\infty} (-1)^m H_{n+m}^{(2)}(\Lambda r_o) \exp(jm\phi_o) 
\]
\[\exp(jhz_o) M_{mh}^{1}(r,\phi, z)\]
\[
N_{nh}^{4}(r',\phi', z') = \sum_{m=-\infty}^{\infty} (-1)^m H_{n+m}^{(2)}(\Lambda r_o) \exp(jm\phi_o) 
\]
\[\exp(jhz_o) N_{mh}^{1}(r,\phi, z)\]

These are desired translation theorems. They are valid for all \(r \leq r_o\).
APPENDIX C

ORTHOGONALITY PROPERTIES OF CYLINDRICAL WAVE VECTORS

Property C1

\[ \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} M_{nh}^i \times M_{mg}^j \cdot \hat{r} \, d\phi \, dz = 0 \]  \hspace{1cm} (C.1)

This property follows from the fact that the vector \( \mathbf{M} \) contains no \( z \) component. Thus the product \( M_{nh}^i \times M_{mg}^j \) has only a \( z \) component which is zero when a scalar multiplied by the unit vector \( \mathbf{r} \).

Property C2

\[ \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} N_{nh}^i \times N_{mg}^j \cdot \hat{r} \, d\phi \, dz = 0 \]  \hspace{1cm} (C.2)

The integrand in (C.2) can be simplified as follows:

\[ \begin{aligned}
    &N_{nh}^i \times N_{mg}^j \cdot \hat{r} = N_{mg}^j \cdot (\hat{r} \times N_{nh}^i) \\
    &=\left( \frac{nh \chi^2}{k^2 r} Z_n^i(\Lambda r) Z_m^j(\chi r) - \frac{mg \Lambda^2}{k^2 r} Z_n^i(\Lambda r) Z_m^j(\chi r) \right) \exp(j(m+n)\phi) \exp(-j(g+h)z) \\
    &\text{(C.3)}
\end{aligned} \]

This is identically zero when integrated with respect to \( \phi \) and \( z \) unless \( m=n \) and \( g=h \). However, under these conditions, the term in the big parentheses is identically zero since \( \chi=\Lambda \) when \( g=h \). Thus property C2 holds.
Property C3

\[
\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} M_{nh}^i \times M_{mg}^j \cdot \hat{r} \ d\phi \ dz = \begin{cases} 
\frac{4 \pi^2 \Lambda^3}{k} Z_n^i(\Lambda r) \frac{\partial Z_m^j(\Lambda r)}{\partial \Lambda} \delta(g + h), & \text{for } m = -n \\
0, & \text{otherwise}
\end{cases}
\]  

(C.4)

The integrand of the preceding integral can be simplified as follows:

\[
N_{nh}^i \times M_{mg}^j \cdot \hat{r} = -N_{nh}^i \cdot (\hat{r} \times M_{mg}^j)
\]

(C.5)

\[
= \frac{\Lambda^3}{k} Z_n^i(\Lambda r) \frac{\partial Z_m^j(\chi r)}{\partial \chi} \exp(j(m + n)\phi) \exp(j(g + h)z)
\]

This equation is zero when integrated with respect to \( \phi \) unless \( m = -n \). In this case the integral of the exponential term involving \( \phi \) is \( 2\pi \). The integral of the exponential term involving \( z \) results in the factor \( 2\pi \delta(g + h) \). Since this is zero for \( g + h \neq 0 \), it follows that the substitution \( g = -h \) can be made in the rest of the expression.
REFERENCES


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[69] W. K. Kefauver, “State of near-field metrology using the Near-Field Measurement Laboratory at Lockheed Martin, Denver,” *IEEE Antennas and


errors in planar near-field measurements to arbitrary accuracy.” *IEEE

[92] K. Nagai, Y. Aoki, and M. Suzuki, “On the numerical reconstruction of
images from a microwave hologram,” *IEEE, Trans. Microwave Theory and


[95] A. C. Newell, R. C. Baird and P. F. Wacker, “Accurate measurement of
antenna gain and polarization at reduced distances by an extrapolation

[96] A. C. Newell and D. M. Kerns, “Determination of both polarization and
power gain of antennas by a generalized 3-antenna measurement method, “

field and experimentally determine the near-field phase center of a horn
antenna,” *IEEE Transactions on Instrumentation and Measurement*, Vol. 42,
No. 1, Feb 1993, pp. 51-53.


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