CHAPTER 1:
INTRODUCTION

1.1 BACKGROUND

Concrete filled steel tubular (CFST) members utilise the advantages of both steel and concrete. They comprise of a steel hollow section of circular or rectangular shape filled with plain or reinforced concrete. They are widely used in high-rise and multi-storey buildings as columns and beam-columns, and as beams in low-rise industrial buildings where a robust and efficient structural system is required.

There are a number of distinct advantages related to such structural systems in both terms of structural performance and construction sequence. The inherent buckling problem related to thin-walled steel tubes is either prevented or delayed due to the presence of the concrete core. Furthermore, the performance of the concrete in-fill is improved due to confinement effect exerted by the steel shell. The distribution of materials in the cross section also makes the system very efficient in term of its structural performance. The steel lies at the outer perimeter where it performs most effectively in tension and bending. It also provides the greatest stiffness as the
material lies furthest from the centroid. This, combined with the steel’s much greater modulus of elasticity, provides the greatest contribution to the moment of inertia. The concrete core gives the greater contribution to resisting axial compression.

The use of concrete filled steel tubes in building construction has seen resurgence in recent years due mainly to its simple construction sequence, apart from its superior structural performance. Typically, it was used in composite frame structures. The hollow steel tubes that are either fabricated or rolled were erected first to support the construction load of the upper floors. The floor structures consist of steel beams supporting steel sheeting decks on which a reinforced concrete slab is poured. Such structural system has the advantage of both steel and reinforced concrete frame. It has the structural stiffness and integrity of a cast-on-site reinforced concrete building, and the ease of handling and erection of a structural steelwork. The hollow tubes alone were designed in such a way that they are capable of supporting the floor load up to three or four storey height. Once the upper floors were completed, the concrete was pumped into the tubes from the bottom. To facilitate easy pumping the tubes were continuous at the floor level. Modern pumping facility and high performance concrete make pumping three or four storey readily achievable. Due to the simplicity of the construction sequence, the project can be completed in great pace.

The structural behaviour of concrete-filled tubes is complex because of the interaction between the steel tube and the in-fill concrete. The pioneer research effort on the structural behaviour of CFST members was first made by Kloppel and Goder (1957); Salani and Sims (1964); Furlong (1967); Gardner and Jacobson (1967);
ner (1968); Neogi, Sen and Chapman (1969); Knowles and Park (1969); 
Knowles and Park (1970); Sen (1972). These researches were mainly concentrated 
the fundamental behaviour of short columns with the slenderness being largely 
sored. The design equations were also derived based on these researches. After a 
ade of relative tranquillity, there has been resurgence in CFST research in recent 
rs all over the world. These were represented by groups in Australia, China, 
pe, Japan, and North America [Roik and Bergmann, 1984; Cai, 1991; Zhong, 
1; Sakino and Hayashi, 1991; Masuo et al., 1991; Luksha and Nesterovich, 1991; 
amura et al., 1994; Bridge et al., 1995; Matsui et al., 1995; Zhang and Shahrooz, 
99]. A number of major design codes was produced as a result of such extensive 
d AISC-LRFD (1993). In recent years, research on CFST is concentrated on the 
plex issues such as interaction effect, slenderness effect and the use of high 
ength concrete [Rangan and Joyce, 1992; Kilpatrick and Rangan, 1997; Chung, et 
., 1999]. There was also limited research effort on attempting to develop simplified 
equations and procedures for design [Chen and Atsuta, 1976; Tomii and Sakino, 
979a; Gourley and Hajjar, 1996].

Different approaches and design philosophies were adopted in different design 
codes. In Japan, Standard for the design of composite columns in the Steel 
reinforced construction published by Architectural Institute of Japan is 
based on a simple method of superposition that uses the allowable stresses of the 
materials or the working stress method. A modified method of the superposed
strength of a column section is adopted for slender columns considering the effect of additional bending moment.

The design codes ACI-318 and AISC-LRFD are different in concept because of the industrial sectors they were evolved from. ACI-318 adopts the traditional reinforced concrete approach. AS 3600-1994 also uses the concept of reinforced concrete design. The AISC-LRFD uses the concept of structural steel.

The Eurocode 4, being a dedicated code for composite construction combined the design approach of both structural steelwork and reinforced concrete for the design of columns. It uses the concept of a rectangular stress block to work out the section strength interaction diagram, then use the buckling curve to reduce the section strength interaction diagram to a member strength interaction diagram.

The approach for reinforced concrete column design is quite different from that for steel columns. The strength is based on the cross-section interaction curve. The main difficulty is the amount of algebraic work required to derive this curve accurately. A number of different methods have been used to analyse the cross-section strength of a CFST. Tomii and Sakino (1979a) computed CFST cross-section strength using a simplified stress block method. Chen and Atsuta (1976) derived parametric equations to describe the moment-curvature-thrust relationship of CFST sections. Hajjar and Gourley (1996) described a set of expressions to compute capacity of cross-section strength.
There are different variations of the theory that has been proposed to calculate strength of a composite column. A finite difference method with fibre element is based on numerical integration techniques. This method of analysis entails relating stresses to strains at discrete points on the cross-section and obtaining resultant internal forces by integrating the stresses. The fibre element approach have been used by Shakir-Khalil and Zeghiche (1989) and Kawaguchi, et al. (1991) for uniaxial bending of rectangular CFTS and by Bode (1976) and Ichinohe, et al. (1991) for uniaxial bending of circular CFTS, and by Muñoz and Hsu (1997) for biaxial bending of concrete encased composite columns.

1.2 STATEMENT OF THE PROBLEM

Although there are various codes of practice available for the design of concrete-filled tubular member, these codes, in most cases cover only the normal strength concrete. Furthermore the procedure of deriving the strength material diagram requires, a large amount of algebraic work, which makes most of the codes difficult to interpret and follow.

High strength concrete became more widely used in combination with thin-walled steel tubes. The existing codes of practice should not be used without precautions, as high strength concrete behaves distinctly different from normal strength concrete in certain areas.
In this thesis, in-depth studies are attempted to provide a thorough investigation for the behaviour and design of concrete-filled tubular members in the following areas:

1. A general numerical analysis procedure is provided, based on finite difference method to study the behaviour of concrete filled members.

2. A set of simplified, yet accurate constitutive relationships are proposed for steel and concrete in concrete filled steel tubes. Effect, such as confinement is included in the formulation.

3. High strength concrete and the effect on the behaviour of concrete filled tubular columns is thoroughly investigated.

4. A set of numerical expressions for the prediction of strength of concrete-filled tubular section and member are proposed for design purposes.

5. Equations are also proposed for biaxial bending of concrete filled tubes.

1.3 STRUCTURE OF THE THESIS

In this thesis the studies are focused on the behaviour of CFST columns including both rectangular and circular sections. Both normal strength and high strength concrete are considered in the investigations. The complete load deformation behaviour of the columns was studied using a general analytical procedure based on the fibre element analysis approach. From these analyses, the cross-section strength and member strength of CFST columns subjected to axial load and biaxial bending were determined. The effects of concrete confinement and the effects of second order (p-δ) on the strength were also considered in a more rational manner.
Design recommendations were proposed based on the extensive studies which were verified with existing test results from published sources and were compared with design rules from other design codes, in particular, ACI, AISC, Australian Standard and Eurocode.

The present studies aim at developing new design interaction equations to simplify the complex calculation procedures as seen in most of the design codes for CFST members. The proposed equations must provide an accurate solution for practical design tasks having a wide range of material strengths, cross section dimensions and lengths. These equations include (1) a series of equations representing the cross-section strength of a CFST section, (2) a simple mathematical expression that can represent the strength interaction diagrams for CFST beam-columns under combined bending moments and axial load and (3) a design equation to predict the ultimate load capacity of slender CFST columns subject to biaxial bending moments and axial load.

In Chapter 2 an extensive literature review related to CFST beam and beam column and their characteristic were discussed. The behaviour was described for members on confinement effects, ductility and strength and local buckling of steel shell. The significant CFST papers were individually summarised in detail following the general overview. Discussions were presented on the design formulae and guidelines proposed by different researches. Design procedures suggested in the current codes of practices were also discussed.
In Chapter 3 a general numerical analysis procedure was presented to calculate the complete load-deflection and moment-curvature relationship for concrete steel composite columns under combined axial force and biaxial bending. The analytical method was introduced to calculate the deformation which satisfies the compatibility equations. The method was based on fibre analysis formulation and used an iteration procedure to obtain the solution. The p-δ effect due to the slenderness of the column was also included in the formulation.

In Chapter 4 a set of effective stress-strain relationships for steel and concrete is presented. The confinement effect on the concrete core developed by the steel tube that leads to the enhancement of the strength of the concrete core is included in the formulation for high and normal strength concrete. Steel strength reduction due to triaxial state is also considered. The results were verified with a large collection of published test results. The results from the proposed method were also compared with that of Eurocode 4 and AISC-LRFD.

Chapter 5 analyses presents a series of cross section strength interaction equations for CFST members. The equations were determined empirically based on the numerical analyses. The parameter analysis covers a wide range of concrete strength from 20 MPa to 120 MPa, and steel strength from 200 MPa to 600 MPa, and tube depth-to-thickness ratio up to approximately 100. The numerical results were also verified with the available test data. Comparison was also made with other published numerical formulations.
Chapter 6 presents an integrated design methodology. The Eurocode 4 design principle was modified, so that more accurate numerical expressions based on fibre element method were given for pure axial loads under triaxial state for both normal strength and high strength concrete. The design predictions were compared with the published test results and results from other design code recommendations.

In Chapter 7 analyses were presented of a set of analysis and design equations representing member strength capacity of CFST beam-columns. A load line considering the second order effects on slender columns was determined by incorporating a moment magnification factor. The intersection between the member strength equation and the load line was used to predict the capacity of the column. The numerical results were verified with the available test data.

In Chapter 8 analyses were presented a design equation to investigate the ultimate load capacity of slender CFST columns subject to biaxial bending moments and axial load. Numerous computer calculations were performed by using fibre analysis formulation to study the load contour diagram. A verification of the proposed interaction equation was made by comparing the actual test results and current ACI, AISC and Eurocode 4 design codes.
CHAPTER 2:

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter describes a review of the behaviour of concrete-filled steel tubular (CFST) members subject to axial load, bending moment or a combination. The discussion is divided in two parts. In the first part the discussion is focussed on the characteristic behaviour of columns, beam columns, and beams of varying length and it provides a proper background for the research that is going to be dealt with in this thesis.

The second part summarises the major theoretical and experimental researches performed throughout the world over the past several decades on CFST. The studies on analysis and design of the CFST sections are reviewed and discussed. The design rules for the analysis of steel concrete composite columns provided in different codes of practice are also discussed.
2.2 BEHAVIOUR OF CONCRETE FILLED STEEL TUBE COLUMNS

2.2.1 Columns under Axial Compression

Some of the earliest research on concrete filled steel tubular columns subjected to concentric compression was carried out by Gardner and Jacobson (1967), Knowles and Park (1969) and Sen (1972). In the investigations into the behaviour of concrete filled circular tubes, they found that the concrete containment results in an enhancement of the compressive strength, and also in the development of hoop stresses in the steel tube which cause a reduction in the effective yield strength of steel. Then, more experimental and theoretical studies were performed by other researchers [ie. Knowles and Park, 1970; O’Shea and Bridge, 1994].

Tomii (1991) found that the measured ultimate load of circular CFSTs is considerable larger than the nominal load, which is the sum of the two component strengths. This is due to strain hardening of the steel and the confinement of the concrete. Although the confinement effect diminishes with increasing column length and is generally neglected for columns of practical length, it ensures that the column behaves in a ductile manner, a distinct advantage in seismic applications. Tests on approximately 270 stub columns [Tomii et al., 1977] showed that axial load versus longitudinal strain relationships in a classification based on test parameters including cross-section shape, diameter to wall thickness ratio \((D/t)\) and concrete and steel strength.
For CFST slender column, stability rather than strength will govern the ultimate load capacity. Overall column buckling will precede strains of sufficient magnitude to allow large volumetric expansion of the concrete to occur. Hence, for overall buckling failures there is no confinement of the concrete and thus no additional strength gain. Many authors have agreed that a slenderness ratio \((L/D)\) equal to 15 generally marks an approximate boundary between short and long column behaviour. Neogi, Sen and Chapman (1969) originally proposed this value for eccentrically loaded columns. Chen and Chen (1973), Bridge (1976), and Prion and Boehme (1989) confirmed the \(L/D\) value of 15. Knowles and Park (1969) proposed a \(KL/r\) (the ratio of effective length to radius of gyration), value of 44 (approximately equal to an \(L/D\) of 12) above which confinement does not occur. However, Zhong et al. (1991) specified a lower value of \(L/D\) equal to 5 above which confinement does not occur.

### 2.2.2 Concrete Filled Steel Tube Beam (Pure Bending)

For the derivation of ultimate moment capacity of the concrete-filled steel tubular sections, the reinforced concrete theory was considered by most of the researchers. In some of codes of practices [ACI 318, 1995; AS3600, 1994], concrete failure is considered at a limiting concrete strain of 0.3% and carries no strength in the tensile zone, and the tensile resistance of a CFST depends on the steel alone. Therefore, moment resistance is highly influenced by the steel tube. The only contribution of the concrete to moment resistance occurs due to the movement of the neutral axis of the cross section toward the compression face of the beam with the addition of concrete.
This effect can be enhanced by using thinner tubes or higher strength concrete [Furlong, 1967]. Tests by Bridge (1976) showed that the concrete core only provides about 7.5% of the capacity in member under pure bending. However, Kitada (1992) and Brahmachari (1997) reported that substantial increases in strength, and especially ductility, could be achieved by filling hollow steel tubes with concrete.

For the steel hollow sections, most of the studies assumed the steel section is fully plastic at the time of failure for the simplification of the analysis. Except in some of the studies [Barber et al., 1987; Lu and Kennedy, 1994] the stresses in steel were derived from corresponding strain values obtained during experiments to compare test results with the theory.

2.2.3 Combined Axial Load and Bending

The parameters influencing the behaviour of beam-columns include (1) D/t ratio, (2) axial load ratio (N/N₀), and (3) L/D ratio or the slenderness of the member. Firstly, the D/t ratio determines the point of local buckling and it affects the section's ductility. A smaller D/t ratio delays the onset of local buckling of the steel tube. Tubes with high D/t ratios (above 44) [Tomii and Sakino, 1979a] will often exhibit local buckling even before yielding of the section occurs. A low D/t provides greater ductility, illustrated by the long plateau in the moment-curvature diagrams for such columns. Tomii and Sakino also showed that beam-columns with low D/t ratios (24 and 33) could sustain the maximum moment after local buckling. Beam-columns with high D/t ratio (44) began to lose capacity as the curvature increased, although
only under large axial loads did the capacity drop significantly. Ichinohe, *et al.* (1991) determined that for specimens with a $D/t$ less than 53, the moment increases after local buckling without strength degradation.

Studies on concrete-filled steel rectangular hollow section columns by Tsuji *et al.*, (1992) demonstrated that for columns with larger $D/t$ ratio, local buckling appeared first, then yielding occurred whilst for the columns with smaller $D/t$ ratio failure occurred first, followed by local buckling.

Prion and Boehme (1989) found on the tests of CFSTs using high strength concrete (73 MPa to 92 MPa), that the steel yielded first, then the concrete failed by shearing, after which the capacity of the section fell off considerably, and local buckling occurred in the steel tube at the location of the shear failure in the concrete. The columns in their tests had a $D/t$ ratio of 92. Luksha and Nesterovich (1991) noticed a similar type of failure in their tests of large diameter tubes. Tubes with $D/t$ ranging from 54 to 104 began failing at 90% of ultimate load with the formation of buckles along the tube. At incipient of failure, the concrete sheared causing the steel to buckle completely. This shearing / local buckling type of failure is undesirable because it is often sudden and could be catastrophic [Knowles and Park, 1969]. Other composite beam-column studies [Neogi *et al.*, 1969; Prion and Boehme, 1994] indicated that local buckling can either be prevented or delayed and the yield stress of steel is more effectively utilised.
The axial load ratio ($N/N_0$) is a second important parameter in CFST beam-column behaviour. The relationship between the moment and axial load, as illustrated by interaction diagrams, is typically a curve that bulge outward for low axial loads (maximum moments exceed $M_0$ computed ultimate moment assuming no axial force) and then linearly approaches $N_0$ showing a rapid decrease in the moment capacity for high axial load ratios. As axial compressive load is added to a member in pure bending, the contribution of the concrete begins to increase, utilising the composite action of the section to a greater extent. Like the $D/t$ ratio, the axial load ratio has an effect on ductility. Large values of $N/N_0$ lead to rapid moment capacity deterioration and brittle failures. In these cases, the tensile side of the steel tube will be elastic as the concrete on the compression side crushes. If the steel on the compression side has buckled (more likely as the $D/t$ ratio increases), a brittle failure ensues. It can be seen that a combination of a high $D/t$ and high $N/N_0$, leads to undesirable modes of failure. Because of this, Tomii and Sakino (1979a,b,c) limited their studies to beam-columns with $N/N_0$ less than or equal to 0.5.

Finally, the $L/D$ ratio has significant effect on the member's performance. A number of researchers including Chen and Chen (1973), Bridge (1976), Tomii and Sakino (1979a) and Galambos (1988) have presented interaction charts which show the moment-axial load relationship for different $L/D$s. For a given cross-section, the computed ultimate moment assuming no axial force ($M_0$) remains the same with an increase in length, but the maximum axial load ($N_0$) drops markedly. These results
are for the most part analytical and only a limited number of tests covering a small range of beam-column slenderness ratios have been performed.

2.3 STRESS-STRAIN BEHAVIOUR OF CFST COLUMNS

The stress strain behaviour of the steel and the concrete in a CFST is multiaxial because of the interaction between the two materials. The nonlinear relationship to be used depends on the degree of confinement anticipated in the CFST section. The interaction between the concrete and the steel results in additional strength and ductility in the concrete. Circular CFSTs show a greater increase in strength [Knowles and Park, 1970] and greater enhancement of ductility [Tomii et al., 1973].

Previous researches have been conducted in reinforced concrete using steel ties to provide improvements in ductility and strength for reinforced concrete columns subjected to concentric axial load. As a steel tube can be modelled as a limiting case of ties with zero spacing, researches conducted on reinforced concrete are directly applicable. However care needs to be taken to ensure issues such as pitch spacing and concrete cover are correctly considered [O’Shea and Bridge, 1997].

Although some researches [Ahmad and Shah, 1987; Fafitis and Shah, 1985] have studied the response of normal or high strength concrete confined with steel ties, difficulties often arise in the accurate determination of the confining pressures.
Kavoossi and Schmidt (1993) conducted a number of tests on circular stub CFST columns filled with 60 to 70 MPa concrete where axial load was applied only to the concrete, it was found that the ultimate strength of the concrete was increased by ratios ranging from 1.84 to 2.52 based on the assumption that the steel tube yields in the circumferential direction. O’Shea and Bridge (1996) conducted a number of tests with 113 MPa high strength concrete, it was found that the enhancement ratios were much lower, the ranging from 1.02 to 1.19, depended upon the thickness of the steel tubes.

Tests have also been conducted on active confining pressure provided by using a pressurised triaxial cell and passive confining pressure provided by ties and steel tubes, as well as the selection of an appropriate load path. Attard and Setunge (1996) conducted a number of triaxial tests on high and very high strength concrete (60 to 130 MPa) subjected to low confining pressures (1 to 20 MPa) and, it was mentioned that the behaviour obtained from active and passive pressure are similar, while the load path only becomes important for high confining pressure with ratio of confining stress to peak confined axial stress greater than 0.15.

O’Shea and Bridge (1997) used the test results from Attard and Setunge (1996) and developed an analytical model to predict complete response of the confined concrete under slow strain conditions.
2.4 DESIGN AND NUMERICAL METHODS

Researches on concrete filled steel tubes have been continuing over 40 years. It is generally agreed by research workers that, using numerical analysis, it is now possible to predict the behaviour up to failure of CFST columns. This section reviews the results of such predictions to validate for the theoretical studies and the experimental investigations in the behaviour of CFST columns.

Furlong (1967; 1968) measured surface strains on stub-column specimens under axial force and increasing bending moments, which led him to formulate a tangent modulus theory to predict the axial load capacity of slender columns. A very extensive testing program of concrete-filled steel tubes of both circular and square cross section was carried out by Furlong which resulted in the development of an interaction equation for combined axial load and bending moment. The interaction equation of a lower bound of the strength for a column is given by the equation:

\[
\left( \frac{N_u}{N_0} \right)^2 + \left( \frac{M_u}{M_0} \right)^2 \leq 1
\]

where \( N_u \) and \( M_u \) are the measured ultimate loads and moment, respectively. \( N_0 \) is the lower limit of axial load estimated by summing the yield strength of the steel and the load on the concrete at the time of the steel yield (based on the strain at the yield load). \( M_0 \) is the lower limit value for moments, and is the plastic moment capacity of the steel tube alone.
Neogi, Sen and Chapman (1969) reported that the apparent increase in concrete strength due to the confinement by the steel section, was not that significant for specimens with high slenderness ratio and considerable eccentricities of the axial load. A computer model was developed to calculate the strength of slender columns and was successfully used to estimate the strength.

Bridge (1976) performed an elastic-plastic column stability analysis based on moment-thrust-curvature relationships. Simplified stress-strain relationships were used for steel and concrete. The analytical predictions matched the experimental deflections exactly up to the yield point of the specimens. After yielding there was some discrepancy. The author attributed these variations to a number of possibilities: errors in determining the steel tube yield stress, variation of yield across the cross section, a different concrete strength than assumed, or a residual stress pattern that was not accurately simulated by the stress-strain curve.

Cai and Jiao (1984) at the China Academy of Building Research, presented the test results of 57 concentrically loaded short columns. Cai and Jiao (1985) also presented the test results of 51 eccentrically-loaded concrete-filled steel tubular columns with eccentricity ratios \( e/r \) varying from 0 to 1.3 and length over diameter ratio \( L/D \) from 4 to 22. The test results showed that the ultimate strength of the column decreased significantly as its slenderness ratio and eccentricity ratio increased. They proposed a formula based on limit analysis where the overall strength reduction factor of the column was expressed as the product of a reduction due to slenderness ratio \( \phi_l \) and that due to eccentricity ratio (the ratio of load eccentricity to the radius
of the concrete core) $\phi_c$. Cai and Gu (1985) presented the test results of 26 concentrically loaded long concrete filled tubular columns with length over diameter ratio ($L/D$) varying from 3 to 50. An empirical formula for the strength reduction coefficient based on the test data was proposed. Cai (1987) proposed the following formula to determine the ultimate load of a column that fails by overall inelastic buckling:

$$N_u = \phi \phi_c N_0$$

where $N_0$ is the ultimate strength of axially loaded stub column, given as:

$$N_0 = A_c f_c (1 + \sqrt{\phi} + 1.1\phi)$$

where the confinement ratio $\phi = \frac{A_s f_s}{A_c f_c}$ and $A$ and $f$ are the respective areas and ultimate stress in the two material. For combined axial load and bending, the author presented an interaction equation based on the above parameters:

for eccentricity ratios less than or equal to 1.55

$$\frac{N_u}{N_0} + 0.74 \frac{M_u}{M_0} = \phi_i$$

for eccentricity ratios greater than 1.55

$$\frac{M_u}{M_0} = \phi_i$$
Shakir-Khalil and Zeghiche (1989) presented a numerical analysis for the ultimate load of a column. The first step in the numerical analysis was to develop moment-thrust-curvature diagrams. Following that, the analysis was carried out by a finite element method and also by an approximate method using a sine curve to model the deflection with equilibrium maintained at mid-height.

Rangan and Joyce (1992) conducted a series of tests on normal strength steel tubes filled with high strength concrete. The specimens were eccentrically loaded and were bent in symmetrical single curvature. The parameters examined included column slenderness and load eccentricity. A numerical method to calculate the strength of columns was presented, and the failure load was determined by iteratively computing the internal and external moments until equilibrium was established. The computation of the external moment included the effects of creep, initial eccentricity, and initial imperfections. To relate the internal and external moments, the authors assumed a deflected shape in the form of a sine curve. Curvature was calculated by dividing the extreme fibre strain by the neutral axis distance. From this, deflection was calculated using the sine curve assumption and the mid-height relationship between curvature and deflection. The neutral axis was adjusted until moment equilibrium was achieved.

Bridge and Yeung (1993) presented a design formula to determine the limiting slenderness ($L/r$) of a ‘short’ composite column. Columns defined as ‘short’ are columns which can be designed for their full cross-section strength without considering the strength reducing effects of stability. Their formulation considered
the effects of cross section type (CFSTs and SRCs of different size), slenderness ratio, initial imperfections, loading eccentricities, and the ratio of applied end moments. An inelastic non-linear column analysis was used to verify the proposed equation.

Hajjar and Gourley (1996) used fibre element method to develop of a polynomial equation to represent cross section strength of a CFST subjected to a combination of axial force and bending moment. The equation is a function of two, the width to thickness ratio of steel tube, and the ratio of the concrete compressive strength to the yield stress of the steel tube.

Kilpatrick and Rangan (1997) performed a series of tests on concrete filled steel tubular slender columns with concrete strengths of 57 or 96 MPa. The specimens were with a D/t ratio of 35 or 42. The high strength steel was used for the tube with the yield strength of 450 MPa. The first group of nine columns were bent into single curvature with equal end eccentricities of +15 mm. The second group of eight columns bent into single curvature as the eccentricity varied between +5 mm and +50 mm. The third group of twenty-four columns for the eccentricity varied between +0 mm and +50 mm at the top of column and −50 mm and +50 mm at the bottom of the column.

Zhang and Shahrooz (1999) used the test data from past studies and three additional specimens test. The test specimens include short and slender CFSTs made with normal and high strength steel tubes filled with normal and high strength concrete.
Comparison was made with ACI and AISC codes for calculating the capacity of the CFST columns. It was found the capacities from ACI and AISC codes varied significantly. However the results from the ACI method are generally closer, and the both codes were reasonable for slender CFST made with normal strength steel tubes. Neither ACI nor AISC method is applicable for case in which high strength tubes are used.

2.5 REVIEW OF DESIGN CODES

Two design methods are most widely used in codes of practices for the design of steel-concrete composite members. These are either reinforced concrete and structural steel approaches. The design approaches described in ACI-318 and AISC-LRFD are different in concept. The concept of ACI-318 is the traditional reinforced concrete approach. Australian code AS 3600 also uses the concept of reinforced concrete design. The AISC-LRFD uses the concept of structural steel approach. The Eurocode 4 uses a logical design procedure, where the behaviour of composite columns is treated by a combination of both approaches.

In the following sections a review of design methods is provided in different codes of practices on steel-concrete composite sections. Review mainly focuses on four design codes. They are ACI-318 (1995), AISC-LRFD (1994), AS 3600 (1994) and Eurocode 4 (1994).
2.5.1 ACI 318

The ACI rules for composite columns follow the same procedures as for reinforced concrete columns. The ACI code provides the method to evaluate the strength of the cross section and the effects of slenderness and it requires a minimum eccentricity of axial loads for column design.

Limitations

In the ACI design method the yield stress of structural steel used in calculating the strength of composite columns should not exceed 414 MPa, and it does not permit column design given for high strength concrete. The secant modulus of elasticity of concrete is calculated from the formula:

\[ E_c = 0.043 \rho_c^{1.5} \sqrt{f'_c} \]

where \( \rho_c \) is the density of concrete in kg/m\(^3\) and \( E_c \) and \( f'_c \) are expressed in MPa.

This formula is not adjusted for high strength concrete, and overestimates the modulus of elasticity for concrete strength above 40 MPa.

Elastic Flexural Stiffness

The actual elastic stiffness of the steel-concrete composite section \((ED)_s\) in the ACI-318 is given as,

\[(ED)_s = E_s I_s + 0.75(0.2E_c I_c)\]
or

\[(EI)_e = 0.75(0.4E_c I_g)\]

where \(I_g\) is the second moment of area of the gross composite cross-section ignoring steel; \(I_s\) is the second moment of area of the steel; \(E_s\) is the modulus of elasticity of the steel and \(E_c\) is the elastic modulus of concrete.

**Axial Compression**

The axial resistance \(N_n\) is limited to 85 percent of the theoretical squash load \(N_0\) for concrete-filled steel tubes in ACI-318 is given as

\[N_n = 0.8 \phi N_0\]

where \(\phi\) is the capacity reduction factor, a value of 0.75 for concrete-filled steel tube and the value of \(N_0\) is given

\[N_0 = A_s f_y + 0.85 A_c f'_c\]

where \(A_c\) = area of concrete in cross section

\(A_s\) = area of structural steel in cross section

\(f'_c\) = strength of concrete from standard cylinder tests

\(f_y\) = yield strength of structural steel

**Moment Capacity without Axial Load**

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The calculation of the ultimate moment capacity of the section $M_e$ was similar to that used by the stress block analysis, assuming concrete rectangular stress block equal to 0.85 times the compressive area of concrete bounded by edges of the cross-section and a straight line parallel to the neutral axis at a distance $\beta_1$ from the extreme fibre concrete. The factor $\beta_1$ depends on the concrete strength. It is 0.85 for concrete strength less than 30 MPa and is reduced continuously at a rate of 0.05 for each 7 MPa of strength in excess of 30 MPa, but must not be taken to be less than 0.65. It is also assumed that tensile concrete carries no strength. The stress in the steel below the yield strength $f_y$ is taken as $E_s$ (elastic modulus of steel) times the steel strain whilst the steel stress is considered as $f_y$, when the strains are greater than yield strain.

**Axial Load and Bending**

ACI 318-95 used a uniform stress specified for the concrete. However, the ultimate concrete stress is taken as $f_e'$ instead of 0.85 $f_e'$ to reflect that concrete inside tubes does not split and the environment is better protected. The steel tube in a CFST is converted into equivalent “reinforcing bars” that are distributed around the concrete infill. The steel is assumed to follow an elastic-perfectly plastic stress-strain relationship. The moment strength corresponds to developing a compressive strain of 0.3% in the concrete. Using the assumptions, closed-form expressions may also be derived to generate the axial load moment interaction diagrams.
For slender CFST column, the buckling effects are accounted for by the use of a moment magnifier $\delta_b$, calculated by multiplying the maximum first order bending moment $M_{Sd}$ with the moment magnifier $\delta_b$. In ACI-318, this is given as:

$$M^* = \delta_b M_{Sd}$$

where

$$\delta_b = \frac{C_m}{N_{Sd}} \frac{1}{1-N_{cr}} \geq 1.0$$

in which $N_{cr}$ is the critical buckling stress resultant given by:

$$N_{cr} = \frac{\pi^2 (EI)_{ce}}{L_e^2}$$

where $L_e$ is the effective length and $N_{Sd}$ is the axial force for end-loaded braced members; $C_m$ is the moment factor, computed as:

$$C_m = 0.6 + 0.4 r \geq 0.4$$

where $r$ is the ratio of smaller to larger end moment and is positive when the member is bent in single curvature.

**Biaxially Eccentric Loading**

The ACI-318 suggests adequately a biaxial-strength analysis given as

$$\frac{1}{N_i} = \frac{1}{N_x} + \frac{1}{N_y} - \frac{1}{N_0}$$
where \( N_i \) = capacity for biaxially eccentric axial force

\[ N_x = \text{axial force capacity for eccentricity } e_x \text{ in the plane of the } x \text{ axis.} \]

\[ N_y = \text{axial force capacity for eccentricity } e_y \text{ in the plane of the } y \text{ axis.} \]

2.5.2 AISC-LRFD

The concept of applying AISC column design methodology to composite columns by the use of modified properties was presented by Furlong [Furlong, 1974]. Modified yield stress \( F_{my} \), modified modulus of elasticity \( E_m \) and modified radius of gyration \( r_m \) were incorporated into an allowable stress procedure that was published by Task Group 20 of the Structural Stability Research Council, and the modified properties were adopted into the LRFD procedure which became a part of the 1986 AISC-LRFD code.

Limitations

The AISC-LRFD recommended the concrete strength be limited to 69 MPa. A lower limit of 17 MPa is recommended. The yield stress in calculating the strength of composite columns should not be taken greater than 414 MPa.

Axial Strength

In the AISC-LRFD code, the design strength of CFST columns is determined from the same equation as those applicable to steel columns except that the formulas are
entered with modified properties $F_{my}$, $E_m$ and $r_m$. The axial design strength is given as

$$N_d = 0.85 \, A_s \, F_{cr}$$

where $F_{cr}$ is the critical stress of the column given as

$$F_{cr} = (0.658 \lambda_c^2) \, F_{my} \quad \lambda_c^2 \leq 2.25$$

and

$$F_{cr} = \frac{0.877}{\lambda_c^2} \, F_{my} \quad \lambda_c^2 > 2.25$$

where

$$\lambda_c^2 = \left(\frac{L_e}{\pi r_m^2}\right)^2 \left(\frac{F_{my}}{E_m}\right)$$

in which

$$F_{my} = f_y + 0.85\left(\frac{A_e}{A_s}\right) f_c$$

and

$$E_m = E_x + 0.4\left(\frac{A_e}{A_s}\right) E_c$$

where $F_{my}$ is modified yield stress, $E_m$ is modified modulus of elasticity and the modified radius of gyration $r_m$ is the larger of (1) the radius of gyration of the steel section $r_s$ and (2) 30 percent of the gross CFST section in the plane of bending.
Moment Capacity without Axial Load

The nominal flexural strength $M_0$ of a column cross section may be determined from the plastic state of stress or from an analysis of flexural strength at the ultimate state of strain. For simplicity, the AISC-LRFD commentary offers an approximate equation for moment capacity of doubly symmetric sections given as

$$M_0 = Z f_y + \left( \frac{h_2}{2} - \frac{A_w}{1.7 h_1} \frac{f_y}{f'_e} \right) A_w f_y$$

where $Z =$ plastic section modulus of steel shape

$h_1 =$ concrete width perpendicular to the plane of bending

$h_2 =$ concrete thickness in the plane of bending

$A_w =$ web area of steel shape plus any longitudinal bars at centre of section

Axial Load and Bending

For CFST columns symmetrical about the plane of bending, the interaction of compression and flexure should be limited by the following bilinear relationship

$$\frac{N_u}{\phi_c N_n} + \frac{8 M_u}{9 \phi_b M_0} \leq 1.0 \quad \text{for } N_u \geq 0.2 \phi_c N_n$$

and

$$\frac{N_u}{2 \phi_c N_n} + \frac{8 M_u}{\phi_b M_0} \leq 1.0 \quad \text{for } N_u < 0.2 \phi_c N_n$$

where $N_u =$ factored axial force

$M_u =$ factored moment increased for slenderness effects
\( N_n \) = nominal thrust capacity including slenderness effects

\( M_n \) = ultimate moment capacity without axial force

\( \phi_b \) = resistance factor for bending = 0.85

\( \phi_c \) = resistance factor for compression = 0.85

2.5.3 AS 3600-1994

The design recommendations described in AS 3600-1994 is mostly similar to that of ACI-318 by using the reinforced concrete design approach. Thus, the review in this section specifically focused on the minimum load eccentricity and elastic flexural stiffness, and biaxially eccentric loading for the composite section.

In AS 3600 it requires that a minimum bending moment be considered in the design of CFST columns, the minimum end eccentricity \( e_{min} = 0.05 \ D \) about each principal axis. This compares with the ACI-318 end eccentricity \( e_{min} = (15.2 + 0.03D) \), where \( D \) is the depth of the gross cross-section of the CFST member (in \( \text{mm} \)).

The AS 3600 uses the assumption that elastic flexural stiffness corresponds to maximum moment capacity \( M_{ub} \) of the cross-section. The secant flexural stiffness in the AS 3600 is given as,

\[
(ED)_e = 167M_{ub}D
\]
The design rule for the biaxially eccentric loading in the AS 3600 is given as

$$\left( \frac{M_{Sdx}}{\phi_x M_{0x}} \right)^{\alpha_n} + \left( \frac{M_{Sdy}}{\phi_y M_{0y}} \right)^{\alpha_n} \leq 1.0$$

where $M_{0x}$ and $M_{0y}$ are design strengths in uniaxial bending about the principal axes $x$ and $y$, respectively; $\phi_x$ and $\phi_y$ are the capacity reduction factor about the principal axes $x$ and $y$, respectively; and $M_{Sdx}$ and $M_{Sdy}$ are the design moments and $\alpha_n$ is given as

$$\alpha_n = 0.7 + 1.7 \frac{N_t}{0.6 N_0} \quad 1.0 \leq \alpha_n \leq 2.0$$

where $N_0$ is the axial force capacity for concentric load and $N_t$ is the design axial force.

2.5.4 Eurocode 4-1994

In this section the design recommendations in Eurocode 4 on concrete-filled steel tubes are discussed in detail.

Limitations

The Eurocode 4 places the limitations of concrete strengths which should not exceed 50 MPa and the diameter (or width) to wall thickness should be limited by

$$\frac{D}{t} \leq 52$$

for rectangular sections
where \( D / t \leq 90 \varepsilon^2 \) for circular sections

where \( D \) is the greater overall dimension of the section parallel to a principal axis, \( t \) is the thickness of the wall of a concrete-filled hollow section, \( \varepsilon = \frac{235}{f_y} \) and \( f_y \) is the yield strength of the steel.

**Elastic Flexural Stiffness**

The elastic flexural stiffness of the composite section is given in the Eurocode 4 as,

\[
(ED)_e = E_s I_s + 0.8E_{cd} I_c
\]

where \( I_s \) and \( I_c \) are the second moment of areas of structural steel and the concrete, respectively. \( E_s \) is the elastic modulus for the structural steel and \( E_{cd} = \frac{E_{cm}}{\gamma_{ce}} \) where \( E_{cm} \) is the secant modulus of elasticity of concrete and \( \gamma_{ce} = 1.35 \) is the safety factor for stiffness.

**Axial Compression**

The Eurocode 4 code provides a formula to predict the axial load for CFST columns including the effect of confinement:

\[
N_e = A_s \delta_s \frac{f_y}{\gamma_s} + A_e \frac{f_y}{\gamma_c} \left( 1 + \eta_1 \frac{t}{D} \frac{f_y}{f_c} \right)
\]

where \( \gamma_c \) and \( \gamma_s \) are the partial safety factors and the value of \( \eta_1 \) and \( \eta_2 \) are given as for \( 0 \leq \varepsilon \leq 0.1D \)
\[ \eta_1 = \eta_{10}(1 - \frac{10e}{D}) \]

\[ \eta_2 = \eta_{20} + (1 - \eta_{20}) \frac{10e}{D} \]

and the value of \( \eta_{10} \) and \( \eta_{20} \) are given as

\[ \eta_{10} = 4.9 - 18.5 \bar{l} + 17 \bar{l}^2 \geq 0 \]

\[ \eta_{20} = 0.25(3 + 2 \bar{l}) \leq 1.0 \]

The relative slenderness of \( \bar{l} \) is given as

\[ \bar{l} = \sqrt{\frac{N_0}{N_{cr}}} \]

**Buckling and Imperfection Effects**

The buckling effects are accounted for by the use of a moment magnifier \( \delta_b \) which can be calculated by multiplying the maximum first order bending moment \( M_{sd} \) with a magnifier \( \delta_b \) given as:

\[ M^* = \delta_b M_{sd} \]

where

\[ \delta_b = \frac{C_m}{N_{sd} / N_{cr}} \geq 1.0 \]

in which
\[ N_{cr} = \frac{\pi^2 (EI)_{e}}{L_e^2} \]

where \( C_m \) is the moment factor, is given as

\[ C_m = 0.66 + 0.44 \, r \geq 0.44 \]

Accounting for slenderness and imperfections, the column strength \( N_c \) in axial compression is given by

\[ N_c = \chi_k N_0 \]

\( \chi_k \) is the reduction factor due to buckling, equal to

\[ \chi_k = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}} \]

where

\[ \phi = 0.5 \left[ 1 + \alpha (\lambda - 0.2) + \lambda^2 \right] \]

where \( \alpha \) depends on the buckling effects, a value of 0.21 was adopted for CFST column (Eurocode 3).

The member strength interaction diagram for a composite column is constructed based on the section strength interaction as indicated in Fig. 2.1. The value of \( \mu_k \) corresponding to \( \chi_k \) can be determined on the interaction curve.
The influence of this imperfection is assumed to decrease linearly to the value \( \chi_n \).

For end moments, the expression is given as

\[
\chi_n = \chi_k \frac{(1-r)}{4}
\]

For an acting axial force \( \chi_d = \frac{N_{sd}}{N_0} \) the moment factor is \( \mu_d \). The remaining bending capacity is measured by the distance \( \mu \) given as

\[
\mu = \mu_d - \mu_k \left( \frac{\chi_d - \chi_n}{\chi_k - \chi_n} \right)
\]

The bending resistance is

\[
M = 0.9 \ \mu M_0
\]

where the reduction of the moment resistance is the value of 0.9, because of the unconservative assumption that the concrete block is fully plastic to the neutral.
CHAPTER 3:
A GENERAL NUMERICAL PROCEDURE
FOR THE ANALYSIS OF STEEL CONCRETE
COMPOSITE COLUMNS

3.1 INTRODUCTION

For the design of steel-concrete composite columns subject to an eccentric load which causes uniaxial or biaxial bending, the first task is commonly to generate the axial force and moment strength interaction curves. Based on the section strength interaction diagram the member strength is obtained by considering the effect of member buckling. The strength checking is then made by comparing the applied load and member strength. Accurate numerical methods have long been proposed to calculate the section strength of a composite column. There are different variations of the theory, and all of these are based on the principle of classic mechanics.

Two typical representations of these methods are the strip method and the fibre method [Bode, 1976; Shakir-Khalil and Zeghiche, 1989; Kawaguchi et al., 1991;
Bridge, et al., 1997; Zhang and Chao, 1998; Zhang and Shahrooz; 1999]. In the *strip* method the whole composite cross-section is divided into a number of strips by a group of lines parallel to the axis of bending. The formulation is obtained for each strip based on the principle of equilibrium and compatibility. In the *fibre* method, a grid of fibre monitoring points over the section is established.

This chapter presents a general numerical analysis procedure to calculate the complete load-deflection and moment-curvature relationship for concrete filled steel tubular columns (CFST) under combined axial force and biaxial bending. The method was based on fibre analysis formulation. It first calculates the deformation which satisfies the compatibility equations and uses an iteration procedure to obtain the solution. A computer program was developed to evaluate the strength in accordance with design codes, the types of cross sections including typical shapes for both concrete encased and concrete filled steel tubes.

### 3.2 BASIC ASSUMPTIONS

In the study of CFST subjected to axial load and biaxial bending, the following assumptions have been made:

1. Plane section remains plane after loading;
2. Perfect bond exists between the concrete core and the steel shell at the material interface;
3. Monotonic loading;
4. Effect of creep and shrinkage is neglected; and

5. The shear deformation and torsional effect are all neglected.

3.3 TYPES OF ANALYTICAL MODEL AND DISCRETIZATION OF CROSS SECTIONS

3.3.1 Types of Composite Column and their Analytical Model

There are two basic types of composite column, namely the concrete filled tube and the concrete-encased. The concrete filled tubular columns have the advantages that they require no formwork and no reinforcement. The most common steel sections used are rectangular and circular hollow sections. There are two disadvantages for the concrete-filled columns compared with the concrete-encased steel columns. The concrete inside the tube does not provide the same degree of protection to the steel against fire as in the case of encased columns. As a norm, an alternative form of fire proofing may be required. The other disadvantage is the difficulty of effecting the connections.

In the case of concrete encased columns, the concrete encasement enhances the behaviour of the structural steel core by stiffening it, and making it more effective against both local and overall buckling. The load-bearing concrete encasement performs the additional function of fire proofing the steel core. The main disadvantage of this type of composite column is that it requires a complete formwork. Furthermore, a reinforcement cage is required in order to prevent the
concrete cover from spalling at low load levels, especially in the case of eccentrically loaded columns.

On the other hand, there are a lot of advantages in the concrete-filled tube system. Apart from the fact there is no need of any column formwork, the steel tube often has sufficient strength to support the construction load and floor dead load. The concrete core provides the support for steel tubes, which often prevents or delays the onset of local buckling. To the concrete core the steel tube provides effective restraint so that the concrete core is in the triaxial stress state, especially at the higher loading range. The triaxial stress state enhances the behaviour of concrete in both terms of stress and ductility.

As a result of the superior performance and ease of construction procedure, the concrete filled tubular columns are widely used in not only high-rise, but also in low-rise commercial construction.

The analytical model as shown in Fig. 3.1 (a)-(b) includes both concrete filled steel tubes and concrete encased columns. When the thickness of the tube equals to zero, the model represents the concrete encased columns, which absention of the rolled steel and reinforcement bars in the section represents concrete filled tubes.
Fig. 3.1(a) Composite column of rectangular section

Fig. 3.1(b) Composite column of circular section
3.3.2 Discretization of the Cross Sections

A separate meshing scheme is used for the rectangular and the circular sections. Figs. 3.2(a) and (b) illustrate a typical discretization of concrete filled steel tubular columns for both rectangular and circular sections with steel reinforcement.

In Fig. 3.2 (a), the rectangular section is divided into four meshing zones. The symmetrical web and flange of the steel tube comprise two of the zones, with the concrete core comprising the third. In situation where steel reinforcement exists, the individual steel reinforcement comprises the fourth zone. Each zone is then subdivided into a number of slices parallel to the x or y axis except for steel reinforcement where one individual reinforcement is taken as one mesh. The cross-sectional meshing developed is similar to that in the fibre method. In Fig. 3.2 (b), the circular section is divided into five meshing zones. The section is composed of a single steel zones and four concentric circular concrete zones. Four concrete zones are used in order to achieve a relatively constant aspect ratio for the concrete element. The cross-sectional meshing developed is similar to that in the fibre method.
Fig. 3.2 (a) Discretization of a composite rectangular section

Fig. 3.2 (b) Discretization of a composite circular section
3.4 ANALYTICAL FORMULATION FOR COMPOSITE COLUMNS

A Cartesian coordinate system is adopted in the analytical method. The coordinates are oriented such that the $x$-axis and $y$-axis lie parallel to the plane of the cross section and the $z$-axis lies in the longitudinal (axial) direction of the member. For convenience, the origin of the coordinate is placed to coincide with one of the end of supports. The column is divided into a series of segment. Each segment is represented at its midlength by a cross section (Fig 3.3). It is assumed that both ends of the column (nodes 0 and $m$) are restrained from all translational movements in the $x$, $y$ and $z$ directions but can rotate freely.

Only the effects of axial stress and strain are included in the analysis. Transverse stresses and strains are considered to have little effect on the overall behaviour of the member and are therefore ignored. It is also assumed that the member does not rotate about the longitudinal axis and, hence, the moment and the rotation about the $z$-axis are not included.
Fig. 3.3 Analytical model for column
3.4.1 Cross-Section Analysis

The equations for the cross-section analysis are derived by considering the cross section that comprises \( n \) elements of appropriate shapes and sizes as described in section 3.3.2. Each element is defined by its area \( a_i \) and the coordinates of its centroid \((x_i, y_i)\), in which the subscript \( i \) denotes the element number (i.e., \( i = 1, 2, \ldots, n \)) (Fig. 3.4). The curvature and the location of the neutral axis can then be calculated. The strain at each element is computed assuming the strain distribution is linear. The strain at any element is computed by multiplying the curvature by the distance from the centroid of the element to the neutral axis, measured with respect to a line perpendicular to the neutral axis.

![Diagram of cross-section analysis](image)

Fig. 3.4 Definition of cross section analysis
The total axial deformation of each element is calculated from the cross-section deformations assuming that plane sections remain plane during deformation. The axial strain at the centroid of an individual element can be expressed as

\[ \varepsilon_i = \varepsilon_o - \phi_y x_i + \phi_x y_i \]  

(3.1)

where \( \varepsilon_o, \phi_x, \phi_y \) = cross-section strain along the z axis and curvatures about the x-axis and y-axis, respectively. \( \varepsilon_i \) = axial strain of the element.

Once the strain at each element has been determined, stresses are computed from the strains using the appropriate constitutive relationships which will be described in the next chapter. For a value of strain \( \varepsilon_o \), a value of the secant modulus of elasticity \( \varepsilon_i \) for steel or concrete elements can be obtained from the constitutive relationships. The secant modulus of elasticity can be assured to give the positive values and to prevent the singularity problem in the matrix operation.

The axial force in each element is simply calculated by

\[ f_i = \sigma_i a_i \]  

(3.2)

where \( \sigma_i \) = axial stress and \( f_i \) = axial force of the element.

The resultant internal axial force on the cross-section is computed by numerically integrating the stresses over the entire cross-section, which is accomplished by
summing the force in each element:

\[ p_z = \sum_{i=1}^{n} f_i = \sum_{i=1}^{n} s_i \varepsilon_i a_i \quad (3.3a) \]

For biaxial loading, the correct orientation of the neutral axis must also be established. Once the axial force equilibrium is established, moments in the major \( x \) and minor \( y \) axis directions are calculated by summing the moment produced by the force in each element (the product of the element's force and its distance to the neutral axis):

\[ m_x = \sum_{i=1}^{n} s_i \varepsilon_i a_i y_i \quad (3.3b) \]

\[ m_y = -\sum_{i=1}^{n} s_i \varepsilon_i a_i x_i \quad (3.3c) \]

\( p_z, m_x, m_y \) = cross-section force along the z-axis (i.e., at \( x = 0, y = 0 \)) and bending moments about the x-axis and y-axis, (Fig. 3.5 (a),(b)). Combining Eqn. (3.1) - (3.3) into a single matrix equation produces

\[
\begin{bmatrix}
p_z \\
m_x \\
m_y
\end{bmatrix} =
\begin{bmatrix}
\sum_{i=1}^{n} s_i a_i & -\sum_{i=1}^{n} s_i a_i x_i & \sum_{i=1}^{n} s_i a_i y_i \\
-\sum_{i=1}^{n} s_i a_i x_i & \sum_{i=1}^{n} s_i a_i x_i^2 & -\sum_{i=1}^{n} s_i a_i x_i y_i \\
\sum_{i=1}^{n} s_i a_i y_i & -\sum_{i=1}^{n} s_i a_i x_i y_i & \sum_{i=1}^{n} s_i a_i y_i^2
\end{bmatrix}
\begin{bmatrix}
\varepsilon_0 \\
\phi_y \\
\phi_x
\end{bmatrix}
\quad (3.4)
\]

which is the force and strain relationship at a particular section.
Fig. 3.5 (a) Definition of $M_x$

Fig. 3.5 (b) Definition of $M_y$
3.4.2 Member Analysis

The equations for member analysis are derived by considering a simply supported column of length \( L \), which comprises \( m \) small segments of lengths (Fig. 3.3). Each segment is defined by its length \( l \). For the slender column, the second-order effect may be critical for the biaxial bending calculations. It should therefore be included in the formulation. Eqn. (3.4) can be simplified in the following form

\[
\begin{bmatrix}
    p_x \\
    p_z (e_x - \delta_{x}^j) \\
    p_z (e_y - \delta_{y}^j)
\end{bmatrix} =
\begin{bmatrix}
    C_{11} & -C_{12} & C_{13} \\
    C_{21} & -C_{22} & C_{23} \\
    C_{31} & -C_{32} & C_{33}
\end{bmatrix}
\begin{bmatrix}
    \varepsilon_0 \\
    \phi_x \\
    \phi_y
\end{bmatrix}
\]  
(3.5)

The relationship between axial force, eccentricity and bending moment are illustrated in Fig. 3.5. In the above \( \delta_{x}^j, \delta_{y}^j = \) nodal \((j)\) displacements in the \( x, y \) directions and

\[
C_{11} = \sum_{i=1}^{n} s_i a_i
\]

\[
C_{12} = C_{21} = \sum_{i=1}^{n} s_i a_i x_i
\]

\[
C_{13} = C_{31} = \sum_{i=1}^{n} s_i a_i y_i
\]

\[
C_{22} = \sum_{i=1}^{n} s_i a_i x_i^2
\]

\[
C_{23} = C_{32} = \sum_{i=1}^{n} s_i a_i x_i y_i
\]

\[
C_{33} = \sum_{i=1}^{n} s_i a_i y_i^2
\]
To study the three dimensional behaviour of a slender column, each nodal \((j)\) displacement of the slender column can be expressed by a third order Taylor series expansion:

\[
S_j(\xi) = b_0 + b_1 \xi + b_2 \xi^2 + b_3 \xi^3 \tag{3.6a}
\]

where \(\xi\) : local coordinate within each element, it has a value of zero at node \((j-1)\) and a value of \(l\) at node \(j\) for the element. Differentiating Eqn. (3.6a) with respect to \(\xi\), one gets

\[
S'_j(\xi) = b_1 + 2b_2 \xi + 3b_3 \xi^2 \tag{3.6b}
\]

\[
S''_j(\xi) = 2b_2 + 6b_3 \xi \tag{3.6c}
\]

The boundary conditions of node \((j-1)\) and \((j)\) are as follows:

\[
\delta^{j-1} = S_j(0) = b_0 \tag{3.7a}
\]

\[
\theta^{j-1} = S'_j(0) = b_1 \tag{3.7b}
\]

\[
\delta^j = S_j(l) = b_0 + b_1 l + b_2 l^2 + b_3 l^3 \tag{3.7c}
\]

\[
\theta^j = S''_j(l) = b_1 + 2b_2 l + 3b_3 l^2 \tag{3.7d}
\]

where \(\delta^{j-1}, \delta^j\) = nodal \((j-1)\) and \((j)\) displacements, respectively; \(\theta^{j-1}, \theta^j\) = nodal \((j-1)\) and \((j)\) rotations, respectively (Fig. 3.6).
And

\[ S'_j(l) = 2b_2 + 6b_3l \]  

(3.7e)

Eqn. (3.7 (a)–(d)) can be simplified in the following form:

\[
\begin{bmatrix}
\delta^{j-1} \\
\theta^{j-1} \\
\delta^j \\
\theta^j
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
1 & l & l^2 & l^3 \\
0 & 1 & 2l & 3l^2
\end{bmatrix}
\begin{bmatrix}
b_0 \\
b_1 \\
b_2 \\
b_3
\end{bmatrix}
\]

(3.8)

Rearrange Eqn (3.8), then the form is expressed as follow:

\[
\begin{bmatrix}
b_2 \\
b_3
\end{bmatrix} =
\begin{bmatrix}
l^2 & l^3 \\
2l & 3l^2
\end{bmatrix}^{-1}
\begin{bmatrix}
\delta^j \\
\theta^j
\end{bmatrix} -
\begin{bmatrix}
1 & l \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\delta^{j-1} \\
\theta^{j-1}
\end{bmatrix}
\]

(3.9)
Solving Eqn. (3.9) and substituting into Eqn. (3.7e), it becomes

\[
S_j^*(l) = \frac{1}{l^2} \left( 6\delta_{j-1}^i + 2l\theta_{j-1}^i - 6\delta_j^i + 4l\theta_j^i \right) \quad (3.10)
\]

And the physical boundary condition, the curvature, is related to \( S_j^*(l) \) as

\[
\phi_x^j = S_{j,x}^*(l) \quad (3.11a)
\]

\[
\phi_y^j = -S_{j,x}^*(l) \quad (3.11b)
\]

Then

\[
\phi_x^j = \frac{1}{l^2} \left( 6\delta_{j-1}^{i-1} + 2l\theta_{j-1}^{i-1} - 6\delta_j^{i-1} + 4l\theta_j^{i-1} \right) \quad (3.12a)
\]

\[
\phi_y^j = -\frac{1}{l^2} \left( 6\delta_{j-1}^i + 2l\theta_{j-1}^i - 6\delta_j^i + 4l\theta_j^i \right) \quad (3.12b)
\]

where \( \delta_{j-1}^i, \delta_{j-1}^{i-1} \) = nodal \((j-1)\) displacements in the \(x\) axis and \(y\) axis, respectively,

\( \theta_{j-1}^i, \theta_{j-1}^{i-1} \) = nodal \((j-1)\) rotations about the \(x\) axis and \(y\) axis, respectively, and \( \phi_x^j, \)

\( \phi_y^j \) = nodal \((j)\) curvatures about the \(x\) axis and \(y\) axis, respectively.
Substituting the above equations into Eqn. (3.5) and rearranging it, and for the segment \((j)\)

\[
I^2 p_s \begin{bmatrix}
    p_x \\
    p_y \\
    e_x - \delta_x^i \\
    e_y - \delta_y^i
\end{bmatrix} = \begin{bmatrix}
    I^2 C_{11} & 6C_{12} & 6C_{13} & 2IC_{12} & 2IC_{13} & -6C_{12} & -6C_{13} & 4IC_{12} & 4IC_{13} \\
    I^2 C_{21} & 6C_{22} & 6C_{23} & 2IC_{22} & 2IC_{23} & -6C_{22} & -6C_{23} & 4IC_{22} & 4IC_{23} \\
    I^2 C_{31} & 6C_{32} & 6C_{33} & 2IC_{32} & 2IC_{33} & -6C_{32} & -6C_{33} & 4IC_{32} & 4IC_{33}
\end{bmatrix}_{(j)} \begin{bmatrix}
    \varepsilon_0^i \\
    \delta_x^{i-1} \\
    \delta_y^{i-1} \\
    \theta_x^{i-1} \\
    \theta_y^{i-1}
\end{bmatrix}
\]  

(3.13)

\[
I^2 p_s \begin{bmatrix}
    1 \\
    e_x - \delta_x^i \\
    e_y - \delta_y^i
\end{bmatrix} = \begin{bmatrix}
    0 & 6C_{12} & 6C_{13} & 2IC_{12} & 2IC_{13} \\
    0 & 6C_{22} & 6C_{23} & 2IC_{22} & 2IC_{23} \\
    0 & 6C_{32} & 6C_{33} & 2IC_{32} & 2IC_{33}
\end{bmatrix}_{(j)} \begin{bmatrix}
    \varepsilon_0^{i-1} \\
    \delta_x^{i-1} \\
    \delta_y^{i-1} \\
    \theta_x^{i-1} \\
    \theta_y^{i-1}
\end{bmatrix}
\]

(3.14)

\[
\begin{bmatrix}
I^2 C_{11} & -6C_{12} & -6C_{13} & 4IC_{12} & 4IC_{13} \\
I^2 C_{21} & -6C_{22} & -6C_{23} & 4IC_{22} & 4IC_{23} \\
I^2 C_{31} & -6C_{32} & -6C_{33} & 4IC_{32} & 4IC_{33}
\end{bmatrix}_{(j)} \begin{bmatrix}
    \varepsilon_0^i \\
    \delta_x^i \\
    \delta_y^i \\
    \theta_x^i \\
    \theta_y^i
\end{bmatrix}
\]
and let

\[
K_a' = \begin{bmatrix}
0 & 6C_{12} & 6C_{13} & 2IC_{12} & 2IC_{13} \\
0 & 6C_{22} & 6C_{23} & 2IC_{22} & 2IC_{23} \\
0 & 6C_{32} & 6C_{33} & 2IC_{32} & 2IC_{33}\end{bmatrix}_{(j)}
\]

(3.15)

\[
K_b' = \begin{bmatrix}
l^2C_{11} & -6C_{12} & -6C_{13} & 4IC_{12} & 4IC_{13} \\
l^2C_{21} & -6C_{22} & -6C_{23} & 4IC_{22} & 4IC_{23} \\
l^2C_{31} & -6C_{32} & -6C_{33} & 4IC_{32} & 4IC_{33}\end{bmatrix}_{(j)}
\]

(3.16)

Adding segments \( j=1 \) to \( m \), together, it results in the following system matrix

\[
\begin{bmatrix}
1 \\
(e_x - \delta_x) \\
(e_y - \delta_y) \\
\vdots \\
1 \\
(e_x - \delta_x) \\
(e_y - \delta_y)
\end{bmatrix}
= \begin{bmatrix}
K_a' & K_b' & K_a' \\
K_a' & K_b' & K_a' \\
\vdots & \vdots & \vdots \\
K_a' & K_b' & K_a'
\end{bmatrix}
\begin{bmatrix}
\delta^x_0 \\
\delta^y_0 \\
\delta^x_0 \\
\delta^y_0 \\
\delta^x_0 \\
\delta^y_0
\end{bmatrix}
\]

(3.17)

For columns with pinned ends
at node 0, \[ \delta_x^0 = \delta_y^0 = 0 \] (3.18)

at node m, \[ \delta_x^m = \delta_y^m = 0 \] (3.19)

Substituting the above boundary conditions into Eqn (3.17) and selecting the rotation at the column end \( \theta_x^0 \) as the control increment for each iteration step and interchange \( \theta_x^0 \) and \( p_x \) in Eqn (3.17), rearranging it, Eqn (3.17) will be in the following form:

\[
\begin{bmatrix}
2IC_{12}^1 \\
2IC_{22}^1 \\
2IC_{32}^1 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
-\lambda^2 & 2IC_{13}^1 \\
-\lambda^2 & 2IC_{23}^1 \\
-\lambda^2 & 2IC_{33}^1 \\
0 \\
0 \\
\vdots \\
\vdots \\
0
\end{bmatrix}
\begin{bmatrix}
\theta_x^0 \\
\theta_y^0 \\
\vdots \\
\delta_x^j \\
\delta_y^j \\
\vdots \\
\delta_x^m \\
\delta_y^m
\end{bmatrix}
\begin{bmatrix}
p_x \\
\theta_y^0 \\
\vdots \\
\varepsilon_0 \\
\varepsilon_j \\
\vdots \\
\varepsilon_m \\
\theta_y^m
\end{bmatrix}
\] (3.20)

or simply

\[ -\theta_x^0 \{ C \} = [K_c] \{ D \} \] (3.21)

3.4.3 Computational Procedure

When analysing the behaviour of a column, a value of \( \theta_x^0 \) is initially guessed, and
then successively improved by an iterative procedure. The value of deformation calculated in one iteration is checked against those calculated in the previous iteration. The solution is considered to have converged when the difference in the deformations calculated in the iteration is within a pre-assigned tolerance. In the analysis, \( \{C\} \) and \([K_c]\) are treated as known and \( \{D\} \) as unknown.

The computational procedure for calculating the load-deflection curve consists of the following steps and the flow chart is given in Fig. 3.7:

1. Choose an initial value for the end nodal rotation about \( x \)-axis \( \theta_x^0 \).
2. Select a set of trial value of \( \varepsilon_x^i, \varepsilon_y^i, \delta_x^i, \theta_x^i \) and \( \theta_y^i \).
3. Calculate the value of \( \{C\} \) and \([K_c]\) corresponding to the trial value.
4. Using Eqn. (3.20) to calculate a set of new deformation value \( \{D\} \).
5. If the calculated value of deformation \( \{D\} \) satisfies convergence criteria when compared with the initial trial value of deformation, proceed to step (6).
   Otherwise use the calculated value of deformation and repeat step (3) to (4) until the value are satisfied to pre-assigned tolerance. The corresponding values of \( p_x \) and \( \theta_x^0 \) (or \( \varepsilon_x^i \)) represent a point on the load-deflection (curvature) path of the column.
6. By successively incrementing the value of the rotation about \( x \) axis \( \theta_x^0 \) and repeat step (2) to (5) for each value of \( \theta_x^0 \) the complete load-deflection curve is obtained.
Fig 3.7 Flow chart of the numerical analysis
3.5 CONCLUSIONS

This chapter described a numerical procedure and computational formulation used to study the behaviour of steel-concrete composite columns under a general axial load condition. It forms the basis of this thesis. The procedure will be used to generate the complete load-deflection and moment-curvature path of slender concrete-filled steel tubular columns subjected to the combined biaxial bending and axial load in various chapter of this thesis. The p-δ effect due to the slenderness of the column is included in the formulation.

The results of applying this procedure to investigate the behaviour of different CFST configuration are also presented in the following chapters.
CHAPTER 4:

CONSTITUTIVE RELATIONSHIPS

FOR CONCRETE FILLED STEEL TUBES (CFST)

4.1 INTRODUCTION

The analysis and design of steel-concrete tubular columns often involve complex calculation. As mentioned earlier the section analysis is commonly performed first to generate section strength interaction diagrams, based on which member strength interaction diagrams are produced [ACI 318, 1995; AS 3600, 1994; Eurocode 4, 1994]. For the section analysis, both strip and fibre methods [Bode, 1976; Shakir-Khalil and Zeghiche, 1989; Kawaguchi et al., 1991; Bridge, et al., 1997; Zhang and Chao, 1998; Zhang and Shahrooz, 1999] can be used.

In Chapter 3 a general numerical procedure was given. It utilised the equilibrium and strain compatibility conditions to determine the strain. Stress-strain relationships were then used to generate the stresses for concrete and steel, respectively. At
different stage of the analysis, the section strength and member strength interaction
diagram can be generated in addition to the load deformation path of the column.

As discussed previously the accuracy of numerical analyses relies on the accuracy of
constitutive relationships. The accuracy of constitutive relationships is therefore
important to the accuracy of the design solution. Due to confinement, concrete and
steel in CFST behave differently from their normal uniaxial behaviour. The normal
constitutive relationships for steel and concrete require modification if they be used
successfully in concrete filled steel tubes. This is because in concrete-filled steel
tubular columns, there is significant confinement effect in circular sections, therefore
the concrete behaves differently from uniaxial behaviour; similarly steel tubes
behaves differently from the normal uniaxial behaviour for steel materials.

In this chapter, a set of modified uniaxial constitutive relationships was presented for
the analysis of concrete-filled steel tubular sections. Both ductility and strength
enhancement were considered, depending upon the slenderness of the section, the
type of the section and the concrete strength. The proposed relationships were used to
calculate the ultimate axial load capacity of the composite column. The results from
the proposed relationships were verified with published experimental results and
good agreement was obtained for the proposed relationships.
4.2 CONSTITUTIVE MODELS FOR STEEL AND CONCRETE

To calculate element stresses from element strains is the essence of the finite difference approach. The accuracy of the finite difference solution relies on the accurate formulation of the concrete and the steel stress-strain constitutive relationships. The stress-strain calculation in the fibre method permits nonlinear relationships to be employed and allows different relationships to be used depending on the degree of confinement anticipated in the CFST section. In this research, four different empirical nonlinear constitutive models were formulated to represent the uniaxial stress-strain behaviour in CFST. These are steel and concrete models for circular CFST and steel and concrete models for rectangular CFST.

4.2.1 The Confinement Effect at Different Load Level

The stress-strain behaviour of the steel and concrete in a CFST section is multiaxial because of the interaction between the two materials. This may be illustrated by considering a CFST section subjected to a pure axial load (Fig 4.1). As axial compressive load increases from zero (Fig 4.1a), the steel and the concrete deform longitudinally at the same rate, but expand laterally at different rates due to different Poisson’s ratio effect. At the initial strain, the Poisson's ratio of the steel ($\nu_s$) being approximately 0.3 exceeds the Poisson's ratio of the concrete ($\nu_c$) of 0.15 to 0.25, resulting in a greater lateral expansion of the steel, and little or no interaction exists between the two materials [Gardner and Jacobson, 1967; Tsuji, et al., 1992]. During this stage of loading, the steel and the concrete sustain load independently of one
another (Fig 4.1b). At an axial strain of about 0.001, microcracking in the concrete begins to occur and the lateral expansion of the concrete increases and begins to approach the lateral expansion rate of the steel (Fig 4.1c). Between a strain of 0.001 and 0.002, the concrete expansion re-initiates full interactive contact between the two materials (Fig 4.1d), which induces biaxial stresses in the steel and triaxial stresses in the concrete [Knowles and Park, 1970; Bode, 1976; Cai, 1987; Tsuji, et al., 1991; Zhang, et al., 1991].

4.2.2 The Confinement Effect of Different Sectional Shapes

The interaction between the concrete and the steel results in additional strength and ductility in the concrete. Circular CFSTs show a greater increase in strength [Knowles and Park, 1970] and a greater enhancement of ductility [Tomii, et al., 1973] than rectangular CFSTs. A circular tube develops circumferential or hoop tension to exert lateral pressure on the concrete, but the flat sides of a rectangular tube provide little perpendicular pressure to restrain the expanding concrete. Therefore, the two shapes must be modelled using different concrete and steel constitutive relationships.
Fig 4.1 The development process of confinement effect
4.2.3 The Effect of Length to Diameter Ratio on the Confinement

Chen and Chen (1973) compared the numerical axial compressive strength results with test data. It was found that for length/diameter (depth) ratio of $L/D > 15$, the results were in good agreement; and for $L/D < 15$ the numerical results became too conservative. This is due to the fact that for shorter column, ductility and strength enhancement on the composite columns become significant. The confinement effect of the concrete reduces with increase of slenderness ratio $L/D$. The slenderness ratio for the composite column can also be defined as [Sen, 1972]:

$$\bar{\lambda} = \frac{L}{L_c}$$

where

$$L_c = \pi \sqrt{\frac{E_s I_s + E_c I_c}{A_s f_{yd} + A_c f_{cd}}}$$

$E_s I_s$ and $E_c I_c$ are the bending stiffness of the steel and concrete section, respectively; and $A_s f_{yd}$ and $A_c f_{cd}$ are the strength contribution of the steel and concrete section, respectively. Virdi and Dowling (1975) suggested that if $\bar{\lambda} \geq 1$, the confinement effect be neglected. The range of the length/diameter of CHS ratio are 24~29 when $\bar{\lambda} = 1$. Therefore the confinement effect should be considered for $L/D = 5$~25.
4.3 CONCRETE CONSTITUTIVE RELATIONSHIPS

The stress-strain behaviour of concrete is a function of its unconfined uniaxial behaviour, the shape of the cross-section, the relative ratio of length to diameter or depth \((L/D)\), and the relative ratio of concrete to steel strength, commonly expressed as the diameter or depth/thickness ratio \((D/t)\). The latter two factors affect the degree of concrete confinement provided by the steel. The constitutive relationships for circular CFST sections should be able to model both an increase in concrete strength and an increase in concrete ductility due to the confining action of the steel [Tomii, et al., 1973; Ichinohe, et al., 1991]. For conservative purpose both of the ductility enhancement and confine effect of the concrete are not considered for rectangular CFST in this research.

4.3.1 Rectangular CFST Sections for Concrete

Normal strength concrete and high strength concrete are considered separately because of their distinct behaviour. For normal strength concrete, CEB-FIP (1978) model was used. The CEB-FIP model for the stress-strain relationship of concrete defines the stress as a function of strain by:

\[
\frac{\sigma}{\sigma_c} = \frac{\varepsilon(a - 206600\varepsilon)}{1 + b\varepsilon}
\]  

(4.1)

Where \(\sigma\): the stress at strain \(\varepsilon\)

\(\sigma_c\): the maximum compressive stress in MPa
$a$ and $b$ are constants given as

$$a = 39,000(\sigma_c + 7.0)^{-0.953}$$

$$b = 65,600(\sigma_c + 10.0)^{1.085} - 850.0$$

For high strength concrete, those proposed by Admad and Shah (1987) were used, stress-strain curve is given by:

For the ascending part:  \[ \sigma = \sigma_c \left[ 1 - (1 - \varepsilon / \varepsilon_0)^A \right] \] \hspace{1cm} (4.2a)

For the descending part:  \[ \sigma = \sigma_c \exp[-24.6\sigma_c (\varepsilon - \varepsilon_0)^{1.15}] \] \hspace{1cm} (4.2b)

where $\sigma_c$ and $\varepsilon_0$ = the peak (maximum) stress and the corresponding strain.

$$A = E_c \left( \varepsilon_0 / \sigma_c \right)$$

$$\varepsilon_0 = 0.001648 + 0.165 \times 10^{-5} \times \sigma_c$$

$E_c$ = the elastic modulus of concrete

$\rho$ = density of concrete in kg/m$^3$

The tensile behaviour of the concrete used in this research for rectangular CFST section is adopted from the formulation proposed by Vecchio and Collins (1986) and used by Sanz-Picon (1992) in the analysis of reinforced concrete cross-sections. The concrete is assumed to follow a linear, elastic curve given by the equation:

$$\sigma_{ct} = E_{ct} \varepsilon_t$$ \hspace{1cm} (4.3)
in which $E_t$ is the modulus of elasticity of concrete under tensile stress, $E_t = \frac{\sigma_c}{0.001}$, until tensile concrete stress reaches the flexural tensile strength of the concrete. The flexural tensile strength is given by AS3600 (1994):

$$\sigma_{cr} = 0.6\sqrt{\sigma_c} \quad (4.4)$$

Once the concrete reaches this stress, its strength falls off according to the following equation:

$$\sigma_{cr} = \frac{\sigma_{cr}}{1 + \sqrt{200}\varepsilon_i} \quad (4.5)$$

Fig. 4.2 shown complete stress strain expression for concrete in rectangular CFSTs for three different unconfined concrete cylinder strengths equal to 40 MPa, 80 MPa and 100 MPa.

![Fig. 4.2 Concrete stress strain curves for rectangular CFSTs](image-url)
4.3.2 Circular CFST Section for Concrete

The additional strength of concrete under confinement was first recognised in reinforced concrete members. Concrete confined by steel hoops develops a greater strength than unconfined concrete [Kent and Park, 1971]. Correspondingly, the strength enhancement due to confinement of the concrete in circular CFST is considered in the triaxial state of stress. The confined concrete strength, $\sigma_{cc}$, is defined in the following expression:

$$\sigma_{cc} = \sigma_c + k\sigma_{cl}$$  \hspace{1cm} (4.6)

where $\sigma_{cl}$ is the lateral pressure exerted by the steel tube on the concrete core and $k$ is the triaxial factor that has a value of 4 to 6 in the low strength domain and a value of 3 to 4 for the high strength domain [Cederwall, 1988]. The lateral pressure, $\sigma_{cl}$, can be derived from mechanics by equilibrating forces on a semicircular portion of the steel tube. The circumferential tensile force in the tube is $2t\sigma_{sh}$, where $\sigma_{sh}$ is the hoop or circumferential stress in the steel tube. The equilibrating force exerted by the lateral pressure of the concrete, assuming a thin-walled tube, i.e. $D \gg t$; is $D \sigma_{cl}$. The resulting expression for the lateral pressure is:

$$\sigma_{cl} = \frac{2t}{D}\sigma_{sh}$$  \hspace{1cm} (4.7)

The derivation of the value for hoop stress $\sigma_{sh}$ is discussed in next section.
According to Chen’s research [Chen, 1972], it was found that the confined effect on the concrete will be reduced by the increase of end moment and the ratio of length to diameter \((L/D)\). Two models are used in this research: (1) with strength enhancement, for \(L \leq 0.5 \, L_c\), (2) without strength enhancement, for \(L > 0.5 \, L_c\) [Eurocode 4, 1994].

The tensile region for the concrete in a circular CFST is identical to the formulation developed for the concrete in a rectangular CFST. The steel does not markedly influence behaviour of the concrete in tension. Therefore, the shape of the cross-section will not affect tensile behaviour.

**4.3.3 Normal Strength Concrete in Circular CFSTs**

**Concrete with ductility enhancement only**

The portion of the circular concrete stress-strain expression beyond the strain at the maximum concrete strength is based on the curve proposed by Tomii and Sakino (1979a). Beginning at \(\varepsilon_0\) (the strain at \(\sigma_c\)), the stress remains constant until a strain of 0.005 is reached. This plateau region represents the added ductility provided by the confining action of the steel tube.

The descending branch of the concrete curve extends from a strain of 0.005 to 0.015 at a slope which is a function of the \(D/t\) ratio. The larger the \(D/t\) ratio, the steeper the slope. This reflects the loss of concrete ductility with a decrease in the amount of
confinement provided by the steel. This linear segment descends from a stress of $\sigma_c$ to $\sigma_{cp}$, where $\sigma_{cp}$ is given by the following formula:

$$\sigma_{cp} = (1.6 - 0.025 \frac{D}{t}) \sigma_c$$  \hspace{1cm} (4.8)

Equation (4.8) was developed by Tomii and Sakino (1979a) using experimental data from rectangular CFST with $D/t$ ratios between 24 and 44. To encompass a comprehensive range of $D/t$ ratios, the Tomii and Sakino model for strains beyond 0.005 was extended as follows. For a $D/t$ ratio of 24, Eqn. (4.8) equals $\sigma_c$. It is assumed in this work that, regardless of the $D/t$ ratio, the concrete does not exceed its cylinder strength ($\sigma_c$). Therefore, sections with a $D/t$ ratio less than 24 maintain a constant concrete stress of $\sigma_c$ as the strain increases from 0.005 to 0.015. Sections with $D/t$ ratios greater than 44 are governed by Eqn. (4.8) up to a $D/t$ ratio of 64. At a $D/t$ ratio of 64, $\sigma_{cp}$ equals zero. All sections with $D/t$ ratios larger than this follow the same curve as the section with a $D/t$ ratio of 64, dropping from $\sigma_c$ at a strain of 0.005 to zero stress at a strain of 0.015. For strain values beyond 0.015, the stress in all sections is assumed to remain at a constant value ($\sigma_{cp}$) for any subsequent value of strain [Tomii and Sakino, 1979a]. The complete stress strain expression for normal strength concrete in a circular CFSTs without strength enhancement is illustrated in Figure 4.3.
Concrete with both strength and ductility enhancement

For concrete with both strength and ductility enhancement, the approach is similar to that used in the circular CFST model in the stress-Strain formulation. The only difference is that the confined stress $\sigma_{ce}$ substitutes $\sigma_c$ for the formulation. A series of curves for concrete strength of 40 MPa under different lateral pressure is illustrated in Fig. 4.4.
Fig 4.4 Concrete stress strain curves for circular CFSTs ($L \leq 0.5 \ L_c$)

4.2.4 High Strength Concrete in Circular CFST Section

The stress-strain behaviour of high strength concrete used in this chapter for circular CFST section is adopted from the proposed equation by O’Shea and Bridge (1997) with modification. The equation is selected because the analytical model has developed from a number of triaxial test on high and very high strength concrete (60-130 MPa) and relies on the confinement to concrete by steel.

\[
\sigma = \sigma_{cc} \frac{AX + BX^2}{1 + CX + DX^2}
\]

(4.9)

where $A$, $B$, $C$, and $D$ are coefficients of stress and
\[ X = \frac{\varepsilon}{\varepsilon_{cc}} \]

For the ascending region

\[ A = \frac{E_c \varepsilon_{cc}}{\sigma_{cc}} \]

\[ B = \frac{(A-1)^2}{1 - \left(\frac{0.45\varepsilon_c}{\sigma_{cc}}\right)} \]

\[ C = A - 2 \]

\[ D = B + 1 \]

where \( E_c \) is the elastic modulus of concrete.

The confined concrete strength \( \sigma_{cc} \) is calculated from Eqn (4.6) and described in the next section. For concrete with silica fume, O'Shea defined the strain at the confined peak stress, \( \varepsilon_{cc} \) as

\[ \varepsilon_{cc} = \varepsilon_c \left[ 1 + (17 - 0.06\sigma_c) \left( \frac{\sigma_{cl}}{\sigma_c} \right) \right] \]

and

\[ \varepsilon_c = \frac{353\sigma_c}{E_c \sqrt{\sigma_c}} \]

where \( \sigma_{cl} \) is lateral pressure.

For the descending region, the coefficients are given as:
\[ A = \left( \varepsilon_{2i} - \varepsilon_i \right) \left[ \frac{\varepsilon_{2i} E_i}{\varepsilon_{cc}} - \frac{4 \varepsilon_i E_{2i}}{\sigma_{cc} - \sigma_{2i}} \right] \]

\[ B = \left( \varepsilon_i - \varepsilon_{2i} \right) \left[ \frac{E_i}{\sigma_{cc} - \sigma_i} - \frac{4 E_{2i}}{\sigma_{cc} - \sigma_{2i}} \right] \]

\[ C = A - 2 \]

\[ D = B + 1 \]

where

\[ E_i = \frac{\sigma_i}{\varepsilon_i} \]

\[ E_{2i} = \frac{\sigma_{2i}}{\varepsilon_{2i}} \]

\[ \sigma_{i_c} = \frac{\sigma_{i_c} - 1}{5.06 \left( \frac{\sigma_{cl}}{\sigma_c} \right)^{2.06} + 1} + 2 \]

\[ \varepsilon_{i_c} = \frac{\varepsilon_{i_c} - 2}{1.12 \left( \frac{\sigma_{cl}}{\sigma_c} \right)^{0.26} + 1} + 2 \]

where \( \sigma_{i_c} \) and \( \varepsilon_{i_c} \) are the stress and strain at the uniaxial point of inflection. These can be calculated using

\[ \frac{\varepsilon_{i_c}}{\varepsilon_c} = 2.5 - 0.3 \ln(\sigma_c) \]

\[ \frac{\sigma_{i_c}}{\sigma_c} = 1.41 - 0.17 \ln(\sigma_c) \]

While the stress at a strain of \( \varepsilon_{2i} = 2 \varepsilon_{i_c} \) is calculated using
\[
\frac{\sigma_{2l}}{\sigma_{ce}} = \frac{\sigma_{2le}}{\sigma_c} - 1 + 2 \frac{6.35(\sigma_{cl}^{cl})^{0.62}}{\sigma_c} + 1
\]

with the uniaxial value calculated from

\[
\frac{\sigma_{2le}}{\sigma_c} = 1.45 - 0.25\ln(\sigma_c)
\]

The strength of the concrete is dependent on the amount of confinement afforded by the steel. Thus, the level of confinement should be an input to the equation. The value of confining pressure for normal and high strength concrete is discussed in the next section.

The tensile region of the concrete in a circular CFST section is identical to the formulation developed for the concrete in a rectangular CFST section described previously. The steel does not markedly influence on the behaviour of the concrete in tension. Therefore, the shape of the cross-section will not affect tensile behaviour. The complete stress-strain expression for concrete in a circular CFST is illustrated in Fig. 4.5 for 100 MPa of concrete strength under different lateral pressure.
Fig. 4.5 High strength concrete stress strain curves for circular CFSTs

4.4 STEEL CONSTITUTIVE RELATIONSHIPS

Based on the discussion at the beginning of this section, the steel in a concrete filled steel tube may be in a state of either uniaxial or biaxial stress. The effect of confinement in CFST subjected to compressive loads will result in different behaviour for circular and rectangular tubes. The hoop stress induced in circular tubes by the expanding concrete results in a loss of longitudinal capacity which must be modelled in the compressive stress-strain curve of the steel. The relatively smaller effects of confinement in rectangular CFST member are modelled accordingly.

Any interaction that may occur between the steel and the concrete in the tension region is neglected due to the concrete in tension offering little resistance. Therefore,
steel in tension acts independently of the concrete, and is modelled as if it were a hollow tube.

4.4.1 Rectangular CFST Sections for Steel

In the stress-strain formulation contained in this work, the effect of confinement of the concrete in a rectangular CFST is assumed to be on the ductility and the increase in the concrete strength is not considered. For steel in a rectangular CFST any biaxial stresses are assumed to be negligible. Therefore, the stress-strain expression for steel in a rectangular CFST is represented by a curve similar to that used for hollow tubes or I-sections.

The uniaxial stress-strain curve for steel is illustrated in Fig. 4.7. The steel exhibits linear elastic behaviour up to the yield point of the material ($\sigma_y$) and then follows a perfectly plastic plateau.

4.4.2 Circular CFST Sections for Steel

The concrete in a circular CFST is fully confined, therefore both strength and ductility are enhanced. The steel stress-strain relationship is modelled accordingly by approximately including in the formulation the biaxial stresses induced by the lateral pressure of the confined concrete. When the steel reaches yield stress at which the large plastic strains occur, part of the load resisted by the steel is transferred to the concrete core. According to van Mises yield criteria, the longitudinal stress in the
steel decreases with increasing lateral or hoop tensile stress. Therefore, the strength of the concrete core is continuously enhanced. The enhancement is also caused by the dramatic lateral expansion of concrete due to the development of cracks at the late stage of loading. Finally, the failure of the column occurs when the resultant compressive force carried by the steel tube and the concrete core reaches the ultimate value.

The total axial load $N_0$ carried by a concrete-filled tubular column with a vertical stress $\sigma_{sv}$ in the steel tube and a vertical stress $\sigma_{cv}$ in the concrete core can be approximately as:

$$N_0 = A_c \sigma_{cv} + A_s \sigma_{sv}$$  \hspace{1cm} (4.10)

where $A_c$ is the area of concrete, $A_c = \frac{\pi D_c^2}{4}$ and $A_s$ is the area of steel, $A_s = \pi t D_c$.

Substituting into Eqn. (4.7) then

$$\sigma_{cl} = \frac{2t}{D_c} \sigma_{sh} = \frac{A_s}{2A_c} \sigma_{sh}$$  \hspace{1cm} (4.11)

![Fig 4.6 Definition of concrete and steel stress](image-url)
Fig. 4.6 defines the relationship between steel hoop stress and concrete lateral pressure. Substituting Eqn. (4.11) and (4.6) into Eqn. (4.10).

\[ N_0 = A_c \sigma_c + A_s (\sigma_{sv} + 0.5 k \sigma_{sh}) \]  \hspace{1cm} (4.12)

the maximum value of \( N_0 \) can be achieved when

\[ \frac{\partial N_0}{\partial \sigma_{sv}} = 1 + \frac{k}{2} \frac{\partial \sigma_{sh}}{\partial \sigma_{sv}} = 0 \]

or

\[ \frac{\partial \sigma_{sh}}{\partial \sigma_{sv}} = -\frac{2}{k} \]  \hspace{1cm} (4.13)

The state of stress in the tube at the yield point may be analysed by considering the von Mises yield criterion [Cai, 1987; Ichinohe, et. al., 1991]. The von Mises yield criterion is expressed by the following equation:

\[ \sigma_{sv}^2 + \sigma_{sh}^2 - \sigma_{sv} \sigma_{sh} = \sigma_y^2 \]  \hspace{1cm} (4.14)

Differentiating Eqn. (4.14) with respect to \( \sigma_{sv} \)

\[ \sigma_{sv} \left( 2 - \frac{\partial \sigma_{sh}}{\partial \sigma_{sv}} \right) - \sigma_{sh} \left( 1 - 2 \frac{\partial \sigma_{sh}}{\partial \sigma_{sv}} \right) = 0 \]  \hspace{1cm} (4.15)

Substituting Eqn. (4.13) into Eqn. (4.15) and rearranging it,

\[ 2(k + 1) \sigma_{sv} - (4 + k) \sigma_{sh} = 0 \]  \hspace{1cm} (4.16)
Substituting Eqn. (4.16) into Eqn. (4.14), then Eqn. (4.14) can be rearranged to solve for the ratio of the longitudinal steel stress to the yield stress in terms of triaxial factor, $k$:

$$\sigma_{\gamma} = \gamma_{\nu} \sigma_{\gamma}$$

where

$$\gamma_{\nu} = \frac{k + 4}{\sqrt{3(k+1)^2 + 9}}$$

(4.17)

If a triaxial factor, $k$, of 4 is assumed, Eqn. (4.17) produces a reduction value of $\gamma_{\nu}$ equal to 0.873. Therefore, the longitudinal stress that causes yielding in the steel tube is reduced to $0.873 \sigma_{\gamma}$ due to the lateral confinement effect value of $k$. The corresponding yield strain is also reduced by the same factor. Once the compressive stress reaches the reduced yield strength, the steel enters the perfectly plastic region, remaining at the reduced stress (Fig. 4.7).

The hoop stress ratio ($\gamma_h$) may be computed by using the reduction factor for the compressive stress. This value is defined as the circumferential stress in the steel tube to the yield stress of the steel tube. To solve for the ratio of the steel hoop stress to the yield stress in terms of the longitudinal steel stress, $\gamma_h$, substituting Eqn. (4.17) into Eqn. (4.14), then

$$\sigma_{sh} = \gamma_h \sigma_{\gamma}$$

where

$$\gamma_h = 0.5(\sqrt{4 - 3\gamma_{\nu}^2} - \gamma_{\nu})$$

(4.18)
When $\gamma_v$ equal to 0.873, Eqn. (4.18) produces a reduction value of $\gamma_h$ equal to 0.22. In past research, the hoop stress, which arises from the outward pressure of the concrete, has been assigned values ranging from 0.2 $\sigma_y$ to 0.5 $\sigma_y$ [Knowles and Park, 1970; Ichinohe, et al., 1991; Zhang, et al., 1991]. Substituting Eqn. (4.18) into Eqn. (4.7), the lateral pressure exerted by the steel tube on the concrete core at the ultimate load may be shown as follows:

$$\sigma_d = \frac{2t}{D_c} \gamma_h \sigma_y$$  \hspace{1cm} (4.19)

The stress-strain constitutive formulation for steel in tension in circular CFST is identical to that described in Section 4.2.2 for rectangular CFST. Fig. 4.7 illustrates the steel stress-strain behaviour for both rectangular and circular CFST section.

![Steel stress-strain curves with or without confinement effect](image-url)

Fig. 4.7 Steel stress-strain curves with or without confinement effect
4.5 ULTIMATE AXIAL COMPRESSIVE STRENGTH

The nominal axial load carried by circular CFST column can be adopted from Eqn. (4.12) expressed as follow:

\[ N_0 = A_s \gamma_v f_{yd} + A_s f_{cd} \left(1 + 2k \gamma_h \frac{t}{D} f_{yd} \right) \]  \hspace{1cm} (4.20)

where \( \gamma_v \) is the vertical steel reduction ratio, and \( \gamma_h \) is the hoop steel reduction ratio, derived in the previous section:

\[ \gamma_v = \frac{k + 4}{\sqrt{3(k+1)^2 + 9}} \]  \hspace{1cm} (4.21)

\[ \gamma_h = 0.5(\sqrt{4 - 3\gamma_v^2} - \gamma_v) \]  \hspace{1cm} (4.22)

where \( k \) is triaxial factor that has a value of 4.0 for normal strength. For high strength concrete, Cederwall [Cederwall, 1988] recommended a lower value of \( k \) should be used. In this research a \( k \) value of 3 is adopted as suggested by Setunge [Setunge, et al., 1992] for high strength concrete.

Eurocode 4 also provides a formula to predict the axial load for CFST columns including the effect of confinement:

\[ N_0 = A_s \eta_3 f_{yd} + A_c \left( f_{cd} + \eta_1 \frac{t}{D} f_{yd} \right) \]  \hspace{1cm} (4.23)

where
for $0 \leq e \leq 0.1D$

$$\eta_1 = \eta_{10} \left(1 - \frac{10e}{D}\right) \quad (4.24a)$$

$$\eta_2 = \eta_{20} + (1 - \eta_{20}) \frac{10e}{D} \quad (4.24b)$$

and

$$\eta_{10} = 4.9 - 18.5\bar{x} + 17\bar{x}^2 \geq 0 \quad (4.25a)$$

$$\eta_{20} = 0.25(3 + 2\bar{x}) \leq 1.0 \quad (4.25b)$$

For slender columns, $\bar{x} \geq 0.5$, with large eccentricity, $e \geq 0.1D$, $\eta_1 = 0$ and $\eta_2 = 1$.

For a short column, the relative slenderness, $\bar{x}$, is close to zero, the above coefficients become $\eta_1 = 4.9$ and $\eta_2 = 0.75$. Comparing Eqs. (4.20) and (4.23), an equivalent group of coefficients is proposed,

$$\eta_{10} = 2k\gamma_h = k(\sqrt{4 - 3\gamma_v^2} - \gamma_v) \quad (4.26a)$$

$$\eta_{20} = \gamma_v = \frac{k + 4}{\sqrt{3(k+1)^2 + 9}} \quad (4.26b)$$

By trial and error, for a $k$ of 6.9, the proposed coefficients become $\eta_1 = 4.83$ and $\eta_2 = 0.78$. These are very close to those given in Eurocode 4 when $\bar{x} = 0$. This also implies that an implicit $k$ value of 6.9 is used when the Eurocode 4 procedure is followed.
An empirical expression of \( k \) can be proposed by best-fitting the test data from three different sources [Gardner and Jacobson, 1967; Kloppel and Goder, 1957; Knowles and Park, 1970] in Table 6.1-3. It gives a triaxial factor,

For \( \overline{\lambda} \leq 0.2 \)

\[
k = 6.9 + 6.3 \overline{\lambda} \tag{4.27a}
\]

For \( 0.2 < \overline{\lambda} \leq 0.5 \)

\[
k = 34\overline{\lambda}^2 - 26\overline{\lambda} + 12 \tag{4.27b}
\]

Figs. 4.8 and 4.9 shown the proposed coefficients \( \eta_1 \) and \( \eta_2 \) in term of \( \overline{\lambda} \) when the proposed \( k \) value is used. They are compared with Eurocode 4 recommendations.

Fig. 4.8 Comparison with Eurocode 4 and proposed method for \( \eta_1 \)
Fig. 4.9 Comparison with Eurocode 4 and proposed method for $\eta_2$

4.6 COMPARISONS OF RESULTS

4.6.1 Comparison with O'Shea's Confinement Test

The results from the proposed equations are compared with O'Shea's test results [O'Shea and Bridge, 1994; 1997] in Table 4.1. The predicted strengths for $k$ of 3 for high strength concrete and $k$ of 4 for normal strength concrete are listed in the table. The normalised predicted concrete strength with enhancement using the proposed equation, $f_{cd}/f_c$ has been shown in Fig 4.10 with the measured value from the confinement tests. For specimens filled with nominal 50 and 80 MPa concrete proposed method accurately predicts the confined concrete strength for thin tubes, but differs significantly from the test results obtained for the thicker tube. However generally good agreement is obtained for the method. For specimens filled with nominal 110 MPa concrete and high yield stress, the proposed method conservatively estimates the confined concrete strength.
Table 4.1 Comparison with O'Shea Confinement Test

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Diam. D (mm)</th>
<th>Thick t (mm)</th>
<th>Ratio D/t</th>
<th>Steel $f_y$ (MPa)</th>
<th>Conc. $f_c$ (MPa)</th>
<th>Conf. Test Strength</th>
<th>Prop. $f_{cl}^*$</th>
<th>Prop. $f_{ce}$</th>
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<td>S20CL</td>
<td>190</td>
<td>2.0</td>
<td>95</td>
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<td>113.5</td>
<td>135.6</td>
<td>0.758</td>
<td>115.7</td>
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<td>190</td>
<td>1.15</td>
<td>165.2</td>
<td>184.8</td>
<td>113.5</td>
<td>116.32</td>
<td>0.296</td>
<td>114.39</td>
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<tr>
<td>S30CS50B</td>
<td>165</td>
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<td>58.6</td>
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<td>48.3</td>
<td>59.89</td>
<td>2.71</td>
<td>59.12</td>
</tr>
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<td>41.0</td>
<td>51.66</td>
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<td>306.1</td>
<td>48.3</td>
<td>52.65</td>
<td>1.07</td>
<td>52.57</td>
</tr>
<tr>
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<td>168.1</td>
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<td>45.51</td>
<td>0.48</td>
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<td>45.51</td>
<td>0.42</td>
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<td>1.64</td>
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<td>98.2</td>
<td>256.4</td>
<td>74.7</td>
<td>84.41</td>
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<td>74.7</td>
<td>85.16</td>
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<td>75.46</td>
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*$f_{cl}$: the lateral pressure exerted by the steel tube on the concrete core

The errors of predictions for column strength versus dimensionless ratio of diameter to thickness and ratio of concrete strength to steel strength ($D/l)(f_{cl}/f_y)$ and column slenderness $L/D$ are presented graphically in Fig. 4.11 and Fig. 4.12. Also shown are the respective linear trend lines, together with the coefficient of determination $R^2$, where $R$ is the correlation coefficient. In both instances the coefficient of determination is quite low which in turn indicates that the majority of the variation is stochastic. Other non-linear trend lines yielded a negligible improvement in correlation. Consequently it was concluded that there was no significant correlation.

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between the dimensionless coefficient and the accuracy of the prediction of confinement strength.

Fig 4.10 Comparison of confinement effect versus ratio of diameter to thickness

Fig 4.11 Errors versus dimensionless coefficient
Fig 4.12 Errors versus lateral pressure

4.6.2 Comparisons with Tests and Code Recommendations

In order to verify the accuracy of the expression for circular CFST cross section, the axial strength from the proposed method and the results from AISC code, and Eurocode 4 are all compared with the test results of short columns. The relative slenderness, $\bar{\lambda}$, of all the test specimen, is under 0.5 [Eurocode 4, 1994]. These comparisons are illustrated in Table 4.6. The tables are referenced by the notation used by the respective authors in their literature, as listed in column 1 of Table 4.2, 4.4 and 4.5. It can be seen from Table 4.6 that the proposed method gives a much better agreement than AISC and Eurocode 4.
Table 4.2 The test results of Gardner and Jacobson [Gardner and Jacobson, 1968]

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Diameter ($D 	imes t$ mm)</th>
<th>Ratio ($L/D$)</th>
<th>Conc. ($f_{cd}$ MPa)</th>
<th>Steel $f_{yd}$ (MPa)</th>
<th>$E_s$ (MPa)</th>
<th>Test $N_{test}$ (kN)</th>
</tr>
</thead>
<tbody>
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<td>Gard68-1</td>
<td>101.68x3.07</td>
<td>14.99</td>
<td>33.95</td>
<td>601.95</td>
<td>205975</td>
<td>818.4</td>
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<tr>
<td>Gard68-2</td>
<td>101.73x3.10</td>
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<td>31</td>
<td>601.95</td>
<td>205975</td>
<td>800.64</td>
</tr>
<tr>
<td>Gard68-3</td>
<td>120.65x4.09</td>
<td>8.7</td>
<td>34.23</td>
<td>449.26</td>
<td>190543</td>
<td>1156.48</td>
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<td>Gard68-4</td>
<td>120.83x4.09</td>
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<td>29.43</td>
<td>449.26</td>
<td>190543</td>
<td>1092.43</td>
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<tr>
<td>Gard68-5</td>
<td>120.83x4.09</td>
<td>8.69</td>
<td>25.79</td>
<td>449.26</td>
<td>190543</td>
<td>949.65</td>
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<tr>
<td>Gard68-6</td>
<td>152.60x3.15</td>
<td>10.98</td>
<td>20.78</td>
<td>412.98</td>
<td>202340</td>
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<td>Gard68-7</td>
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<td>25.79</td>
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<td>196716</td>
<td>330.264</td>
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</table>

Table 4.3 The test result [Housing and Liome Financed Agency, 1967]

<table>
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<tr>
<th>Specimen No.</th>
<th>Diameter ($D 	imes t$ mm)</th>
<th>Ratio ($L/D$)</th>
<th>Conc. ($f_{cd}$ MPa)</th>
<th>Steel $f_{yd}$ (MPa)</th>
<th>$E_s$ (MPa)</th>
<th>Test $N_{test}$ (kN)</th>
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<td>198911</td>
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<td>Hous67-3</td>
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<td>198911</td>
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Table 4.4 The test result of Kloppel and Godner [Kloppel and Godner, 1957]

<table>
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<tr>
<th>Specimen No.</th>
<th>Diamen. ( D x t ) (mm)</th>
<th>Ratio ( L/D )</th>
<th>Conc. ( f_{cd} ) (MPa)</th>
<th>Steel ( f_{yd} ) (MPa)</th>
<th>( E_s ) (MPa)</th>
<th>Test ( N_{test} ) (kN)</th>
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Table 4.5 The test result of Knowles and Park [Knowles and Park, 1971]
Table 4.6 Comparison of test results with AISC code, Eurocode4 and Proposed method

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The accuracy of the expression for axial strength in circular CFST and the results from AISC code and proposed method are also verified by comparing them to the available results of slender columns for $\bar{\lambda} > 0.5$. The method is described in Chapter 6. These comparisons are illustrated in Table 4.7. The table are also referenced by the notation used by the respective authors in their literature, as listed in column 1 of Table 4.2 - 4.5. It can be seen from Table 4.7 that the proposed method gives a better agreement than AISC. But both of the methods were slightly conservative.
Table 4.7 Comparison of test results with AISC code and Proposed method

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4.6.3 Experimental Verification

From the literature review conducted, over 89 individual tests on circular CFST’s were identified in which the slenderness ratios \(L/D\) ranged from 3 to 34. Fifty-four or 61% of the columns had a slenderness ratios in excess of 12 which is defined as slender columns [AIJ, 1994]. The design methods have been compared in Table 4.5 and 4.6. For proposed method, the mean values of the ratio of measured/predicted column strength and corresponding standard deviations were 1.158 and 0.148 respectively; For AISC-LRFD, 1.256 and 0.177 respectively. For Eurocode 4, 1.190 and 0.147 respectively.

Clearly the proposed method provides the best estimate, with the lowest median and standard deviation. The errors of predictions for column strength versus ratio of diameter to thickness \((D/t)\), column slenderness \((L/D)\) and ratio of steel strength to concrete strength are presented graphically in Fig. 4.13 to Fig. 4.15. The results clearly show the low scatter when compared to the other methods in Fig 4.16 to Fig 4.21. This indicates that the other methods can be quite conservative and the accuracy of the prediction of column strength is not significantly influenced by the column slenderness \(L/D\), ratio of diameter to thickness \(D/t\) and ratio of steel strength to concrete strength.
Fig 4.13 Errors versus ratio of diameter to thickness (proposed method)

Fig 4.14 Errors versus column slenderness (proposed method)
Fig 4.15 Errors versus ratio of steel strength to concrete strength (proposed method)

Fig 4.16 Errors versus ratio of diameter to thickness (AISC Code)
Fig 4.17 Errors versus column slenderness (AISC Code)

Fig 4.18 Errors versus ratio of steel strength to concrete strength (AISC Code)
Fig 4.19 Errors versus ratio of diameter to thickness (Eurocode 4)

Fig 4.20 Errors versus column slenderness (Eurocode 4)
4.7 CONCLUSIONS

The effective uniaxial stress-strain behaviour for steel and concrete was studied in this chapter. The confinement effect on the concrete core developed by the steel tube that leads to the enhancement of the strength of the concrete core was included in the formulation for high and normal strength concrete steel strength reduction due to triaxial state is also considered.

Different stress-strain constitutive relationships were proposed for steel and concrete with or without confinement effect, in particular, in relation to the design of circular concrete filled steel tubes (CFSTs) with normal/high strength concrete. An effective uniaxial stress-strain relationship was derived to represent the realistic triaxial or biaxial stress state in steel and concrete. The proposed constitutive relationships were used to calculate the strength of a circular CFST.
CHAPTER 5:

ACCURACY OF NUMERICAL EXPRESSIONS

FOR THE SECTION ANALYSIS

OF CFST BEAM-COLUMNS

5.1 INTRODUCTION

The analysis and design of CFSTs subjected to combine axial compression and biaxial bending require a comprehensive knowledge of the behaviour of the CFST cross sections. Different procedures were proposed by different codes of practice to generate section strength interaction diagrams [i.e., ACI 318, 1995; AS3600, 1994; Eurocode 4, 1994; AIJ-CFT, 1994]. There has been attempt to represent the strength interaction diagrams by a set of simple numerical equations [Hajjar and Gourley, 1996].

In this chapter, a set of numerical expressions was developed to represent the strength interaction curves for rectangular or circular CFSTs having a wide range of material strengths and cross section geometry. The whole cross-section strength
surface can be calibrated to failure surfaces generated from an accurate nonlinear fibre element analysis of CFST beam-columns [Bode, 1976; Shakir-Khalil and Zeghiche, 1989; Kawaguchi et al., 1991; Bridge, et al., 1997; Zhang and Chao, 1999] as presented in Chapter 3. This numerical expression provided a compact and efficient interaction diagram representation for the resulting strength of CFST sections. The accuracy of the proposed expressions was verified by the available test results and other published results. The expressions can be applied to CFST sections with concrete compression design strength $f_{cd}$ up to 120 MPa, steel yield design strength $f_{yd}$ up to 600 MPa, and tube depth-to-thickness ratios ($D/t$) up to approximately 100.

5.2. FIBRE ELEMENT ANALYSIS

The fibre element analysis used to analyse the behaviour of CFST cross sections had been used by many researchers [Hajjar and Gourley, 1996; Bridge et al., 1997]. The procedure was described in detail in Chapter 3. After the cross section had been meshed, the axial strain of each element was calculated from the cross section. For each curvature increment, the analysis was carried out to obtain the cross section moment. This iterative procedure began with the initialisation of the curvature which defined the strain distribution over the cross section. The curvature $\phi$ was incremented successively until the value of bending moment changed to zero which reflects the pure axial load case. The resultant axial force on the cross section and moments in the $x$ and $y$ direction $m_x$, $m_y$ were calculated.
5.3 EQUATION OF CFST CROSS SECTION STRENGTH SURFACE

In this chapter, the calibrated fibre element analysis described in the preceding section was used to generate force point data for a wide range of CFST sections. An empirical numerical expression representing a general cross-section strength surface for rectangular and circular CFST was then developed using this fibre element analysis data.

The first step in the development of an expression for the CFST cross-section strength surface entails selecting a wide range of cross-sections representative of CFST sections. There are two properties which most directly affect the behaviour of the CFST cross section. These may be identified by two dimensionless ratios: the ratio of the section depth (or diameter for circular section) to wall thickness ($D/t$) and the ratio of the steel yield design strength to the concrete compression design strength ($f_{yd}/f_{cd}$).

Five series of sections were selected, with $D/t$ ratios of 20, 40, 50, 80 and 100. Within each series, different $f_{yd}/f_{cd}$ ratios were chosen. The design concrete strength $f_{cd}$ ranges from 20 MPa to 120 MPa with a uniform increment of 10 MPa and the design steel yield strength $f_{yd}$ ranges from 200 MPa to 600 MPa with an increment of 50 MPa. This results in $f_{yd}/f_{cd}$ ratio ranging from 2 to 25. Hundreds of sections were analysed with a wide range of section and material properties.
5.3.1 Nominal Strength of CFST Sections

The first requirement necessary to calculate the equation to represent strength interaction diagrams of a CFST member is a method for calculating the nominal axial and bending moment capacity of the section. In the previous analysis, these values were computed by the fibre analysis procedure. However, for designing purpose, these values must be computed explicitly by a simple, yet accurate set of equations.

**Squash Load**

**Rectangular CFST Section**

The nominal axial force capacity may be calculated directly as it is done in the fibre analysis program. The squash load values, \( N_0 \), has been given in various codes such as ACI-318, AS3600 and Eurocode 4. The predicted \( N_0 \) values are given by the following expression:

\[
N_0 = A_f f_{yd} + \alpha I A_c f_{cd}
\]  

(5.1)

where \( \alpha I = 0.85 \) for rectangular block analysis on normal and high strength concrete [ACI 318-1995; AS 3600-1994].

**Circular CFST Section**

The nominal axial load carried by circular CFST column can be adopted from Eqn. (3.20) with the enhanced concrete strength as follows:
\[ N_0 = A_i \gamma_v f_{yd} + A_v \alpha_1 (f_{cd} + 2k \gamma_h \frac{t}{D} f_{yd}) \]  

(5.2)

where

\[ \gamma_v = \frac{k + 4}{\sqrt{3(k + 1)^2 + 9}} \]  

(5.3)

\[ \gamma_h = 0.5(\sqrt{4 - 3\gamma_v^2} - \gamma_v) \]  

(5.4)

where \( k \) is triaxial factor that has a value of 4.0 for normal strength and a \( k \) value of 3 for high strength concrete.

**Ultimate Moment of Resistance**

**Rectangular CFST Section**

In this work, the calculation of the ultimate moment capacity of the section \( M_u \) was similar to that used by the stress block analysis, assuming concrete rectangular stress blocks equal to \( \alpha_I \) times the compressive area of concrete and triangle stress blocks equal to the tensile area of concrete. The tensile moment resistance was included because for larger CFST sections with a high \( D/t \) ratio and a high concrete strength, the neutral axis is very close to the top fibre of the section and a large portion of the concrete is subjected to tension. Although the concrete strength in tension is only approximately one-tenth of the compression strength, the large percentage of concrete in tension will nevertheless have some effects on the moment resistance of the CFST. Neglecting this effect produced significantly less accurate results [Hajjar and Gourley, 1996].
For a rectangular CFST cross section with width $b$ and depth $D$ (Fig. 5.1), the neutral axis, $y_0$ (measured from the centroid of the cross section), is computed by the following equation:

$$y_0 = \frac{0.5(\bar{C}_c - \bar{T}_c)d_c}{\bar{C}_c - \bar{T}_c + \bar{T}_s} \quad (5.5)$$

where

$$\bar{C}_c = \alpha_i f_{cd} b_c \beta_i$$

$$\bar{T}_c = 0.5 f_{erd} b_c \bar{\varepsilon}_{cr}$$

$$\bar{T}_s = 4 t f_{yd}$$

$$b_c = b - 2 t$$

$$d_c = D - 2 t$$

$$f_{erd} = 0.6 \sqrt{f_{cd}}$$

$$\bar{\varepsilon}_{cr} = \frac{f_{ord}}{0.003E_c}$$

$$\beta_i = 1.09 - 0.008 f_{cd} \quad 0.85 \geq \beta_i \geq 0.65$$

Given this neutral axis location, the nominal moment for a general rectangular CFST cross section may be calculated by

$$M_0 = 0.5 \bar{C}_c \gamma_c (d_c - \beta_1 y_c) + \bar{T}_c \gamma_c (\frac{2}{3} \bar{\varepsilon}_{cr} y_c - y_0) + \frac{1}{3} \bar{T}_s y_s^2$$

$$+ (t b) f_{yd} (D - t) + (2 t y_p) f_{yd} (d_c - y_p) \quad (5.6)$$

where

$$\gamma_c = 0.5 d_c - y_0$$

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\[ y_s = y_c - \frac{f_{yd}}{0.003E_s} \]

\[ y_p = y_c - y_s \]

The physical meaning of the symbols not given in Fig. 5.1 is listed under notation.

![Composite Section Strain Diagram Concrete Portion Steel Portion](image)

Fig. 5.1 Strain and stress distribution for rectangular section

**Circular CFST Section**

The method for the ultimate strength analysis of rectangular CFST cross sections was described in the last section can be applied to circular sections. Fig. 5.2 shows the effective area of the stress block for concrete of depth \( y_{c0} \) and maximum width \( b_{c0} \) with \( r_c \) representing the radius of the section and \( \alpha_c \) the angle subtended at the centre of the circle by \( b_{c0} \). For a circular CFST cross section with diameter \( D \) (Fig. 5.2), the
neutral axis, $y_0$ (measured from the centroid of the cross section), may be computed from the following equation:

$$C_c - C_r + T_{sc} - T_s = 0 \quad (5.7)$$

where

$$C_c = 0.425 f_{cd} r_c^2 (\alpha_c - \sin \alpha_c)$$

$$C_r = 0.5 f_{crd} b_{cr} y_c$$

$$T_{sc} = f_{yd} r_s \alpha_{sc} t$$

$$T_s = f_{yd} r_s \alpha_{s} t$$

in which

$$b_{co} = 2 \sqrt{2 r_c y_{co} - y_{co}^2}$$

$$y_{co} = \beta_1 (r_c - y_0)$$

$$\alpha_c = 4 \tan^{-1} \left( \frac{2 y_{co}}{b_{co}} \right)$$

$$f_{crd} = 0.6 \sqrt{f_{cd}}$$

$$y_c = (r_c - y_0) \frac{f_{crd}}{0.003 E_c}$$

$$y_{cr} = y_c - y_0$$

$$b_{cr} = \sqrt{r_c^2 - y_{cr}^2} + \sqrt{r_c^2 - y_0^2}$$

$$b_{sc} = 2 \sqrt{2 r_s y_{sc} - y_{sc}^2}$$

$$y_{sc} = r_s - y_0 - y_s$$

$$\alpha_{sc} = 4 \tan^{-1} \left( \frac{2 y_{sc}}{b_{sc}} \right)$$
\[ b_{sc} = 2\sqrt{2r_s y_{st} - y_{st}^2} \]

\[ y_{st} = r_s + y_0 - y_s \]

\[ \alpha_{st} = 4 \tan^{-1}\left( \frac{2y_{st}}{b_{st}} \right) \]

\[ r_c = 0.5(D - 2t) \]

\[ r_s = r_c + 0.5t \]

\[ y_s = (r_c - y_0) \frac{f_{yd}}{0.003E_s} \]

The physical meaning of the symbols not given in Fig. 5.2 is listed under notation.

Given the neutral axis location, the nominal moment for a general circular CFST cross section may be calculated by:

\[
M_0 = C_c\left[\frac{4}{3} r_c \sin\left(\frac{\alpha_c}{2}\right) - \sin\alpha_c\right] + C_t\left(\frac{2}{3} y_{cr} - y_0\right) + \frac{4}{3} f_{yd} y_s^2
\]

\[+ T_{sc}\left[\frac{2r_s \sin(\alpha_{sc})}{\alpha_{sc}}\right] + T_{st}\left[\frac{2r_s \sin(\alpha_{st})}{\alpha_{st}}\right] \quad (5.8)\]
Fig. 5.2 Strain and stress distribution for circular section

5.3.2 Development of the Coefficients at the Special Force Points

The calculation of cross section interaction diagrams is needed in the design of steel-concrete composite columns. A general numerical expression describing the strength interaction diagram will use certain points on the interaction curve. These force points (A-E) are marked in the interaction diagram in Fig. 5.3. For each point it is
defined as following:

Point A marks the axial resistance where,

\[ N_a = N_0 \]
\[ M_a = 0.0 \]

Point B is under pure bending where,

\[ N_b = 0.0 \]
\[ M_b = M_0 \]

Point C marks the point where axial strength contribution is due to the concrete core only without considering the contribution of steel. It can be easily calculated as:

\[ N_c = A_c f_{cd} \]
\[ M_c = \alpha_{cm} M_0 \]

Point D provides a point where the bending moment resistance is equal to that of pure bending situation, where

\[ N_d = \alpha_{dn} N_0 \]
\[ M_d = M_0 \]

Point E represents the point where the axial strength is the half of that at Point D, i.e.

\[ N_e = 0.5 N_d \]
\[ M_e = \alpha_{em} M_0 \]

where \( N_0, M_0 \) = axial compression and ultimate moment strengths of the cross section, respectively. Their computation is described in previous sections, \( \alpha_{em}, \alpha_{dn} \) and \( \alpha_{em} \) are coefficients to be determined.
The coefficients $\alpha_{cm}$, $\alpha_{dn}$ and $\alpha_{en}$ were determined by a least-squares procedure using the large amount of axial force and moment data from the fibre analysis. The least squares analysis was based on 385 analysis data. The resulting expression are given in terms of the $D/t$ and $f_{yd}/f_{cd}$ ratio of the cross section. The resulting expression of the coefficients for both rectangular and circular sections are listed below.

![Interaction curve with polygonal approximation](image)

**Fig. 5.3 Interaction curve with polygonal approximation**

**Rectangular CFST sections**

**Normal Strength Concrete ($f_{cd} \leq 50$ MPa)**

$$
\alpha_{cm} = (0.0026f_{yd} - 1)\left(\frac{N_c}{N_0}\right)^2 - (0.0022f_{yd} - 0.4143)\left(\frac{N_c}{N_0}\right) + 1.0
$$
\[ \alpha_{dn} = [-0.0004 \left( \frac{f_{yd}}{f_{cd}} \right)^2 + 0.007 \left( \frac{f_{yd}}{f_{cd}} \right) + 0.21] \log \left( \frac{D}{t} \right) + [0.0027 \left( \frac{f_{yd}}{f_{cd}} \right)^2 - 0.076 \left( \frac{f_{yd}}{f_{cd}} \right) - 0.05] \]

\[ \alpha_{en} = 0.047 \left( \frac{D}{t} \right)^{-1.09} + 0.874 \left( \frac{f_{yd}}{f_{cd}} \right)^{0.029} \quad (5.9 \ a-c) \]

High Strength Concrete \((f_{cd} > 50 \text{ MPa})\)

\[ \alpha_{en} = -2.04 \left( \frac{N}{N_0} \right) \left( \frac{f_{yd}}{f_{cd}} \right)^{-0.67} + 1.87 \left( \frac{f_{yd}}{f_{cd}} \right)^{-0.35} \leq 1.0 \]

\[ \alpha_{dn} = [-0.005 \left( \frac{f_{yd}}{f_{cd}} \right)^2 + 0.058 \left( \frac{f_{yd}}{f_{cd}} \right) + 0.09] \log \left( \frac{D}{t} \right) + (0.028 f_{yd} - 15.2) \left( f_{cd} \right)^{0.0028 f_{yd}^{-1.32}} \leq 1.0 \]

\[ \alpha_{en} = [0.0006 \left( \frac{f_{yd}}{f_{cd}} \right)^2 - 0.0073 \left( \frac{f_{yd}}{f_{cd}} \right) + 0.03] \left( \frac{D}{t} \right) \\
[0.004 \left( \frac{f_{yd}}{f_{cd}} \right)^2 - 0.035 \left( \frac{f_{yd}}{f_{cd}} \right) + 0.95] \quad (5.10 \ a-c) \]

Figs. 5.4(a-c) and 5.5(a-c) shows the errors for the points obtained using Eqn. (5.9 a-c) and Eqn. (5.10 a-c) for each of the 385 CFST cross-sections. The errors are computed based upon the comparison with the values obtained from the fibre element analysis of the individual cross-sections, which is regarded the accurate solution.
Fig 5.4 (a) Errors in coefficient of force point C for normal strength concrete

Fig 5.4 (b) Errors in coefficient of force Point D for normal strength concrete
Fig 5.4 (c) Errors in coefficient of force point E for normal strength concrete

Fig 5.5 (a) Errors in coefficient of force point C for high strength concrete
Fig 5.5 (b) Errors in coefficient of force point D for high strength concrete

Fig 5.5 (c) Errors in coefficient of force point E for high strength concrete
Circular CFST Sections

Normal Strength Concrete \((f_{cd} \leq 50 \text{ MPa})\)

(1) Without Strength Enhancement:

\[
\alpha_{cu} = \left[ a_2 \left( \frac{D}{t} \right)^2 + a_1 \left( \frac{D}{t} \right) + a_0 \right] \leq 1.0
\]  
(5.11a)

\[
a_0 = 0.5 \times 10^{-3} \left( \frac{f_{ydl}}{f_{cd}} \right)^2 - 0.97 \times 10^{-2} \left( \frac{f_{ydl}}{f_{cd}} \right) - 1.0
\]

\[
a_1 = -0.37 \times 10^{-4} \left( \frac{f_{ydl}}{f_{cd}} \right)^2 - 0.83 \times 10^{-3} \left( \frac{f_{ydl}}{f_{cd}} \right) + 0.88 \times 10^{-2}
\]

\[
a_2 = \left[ 0.43 \left( \frac{f_{ydl}}{f_{cd}} \right)^2 - 8.51 \left( \frac{f_{ydl}}{f_{cd}} \right) + 61.6 \right] \times 10^{-6}
\]

\[
\alpha_{du} = \left[ a_2 \left( \frac{D}{t} \right)^2 + a_1 \left( \frac{D}{t} \right) + a_0 \right] \leq 1.0
\]  
(5.11b)

\[
a_0 = \left[ 0.23 \left( \frac{f_{ydl}}{f_{cd}} \right)^2 - 6.3 \left( \frac{f_{ydl}}{f_{cd}} \right) - 40.2 \right] \times 10^{-2}
\]

\[
a_1 = 0.7 \times 10^{-7} \left( \frac{f_{ydl}}{f_{cd}} \right)^2 - 0.5 \left( \frac{f_{ydl}}{f_{cd}} \right) + 0.015
\]

\[
a_2 = \left[ -0.3 \left( \frac{f_{ydl}}{f_{cd}} \right)^2 + 1.14 \left( \frac{f_{ydl}}{f_{cd}} \right) + 11.24 \right] \times 10^{-5}
\]

\[
\alpha_{en} = \left[ a_2 \left( \frac{D}{t} \right)^2 + a_1 \left( \frac{D}{t} \right) + a_0 \right]
\]  
(5.11c)

\[
a_0 = -0.7 \times 10^{-3} \left( \frac{f_{ydl}}{f_{cd}} \right)^2 + 0.21 \left( \frac{f_{ydl}}{f_{cd}} \right) + 0.82
\]

\[
a_1 = 0.87 \times 10^{-2} \left( \frac{f_{ydl}}{f_{cd}} \right)^2 - 0.25 \left( \frac{f_{ydl}}{f_{cd}} \right) + 1.79
\]

\[
a_2 = \left[ -0.43 \left( \frac{f_{ydl}}{f_{cd}} \right)^2 + 8.5 \left( \frac{f_{ydl}}{f_{cd}} \right) - 21.6 \right] \times 10^{-6}
\]
(2) With Strength Enhancement:

\[ \alpha_{cw} = a_2 \left( \frac{D}{t} \right)^2 + a_1 \left( \frac{D}{t} \right) + a_0 \leq 1.0 \]  
\[ \alpha_{sh} = [a_2 \left( \frac{D}{t} \right)^2 + a_1 \left( \frac{D}{t} \right) + a_0] \leq 1.0 \]  
\[ \alpha_{cm} = [a_2 \left( \frac{D}{t} \right)^2 + a_1 \left( \frac{D}{t} \right) + a_0] \leq 1.0 \]

\[ a_0 = 0.15 \times 10^{-2} \left( \frac{f_{yd}}{f_{cd}} \right)^2 - 0.034 \left( \frac{f_{yd}}{f_{cd}} \right) + 1.18 \]

\[ a_1 = [-0.6 \left( \frac{f_{yd}}{f_{cd}} \right)^2 + 13.59 \left( \frac{f_{yd}}{f_{cd}} \right) - 83.52] \times 10^{-4} \]

\[ a_2 = [0.213 \left( \frac{f_{yd}}{f_{cd}} \right)^2 - 4.43 \left( \frac{f_{yd}}{f_{cd}} \right) + 27.24] \times 10^{-6} \]

\[ a_0 = [0.25 \left( \frac{f_{yd}}{f_{cd}} \right)^2 - 6.46 \left( \frac{f_{yd}}{f_{cd}} \right) - 46.66] \times 10^{-2} \]

\[ a_1 = [-0.2 \left( \frac{f_{yd}}{f_{cd}} \right)^2 - \left( \frac{f_{yd}}{f_{cd}} \right) + 117] \times 10^{-4} \]

\[ a_2 = [0.34 \left( \frac{f_{yd}}{f_{cd}} \right)^2 + 33.68 \left( \frac{f_{yd}}{f_{cd}} \right) - 680] \times 10^{-7} \]

\[ a_0 = [-0.6 \left( \frac{f_{yd}}{f_{cd}} \right)^2 + 16.4 \left( \frac{f_{yd}}{f_{cd}} \right) + 862.1] \times 10^{-3} \]

\[ a_1 = [0.84 \left( \frac{f_{yd}}{f_{cd}} \right)^2 - 23.64 \left( \frac{f_{yd}}{f_{cd}} \right) + 181.1] \times 10^{-2} \]

\[ a_2 = 1 \]
Figs. 5.6(a-c) and 5.7(a-c) shows the errors in the points obtained using Eqn. (5.11 a-c) and Eqn. (5.12 a-c) for each of the CFST cross-sections. The errors are computed based upon the comparison with the values obtained from the fibre element analysis of the individual cross-sections, which is regarded as the accurate solution.

Fig 5.6 (a) Errors in coefficient of force point C without strength enhancement

Fig 5.6 (b) Errors in coefficient of force point D without strength enhancement
Fig 5.6 (c) Errors in coefficient of force point E without strength enhancement

Fig 5.7 (a) Errors in coefficient of force point C with strength enhancement
Fig 5.7 (b) Errors in coefficient of force point D with strength enhancement

Fig 5.7 (c) Errors in coefficient of force point E with strength enhancement
High Strength Concrete (\( f_{cd} > 50 \text{ MPa} \))

\[
\alpha_{cn} = a_2 \left( \frac{D}{t} \right)^2 + a_1 \left( \frac{D}{t} \right) + a_0 \leq 1.0 \quad (5.13a)
\]

\[
a_0 = [0.14\left( \frac{f_{yd}}{f_{cd}} \right)^2 + 0.48\left( \frac{f_{yd}}{f_{cd}} \right) + 73.65] \times 10^{-2}
\]

\[
a_1 = [-0.58\left( \frac{f_{yd}}{f_{cd}} \right)^2 + 6.09\left( \frac{f_{yd}}{f_{cd}} \right) - 41.92] \times 10^{-4}
\]

\[
a_2 = 2 \times 10^{-5}
\]

\[
\alpha_{cn} = [a_2 \left( \frac{D}{t} \right)^2 + a_1 \left( \frac{D}{t} \right) + a_0] \leq 1.0 \quad (5.13b)
\]

\[
a_0 = [0.21\left( \frac{f_{yd}}{f_{cd}} \right)^2 - 2.65\left( \frac{f_{yd}}{f_{cd}} \right) + 7.50] \times 10^{-1}
\]

\[
a_1 = [-0.43\left( \frac{f_{yd}}{f_{cd}} \right)^2 + 3.8\left( \frac{f_{yd}}{f_{cd}} \right) + 6.3] \times 10^{-3}
\]

\[
a_2 = [0.28\left( \frac{f_{yd}}{f_{cd}} \right)^2 - 2.20\left( \frac{f_{yd}}{f_{cd}} \right) - 3.36] \times 10^{-5}
\]

\[
\alpha_{cn} = [a_2 \left( \frac{D}{t} \right)^2 + a_1 \left( \frac{D}{t} \right) + a_0] \quad (5.13c)
\]

\[
a_0 = [-0.9\left( \frac{f_{yd}}{f_{cd}} \right)^2 + 31.7\left( \frac{f_{yd}}{f_{cd}} \right) + 679.9] \times 10^{-3}
\]

\[
a_1 = [0.18\left( \frac{f_{yd}}{f_{cd}} \right)^2 - 5.17\left( \frac{f_{yd}}{f_{cd}} \right) + 38.89] \times 10^{-3}
\]

\[
a_2 = [-0.25\left( \frac{f_{yd}}{f_{cd}} \right)^2 + 5.88\left( \frac{f_{yd}}{f_{cd}} \right) - 12.80] \times 10^{-6}
\]
Figs. 5.8(a-c) shows the errors in the points obtained using Eqn. (5.13 a-c) for each of the CFST cross-sections. The errors are computed based upon the comparison with the values obtained from the fibre element analysis of the individual cross-sections.

Fig 5.8 (a) Errors in coefficient of force point C for high strength concrete

Fig 5.8 (b) Errors in coefficient of force point D for high strength concrete
Fig 5.8 (c) Errors in coefficient of force point E for high strength concrete

5.3.3 Determination of CFST Cross-Section Strength Equation

Once the values of the special points were determined empirically as in previous section. The expression describes the cross-section strength surface of rectangular or circular CFST sections, can obtained by using curve-fitting method. The expression takes the following general form, where

\[ M_i = f(N_i) = \sum_{j=1}^{5} c_j p_j(N_i) \]  
(5.14)

where \( p_j \) is a continuous polynomial equation, \( c_j \) are coefficients. For the polynomial equation \( p_j \), a Lagrange polynomial was found to be the simplest yet accurate expression. It is given by the following equation:
\[ p_j(N_i) = \sum_{k=1}^{5} (N_j - N_k) \] (5.15)

Substituting the values of the force points shown in previous section and Eqn. (5.15) into Eqn. (5.14), then

\[ M_i = \sum_{j=1}^{5} M_j \sum_{k=1}^{5} \frac{(N_j - N_k)}{(N_j - N_k)} \] (5.16)

or simply as following:

For \( N_c \leq N_i < N_a \)

\[ M_i = M_c \frac{(N_a - N_i)}{(N_a - N_c)} \] (5.17a)

and for \( N_i < N_c \)

\[ M_i = M_b \frac{(N_i - N_c)(N_i - N_d)(N_i - N_e)}{(N_b - N_c)(N_b - N_d)(N_b - N_e)} + M_c \frac{(N_i - N_b)(N_i - N_d)(N_i - N_e)}{(N_c - N_b)(N_c - N_d)(N_c - N_e)} \]

\[ + M_d \frac{(N_i - N_b)(N_i - N_e)(N_i - N_d)}{(N_d - N_b)(N_d - N_e)(N_d - N_b)} + M_e \frac{(N_i - N_b)(N_i - N_c)(N_i - N_d)}{(N_c - N_b)(N_c - N_e)(N_c - N_d)} \] (5.17b)
5.4 COMPARISONS OF RESULTS

5.4.1 Comparison of Ultimate Axial Compressive Strength

The results of the axial load computations from Eqn. (5.2) were compared with O’Shea’s test result [O’Shea and Bridge, 1996; 1997]. The comparison is listed in Table 5.1. There were over 22 individual tests with the concrete cylinder strength ranging from 41 MPa to 113 MPa. The mean values of the ratio of measured/predicted column strength and corresponding standard deviations were calculated. For the 5 columns with cylinder strength of less than 50 MPa, the measured/predicted strength ratio is 1.06 and standard deviations is 4.33% respectively; for the remaining 17 columns, 1.02 and 5.63% respectively. On an average for all 22 columns, they are 1.03 and 5.56% respectively. It is therefore concluded that good agreement was observed between the strength of columns obtained from the tests with normal and high strength concrete and the proposed method with or without the effect of confinement.
Table 5.1 Comparison with O'Shea test

<table>
<thead>
<tr>
<th>No.</th>
<th>Diam. Dxt(mm)</th>
<th>Length L(mm)</th>
<th>Ratio D/t</th>
<th>Steel $f_{yd}$ (MPa)</th>
<th>Conc. $f_{cd}$ (MPa)</th>
<th>Max. Load $N_{Test}$ (kN)</th>
<th>Prop. $N_{Prop}$ (kN)</th>
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<td>S30CS50B</td>
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<td>580.5</td>
<td>58.6</td>
<td>363.3</td>
<td>48.3</td>
<td>1662</td>
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<td>256.4</td>
<td>41.0</td>
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<td>1695</td>
<td>1664.1</td>
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<td>41</td>
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<td>1275.7</td>
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<td>58.6</td>
<td>363.3</td>
<td>80.20</td>
<td>2295</td>
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</tr>
<tr>
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<td>663.5</td>
<td>98.2</td>
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<td>74.7</td>
<td>2592</td>
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</table>
The errors of predictions for column strength versus ratio of steel yield strength to concrete cylinder strength \((f_{yd}/f_{cd})\) and ratio of diameter to thickness \((D/t)\) are presented graphically in Fig. 5.9 (a) and Fig. 5.9(b). It also shows the respective linear trendlines together with the coefficient of determination \(R^2\), where \(R\) is the correlation coefficient. The coefficient of determination is quite low which in turn indicates that the majority of the variation is stochastic.

![Graph showing errors versus ratio of diameter to thickness](image)

**Fig 5.9 (a)** Errors versus ratio of diameter to thickness
Fig 5.9 (b) Errors versus ratio of steel strength to concrete strength

5.4.2 Comparison of Ultimate Bending Moment

The results of the moment computations from Eqn. (5.6) were compared with the results from published papers [Chen and Chen (1973); Tomii and Sakino (1979a); Lu and Kennedy (1994)] and AISC code. These comparisons are listed in Table 5.2. The comparisons with experimental results of CFST produced somewhat larger errors. However, Eqn. (5.6) almost always yields conservative results, due largely to the fact that, for simplicity, this equation neglects strain hardening of the steel tube, as well as the effects of confinement in the CFST.
### Table 5.2 Comparison of tests to proposed method and AISC code

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Dimension $B \times D \times t$</th>
<th>$f_{yd}$</th>
<th>$f_{cd}$</th>
<th>$M_{0,\text{Test}}$</th>
<th>$M_{0,\text{AISC}}$</th>
<th>$M_{0,\text{Pr}}$</th>
<th>$M_{0,\text{Pr}} / M_{0,\text{Test}}$</th>
<th>Note</th>
</tr>
</thead>
<tbody>
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</tr>
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<td>1.06</td>
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<td>13.34</td>
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<td>17.48</td>
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<td>-</td>
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<tr>
<td>Lu94-1</td>
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<td>389</td>
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<td>0.95</td>
<td>-</td>
</tr>
<tr>
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<td>46.9</td>
<td>147</td>
<td>109.8</td>
<td>125.7</td>
<td>0.86</td>
<td>-</td>
</tr>
<tr>
<td>Lu94-3</td>
<td>152x254x6.4</td>
<td>377</td>
<td>46.7</td>
<td>211</td>
<td>161</td>
<td>181.2</td>
<td>0.86</td>
<td>Strong axis flex.</td>
</tr>
<tr>
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<td>112.6</td>
<td>123.46</td>
<td>0.86</td>
<td>Weak axis flex.</td>
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### 5.4.3 Comparison of Combined Compression and Bending

The accuracy of the numerical expression for CFST cross section strength and the results from American Structural Specifications Liaison Committee (SSLC) code [Task Group 20, SSLC, 1979] were both verified by comparing them with the test results of short columns. All of the ratio of the column length to the tube depth $L/D$ is under 12 [AIJ, 1994]. These comparisons are illustrated in Figs. 5.10 to 5.14 for the experiments listed in Table 5.3.
Table 5.3 List of test data

<table>
<thead>
<tr>
<th>No.</th>
<th>Dimension (Bx Dx t)</th>
<th>Length (L)</th>
<th>Conc. $(f_{cd})$</th>
<th>Steel $(f_{yd})$</th>
<th>Ratio $D/t$</th>
<th>Ratio $L/D$</th>
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<tbody>
<tr>
<td>Furl67-1</td>
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<td>484.7</td>
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<td>330.96</td>
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<td>Know69-1</td>
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<td>10.7</td>
</tr>
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<td>38.24</td>
<td>194.2</td>
<td>43.67</td>
<td>3</td>
</tr>
<tr>
<td>Tomi79-I1</td>
<td>100 x 100 x 2.27</td>
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<td>305</td>
<td>44.05</td>
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<td>Tomi79-I1</td>
<td>100 x 100 x 2.98</td>
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<td>20.6</td>
<td>289.3</td>
<td>33.56</td>
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</tr>
<tr>
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<td>20.6</td>
<td>288.3</td>
<td>33.44</td>
<td>3</td>
</tr>
<tr>
<td>Tomi79-I1</td>
<td>100 x 100 x 4.25</td>
<td>300</td>
<td>18.63</td>
<td>284.4</td>
<td>23.53</td>
<td>3</td>
</tr>
<tr>
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<td>19.81</td>
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<tr>
<td>Mats95-1</td>
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<td>418.5</td>
<td>23.33</td>
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<td>Mats95-1C</td>
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<td>660</td>
<td>40.9</td>
<td>358.7</td>
<td>36.7</td>
<td>4</td>
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<tr>
<td>Chung99-1</td>
<td>125 x 125 x 3.2</td>
<td>500</td>
<td>86</td>
<td>450</td>
<td>39</td>
<td>4</td>
</tr>
</tbody>
</table>

Fig. 5.10 (a)–(c) shown the comparison with the test results by Furlong (1967). The results of SSLC [Task Group 20, SSLC, 1979] and the results from the proposed numerical expression analysis for three different specimens are also given in Fig. 5.10 (a)-(c). It can be seen that the results from the SSLC are significantly lower than those predicted by the sectional analysis. The test results are in good agreement with
the numerical expression results. In Fig. 5.10 (a) and (b), large discrepancy exists between the test result and both the proposed numerical expression and the SSLC code recommendation, indicating the test results may be unreliable.

Fig. 5.10(a) Comparisons with Furlong’s result

\[(D/t = 26.5, f_{cd} = 44.8 \text{ MPa}, f_{yd} = 485 \text{ MPa})\]

Fig. 5.10 (b) Comparisons with Furlong’s result

\[(D/t = 47.6, f_{cd} = 23.4 \text{ MPa}, f_{yd} = 330 \text{ MPa})\]
Fig. 5.10(c) Comparisons with Furlong’s result

\( (D/t = 32.0, f_{cd} = 29.0 \text{ MPa}, f_{yd} = 330 \text{ MPa}) \)

Fig. 5.11 also illustrates the cross-section strength equation and SSLC code results versus the limited cross-section strength test data presented by Knowles and Park (1969). It shows strong correlation with the experimental results.

Fig. 5.11 Comparisons with Knowles’s result

\( (D/t = 25, f_{cd} = 40.7 \text{ MPa}, f_{yd} = 320 \text{ MPa}) \)
Similarly, the analytical results were also compared with the test results for rectangular section by Tomii et al., (1979a) in Fig. 5.12 (a)-(d). It is clear that the test results are in reasonable agreement with the numerical expression results. Error between the two results becomes greater as the wall of the steel shell becomes thicker. This is possibly due to the confinement effect provided by the steel shell to the concrete. The confinement effect increases as the wall becomes more rigid. However it should be borne in mind that the confinement is only effective for squat specimens, it reduces as the specimens become more slender.

![Graph showing comparisons with Tomii's result](image)

Fig. 5.12(a) Comparisons with Tomii's result

\[(D/t= 44.0, f_{cd} = 38.3 \text{ MPa}, f_{yd} = 195 \text{ MPa})\]
Fig. 5.12(b) Comparisons with Tomii's result

\[(D/t = 44.0, f_{cd} = 21.6 \text{ MPa}, f_{yd} = 305 \text{ MPa})\]

Fig. 5.12(c) Comparisons with Tomii's result

\[(D/t = 45, f_{cd} = 21.6 \text{ MPa}, f_{yd} = 340 \text{ MPa})\]
Fig. 5.12(d) Comparisons with Tomii's result

\[(D/t = 45, f_{cd} = 21.6 \text{ MPa}, f_{yd} = 290 \text{ MPa})\]

Fig. 5.12(e) Comparisons with Tomii's result

\[(D/t = 34, f_{cd} = 20.6 \text{ MPa}, f_{yd} = 290 \text{ MPa})\]
Fig. 5.12(f) Comparisons with Tomii’s result

\(D/t = 34, f_{cd} = 20.6 \text{ MPa}, f_{yd} = 289 \text{ MPa}\)

Fig. 5.12(g) Comparisons with Tomii’s result

\(D/t = 24.0, f_{cd} = 18.6 \text{ MPa}, f_{yd} = 285 \text{ MPa}\)
Analysis were carried out for specimens tested by Matsui [Matsui et al., 1995]. The specimen sectional dimensions are 150 × 150 mm with a wall thickness of 4.5 mm for rectangular CFST section and 165 × 165 mm with a wall thickness of 4.5 mm for circular CFST section. The concrete strength is 32 MPa and the yield stress of the steel is 410 MPa for rectangular section and the concrete strength is 41 MPa and the yield stress of the steel is 360 MPa for circular section. The results for the short column are shown in Fig. 5.13(a-b). It is found that the test results agree well with that of the numerical expression proposed in this research. Similarly for rectangular sections, the predicted results are more conservative due to possibly the fact that the confinements in rectangular section are not considered in the analysis.
Fig. 5.13(a) Comparisons with Matsui's result

\( D/t = 33.3, f_{cd} = 31.9 \text{ MPa}, f_{yd} = 412 \text{ MPa} \)

Fig. 5.13(b) Comparisons with Matsui's result

\( D/t = 36.7, f_{cd} = 40.9 \text{ MPa}, f_{yd} = 358 \text{ MPa} \)
The analytical results were also compared with the test results for rectangular section by Chung [Chung et al., 1999] in Fig. 5.14. The specimen sectional dimensions are 125 × 125 mm with a wall thickness of 3.2 mm. The concrete strength is 86 MPa and the yield stress of the steel is 450 MPa. Again, it is found the numerical expression results are in good agreement with test results, but slightly conservative.

![Graph](image)

Fig. 5.14 Comparisons with Chung’s result

\[(D/t = 39, f_{c d} = 86 \text{ MPa}, f_{yd} = 450 \text{ MPa})\]

5.5 CONCLUSIONS

This chapter presents a set of empirical equations to represent the cross-section strength of circular, rectangular or square CFST beam-columns. The accurate equations provide a simple representation of the cross-section strength of a CFST section, having a wide range of material strengths and cross-section dimensions. The equation is a function of two fundamental parameters governing the behaviour of
CFST sections: the width-to-thickness ratio of the steel tube, and the ratio of the concrete compressive strength to the yield stress of the steel tube.

The equation are empirical based on some of special points on the strength interaction diagram. The strength values of these special points can be easily calculated from the principle of the mechanics. The coefficients were derived by curve-fitting large amount of analysis data using the fibre analysis procedure. The resulting equations were verified by the available test data from published sources. The results from the proposed equations are also compared with those from the existing design codes. It was found that the proposed in most cases agrees well with the test data, though slightly conservative. The proposed compared with the existing design code recommendations offered more accurate prediction of the strength.

The equations though lengthy in its current forms, can be readily coded into a spreadsheet program, or any programmable calculator. The equations are also capable of covering a great range of design parameter, making them an effective design tool.
CHAPTER 6:

THE DESIGN OF CONCRETE FILLED STEEL TUBULAR BEAM-COLUMNS

6.1. INTRODUCTION

The design of a CFST column may be based on a rigorous analysis of structural behaviour which accounts both for the material non-linearities and for the geometric non-linearities. A procedure for such a rigorous analysis is described in Chapter 3. However, this analysis is intended only for special problems which might arise. The rigorous analysis is generally too complex for routine design.

For routine design, a simple design procedure should be used as provided in some of design codes, *i.e.* in particular AISC-LRFD, ACI-318 and Eurocode 4. There are two
different approaches adopted in the design codes. The AISC code uses steel columns
design approach where buckling functions are used and columns are treated as loaded
concentrically in that they are loaded through their centroids, but with due allowance
being made for residual stresses, initial out-of-straightness and slight eccentricities of the
load. The basis in the design of steel columns is instability or buckling, and any
moments which act at the ends of the column are incorporated by reducing the axial load
by way of an interaction equation.

The ACI-318 method uses the traditional reinforced concrete design approach in that the
design strength is always derived from the section strength. The sectional strength is
calculated from a rectangular stress block concept. The failure is generally, but not
always, attributable to cross-section material failure, and is based on the cross-section
interaction curve. The main difficulty is the amount of algebraic work required to derive
this curve accurately. A set of accurate numerical expressions for the section analysis of
CFST beam-column was described in previous chapters.

Because of the similarity of composite columns to both steel and concrete columns, there
has been a great deal of debate among researchers as to which approach should be
adopted, though, short or stocky composite columns are clearly governed by cross
section failure, while long or slender columns are prone to buckling. A logical design
procedure has been adopted by Eurocode 4, where the behaviour of composite columns
can best be treated by a combination of both approaches. The Eurocode 4 approach is
similar to that for steel column approach whereby a column curve is used to determine the column strength under axial load, and modifies this to handle end moments by applying the reinforced concrete approach.

Although Eurocode 4 is used in many countries for the design of CFST elements, the code does not cover the use of high strength concrete. In this work a procedure was proposed, based on the Eurocode 4 design principle to determine the member strength of CFST columns. It covers both normal strength and high strength concrete. It accounts for effects of concrete confinement and the effects of member imperfections in a more rational manner. The strength models in the proposed method were compared extensively with the results of extensive test results on CFST columns under both axial and eccentric loading over a large range of column slenderness.

6.2 THE CROSS SECTION RESISTANCE FOR AXIAL LOAD

The cross-section strength for axial force is given as follows, which was presented in Chapter 5:

**Rectangular Cross Section**

\[ N_0 = A_x f_{yd} + \alpha_1 A_c f_{cd} \]  \hspace{1cm} (6.1)
where $\alpha_i = 0.85$ for normal strength and high strength concrete [ACI 318, 1995; AS3600, 1994].

**Circular Cross Section**

The nominal axial load carried by circular CFST column can be adopted from Eqn. (5.2) expressed as follow:

$$N_0 = A_r \gamma v f_{yd} + A_c \alpha_1 (f_{cd} + 2k\gamma h \frac{I}{D} f_{yd})$$  \hspace{1cm} (6.2)

where $k$ is 4.0 for normal strength, and a $k$ value of 3 for high strength concrete as discussed in Chapter 4. The above expression includes the confinement effect in the composite column.

**6.3 MEMBER STRENGTH**

The proposed design method for combined compression and bending is similar to that in Eurocode 4. With the end moments and possible horizontal forces within the column length, as well as with the axial force, action effects are determined. For slender columns this must be done considering second order effects.
6.3.1 The Buckling Effect

In this research, similar to Eurocode 4 for end-loaded braced members, the axial force $N_{sd}$ and the maximum end moment $M_{sd}$ are determined from a first order structural analysis. For each of the main bending axes of the column it has to be verified that

\[ N_{sd} \leq \chi_k N_0 \]  \hspace{1cm} (6.3)

where: $\chi_k$ is a reduction factor due to buckling. The buckling curves can also be described in the form of an equation:

\[ \chi_k = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}} \]  \hspace{1cm} (6.4)

where

\[ \phi = 0.5[1 + \alpha(\lambda - 0.2) + \lambda^2] \]  \hspace{1cm} (6.5)

where $\alpha$ depends on the buckling effects, a value of 0.21 was adopted for CFST column (Eurocode 3). The relative slenderness of $\lambda$ is given by:

\[ \overline{\lambda} = \frac{N_0}{\sqrt{N_{cr}}} \]  \hspace{1cm} (6.6)

in which $N_{cr}$ is the critical bucking stress resultant given by:
\[ N_{cr} = \frac{\pi^2 (EI)_{e}}{L_e^2} \]  

(6.7)

where \( L_e \) is the effective length and \((EI)_{e}\) is the actual elastic stiffness.

Eurocode 4 proposed that

\[ (EI)_{e} = E_s I_s + 0.8 \frac{E_c}{1.35} I_c \]  

(6.8a)

In this research it is proposed:

\[ (EI)_{e} = E_s I_s + 0.8 \beta_c E_c I_c \]  

(6.8b)

where: \( \beta_c \) is the load effect;

\( I_c, I_s \) are the concrete, steel moments of inertia;

\( E_s \) is the Young's modules of steel;

\( E_c \) is the secant modulus for the concrete determined from Eurocode 2 for the appropriate concrete grades, equal to \( 9500(f_c' + 8)^{\frac{1}{3}} \) in MPa;

\( f_c' \) is characteristic compressive cylinder strength of concrete at 28 days.

The value of \( \beta_c \) is adopted from Tomii (1979 a) as:

for \( n \leq 0.5 \)
\[\beta_c = 0.311 + 0.619n + 0.457n^2 \quad (6.9)\]

where \(n\) is the ratio of design load to the capacity:

\[n = \frac{N_{sd}}{A_c f_{cd} + A_s f_{yd}} \quad (6.10)\]

and for \(n > 0.5\),

\[\beta_c = 0.735 \quad (6.11)\]

Eqn (6.11) is approximately corresponding to Eurocode 4 for a safety factor of 1.35.

The secondary moment effect due to lateral deflection is accounted for by the use of a moment magnifier \(\delta_b\):

\[M^* = \delta_b M_{sd} \quad (6.12)\]

Where \(M_{sd}\) is the maximum first order bending moment and:

\[\delta_b = \frac{C_m}{1 - \frac{N_{sd}}{N_{cr}}} \geq 1.0 \quad (6.13)\]

where \(C_m\) is the moment factor, equal to

\[C_m = 0.66 + 0.44 r \geq 0.44 \quad (6.14)\]
\( r \) is the ratio of the smaller to larger end moment and is positive when the member is bent in single curvature.

### 6.3.2 Design Principle

The member strength interaction diagram for a composite column is constructed based on the section strength interaction as indicated in Fig. 6.1. The reduction in axial strength and bending moment capacity is due to the influence of imperfections, slenderness and lateral deflection. First of all the bearing capacity of the composite column under axial compression has to be determined according to Eqn. 6.1 or 6.2. The axial strength \( N_k \) is given by writing Eqn. 6.3 as

\[
N_k = \chi_k N_0 \tag{6.15}
\]

This member capacity is represented by the value \( \chi_k \). At the level of \( \chi_k \), a value for the ratio \( \mu_k \) can be read off of the interaction curve, and

\[
M_k = \mu_k M_0 \tag{6.16}
\]

The moment \( \mu_k \) represents the bending moment caused by the axial load that is the bending due to the second order effect, just prior to failure of the column.
The influence of this imperfection is assumed to decrease to zero linearly at the value axial load \(\chi_n\). For end moments, Eqn. 6.17 is shown as follows

\[
\chi_n = \chi_k \frac{(1-r)}{4}
\]  
(6.17)

For an applied axial load \(N\), the ordinate of the design load is given by

\[
\chi_d = \frac{N}{N_0}
\]
(6.18)

as shown in Fig. 6.1, the bending resistance corresponding to axial load \(\chi_d\) is \(\mu_d\). The bending capacity is measured by the distance \(\mu\) in Fig. 6.1, obtained as
\[ \mu = \mu_d - \mu_k \left( \frac{X_d - X_n}{X_k - X_n} \right) \]  
(6.19)

The bending moment resistance is therefore

\[ M = \alpha_m \mu M_0 \]  
(6.20)

where \( \alpha_m \) is the moment reduction factor, in the current proposal equal to 1. In Eurocode 4, the reduction of the moment resistance in Eqn. 6.20 by 10\% (\( \alpha_m = 0.9 \)), because of the unconservative assumption that the concrete block is fully plastic to the neutral axis. But the writer believes that the reduction value appears too conservative for high bending moment and low axial force situations, as discussed in the next section.

For a CFST column, the load-carrying capacity of the cross-section with the influence of imperfection can be represented by an \( N-M \) interaction diagram, as discussed already in Chapter 5, and the reduction due to imperfection is referred to in Fig. 6.2 as the strength line. At each stage of loading the internal force \( N \) in the section is equal to the external applied load \( P \). If it is a pin-ended column with a load applied at a constant eccentricity, Eqn. 6.12 now becomes

\[ M = \frac{C_m}{N} \left( Ne \right) \left( 1 - \frac{N}{N_{cr}} \right) \]  
(6.21)
Equation 6.21 expresses the relation between $N$ and $M$, is therefore the equation of the loading for a particular column with known eccentricity. For the purpose of calculation it is often convenient to arrange Eqn. 6.21 in the form [Warner et al., 1989]

$$\frac{1}{N} = \frac{C_m e}{M} + \frac{1}{N_{cr}} \quad (6.22)$$

In Fig. 6.2, the intersection "A" with the loading line from Eqn. 6.22 and strength line gives the member load capacity of a CFST column.

![Diagram showing calculation of load capacity](image-url)
6.4 COMPARISONS OF RESULTS

6.4.1 Comparison of Combined Compression and Bending

In these research design models were used to determine the member strength, including AISC-LRFD, Eurocode 4 and the proposed method. These were then compared with the test results from published reports for normal strength concrete. Eurocode 4 and AISC-LRFD code limit the concrete cylinder strengths to 50 MPa. Therefore, for high strength concrete comparison was only made between the test results and the proposed method.

Rectangular Cross Section

Comparisons with Grauers (1993)

Analyses were carried out on specimens tested by Grauers (1993). The specimen sectional dimensions are either 120 mm or 250 mm with a wall thickness of either 5 mm or 8 mm. The concrete strength was reported to range from 23 MPa to 103 MPa, and the yield stress of the steel ranges from 300 MPa to 439 MPa. The results of square CFST columns are given in Table 6.1.
<table>
<thead>
<tr>
<th>Spec. No</th>
<th>$D \times B \times t$</th>
<th>$L$</th>
<th>$e$</th>
<th>$f_{yd}$</th>
<th>$f_{cd}$</th>
<th>$N_{test}$</th>
<th>$N_{prop}$</th>
<th>$N_{test} / N_{prop}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gra93-1</td>
<td>120x120x5</td>
<td>3196</td>
<td>16.5</td>
<td>304</td>
<td>47</td>
<td>610</td>
<td>577</td>
<td>1.06</td>
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<td>Gra93-2</td>
<td>120x120x5</td>
<td>3196</td>
<td>15.5</td>
<td>438</td>
<td>46</td>
<td>700</td>
<td>584</td>
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</tr>
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<td>710</td>
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<td>830</td>
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Comparisons with Matsui et al. (1995)

Analyses were also carried out on specimens tested by Matsui et al. (1995). The specimen sectional dimensions are $150 \times 150$ \text{mm} with a wall thickness of 4.5 \text{mm}. Various lengths were tested with a $L/D$ ratio of 4, 8, 12, 18, 24 and 30. The concrete strength is 32 MPa and the yield stress of the steel is 418 MPa. The results of square CFST columns are given in Table 6.2 (a)-(b).
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Table 6.2(b) Comparison of test results with AISC, Eurocode 4 and Proposed method

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Circular Cross Sections

Comparisons with Kilpatrick and Rangan (1997)

Analyses were also carried out on specimens tested by Kilpatrick and Rangan (1997). The specimen was constructed using a 101 mm diameter circular steel tube (CHS) with 2.4 mm wall thickness. The first group with the column length of 1947 mm. The concrete strength is 57 MPa and the yield stress of the steel is 410 MPa. The second group with column length of 2175 mm. The concrete strength is 96 MPa and the yield stress of the steel is 450 MPa. The results of CFST columns are given in Table 6.3.

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161
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| Kil97-15 | 101.7x2.4 | 1947 | +50,+50 | 410 | 57 | 158 | 151 | 1.05 |
| Kil97-16 | 101.5x2.4 | 2175 | +50,+50 | 410 | 96 | 157 | 149 | 1.05 |
| Kil97-17 | 101.5x2.4 | 2175 | +50,+30 | 410 | 96 | 183 | 169 | 1.08 |
| Kil97-18 | 101.5x2.4 | 2175 | +50,+20 | 410 | 96 | 196 | 181 | 1.08 |
| Kil97-19 | 101.5x2.4 | 2175 | +50,+10 | 410 | 96 | 215 | 195 | 1.1 |
| Kil97-20 | 101.5x2.4 | 2175 | +50,+0  | 410 | 96 | 237 | 211 | 1.12 |
| Kil97-21 | 101.5x2.4 | 2175 | +50,-10 | 410 | 96 | 256 | 230 | 1.11 |
| Kil97-22 | 101.5x2.4 | 2175 | +50,-20 | 410 | 96 | 266 | 230 | 1.16 |
| Kil97-23 | 101.5x2.4 | 2175 | +50,-20 | 410 | 96 | 266 | 230 | 1.16 |
| Kil97-24 | 101.5x2.4 | 2175 | +40,+30 | 410 | 96 | 197 | 188 | 1.05 |
| Kil97-25 | 101.5x2.4 | 2175 | +40,+10 | 410 | 96 | 243 | 221 | 1.1 |
| Kil97-26 | 101.5x2.4 | 2175 | +40,+0  | 410 | 96 | 260 | 242 | 1.07 |
| Kil97-27 | 101.5x2.4 | 2175 | +40,-10 | 410 | 96 | 281 | 269 | 1.04 |
| Kil97-28 | 101.5x2.4 | 2175 | +40,-20 | 410 | 96 | 331 | 288 | 1.15 |
| Kil97-29 | 101.5x2.4 | 2175 | +30,+20 | 410 | 96 | 244 | 231 | 1.06 |
| Kil97-30 | 101.5x2.4 | 2175 | +30,+0  | 410 | 96 | 318 | 278 | 1.14 |
| Kil97-31 | 101.5x2.4 | 2175 | +30,-10 | 410 | 96 | 340 | 303 | 1.12 |
| Kil97-32 | 101.5x2.4 | 2175 | +30,-20 | 410 | 96 | 384 | 320 | 1.2 |
| Kil97-33 | 101.5x2.4 | 2175 | +20,+20 | 410 | 96 | 282 | 258 | 1.09 |
| Kil97-34 | 101.5x2.4 | 2175 | +20,+0  | 410 | 96 | 367 | 310 | 1.18 |
| Kil97-35 | 101.5x2.4 | 2175 | +20,-10 | 410 | 96 | 411 | 345 | 1.19 |
| Kil97-36 | 101.5x2.4 | 2175 | +40,-30 | 410 | 96 | 344 | 288 | 1.19 |
| Kil97-37 | 101.5x2.4 | 2175 | +40,-40 | 410 | 96 | 385 | 288 | 1.34 |
| Kil97-38 | 101.5x2.4 | 2175 | +0,+0   | 410 | 96 | 523 | 478 | 1.09 |
| Kil97-39 | 101.5x2.4 | 2175 | +50,-30 | 410 | 96 | 303 | 230 | 1.32 |
| Kil97-40 | 101.5x2.4 | 2175 | +50,-50 | 410 | 96 | 344 | 230 | 1.49 |
Comparisons with Matsui et al. (1995)

Analyses were carried out on specimens tested by Matsui et al. (1995). The specimen sectional dimensions are 165.2 mm for circular sections all with a wall thickness of 4.5 mm. Various lengths were tested with a L/D ratio of 4, 8, 12, 18, 24 and 30. The concrete strength is 41 MPa and the yield stress of the steel is 358 MPa. The results are shown in Table 6.4 (a)-(b).
Table 6.4(a) Predicted column strength - Matsui et al. (1995)

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165
Comparisons with Neogi et al. (1969)

Analyses were carried out on specimens tested by Neogi et al. (1969). The specimen was constructed using the diameters ranged between 127 mm and 69 mm with all thickness between 1 mm to 10 mm. All columns were filled with concrete of strength ranging from 23 MPa to 70.9 MPa. The results of CFST columns are given in Table 6.5(a)-(b).

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### Table 6.5(b) Comparison of test results with AISC code and Proposed method

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<td>804</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>732</td>
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<tr>
<td>Neo69-13</td>
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<td>494</td>
<td>1.06</td>
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<td>1.17</td>
<td>432</td>
<td>1.16</td>
<td>443</td>
<td>1.14</td>
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</table>

**Comparisons with O’Brien and Rangan (1993)**

Analyses were carried out on specimens tested by O’Brien and Rangan (1993). The specimen sectional dimensions are 76.1 mm with either 2.6 mm or 3.2 mm wall thickness. The varying parameters in these tests were the lengths of the columns, which varied between 615 mm and 1755 mm. The concrete strength is 85 MPa and the yield stress of the steel is 341 MPa. The results of CFST columns are shown in Table 6.6.
<table>
<thead>
<tr>
<th>Spec. No</th>
<th>$D_{xt}$ (mm)</th>
<th>$L$ (mm)</th>
<th>$e$ (mm)</th>
<th>$F_{yd}$ (MPa)</th>
<th>$f_{cd}$ (MPa)</th>
<th>$N_{test}$ (kN)</th>
<th>$N_{prop}$ (kN)</th>
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<tr>
<td>Obr93-1</td>
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<td>1755</td>
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<td>190</td>
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</tr>
<tr>
<td>Obr93-2</td>
<td>76.1x2.6</td>
<td>1375</td>
<td>10</td>
<td>341</td>
<td>85</td>
<td>241</td>
<td>206</td>
<td>1.17</td>
</tr>
<tr>
<td>Obr93-3</td>
<td>76.1x2.6</td>
<td>995</td>
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<td>341</td>
<td>85</td>
<td>279</td>
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</tr>
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<td>Obr93-4</td>
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<td>341</td>
<td>85</td>
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<td>76.1x3.2</td>
<td>1755</td>
<td>10</td>
<td>332</td>
<td>85</td>
<td>185</td>
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<td>237</td>
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<td>995</td>
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<td>85</td>
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<td>1.08</td>
</tr>
<tr>
<td>Obr93-9</td>
<td>76.1x3.2</td>
<td>615</td>
<td>10</td>
<td>332</td>
<td>85</td>
<td>388</td>
<td>325</td>
<td>1.19</td>
</tr>
</tbody>
</table>

6.4.2 Experimental Verification

From the literature review conducted, over 131 individual tests on rectangular or circular CFST’s were identified in which the concrete cylinder strength ranged from 23 MPa to 103 MPa. Sixty-three or 48% of the columns had concrete cylinder strength in excess of 50 MPa which are outside the limit of the current design codes. The design methods have been compared in Table 6.1-6.6. The column strength was predicted safely in 121 or 92% of the columns analysed. The mean values of the ratio of measured/predicted column strength and corresponding standard deviations were as follows: for the 68 columns with a cylinder strength less than 50 MPa, 1.07 and 0.0764 respectively; for the
remaining 63 columns, 1.15 and 0.0975 respectively; for all 131 columns, 1.11 and 0.0957 respectively. These values suggest that the accuracy of the prediction of column strength is not significantly influenced by the concrete strength. Clearly the proposed method provides the best estimate, with the lowest median and standard deviation. The results have been plotted in Figure 6.3.

![Graph showing the comparison of test data with proposed method](image)

Fig. 6.3 Comparison of test data with proposed method

The errors of predictions for column strength versus concrete cylinder strength, column slenderness $L/D$ and ratio of diameter to thickness $D/t$ are presented graphically in Fig. 6.4 - Fig. 6.6. Columns with a cylinder strength less than 50 MPa are represented by solid triangles whilst those with a cylinder strength greater than 50 MPa are represented by hollow squares. Also shown are the respective linear trendlines, represented by
dashed and solid lines, together with the coefficient of determination $R^2$, where $R$ is the correlation coefficient. In both instances the coefficient of determination is quite low which in turn indicates that the majority of the variation is stochastic.

Fig 6.4 Errors versus concrete strength
Fig 6.5 Errors versus column slenderness

Fig 6.6 Errors versus ratio of diameter to thickness
6.5 CONCLUSIONS

The proposed strength model indicated that there is no significant relationship between the concrete cylinder strength and the accuracy of the prediction of column strength. Based on this, it is concluded that the current proposed method of design can provide accurate predictions of the strengths of concrete filled tubes for a range of end eccentricities and column slenderness.

From comparisons between AISC-LRFD, Eurocode 4, the proposed method and test results, it was found that for short columns, the test results agree well with Eurocode 4 and proposed method. As the slenderness increases the test results becomes lower due to the slenderness effect. As the slenderness becomes large, it can be seen that the proposed method and the Eurocode 4 method give a better fit to the test data than AISC-LRFD. The Eurocode 4 recommendations when compared with the test data is overly conservative at low axial load range, yet becomes unsafe at the high axial load range. This is particularly dangerous as most of the composite steel concrete columns are used in situations when the axial force is large whilst the bending moment is minimal. The strength model in the proposed method compares reasonably well with the test data, though in most situations slightly conservative.
CHAPTER 7:

NUMERICAL EQUATIONS FOR

MEMBER DESIGN OF CONCRETE FILLED STEEL

TUBULAR COLUMNS

7.1 INTRODUCTION

The calculation of axial force and bending moment strength diagrams plays an important part in the design of steel-concrete composite columns. Although various codes of practice attempt to simplify the complex calculation procedures, most of them still remain tedious and difficult to use.

Design interaction equations for beam-columns member have been proposed in the past by several researchers for steel members and reinforced concrete. Chen and Duan (1989) published interaction equations for steel members (short and slender) under the combined bending moments and axial force. Griffis (1992) examined and
compared the interaction diagrams for composite columns plotted according to the ACI and the AISC-LRFD design approach.

This chapter develops a set of empirical equations for the prediction of member strength of concrete filled tubular columns for both normal and high strength concrete. Firstly, accurate numerical analysis of hundreds of composite columns was performed using a general analysis procedure developed in Chapter 3. Secondly, the coefficients of the proposed numerical equations were derived by curve fitting the large amount of results obtained through numerical analyses. Finally the proposed equations were verified by comparing the results with the published test data from various sources.

It was found that it is possible as for the section design to provide a set of numerical equations to cover all the possible practical design parameters with acceptable accuracy. The equations and the coefficients are lengthy, however they can be readily coded into a programmable calculator or spreadsheet for design purposes.

Based on the proposed equation, a design procedure was also proposed for member design of concrete filled steel tubular (CFST) columns.

7.2 THE EQUATION OF MEMBER STRENGTH LINE

For a CFST beam-column, the load-carrying capacity of the member with the influence of slenderness and imperfection can be represented by an $N-M$ interaction
diagram, as discussed already in Chapter 6, and the resulting diagram is referred to in Fig. 7.1 as the member strength line.

![Member strength diagram](image)

Fig. 7.1 Member strength diagram

### 7.2.1 Equation of CFST Member Strength Line

A general equation representing member strength line of CFST columns subjected to combined bending moment and axial load can be written as follows:

\[
\begin{align*}
N_i & \geq N_{ds} \\
\left(\frac{N_i - N_{ds}}{N_k - N_{ds}}\right)^{\alpha} + \frac{M_i}{M_{ds}} & = 1.0 \quad (7.1a) \\
N_i & < N_{ds} \\
\frac{N_i}{N_{ds}} + \left(\frac{M_i}{M_0 - M_{ds}}\right)^{\beta} & = 1.0 \quad (7.1b)
\end{align*}
\]
where $N_k$, $M_0$ = reduced axial reduction compression and pure bending moment strengths of the cross section, respectively. Their computation is described in the previous chapters; $N_{ds}$, $M_{ds}$ = axial compression and bending moment strengths at point D*, respectively. Their computation is described in the next section; $N_f$, $M_f$ = axial compression and bending moment strengths of a particular point on the member strength line, respectively. And

$$\alpha = 1 + 0.01 \frac{L}{D} \quad (7.2a)$$

$$\beta = 2.95 - 0.095 \frac{L}{D} + 0.0011 \left(\frac{L}{D}\right)^2 \quad (7.2b)$$

### 7.2.2 Development for the Coefficient of the Force Points

The member strength interaction diagram for a CFST column is constructed based on the three basic points as indicated in Fig. 7.1 (1) the reduced maximum axial compressive load, $N_k$; (2) the reduced axial load and moment strength, $N_{ds}$ and $M_{ds}$ at point D and; (3) the pure bending moment, $M_0$.

First of all, the bearing capacity of the CFST column under axial compression has to be determined according to Eqn. 6.1 or 6.3. The concentric strength $N_k$ was given by writing Eqn. 6.10 as
\[ N_k = \chi_k N_0 \]  
(7.3)

The force points D* are marked in the member diagram in Fig. 7.1. It is defined as:

Point D*,

\[ N_{ds} = 0.65 \chi_k N_0 \]  
(7.4)
\[ M_{ds} = \mu_{ds} M_0 \]  
(7.5)

where \( \mu_{ds} \) is a coefficient, and will be discussed in the next section.

The first step in the development of an expression for the CFST member strength line entails selecting a wide range of cross-sections and length of columns representative of CFST members. There are three properties which most directly affect the behaviour of the CFST member strength. These may be identified by three dimensionless ratios: the ratio of the column length to section depth (or diameter for circular section) (\( L/D \)); the ratio of the section depth to wall thickness (\( D/t \)) and the ratio of the steel yield design strength to the concrete compression design strength (\( f_{yd}/f_{cd} \)).

Six Series of length to diameter are selected, with \( L/D \) ratios being 5, 10, 20, 30, 40, 50. Within each series, five series of sections are selected, with \( D/t \) ratios being 20, 40, 50, 80 and 100. Within each of these different \( f_{yd}/f_{cd} \) ratios are chosen. The design concrete strength \( f_{cd} \) ranges from 20 MPa to 120 MPa, a 10 MPa uniform increment in the value. The design steel yield strength of tubes \( f_{yd} \) ranged from 200
MPa to 600 MPa, a 100 MPa uniform increment in the value. Therefore $f_{yd} / f_{cd}$ ranges from 2.5 to 30.

For the selected set of expression, the coefficient $\mu_{ds}$ is determined by a least-squares procedure using the axial force and moment data from the analysis of each member. The coefficient is expressed in terms of the $L/D$, $D/t$ and $f_{yd} / f_{cd}$ ratio of the member:

\[
\frac{M_{ds}}{M_0} = \mu_{ds} = \left[ a_2 \left( \frac{D}{t} \right)^2 + a_1 \left( \frac{D}{t} \right) + a_0 \right] \frac{L}{D}^2 \\
+ \left[ b_2 \left( \frac{D}{t} \right)^2 + b_1 \left( \frac{D}{t} \right) + b_0 \right] \frac{L}{D} \\
+ \left[ c_2 \left( \frac{D}{t} \right)^2 + c_1 \left( \frac{D}{t} \right) + c_0 \right] \leq 1.0
\]  

(7.6)

Rectangular CFST Columns

Normal Strength Concrete ($f_{cd} \leq 50$ MPa)

\[
a_0 = \left[ -0.47 \left( \frac{f_{yd}}{f_{cd}} \right)^2 + 12.89 \left( \frac{f_{yd}}{f_{cd}} \right) - 73.82 \right] \times 10^{-5}
\]

\[
a_1 = \left[ 0.23 \left( \frac{f_{yd}}{f_{cd}} \right)^2 - 5.72 \left( \frac{f_{yd}}{f_{cd}} \right) + 24.94 \right] \times 10^{-6}
\]

\[
a_2 = \left[ -0.21 \left( \frac{f_{yd}}{f_{cd}} \right)^2 + 5.20 \left( \frac{f_{yd}}{f_{cd}} \right) - 22.68 \right] \times 10^{-4}
\]

\[
b_0 = \left[ 0.16 \left( \frac{f_{yd}}{f_{cd}} \right)^2 - 5.15 \left( \frac{f_{yd}}{f_{cd}} \right) + 38.30 \right] \times 10^{-3}
\]

\[
b_1 = \left[ -0.12 \left( \frac{f_{yd}}{f_{cd}} \right)^2 + 3.34 \left( \frac{f_{yd}}{f_{cd}} \right) - 19.35 \right] \times 10^{-4}
\]

\[
b_2 = \left[ 0.13 \left( \frac{f_{yd}}{f_{cd}} \right)^2 - 3.22 \left( \frac{f_{yd}}{f_{cd}} \right) + 15.68 \right] \times 10^{-6}
\]

\[
c_0 = \left[ -0.22 \left( \frac{f_{yd}}{f_{cd}} \right)^2 + 6.28 \left( \frac{f_{yd}}{f_{cd}} \right) + 10.14 \right] \times 10^{-2}
\]
\[ c_1 = [0.2 \left( \frac{f_{yld}}{f_{cd}} \right)^2 - 6.4 \left( \frac{f_{yld}}{f_{cd}} \right) + 45.6] \times 10^{-3} \]
\[ c_2 = [-0.23 \left( \frac{f_{yld}}{f_{cd}} \right)^2 + 5.71 \left( \frac{f_{yld}}{f_{cd}} \right) - 32.6] \times 10^{-5} \quad (7.7) \]

**High Strength Concrete** \((f_{cd} > 50 \text{ MPa})\)

\[ a_0 = [0.16 \left( \frac{f_{yld}}{f_{cd}} \right)^2 - 1.22 \left( \frac{f_{yld}}{f_{cd}} \right) + 2.15] \times 10^{-4} \]
\[ a_1 = [-0.48 \left( \frac{f_{yld}}{f_{cd}} \right)^2 + 3.71 \left( \frac{f_{yld}}{f_{cd}} \right) - 6.91] \times 10^{-6} \]
\[ a_2 = [0.43 \left( \frac{f_{yld}}{f_{cd}} \right)^2 - 4.09 \left( \frac{f_{yld}}{f_{cd}} \right) + 9.47] \times 10^{-8} \]
\[ b_0 = [0.5 \left( \frac{f_{yld}}{f_{cd}} \right)^2 - 10.82 \left( \frac{f_{yld}}{f_{cd}} \right) + 40.97] \times 10^{-3} \]
\[ b_1 = [0.44 \left( \frac{f_{yld}}{f_{cd}} \right)^2 + 12.08 \left( \frac{f_{yld}}{f_{cd}} \right) - 92.38] \times 10^{-5} \]
\[ b_2 = [-0.2 \left( \frac{f_{yld}}{f_{cd}} \right)^2 - 6.59 \left( \frac{f_{yld}}{f_{cd}} \right) + 42.63] \times 10^{-7} \]
\[ c_0 = [-0.93 \left( \frac{f_{yld}}{f_{cd}} \right)^2 + 17.31 \left( \frac{f_{yld}}{f_{cd}} \right) - 18.81] \times 10^{-2} \]
\[ c_1 = [0.8 \left( \frac{f_{yld}}{f_{cd}} \right)^2 - 12.8 \left( \frac{f_{yld}}{f_{cd}} \right) + 50.2] \times 10^{-3} \]
\[ c_2 = [-0.13 \left( \frac{f_{yld}}{f_{cd}} \right)^2 + 1.68 \left( \frac{f_{yld}}{f_{cd}} \right) - 5.25] \times 10^{4} \quad (7.8) \]

**Circular CFST Columns**

**Normal Strength Concrete** \((f_{cd} \leq 50 \text{ MPa})\)

\[ a_0 = [0.68 \left( \frac{f_{yld}}{f_{cd}} \right)^2 + 17.26 \left( \frac{f_{yld}}{f_{cd}} \right) - 422.19] \times 10^{-6} \]
\[ a_1 = [-0.38 \left( \frac{f_{yld}}{f_{cd}} \right)^2 - 9.6 \left( \frac{f_{yld}}{f_{cd}} \right) + 209.64] \times 10^{-7} \]
\[ a_2 = [0.3 \left( \frac{f_{yld}}{f_{cd}} \right)^2 + 12 \left( \frac{f_{yld}}{f_{cd}} \right) - 205] \times 10^{-9} \]

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\[ b_0 = [-0.16\left(\frac{f_{yd}}{f_{cd}}\right)^2 + 1.63\left(\frac{f_{yd}}{f_{cd}}\right) + 16.0] \times 10^{-3} \]

\[ b_1 = [0.5\left(\frac{f_{yd}}{f_{cd}}\right)^2 + 2.6\left(\frac{f_{yd}}{f_{cd}}\right) - 156.9] \times 10^{-5} \]

\[ b_2 = [-0.5\left(\frac{f_{yd}}{f_{cd}}\right)^2 - 3.6\left(\frac{f_{yd}}{f_{cd}}\right) + 133.2] \times 10^{-7} \]

\[ c_0 = [0.5\left(\frac{f_{yd}}{f_{cd}}\right)^2 - 8.72\left(\frac{f_{yd}}{f_{cd}}\right) + 54.64] \times 10^{-2} \]

\[ c_1 = [-0.5\left(\frac{f_{yd}}{f_{cd}}\right)^2 - 11\left(\frac{f_{yd}}{f_{cd}}\right) + 322] \times 10^{-4} \]

\[ c_2 = [-0.8\left(\frac{f_{yd}}{f_{cd}}\right)^2 + 38.1\left(\frac{f_{yd}}{f_{cd}}\right) - 365.6] \times 10^{-6} \quad (7.9) \]

**High Strength Concrete \((f_{cd} > 50 \text{ MPa})\)**

\[ a_0 = [0.37\left(\frac{f_{yd}}{f_{cd}}\right)^2 + 3.97\left(\frac{f_{yd}}{f_{cd}}\right) - 8.77] \times 10^{-4} \]

\[ a_1 = [0.18\left(\frac{f_{yd}}{f_{cd}}\right)^2 - 1.81\left(\frac{f_{yd}}{f_{cd}}\right) + 3.53] \times 10^{-5} \]

\[ a_2 = [-0.17\left(\frac{f_{yd}}{f_{cd}}\right)^2 + 1.59\left(\frac{f_{yd}}{f_{cd}}\right) - 3.16] \times 10^{-7} \]

\[ b_0 = [0.12\left(\frac{f_{yd}}{f_{cd}}\right)^2 - 1.44\left(\frac{f_{yd}}{f_{cd}}\right) + 3.68] \times 10^{-2} \]

\[ b_1 = [-0.9\left(\frac{f_{yd}}{f_{cd}}\right)^2 + 9.69\left(\frac{f_{yd}}{f_{cd}}\right) - 25.45] \times 10^{-4} \]

\[ b_2 = [0.96\left(\frac{f_{yd}}{f_{cd}}\right)^2 - 10.04\left(\frac{f_{yd}}{f_{cd}}\right) + 24.06] \times 10^{-6} \]

\[ c_0 = [0.62\left(\frac{f_{yd}}{f_{cd}}\right)^2 - 5.24\left(\frac{f_{yd}}{f_{cd}}\right) + 35.44] \times 10^{-2} \]

\[ c_1 = [0.85\left(\frac{f_{yd}}{f_{cd}}\right)^2 - 11.65\left(\frac{f_{yd}}{f_{cd}}\right) + 49.48] \times 10^{-3} \]

\[ c_2 = [-0.15\left(\frac{f_{yd}}{f_{cd}}\right)^2 + 1.8\left(\frac{f_{yd}}{f_{cd}}\right) - 5.56] \times 10^{-4} \quad (7.10) \]
7.2.3 Load Capacity of CFST Beam-Columns

For a short column, because of the shortness of the column, the lateral deflection and secondary moment are negligible, and the behaviour is regarded as linear. The path followed by axial load $P$ and bending moment $M$ at column mid-length is shown in Fig 7.2, reaching a maximum at Point 1 of the strength interaction diagram.

For a slender column, the lateral deflection at mid-height, $\delta$, is significant, i.e. the secondary moment is significant, thus there is a progressive increase in lateral deflection with increasing load, until the loading curve meets the member strength line. The behaviour is nonlinear. For very slender columns, the rapid increase in deflection, and hence in secondary moment, is such that the peak load is reached without material failure occurring in the section at mid-height. The failure occurs well within the failure interaction diagram. This corresponds to the buckling of the column.

In the research the principle of Eurocode 4 was adopted. The standard permits a simplified method in the design of CFST beam-columns. The amount by which the column moment is magnified depends very much on the effective length of the column. If the factored design axial force $N_u$ acts in conjunction with the factored, magnified moment $M_{mag}$, the equation of the loading line can be expressed similarly as in Chapter 6:
\[ M_{wag} = N_s (\delta_b e) \]  

(7.11)

where

\[ \delta_b = \frac{C_m}{1 - \frac{N}{N_{cr}}} \]  

(7.12)

The steps in the calculation of the load capacity of column are as follows:

(i) Plot the member strength line for the CFST column by the method discussed in previous section.

(ii) Plot the loading line from Eqn. (7.11). The intersection with the member strength line gives the capacity of the column.

Fig. 7.2 Calculation of load capacity
7.3 RESULTS AND DISCUSSION

In this work the proposed empirical expression was used to predict the member strength of CFST beam-columns. The results were compared with the test results in the published reports.

7.3.1 Normal Strength Concrete

To provide confirmation of the validity of the proposed design equation, comparisons have been made with results of full scale test of rectangular and circular section on 40 uniaxially bent beam-columns as reported by Matsui et al. (1995).

The specimen sectional dimensions are $150 \times 150$ $mm$ with a wall thickness of 4.5 $mm$ for rectangular CFST sections. Various lengths were tested with an $L/D$ ratio of 4, 8, 12, 18, 24 and 30. The concrete strength is 32 MPa and the yield stress of the steel is 410 MPa. The data for the 24 tests reported by Matsui et al. (1995) are shown in Fig 7.3 (a) - (f).
Fig. 7.3(a) Comparisons with Matsui's result

\((L/D = 4, f_{cd} = 32 \text{ MPa}, f_{yd} = 410 \text{ MPa})\)

Fig. 7.3(b) Comparisons with Matsui's result

\((L/D = 8, f_{cd} = 32 \text{ MPa}, f_{yd} = 410 \text{ MPa})\)
Fig. 7.3(c) Comparisons with Matsui’s result

\( L/D = 12, f_{cd} = 32 \text{ MPa}, f_{yd} = 410 \text{ MPa} \)

Fig. 7.3(d) Comparisons with Matsui’s result

\( L/D = 18, f_{cd} = 32 \text{ MPa}, f_{yd} = 410 \text{ MPa} \)
Fig. 7.3(e) Comparisons with Matsui’s result

\[(L/D = 24, f_{cd} = 32 \text{ MPa}, f_{yd} = 410 \text{ MPa})\]

Fig. 7.3(f) Comparisons with Matsui’s result

\[(L/D = 30, f_{cd} = 32 \text{ MPa}, f_{yd} = 410 \text{ MPa})\]
The specimen sectional dimensions are 165 mm in diameter with a wall thickness of 4.5 mm for circular CFST sections. Various lengths were tested with an $L/D$ ratio of 4, 8, 12, 18, 24 and 30. The concrete strength is 42 MPa and the yield stress of the steel is 358 MPa. The data for the 24 tests reported by Matsui et al. (1995) are shown in Fig 7.4 (a)-(f).

![Graph](image)

**Fig. 7.4(a) Comparisons with Matsui’s result**

\[(L/D = 4, f_{cd} = 42 \text{ MPa}, f_{yd} = 358 \text{ MPa})\]
Fig. 7.4(b) Comparisons with Matsui’s result

\[(L/D = 8, f_{cd} = 42 \text{ MPa}, f_{yd} = 358 \text{ MPa})\]

Fig. 7.4(c) Comparisons with Matsui’s result

\[(L/D = 12, f_{cd} = 42 \text{ MPa}, f_{yd} = 358 \text{ MPa})\]
Fig. 7.4 \((d)\) Comparisons with Matsui’s result

\[(L/D = 18, f_{cd} = 42 \text{ MPa}, f_{yd} = 358 \text{ MPa})\]

Fig. 7.4\((e)\) Comparisons with Matsui’s result

\[(L/D = 24, f_{cd} = 42 \text{ MPa}, f_{yd} = 358 \text{ MPa})\]
7.3.2 High Strength Concrete

The test for high strength concrete filled steel tube columns were reported by Chung [Chung et al., 1999]. Seventeen columns with square sections were constructed using a 165.2 × 165.2 mm with a 4.5 mm wall thickness. Various lengths were tested with an $L/D$ ratio of 4, 8, 12, 18, 24 and 30. The concrete strength is 86 MPa and the yield stress of the steel is 450 MPa. The data for the 17 tests reported by Chung et al. (1999) are shown in Fig 7.5 (a)-(f).
Fig. 7.5(a) Comparisons with Chung’s result

\[(L/D = 4, f_{cd} = 86 \text{ MPa}, f_{yd} = 450 \text{ MPa})\]
Fig. 7.5(c) Comparisons with Chung’s result

\( \frac{L}{D} = 12, f_{cd} = 86 \text{ MPa}, f_{yd} = 450 \text{ MPa} \)

Fig. 7.5(d) Comparisons with Chung’s result

\( \frac{L}{D} = 18, f_{cd} = 86 \text{ MPa}, f_{yd} = 450 \text{ MPa} \)
Fig. 7.5(e) Comparisons with Chung’s result

\( L/D = 24, f_{cd} = 86 \text{ MPa}, f_{yd} = 450 \text{ MPa} \)

Fig. 7.5(f) Comparisons with Chung’s result

\( L/D = 30, f_{cd} = 86 \text{ MPa}, f_{yd} = 450 \text{ MPa} \)
Analyses were also carried out on specimens tested by Kilpatrick and Rangan (1997). The specimen was constructed using a 101 \text{mm} diameter circular steel tube (CHS) with 2.4 \text{mm} wall thickness. The first group with the column length at 1947 \text{mm} for the concrete strength is 57 MPa and the yield stress of the steel is 410 MPa. The second group with column length at 2175 \text{mm} and the concrete strength is 96 MPa and the yield stress of the steel is 450 MPa. The results of CFST columns are shown in Figs. 7.6 (a) - (b).

![Graph showing comparisons with Kilpatrick's result](image)

Fig. 7.6(a) Comparisons with Kilpatrick’s result

\[(L/D = 19, f_{cd} = 57 \text{ MPa}, f_{yd} = 450 \text{ MPa})\]
Fig. 7.6(b) Comparisons with Kilpatrick’s result

\[ L/D = 21.5, f_{cd} = 96 \text{ MPa}, f_{yd} = 450 \text{ MPa} \]

7.3.3 Experimental Verification

In Fig. 7.3 ~ 7.6, solid squares represent experimental results, solid lines represent the proposed method presented in Chapter 6 and dash lines show the proposed equation that described in this chapter. In addition to these, the cross-section strength presented in Chapter 5 is showed by dotted line. It is shown that good agreement is observed between the strength of columns obtained from the tests and the proposed equation for slender columns with \( L/D \) ratio greater than 12. However, it appears that the prediction of column strength may become increasingly conservative as \( L/D \) ratio becoming less than 8. This could be due to the proposed methods used Eurocode 4 recommendation with regard to the second order effect and imperfections, and it is overly conservative for short columns.
7.4 CONCLUSIONS

In this chapter a set of empirical equation was presented for the prediction of member strength of CFST columns for both normal and high strength concrete. The proposed equations were derived by accurate numerical method, performed using a general analysis procedure developed in Chapter 3. The numerical equations have also been verified extensively by comparing with 75 full scale CFST tests.

From comparison between the proposed method given in Chapter 6, the proposed equations here and the test results it can be concluded that the proposed method and the proposed equations can provide accurate predictions of the strengths of concrete filled slender tubes with a wide range of end eccentricities and dimension. Short columns with L/D ratio less than 12 can be designed according to the proposed equation given in Chapter 4 without considering the effect of secondary effect.
PLEASE NOTE

The greatest amount of care has been taken while scanning the following pages. The best possible results have been obtained.
CHAPTER 8:

CONCRETE FILLED STEEL TUBULAR BEAM-COLUMNS UNDER BIAXIAL BENDING

8.1 INTRODUCTION

Concrete filled steel tubular (CFST) columns are generally subjected to axial compression and bending about both major axes when located at the corners of a multi-storey building frame. An axial force and moment capacity interaction surface similar to that shown in Fig. 8.1 can be generated. If inelastic response of concrete and steel are represented with an accurate stress-strain function, the procedure produces the most accurate analysis of section strength under biaxial loading. However, the procedure is laborious even when used for only one interaction curve, and defining an interaction surface requires several interaction curves [Furlong, 1979]. The procedure is too tedious for practical applications.
A general analysis procedure with deformation control was described in Chapter 3. The analysis has been successfully used to calculate the strength of a CFST beam-column.

The numerical rules for biaxial bending are difficult to formulate for design purpose. This chapter investigates the design procedure to evaluate the failure surface of beam-column under biaxial bending, incorporating the methods presented in Chapter 6 or 7. Verification of the proposed interaction equation will be made by comparing the analytical results obtained by using the proposed design equation with those of CFST columns tested by Shakir-Khalil and Zeghiche (1989).

![Interaction Diagram for N_n=Constant]

Fig. 8.1 Failure surface and interaction diagram
8.2 DIFFERENT DESIGN RECOMMENDATIONS

For members under biaxial bending at a specified axial force, the strength of a section in biaxial bending can be approximated by observing that the sum of the ratios of the required moment to the nominal uniaxial moment capacity raised to the power of a suitable solve about the two principal axes must be equal to unity: [Gouwens, 1975]

$$\left(\frac{M_{nx}}{M_n}\right)^{\gamma} + \left(\frac{M_{ny}}{M_n}\right)^{\gamma} \leq 1.0 \quad (8.1)$$

where

- $M_{nx}$ = required moment capacity for bending component in the plane of the $x$ axis.
- $M_{ny}$ = required moment capacity for bending component in the plane of the $y$ axis.
- $M_n$ = nominal bending strength with bending only in the plane of the $x$ axis
- $M_{ny}$ = nominal bending strength with bending only in the plane of the $y$ axis
- $\gamma$ = a number between 1 and 2, defining the shape of the biaxial moment contour.

Test results for reinforced concrete columns under biaxially eccentric loading [Furlong, 1979] indicated that strength predictions with Eqn. (8.1) and $\gamma = 2$ are as accurate as the strength predictions for uniaxial bending based on the rectangular stress block for concrete. Trayhair and Bradford (1991) suggested that the member strength changes in relation with the axial force level. The change in the moment contour is reflected in the factor $\gamma$. The suggested expression is:
\[ \gamma = 1.4 + \frac{N_i}{N_0} \]  
(8.2)

where

\(N_i\) = axial force capacity for biaxially eccentric loaded column.

\(N_0\) = maximum axial force capacity under concentric load (the squash load).

The same equation can be applied for composite columns, although sufficient test data are not available [Furlong, 1997].

### 8.2.1 ACI Code

The response to loading of a CFST cross section, for any orientation of the neutral axis, can be obtained by assuming the same strain compatibility and rectangular concrete stress block as those for uniaxial bending. As this procedure is tedious, the Commentary to ACI 318-95 suggests as adequate a biaxial-strength analysis that satisfies an interaction surface. The analysis is based on the following relationship:

\[ \frac{1}{N_i} = \frac{1}{N_x} + \frac{1}{N_y} - \frac{1}{N_0} \]  
(8.3)

with

\(N_x\) = axial force capacity for eccentricity \(e_x\) in the plane of the \(x\) axis.

\(N_y\) = axial force capacity for eccentricity \(e_y\) in the plane of the \(y\) axis.
8.2.2 AISC-LRFD Code

The AISC-LRFD specification uses Eqn. (8.1) with the exponent equal to 1.0. Ratios between the required and actual bending strength considered individually for bending about each principal axis are added to produce the interaction contour which is a straight line. The LRFD procedure is safe, as it underestimates actual capacity of resistance under biaxially eccentric compression.

For symmetrical members, the LRFD specification permits computing the flexural strength ratio \( \frac{M_{fl}}{\phi M_{rs}} \) separately about each of the two major axes, and adding these two components to the axial-force component. Such linear superposition of flexural strength ratios has the effect of producing the same results for biaxially loaded cases as that obtained from Eqn. (8.1) with the exponent \( \gamma = 1 \). Since the capacity of biaxially loaded concrete filled steel columns depends predominantly on the capacity of the steel tube, the use of the LRFD interaction equation is recommended for such columns.

8.2.3 Australian Standard AS 4100

The design rule for the section capacity in the Steel Structure code AS 4100 is given as follow:
\[
\left( \frac{M_{Sdx}}{\phi M_{prx}} \right)^\gamma + \left( \frac{M_{Sdy}}{\phi M_{pry}} \right)^\gamma \leq 1.0 \quad (8.4)
\]

where \( M_{prx} \) is the plastic moment capacity about the \( x \)-axis, reduced for the presence of axial force \( N_i \); \( M_{pry} \) is the corresponding reduced plastic moment about the \( y \)-axis.

It can be approximated very closely by [Trahair and Bradford, 1991]:

\[
M_{prx} = 1.18 M_{plx} \left( 1 - \frac{N_i}{N_0} \right) \leq M_{px} \quad (8.5a)
\]

and

\[
M_{pry} = 1.19 M_{ply} \left[ 1 - \left( \frac{N_i}{N_0} \right)^2 \right] \leq M_{py} \quad (8.5b)
\]

Where \( M_{plx} \) and \( M_{ply} \) are the full plastic moments about the \( x \) and \( y \) axes; respectively. The exponent \( \gamma \) which is based on Eqn. (8.2) and given as

\[
\gamma = 1.4 + \frac{N_i}{\phi N_0} \leq 2.0 \quad (8.6)
\]

where \( \phi = 0.9 \) is the capacity reduction factor. The amplification in bending moment due to axial forces is already accounted for secondary moment effects. The design moment \( M_{Sdx} \) and \( M_{Sdy} \) are given by:

\[
M_{Sdx} = \frac{C_{mx}}{1 - \frac{N_i}{N_{crx}}} M_{mx} \quad (8.7a)
\]

and similarly
\[ M_{Sdy} = \frac{C_{my}}{N_{crx}} M_{my} \quad \text{(8.7b)} \]

where \( M_{mx} \) and \( M_{my} \) are the maximum design moments about the x and y axes respectively as determined from a first order structural analysis, \( N_{crx} \) and \( N_{cry} \) are the elastic buckling loads of the braced member about the x and y-axes respectively, and \( C_{nx} \) and \( C_{ny} \) are reduction effects, due to end moments.

8.2.4 Eurocode 4

For the design of a column under compression and biaxial bending in Eurocode 4, the design procedure described in Chapter 6 again follows. The relative values of moment resistance \( \mu_x \) and \( \mu_y \) are used for a new interaction curve. This linear interaction curve (Fig. 8.2) is cut off at 0.9 \( \mu_x \) and 0.9 \( \mu_y \). The existing moments \( M_{Sdx} \) and \( M_{Sdy} \) related to the respective resistances must lie within the new interaction curve, in the form of the following equations:

\[ M_{Sdx} \leq 0.9 \ \mu_x \ M_{pix} \quad \text{(8.8a)} \]

\[ M_{Sdy} \leq 0.9 \ \mu_y \ M_{ply} \quad \text{(8.8b)} \]

and

\[ \frac{M_{Sdx}}{\mu_x M_{pix}} + \frac{M_{Sdy}}{\mu_y M_{ply}} \leq 1.0 \quad \text{(8.8c)} \]
8.3 DERIVATION OF PROPOSED INTERACTION EQUATION OF

FAILURE SURFACE

Due to research efforts in past decades, design interaction equations have been proposed for reinforced concrete columns, steel, and composite sections by Chen and Duan (1989), Furlong (1979), Hsu (1988), Muñoz and Hsu (1997) and others. A computerised model for predicting strength and behaviour of CFST columns subject to general axial compression and biaxial bending was developed in Chapter 3. The predictive capability of the model has been verified by comparisons with the available test data shown in Table 8.1. Although the model can be used for the analysis of slender columns subjected to biaxial bending and as a tool for the design of members, the rigorous analysis is too complex for routine design. The model can also be used for developing design aids for a simplified design procedure as illustrated in previous chapters.
Table 8.1 Predicted column strength

<table>
<thead>
<tr>
<th>Spec. No</th>
<th>BxDxLx (mm)</th>
<th>Lx, Ly (mm)</th>
<th>ex, ey (mm)</th>
<th>fyld (MPa)</th>
<th>fcld (MPa)</th>
<th>Ntest (kN)</th>
<th>Num. (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shak89-1</td>
<td>80x120x5</td>
<td>3210, 2940</td>
<td>24, 16</td>
<td>343.3</td>
<td>45</td>
<td>268</td>
<td>251.9</td>
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<tr>
<td>Shak89-2</td>
<td>80x120x5</td>
<td>3210, 2940</td>
<td>60, 40</td>
<td>357.5</td>
<td>44</td>
<td>160</td>
<td>156.2</td>
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<tr>
<td>Shak90-1</td>
<td>80x120x5</td>
<td>3210, 2940</td>
<td>12, 8</td>
<td>341</td>
<td>42.6</td>
<td>348</td>
<td>303.5</td>
</tr>
<tr>
<td>Shak90-2</td>
<td>80x120x5</td>
<td>3210, 2940</td>
<td>42, 28</td>
<td>341</td>
<td>46.2</td>
<td>198.5</td>
<td>189.4</td>
</tr>
<tr>
<td>Shak90-3</td>
<td>80x120x5</td>
<td>3210, 2940</td>
<td>24, 40</td>
<td>362.5</td>
<td>42.4</td>
<td>206.8</td>
<td>188.2</td>
</tr>
<tr>
<td>Shak90-4</td>
<td>80x120x5</td>
<td>3210, 2940</td>
<td>60, 16</td>
<td>362.5</td>
<td>40.8</td>
<td>209.8</td>
<td>221.2</td>
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<tr>
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<td>15, 10</td>
<td>346.7</td>
<td>46.2</td>
<td>596.5</td>
<td>560.8</td>
</tr>
<tr>
<td>Shak90-6</td>
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<td>3210, 2940</td>
<td>45, 30</td>
<td>340</td>
<td>46.6</td>
<td>329.2</td>
<td>336.1</td>
</tr>
<tr>
<td>Shak90-7</td>
<td>100x150x5</td>
<td>3210, 2940</td>
<td>75, 50</td>
<td>340</td>
<td>47.2</td>
<td>254.6</td>
<td>244.2</td>
</tr>
</tbody>
</table>

8.4 DERIVATION OF GENERALISED INTERACTION EQUATION OF FAILURE SURFACE

A typical three dimension failure surface of a rectangular CFST column under axial load and biaxial bending is shown in Fig. 8.1. According to Eurocode 4 a nonlinear equation to representing the member strength can be given as follows:

\[
\left(\frac{M_{Sda}}{\mu_x M_{x0}}\right)^\gamma + \left(\frac{M_{Sdy}}{\mu_y M_{y0}}\right)^\gamma \leq 1.0 \quad (8.9)
\]

Where the exponent \(\gamma\) defines the approximate shape of the curve, depending on the level of the axial load. Some numerical studies [Young, 1973; Tebedge and Chen, 1974; Chen and Atsuta, 1976] and Australian code [AS4100, 1994] suggested the
amplification of the bending action be related to the axial force level. In this research, the calibrated fibre element analysis described in the preceding section was used to generate the coefficient for a wide range of load levels. These CFST members are summarised in Table 8.2.

There are two types of sections chosen. The two cross sections in Table 8.2 are $300 \times 300 \text{ mm}$ and $400 \times 400 \text{ mm}$ with the ratio $D/l$ from 30 to 75. All of the sections in Table 8.2 have yield strength of 300 to 400 MPa and concrete strength of 30 to 40 MPa. The ratio of length to height ($L/D$) ranges from 6.7 to 16.7. The load eccentricity increments of 15 degrees, ranging from $15^\circ$ to $45^\circ$ (symmetry). For each three-dimensional CFST column biaxial load strength curve was generated using the fibre element member analysis. From the least squares analysis of Table 8.2, the optimum form of the equation is determined:

$$\gamma = 0.85 + 2.6 \frac{N_L}{N_0}$$

(8.10)

and the value of $\gamma$ is between 1 and 2.
Table 8.2 CFST member used for determining the interaction equation of failure surface

<table>
<thead>
<tr>
<th>Section Type</th>
<th>$t$ (mm)</th>
<th>$L$ (mm)</th>
<th>$e_x$, $e_y$ (mm)</th>
<th>$f_{yd}$ (MPa)</th>
<th>$f_{cd}$ (MPa)</th>
<th>$N_t/N_0$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
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<td>300</td>
<td>6</td>
<td>3500</td>
<td>25, 25</td>
<td>300</td>
<td>30</td>
<td>0.702</td>
<td>2.13</td>
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<tr>
<td>300</td>
<td>6</td>
<td>3500</td>
<td>50, 50</td>
<td>300</td>
<td>30</td>
<td>0.55</td>
<td>2.09</td>
</tr>
<tr>
<td>300</td>
<td>6</td>
<td>3500</td>
<td>100, 100</td>
<td>300</td>
<td>30</td>
<td>0.37</td>
<td>1.87</td>
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<tr>
<td>300</td>
<td>6</td>
<td>3500</td>
<td>150, 150</td>
<td>300</td>
<td>30</td>
<td>0.23</td>
<td>1.416</td>
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<tr>
<td>300</td>
<td>6</td>
<td>3500</td>
<td>200, 200</td>
<td>300</td>
<td>30</td>
<td>0.166</td>
<td>1.213</td>
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<td>300</td>
<td>6</td>
<td>3500</td>
<td>250, 250</td>
<td>300</td>
<td>30</td>
<td>0.128</td>
<td>1.184</td>
</tr>
<tr>
<td>300</td>
<td>6</td>
<td>3500</td>
<td>72.44, 19.41</td>
<td>300</td>
<td>30</td>
<td>0.362</td>
<td>1.776</td>
</tr>
<tr>
<td>300</td>
<td>6</td>
<td>3500</td>
<td>64.95, 37.5</td>
<td>300</td>
<td>30</td>
<td>0.355</td>
<td>1.764</td>
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<tr>
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<td>6</td>
<td>3500</td>
<td>53.03, 53.03</td>
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<td>30</td>
<td>0.352</td>
<td>1.726</td>
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<tr>
<td>300</td>
<td>4</td>
<td>3500</td>
<td>100, 100</td>
<td>300</td>
<td>30</td>
<td>0.299</td>
<td>1.628</td>
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<td>300</td>
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<td>3500</td>
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<td>30</td>
<td>0.384</td>
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<td>2000</td>
<td>100, 100</td>
<td>300</td>
<td>30</td>
<td>0.408</td>
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<td>300</td>
<td>6</td>
<td>5000</td>
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<td>300</td>
<td>30</td>
<td>0.354</td>
<td>1.771</td>
</tr>
<tr>
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<td>6</td>
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<td>100, 100</td>
<td>400</td>
<td>30</td>
<td>0.364</td>
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<td>0.377</td>
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<td>40</td>
<td>0.362</td>
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<td>400</td>
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<td>100, 100</td>
<td>300</td>
<td>30</td>
<td>0.474</td>
<td>1.972</td>
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</table>
8.5 EXPERIMENTAL VERIFICATION

The results of 9 tests on biaxially beam-column were reported by Shakir-Khalil and Mouli (1989; 1990). The strengths were calculated using ACI, AISC, Eurocode 4 and proposed equation listed in Table 8.3. To provide confirmation of the validity of the proposed design equation, the eccentricity of applied load may be replaced as follow:

\[ r_{ecc} = \sqrt{e_x^2 + e_y^2} \]

and

\[ \tan \theta = \frac{e_x}{e_y} \]

The data for the 9 tests with the strength contour were shown in Fig. 8.3 (a)-(i).

The codes and proposed equation predictions are compared with the test results \( (N_{test}) \) in Table 8.3. The mean ratio of test to prediction for the ACI was 0.93 with a standard deviation (stv) of 9.96%. The mean ratio of test to prediction for the AISC was 1.14 with a stv of 13.4%. The mean for the Eurocode 4 predictions was 1.11 with a stv of 4.94%. The mean for the proposed equation predictions was 1.05 with a stv of 7.91%. Clearly, Eurocode 4 and the proposed equation fit the best, all of methods are over 5% on the conservative side on average, except ACI Code 7% on the unconservative side on average.

The design strength of biaxially loaded CFST columns reported from Shakir-Khalil and Mouli (1989; 1990), determined according to the proposed equation, are shown
in Fig. 8.3 (a) - (i) as function of the total eccentricity $r_{ecc}$ and the angle $\theta$. For uniaxially loaded ($\theta=0^\circ$ or $\theta=90^\circ$), the design strength decreases as the eccentricity of load ($r_{ecc}$) increases. For the lower load level, the strength decreases as the angle $\theta$ increases to the value of $45^\circ$ which should be location of the principal axis. The two characteristic effects of a typical strength contour also shown in Fig. 8.4 with concrete strength of 40 MPa, yield strength of steel tube of 400 (MPa). The dimension is $150\times150\times5$ mm and column length of 3000 mm.

Table 8.3 Comparison of test results with codes and proposed equation

<table>
<thead>
<tr>
<th>Spec. No</th>
<th>$N_{test}$ (kN)</th>
<th>$N_{test}/N_0$ (kN)</th>
<th>$N_{ACI}$ (kN)</th>
<th>$N_{test}/N_{ACI}$ (kN)</th>
<th>$N_{AISC}$ (kN)</th>
<th>$N_{test}/N_{AISC}$ (kN)</th>
<th>$N_{Euro}$ (kN)</th>
<th>$N_{test}/N_{Euro}$ (kN)</th>
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<tr>
<td>Shak89-1</td>
<td>268</td>
<td>0.283</td>
<td>285</td>
<td>0.94</td>
<td>257</td>
<td>1.04</td>
<td>236</td>
<td>1.14</td>
<td>248</td>
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<td>0.165</td>
<td>179</td>
<td>0.89</td>
<td>142</td>
<td>1.13</td>
<td>149</td>
<td>1.07</td>
<td>156</td>
<td>1.03</td>
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<td>348</td>
<td>0.378</td>
<td>297</td>
<td>1.17</td>
<td>359</td>
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<td>297</td>
<td>1.17</td>
<td>300</td>
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<td>218</td>
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<td>180</td>
<td>1.10</td>
<td>195</td>
<td>1.01</td>
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<td>206.8</td>
<td>0.214</td>
<td>225</td>
<td>0.92</td>
<td>198</td>
<td>1.04</td>
<td>178</td>
<td>1.16</td>
<td>179</td>
<td>1.16</td>
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<td>Shak90-4</td>
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<td>0.22</td>
<td>215</td>
<td>0.98</td>
<td>176</td>
<td>1.19</td>
<td>192</td>
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<td>204</td>
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<td>540</td>
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<td>231</td>
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Fig. 8.3(a) Shakir-Khalil's specification

\[(B \times D = 80 \times 120, f_{cd} = 45 \text{ MPa}, f_{yd} = 343 \text{ MPa})\]

Fig. 8.3(b) Shakir-Khalil's specification

\[(B \times D = 80 \times 120, f_{cd} = 44 \text{ MPa}, f_{yd} = 357 \text{ MPa})\]
Fig. 8.3(c) Shakir-Khalil's specification

\[ (B \times D = 80 \times 120, f_{cd} = 42 \text{ MPa}, f_{yd} = 341 \text{ MPa}) \]

Fig. 8.3(d) Shakir-Khalil's specification

\[ (B \times D = 80 \times 120, f_{cd} = 46 \text{ MPa}, f_{yd} = 341 \text{ MPa}) \]
Fig. 8.3(e) Shakir-Khalil’s specification

\(B \times D = 80 \times 120, f_{cd} = 42 \text{ MPa}, f_{yd} = 362 \text{ MPa}\)

Fig. 8.3(f) Shakir-Khalil’s specification

\(B \times D = 80 \times 120, f_{cd} = 40 \text{ MPa}, f_{yd} = 362 \text{ MPa}\)
Fig. 8.3(g) Shakir-Khalil’s specification

\[(B \times D = 100 \times 150, f_{cd} = 46 \text{ MPa}, f_{yd} = 346 \text{ MPa})\]

---

Fig. 8.3(h) Shakir-Khalil’s specification

\[(B \times D = 100 \times 150, f_{cd} = 46 \text{ MPa}, f_{yd} = 340 \text{ MPa})\]
Fig. 8.3(i) Shakir-Khalil's specification

\[(B \times D = 100 \times 150, f_{cd} = 47 \text{ MPa}, f_{yd} = 340 \text{ MPa})\]

Fig. 8.4 A typical column specification

\[(B \times D = 150 \times 150, f_{cd} = 40 \text{ MPa}, f_{yd} = 300 \text{ MPa})\]
8.6 CONCLUSIONS

In this chapter, a design failure surface was proposed similar to the interaction equation adopted in AS 4100 for the design of steel structures. The load contour diagram of a CFST column may be represented by a continuous mathematical function, whose shape is controlled by a numerical coefficients, $\gamma$, which depends mainly on the axial force level.

Numerical analysis that determines the biaxial interaction diagrams for CFST column under axial load and bending about two axes was presented in Chapter 3. Comparing with experimental results, it was found that the numerical method is accurate and conservative. Based on the analysis results, an optimum equation of the coefficient for the failure surface was obtained by least square analysis.

The proposed failure surface equation was also validated by the limited available test data. It was concluded that excellent agreement was achieved between the ultimate axial experimental loads and the values predicted by using the author's proposed interaction equation with the appropriate function of $\gamma$. The accuracy of existing code provisions from various design codes was also assessed, and it was found that the Eurocode 4 provision was the most accurate.
PLEASE NOTE

The greatest amount of care has been taken while scanning the following pages. The best possible results have been obtained.
CHAPTER 9:
CONCLUSIONS

9.1 GENERAL

CFST columns provide a flexible, efficient, and economic alternative to traditional structural members. They offer high strength and high stiffness and are cost effective. The research contained in this thesis has been mainly concerned with the behaviour and the design issues for CFST beam-columns. This section summarises the results for this research. Several aspects of the work are highlighted and, comments are made regarding the accuracy of the analytical model.

An extensive literature review was undertaken in Chapter 2 as background work for the analytical model developed in Chapter 3.

In Chapter 3, a computational method was developed for the analysis of CFST beam-columns. It took into account the nonlinear behaviour of the materials that form the CFST section and included the second order effects due to the additional eccentricity
of the applied axial load when the column deforms laterally. The accuracy of the predicted ultimate load depended on the number of segments or subdivisions along the column length and the initial parameters established for the step load increment and the convergence criteria.

In Chapter 4, constitutive relationships were presented for steel and concrete to include the effect of confinement on steel and concrete, in particular, in relation to the design of circular CFSTs with normal or high strength concrete. An effective uniaxial stress-strain relationship was derived to represent the realistic triaxial or biaxial stress state in steel and concrete. The proposed constitutive relationships were used to calculate the strength of a circular CFST. The results were verified with a large collection of published test results. The results from the proposed method were also compared with that of Eurocode 4 and AISC, and it was found that the present formulation offered more accurate results.

In Chapter 5, numerical analyses were carried out of CFST sections. Numerous computer calculations were performed to study the effect of various parameters, including dimensions, material strength, on the strength of the CFST section. The section analysis was based on the fibre analysis formulation. The resulting strength interaction diagram was represented by a numerical expression. The simple numerical expression was derived by curve fitting after hundreds of computer runs to include all the possible practical design solutions. Error analysis was performed of the resulting numerical formulation to ensure the accuracy and consistency of the equations from a designer's point of view. The statistical analysis shown that the
simple numerical expression is accurate. The numerical results were also verified with the available test data. Comparison was also made with results from the code recommendations. Both comparisons demonstrated that the proposed numerical equations are accurate, and slightly conservative. Based on the numerical analysis, a simple and easy to follow calculation procedure was proposed for design application.

Chapter 6 presented an integrated design methodology which can be used for normal and high strength concrete to determine the interaction strength of rectangular and circular shapes under combined compression and bending for CFST columns. The strength model is a modification of Eurocode 4 principle so that it covers both normal strength and high strength concrete. Results from the proposed method were compared with the published test results and results from other design code recommendations. It was found that the proposed method was more accurate than both Eurocode 4 and AISC recommendations.

Numerical equations were presented in Chapter 7 to determine the member capacity of circular and rectangular CFST beam-columns under bending moments and axial load. The proposed design procedure described in the preceding chapter was used to generate force point data for a wide range of CFST beam-columns. The empirical numerical expression representing a general member strength surface for rectangular and circular CFST was then developed using this proposed design data. A load line considering the second order effects on slender columns was determined by incorporating a moment magnification factor. The intersection between the member strength and the load line was used to predict the capacity of the column. The
numerical results were verified with the available test data. Comparison was also made with the proposed method given in Chapter 6. Both comparisons demonstrated that the proposed member strength equations are reasonable accurate, and slightly conservative.

In Chapter 8, a design failure surface equation was proposed for the biaxial bending of CFST columns. Accurate numerical analyses were first performed on columns under biaxial bending. Based on the analysis results, an optimum equation of the coefficient for the failure surface was obtained by least square analysis. The proposed failure surface equation was also validated by the limited available test data. It was concluded that the proposed failure surface equation gives an accurate prediction of strength of columns under biaxial bending. The analytical results were also compared with the ACI, AISC and Eurocode 4 design codes. The comparisons demonstrated that the proposed numerical equations are accurate, and slightly conservative.

9.2 CONCLUSIONS

The last section of this thesis presents the conclusions and the design recommendations applicable to the analysis and design of CFST columns.

A general procedure with deformation control was successfully developed for the analysis of a steel-concrete composite beam-column. The analysis procedure was used to calculate the true strength for a beam-column section and member. The application of the analysis method to a number of CFST columns with a variety of
steel sections filled in concrete proved that it is accurate. Analytical methods were then used to determine the cross section and member strength of CFSTs and for the calibration of the proposed empirical equation.

An extensive verification by the available test results for CFST column specimens under axial compression and bending moments shown that the current design parameters proposed in the Eurocode 4 can be modified to reflect a more consistent approximation and correlation of the design equation with the experimental data. The modifications to the current Eurocode 4 formula for the elastic flexural stiffness, axial load and ultimate bending moment in the design of CFST beam-columns, were suggested in the thesis.

It was recommended that the expression of the elastic flexural stiffness $E I$ used to calculate the critical load and the critical stress of the CFST member should include the effect of the axial load ratio. This is because the axial load ratio has an effect on ductility as the axial load increases the concrete strength in a manner similar to the confinement effect.

With regard to the confinement effect on the concrete core, the calculation of squash loads has accounted for the enhancement of the concrete core strength. A modification was suggested to the Eurocode 4. The proposed expression is applicable to both normal and high strength concrete. Comparing with the test results, the expression proposed gives more accurate results than Eurocode 4.
A simple mathematical formulation was presented for the evaluation of ultimate moment. Analysis was developed considering the concrete tensile strength. Results showed the formulation offered more accurate results, the tensile effect is significantly with a high strength concrete and a high $D/t$ ratio for CFST members.

The research found that the interaction diagrams and load contours of any column cross sections and members can be mathematically represented by a series of continuous equations. These equations incorporates the material and all the geometrical properties of the cross section and the member. They can be particularly defined by interaction coefficients that control the shape of the failure surface diagram of every cross section and member.

The empirical equations for CFST cross section strength surface provided excellent accuracy for a wide range of CFST cross section sizes and material strengths. The equations contain four coefficients represented by five force points, is easily implemented into the analytical model and is based on the ratio of the steel strength to the concrete strength $f_{yd}/f_{cd}$ and the $D/t$ ratio.

Similarly generalised interaction equations of failure surface for CFST beam-columns members were also presented. The interaction equation is applicable to the case of columns under biaxial or uniaxial bending in combination with axial compressive loads. The proposed equations provided predictions of the ultimate strength in close agreement with the experimental results for short, slender, circular,
rectangular, and for normal and high strength concrete column specimens throughout.

Finally, the proposed interaction expressions on the strength of CFST members could be used (1) to calculate the maximum allowable axial load applied to a given CFST column cross section at the specified values of eccentricity about the x and y axis. (2) to predict the adequacy of a particular short or slender CFST column member for a given set of externally applied axial load and uniaxial or biaxial bending moments.

9.3 FUTURE RESEARCH

The research contained in this work forms a part of a major investigation in the analysis and design of CFST member behaviour. A number of additional CFST research topics are either currently being studied or planned to be carried out in the near future. This section lists these further research areas which need to be addressed.

The first immediate research need is additional experimental work. A more comprehensive scope of cross-section geometrise and material strengths should be investigated, especial in the high bending and low axial compression region, where experimental data has been scarce. The results presented in this thesis also indicate a deficiency of experiments for CFSTs with high strength steel and high strength concrete. The numerical models presented in this thesis are suitable for design spectrum which cover almost entire design parameters. This makes these model
lengthy and tedious. These can be further simplified so that it only cover a desired
design spectrum, but with the similar accuracy.

Building structures have to meet fire-safety requirements. Normally these structures
are judged on the basis of standard fire tests. However, such tests are very expensive
and time-consuming. The excellent fire resistance of CFST beam-column warrants
the reduction or elimination of the fire protection sprayed on to the steel tubes.
However, there is limited information in this area. Significant saving can be realised
in terms of material and labours if the fire protection material layer can be reduced. A
mathematical model is currently being developed to predict the fundamental
behaviour of fire-exposed CFST structures.

The next logical step in the research of CFST beam-column, is to investigate the
behaviour of joint, and the behaviour of new structure form where CFST members
are used exclusively.

In summary, there are a number of additional CFST research topics that should be
addressed in the future. Some possibilities include:

- Further parametric studies of CFST structures and the development of
  improved design formulations.

- A test program for high strength concrete CFST columns under axial load
  and bending, and axial load and bi-axial bending, especially for high bending
  region. The variables should include diameter or depth/ thickness ratio \((D/t)\),
column slenderness, eccentricity and combinations of different eccentricities, concrete compressive strength and steel yield strength.

- CFST/steel connections, including topics such as steel/concrete bond at the connection, the development of economic and efficient connections, and the development of an analysis method to model partially-restrained connections.

- A numerical procedure for the thermal analysis of CFST beam-columns.

- Parameter studies of CFST member under fire.

- Experimental verification of the fire model for CFST members.
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THE DESIGN AND BEHAVIOUR OF
CONCRETE FILLED STEEL TUBULAR
BEAM-COLUMNS

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PhD
2000

University of Western Sydney
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THE DESIGN AND BEHAVIOUR OF CONCRETE

FILLED STEEL TUBULAR BEAM-COLUMNS

by

Min Chao

A thesis presented to the

University of Western Sydney

in partial fulfilment of the requirements

for the degree of

Doctor of Philosophy

April, 2000

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Synopsis

Concrete filled steel tubular (CFST) columns are widely used in building and infrastructure projects throughout the world. Compared with other form of construction CFST columns offer superior structural performance and speed and ease of construction.

Design guidelines for CFST columns with normal strength concrete have been recommended in a number of codes of practice. In recent years high strength concrete has been widely used in CFST construction. The codes used for normal strength concrete have also been used in many design situations where high strength concrete is used. Due to the significant difference in the material properties between normal strength concrete and high strength concrete, there is a need to study the behaviour of composite members with higher strength concrete. There has been only limited information in recent years on the use of high strength concrete in CFSTs.

Design procedures and recommendations provided in most of the design codes are tedious and often complex. There have been attempts to simplify the design procedure by providing a simplified expression to predict the capacity of a CFST under a general loading condition.
In this thesis a rigorous analysis procedure was first presented for the analysis of a CFST beam-columns under a general loading condition. The numerical procedure was based on either the strip or the fibre method of analysis. The analysis procedure incorporated both slenderness and secondary moment effects. The realistic stress-strain relationship appropriate for CFSTs was used in the analysis. The constitutive relationship for steel and concrete were developed for different types of sections and for both normal and high strength concrete. In the course of derivation due consideration was given to the effect of confinement on the concrete core and the steel shell. In circular section, the confinement is significant therefore both strength and ductility enhancement was taken into consideration for concrete. For steel due to the same strengthening effect to the concrete the steel is in a biaxial stress state, thus reducing the effective uniaxial yield stress significantly. For rectangular section, the effect of confinement is ignored in the formulation.

The general analysis procedure was verified by many test data and was found to be very accurate to predict the strength of CFST beam-columns. The analysis procedure was then used as the basis in the rest of the thesis for the derivation of various empirical expressions for the prediction of a CFST beam-column either at the section level or at the member level.

The computational procedure is able to determine the complete load-deformation path of a pin-ended CFST column subject to axial compression and biaxial bending. The method has been proved to provide very reasonable and accurate predictions of the ultimate load. At different stages of the analysis, the section strength and member
strength interaction diagram can be generated in addition to the load deformation path of the column.

Based on the accurate computational procedure, a set of mathematical design interaction equations was generated numerically to simplify the complex calculation procedures for the strength of CFST members. The accurate equations satisfy the design conditions having a wide range of material strengths, cross section dimensions and varying lengths. The section strength interaction equations provide the capacity of columns at the section level without considering the effect of slenderness and imperfection.

The accuracy of the proposed empirical equations was verified by comparing with many test results in the existing publications. Comparisons were also made with various design code recommendations and it was found that the proposed offered an accurate solution for practical purpose. The proposed expressions, though lengthy numerically can be easily coded into any spreadsheet program and programmable calculator for routine design.

Similarly, a set of empirical expressions was also presented to represent the member strength interaction diagrams for CFST beam-columns under combined bending moments and the axial load. For the member design the section interaction diagrams are modified to include the effect of slenderness and imperfection as given in Eurocode 4. Based on the proposed equations, a design procedure was also proposed for member design of concrete filled steel tubular columns.
Finally, a design equation was presented to represent the ultimate load capacity of slender CFST columns subject to biaxial bending moments and axial load. Also prosed was the failure surface for beam-column under biaxial bending.

All the analytical results were verified by comparisons with the available test results and current ACI, AISC and Eurocode 4 design codes. The comparisons demonstrated that the proposed numerical equations are accurate, and slightly conservative. Based on the numerical analysis, a simple and easy to follow calculation procedure was proposed for design of CFST columns under either uniaxial or biaxial bending moment and axial load.
Preface

This thesis is submitted to the University of Western Sydney, Australia, for the degree of Doctor of Philosophy. The work described in this thesis was carried out in the School of Construction and Building Sciences at University of Western Sydney, in the period (1995-2000) of candidature.

The thesis addressed comprehensively the design and behaviour of concrete filled steel tubular beam-columns with particular emphasis on simplified design procedures and equations. A general numerical analysis procedure was developed first using the fibre method to study the behaviour of CFST beam-columns. Appropriate constitutive relationships for concrete and steel were employed in the method so that the effect of tube confinement to normal and high strength concrete can be properly modelled.

In subsequent chapters numerical expressions for predicting the sectional and member strength of CFST beam-columns were proposed based on the accurate numerical studies under various loading conditions. Finally the design method was proposed for practical design of CFST subjected to combined axial compression and bending.
The following papers published were derived from the above mentioned research study.


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Statement of Authentication

The work presented in this thesis is, to the best of my knowledge and belief, original except as acknowledged in the text. I hereby declare that I have not submitted this material, either in whole or in part, for a degree at this or any other institution.

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Min Chao
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Fig. 5.10 (b) Comparisons with Furlong’s result ($D/t=47.6, f_{cd} = 23.4$ MPa, $f_{yd} = 330$ MPa)

Fig. 5.10 (c) Comparisons with Furlong’s result ($D/t=32.0, f_{cd} = 29.0$ MPa, $f_{yd} = 330$ MPa)

Fig. 5.11 Comparisons with Knowles’s result ($D/t = 25, f_{cd} = 40.7$ MPa, $f_{yd} = 320$ MPa)

Fig. 5.12(a) Comparisons with Tomii’s result ($D/t = 44.0, f_{cd} = 38.3$ MPa, $f_{yd} = 195$ MPa)

Fig. 5.12 (b) Comparisons with Tomii’s result ($D/t = 44.0, f_{cd} = 21.6$ MPa, $f_{yd} = 305$ MPa)

Fig. 5.12(c) Comparisons with Tomii’s result ($D/t = 45, f_{cd} = 21.6$ MPa, $f_{yd} = 340$ MPa)

Fig. 5.12(d) Comparisons with Tomii’s result ($D/t = 45, f_{cd} = 21.6$ MPa, $f_{yd} = 290$ MPa)

Fig. 5.12(e) Comparisons with Tomii’s result ($D/t = 34, f_{cd} = 20.6$ MPa, $f_{yd} = 290$ MPa)

Fig. 5.12(f) Comparisons with Tomii’s result ($D/t = 34, f_{cd} = 20.6$ MPa, $f_{yd} = 289$ MPa)

Fig. 5.12(g) Comparisons with Tomii’s result ($D/t = 24.0, f_{cd} = 18.6$ MPa, $f_{yd} = 285$ MPa)

Fig. 5.12(h) Comparisons with Tomii’s result ($D/t = 24.0, f_{cd} = 19.8$ MPa, $f_{yd} = 285$ MPa)

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Fig. 8.1 Failure surface and interaction diagram

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Fig. 8.3(a) Shakir-Khalil’s specification ($BxD = 80 \times 120, f_{cd} = 45 \text{ MPa}, f_{yd} = 343 \text{ MPa}$)

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Fig. 8.3(e) Shakir-Khalil’s specification ($BxD = 80 \times 120, f_{cd} = 42 \text{ MPa}, f_{yd} = 362 \text{ MPa}$)

Fig. 8.3(f) Shakir-Khalil’s specification ($BxD = 80 \times 120, f_{cd} = 40 \text{ MPa}, f_{yd} = 362 \text{ MPa}$)

Fig. 8.3(g) Shakir-Khalil’s specification ($BxD = 100 \times 150, f_{cd} = 46 \text{ MPa}, f_{yd} = 346 \text{ MPa}$)

Fig. 8.3(h) Shakir-Khalil’s specification ($BxD = 100 \times 150, f_{cd} = 46 \text{ MPa}, f_{yd} = 340 \text{ MPa}$)

Fig. 8.3(i) Shakir-Khalil’s specification ($BxD = 100 \times 150, f_{cd} = 47 \text{ MPa}, f_{yd} = 340 \text{ MPa}$)

Fig. 8.4 A typical column specification ($BxD = 150 \times 150, f_{cd} = 40 \text{ MPa}, f_{yd} = 300 \text{ MPa}$)
List of Symbols and Notations

$A_c$  
Area of concrete in cross section

$A_s$  
Area of structural steel in cross section

$A_w$  
Web area of steel shape plus any longitudinal bars at centre of section

$C_m$  
Reduction factor

$D$  
Overall depth of rectangular tubes or diameter of circular tubes

$E_c$  
Secant modulus of elasticity of concrete

$E_{cr}$  
Flexural tensile strength

$E_{ct}$  
Modulus of elasticity of concrete under tensile stress

$(E\eta)_t$  
Actual elastic stiffness of the steel-concrete composite section

$E_m$  
Modified modulus of elasticity

$E_s$  
Modulus of elasticity of the steel

$f_c'$  
Strength of concrete from standard cylinder tests

$f_{cd}$  
Concrete compression design strength

$F_{cr}$  
Critical stress of CFST columns

$f_i$  
Axial force of the element

$F_{my}$  
Modified yield stress

$f_y$  
Yield strength of structural steel

$f_{yd}$  
Steel yield design strength

$h_1$  
Concrete width perpendicular to the plane of bending

$h_2$  
Concrete thickness in the plane of bending

$I_g$  
Second moment of area of the gross composite cross-section ignoring steel

$I_s$  
Second moment of area of the steel
$k$ Triaxial factor
$L$ Column of length
$L_e$ Effective length
$M_n$ Ultimate moment capacity without axial force
$M_{nx}$ Nominal bending strength with bending only in the plane of the x axis
$M_{ny}$ Nominal bending strength with bending only in the plane of the y axis
$M_{0x}$ Design strengths in uniaxial bending about the principal axes $x$
$M_{0y}$ Design strengths in uniaxial bending about the principal axes $y$
$M_{plx}$ Full plastic moments about the x - axes
$M_{ply}$ Full plastic moments about the y - axes
$M_{prx}$ Plastic moment about the $x$-axis
$M_{pry}$ Plastic moment about the $y$-axis
$M_{S0d}$ Maximum first order bending moment
$M_u$ Factored moment increased for slenderness effects
$M_{ub}$ Maximum moment capacity
$M_{ux}$ Required moment capacity for bending component in the plane of the x axis
$M_{uy}$ Required moment capacity for bending component in the plane of the y axis
$m_x$ Bending moments about the $x$-axis
$m_y$ Bending moments about the $y$-axis
$N_{cr}$ Buckling critical axial stress resultant
$N_{crx}$ Elastic buckling loads of the braced member about $x$-axes
$N_{cry}$ Elastic buckling loads of the braced member about $y$-axes
$N_i$ Capacity for biaxially eccentric axial force
$N_n$ Nominal thrust capacity
$N_0$ Axial thrust capacity
$N_{Sd}$ Axial force for end-loaded braced members
\( N_u \)  
Measured ultimate loads; Factored axial force

\( N_x \)  
Axial force capacity for eccentricity \( e_x \) in the plane of the \( x \) axis

\( N_y \)  
Axial force capacity for eccentricity \( e_y \) in the plane of the \( y \) axis

\( r \)  
Ratio of smaller to larger end moment

\( r_{ecc} \)  
Eccentricity of applied load

\( r_m \)  
Modified radius of gyration

\( r_s \)  
Radius of gyration of the steel section

\( p_z \)  
Cross-section force along the \( z \)-axis

\( Z \)  
Plastic section modulus of steel shape

\( \alpha \)  
Coefficient of buckling effects

\( \beta_c \)  
Load effect

\( \chi_k \)  
Reduction factor due to buckling

\( \delta_b \)  
Moment magnifier factor

\( \delta^j \)  
Nodal \( (j) \) displacements

\( \varepsilon_i \)  
Axial strain of elements

\( \varepsilon_0 \)  
Cross-section strain along the \( z \) axis

\( \phi \)  
Capacity reduction factor

\( \phi \)  
Confinement ratio

\( \phi_b \)  
Resistance factor for bending

\( \phi_c \)  
Resistance factor for compression
$\phi_e$ Eccentricity ratio

$\phi_l$ Slenderness ratio

$\phi_x$ Curvatures about the $x$-axis

$\phi_x^*$ Capacity reduction factor about the principal axes $x$

$\phi_x^j$ Nodal $(j)$ curvatures about the $x$ axis

$\phi_y$ Curvatures about the $y$-axis

$\phi_y^*$ Capacity reduction factor about the principal axes $y$

$\phi_y^j$ Nodal $(j)$ curvatures about the $y$ axis

$\gamma$ Shape of the biaxial moment contour

$\gamma_c$ Partial safety factors for concrete

$\gamma_h$ Hoop steel reduction ratio

$\gamma_s$ Partial safety factors for steel

$\gamma_v$ Vertical steel reduction ratio

$\lambda$ Relative slenderness

$\theta^j$ Nodal $(j)$ rotations

$\rho$ Density of concrete

$\sigma$ Stress at strain $\varepsilon$

$\sigma_c$ Maximum compressive stress

$\sigma_{cc}$ Confined concrete strength,

$\sigma_{cl}$ Lateral pressure

$\sigma_{cv}$ Vertical stress in the concrete core

$\sigma_{sh}$ Hoop stress

$\sigma_{sv}$ Vertical stress in the steel tube

$\sigma_i$ Axial stress of the element

$\nu_s$ Poisson's ratio of the steel
$\nu_c$  Poisson's ratio of the concrete

$\xi$  Local coordinate within each element